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On the drag on an object immersed in a flowing plasma: the control surface approach

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Abstract. A theory of the drag force acting on an object immersed in a flowing plasma is presented. A control surface method is developed following the approach used in fluid mechanics rather than the linear dielectric response approach found in the literature. The situation examined is one in which the directed velocity of the ions dominates their thermal velocities; a cold ion model is employed.

1. Introduction

The motivation for the present investigation was to determine the drag force on a dust particle in a flowing plasma, but the method adopted may be applicable to other objects in motion in a plasma. In low-pressure plasmas in the laboratory there is always an appreciable flow of ions towards the plasma–sheath boundary. During their travel towards the surrounding space charge sheath, the ions acquire a velocity near to the Bohm velocity, which is equal to the ion acoustic velocity [1]. An early paper by Stangeby and Allen [2] considered the flow around a cylindrical probe, and a typical result is illustrated in Fig. 1. In that paper the probe radius was taken to be large compared with the electron Debye distance, whereas the reverse is true for dust particles. In the present work, use is made of a 'control surface', as employed in the subject of fluid mechanics [3], together with the concept of the Maxwell stress in the electric field [4].

A detailed survey of the existing literature on 'ion drag' will not be presented here since a comprehensive review was published recently [5]. However, some deficiencies of the existing theories should be indicated. Two approaches to the question of ion drag have been made. The traditional method is that of a 'binary collision' approach. Basically, standard Coulomb scattering theory has been adapted to determine the momentum lost by those ions that are deflected by the electric field surrounding the charged dust particle, but which escape capture. One limit to the application of this type of theory can immediately be seen; the potential distribution surrounding the dust particle is 'chosen' and not deduced! It is also assumed to be isotropic in nature, whereas it is known that interesting wake phenomena have been observed in the laboratory [6].

An alternative approach is to assign a permittivity to the plasma and calculate the force on the charged dust particle due to the non-uniform polarization of the

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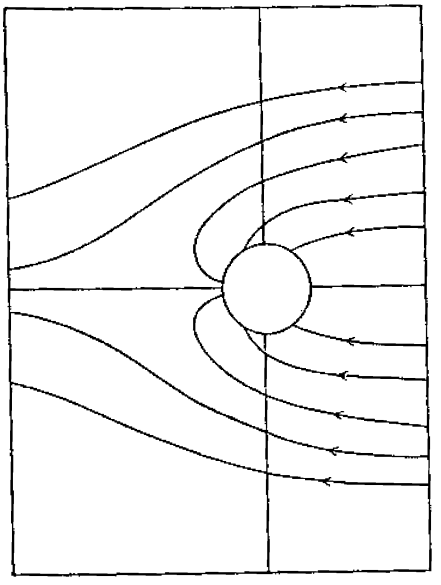


Figure 1. An example of ion flow near an immersed object. The calculation was for a cylindrical probe and small Debye distance. The ion-acoustic Mach number is $v = 0.3$ [2].

surrounding plasma. Clearly this assumes that linear theory will suffice, whereas it is known that the underlying basic set of equations is nonlinear.

Neither of these two theories includes the charging of the dust particle. This is an *ad hoc* addition to both theories, usually employing some version of the orbital motion limited (OML) theory. There are clearly limits to the accuracy of this method; angular momentum is conserved only for central fields of force but in the OML theory it is assumed to be conserved for a given ion. The theory is deficient in any case, since it does not include solving Poisson's equation for the potential distribution in the vicinity of the charged dust particle (or probe) [7].

We can note that the drag force is largely determined by ion collection at relatively high ion velocities. However, there is no reason to suppose that the cross-section for collection is simply given by the size of the grain, as is sometimes suggested [8]. It should also be noted that some authors [5,8] refer to Mach numbers of the flowing plasma in terms of a *thermal* Mach number, i.e. the flow velocity in terms of the thermal velocity of the ions. Other authors [2] compare the flow velocity to the ion-acoustic velocity in defining a Mach number in a plasma (in the absence of a magnetic field), and this appears to be a more natural choice.

Calculations using a particle-in-cell (PIC) code have been carried out by Hutchinson [9]. Boltzmann electrons were assumed, as in the present paper. Hutchinson's model was for hot ions, T_i/T_e was either 0.1 or 1, whereas only cold ions are considered in the theory given below. Most of Hutchinson's results were obtained for relatively large spheres in supersonic flows, although shock phenomena were not discussed, and presumably were not observed. The main result appears to be the surprising one that the OML theory (or an amended version thereof) predicts the floating potential quite accurately. This accuracy is unexpected as the electric potential distribution is *not* spherically symmetrical, so that the angular momentum

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of an ion is not conserved, despite this condition being a requirement of the OML theory [7]. Another result was that an increased ion density was found downstream of the sphere and hence the collected ion flux density was found to depend on the angle θ (in spherical coordinates). This should not be surprising, however, since similar results were found in early work on probes in a flowing plasma [2].

2. The Bohm sheath

The idea developed in the present work stemmed from a study of the well-known Bohm sheath [1]. The assumptions made in the usual model of the sheath are as follows: first, ionization in the sheath is negligible; second, the distribution of ion energies is ignored (monoenergetic ions are considered); and, thirdly, the electric field at the plasma edge is taken to be very small. In addition, it is assumed that the Boltzmann relation holds for the electron density as a function of potential. With these assumptions, Poisson's equation takes the following form:

$$\varepsilon_0 \frac{d^2V}{dx^2} = -n_s e\left\{\left(1-\frac{V}{V_0}\right)^{-1/2} - e^{eV/kT_e}\right\}, \tag{1}$$

where eV_0 represents the kinetic energy of ions when they leave the plasma and enter the sheath (V_0 is not the potential at the sheath edge, which has been taken as zero). This equation, describing the spatial variation of potential, can be integrated once analytically. A second integration must be carried out numerically. The first integration gives the following equation:

$$2n_s eV_0\left(1-\frac{V}{V_0}\right)^{1/2} + n_s kT_e e^{eV/kT_e} - \frac{1}{2}\varepsilon_0\left(\frac{dV}{dx}\right)^2 = 2n_s eV_0 + n_s kT_e. \tag{2}$$

This result is of considerable interest, however, since it can be interpreted physically as a momentum–stress balance [10]. The electric field that accelerates the positive ions through the sheath is associated with a gradient in the electron pressure. The drop in electron pressure in the sheath increases the momentum of the ions as they travel towards the probe or wall. At the same time, however, a stress is developed in the electric field. At the wall the force is the result of ion impact, the (greatly reduced) electron pressure, and the electric force on the surface charge; the last of these acts like a 'negative pressure'.

Whilst it is easy to visualize what is happening at the solid surface, the momentum–stress equation is valid at all points in the sheath. The concept of stress in the electric field (the Maxwell stress) is thereby illustrated by studying this case.

3. A basic momentum–stress theorem

Let us now consider the case in point, that of an object (e.g. a dust particle) immersed in a flowing plasma. The theory adopted assumes that the positive ions behave as a cold gas; the directed motion is assumed to dominate over any random motion and so the ion stress tensor is discarded. The electrons are assumed to be in Boltzmann equilibrium and it is known that this is an excellent approximation in many cases. The assumption may require further justification in the present situation since some electrons may reach regions of positive potential with respect to the ambient plasma. Poisson's equation describes the dependence of potential

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on the space charge and is the only one of Maxwell's equations that enters in the present case. Collisions with neutral atoms or molecules are neglected.

We shall consider three entities, the positive ions, the electrons, and the electromagnetic field (electrostatic field in the present case). The positive ions will be considered as a cold gas, and the ion stress tensor will be discarded so that the momentum equation for the ions can be written as

$$n_{\mathrm{i}} M(\mathbf{v}\cdot\nabla)\mathbf{v} = n_{\mathrm{i}} e\mathbf{E}. \tag{3}$$

The electrons are assumed to be in Boltzmann equilibrium so that

$$n_{\mathrm{e}} = n_0 e^{eV/kT_{\mathrm{e}}}, \tag{4}$$

where e is the magnitude of the electronic charge.

Differentiation of (1.2) gives

$$\nabla p_{\mathrm{e}} = -n_{\mathrm{e}} e\mathbf{E}, \tag{5}$$

since $p_{\mathrm{e}} = n_{\mathrm{e}} kT_{\mathrm{e}}$ and $\mathbf{E} = -\nabla V$.

The space-charge equation for the divergence of $\mathbf{E}$ (it takes the form of Poisson's equation when expressed in terms of V) is

$$\nabla\cdot\mathbf{E} = \frac{(n_{\mathrm{i}} - n_{\mathrm{e}})e}{\varepsilon_0}. \tag{6}$$

Adding (3) and (5) and substituting from (6) we obtain

$$n_{\mathrm{i}} M(\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p_{\mathrm{e}} - \varepsilon_0(\nabla\cdot\mathbf{E})\mathbf{E} = 0. \tag{7}$$

Let us consider first a volume of plasma that does *not* contain an immersed body. A volume integration of (7) reads

$$\iiint n_{\mathrm{i}} M(\mathbf{v}\cdot\nabla)\mathbf{v}\, d\tau + \iiint \nabla p_{\mathrm{e}}\, d\tau - \iiint \varepsilon_0(\nabla\cdot\mathbf{E})\mathbf{E}\, d\tau = 0, \tag{8}$$

which is a statement that the flow of ion momentum out from the volume differs, in general, from that entering, owing to the variation in pressure forces and the (net) electrical forces acting within the volume.

The volume integrals can be converted into surface integrals using Gauss's divergence theorem to give

$$\iint \rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v}\, dS + \iint p_{\mathrm{e}}\mathbf{n}\, dS - \iint \mathbf{T}\cdot\mathbf{n}\, dS = 0, \tag{9}$$

where $\mathbf{n}$ is the unit vector normal to the surface (outwards) and $\mathbf{T}$ is Maxwell's stress tensor where its tensor divergence is a vector, the electric force per unit volume. The last term can be rewritten in the form

$$\iint \mathbf{T}\cdot\mathbf{n}\, dS = \iint \left[\varepsilon_0(\mathbf{E}\cdot\mathbf{n})\mathbf{E} - \frac{1}{2}\varepsilon_0 E^2\mathbf{n}\right] dS. \tag{10}$$

Expressed in words, the electric field can be described as exerting a tension $\varepsilon_0 E^2$ in the direction of the electric field, together with an isotropic pressure of $\frac{1}{2}\varepsilon_0 E^2$. We shall now consider the case of an immersed object, for example a dust particle. Referring to Fig. 2, application of (9) to the volume between S_1 and S_2, where S_1 is a surface near the object and S_2 is any larger radius, yields the result that

$$\iint_{S_1} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p_{\mathrm{e}}\mathbf{n} - \mathbf{T}\cdot\mathbf{n}]\, dS + \iint_{S_2} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p_{\mathrm{e}}\mathbf{n} - \mathbf{T}\cdot\mathbf{n}]\, dS = 0. \tag{11}$$

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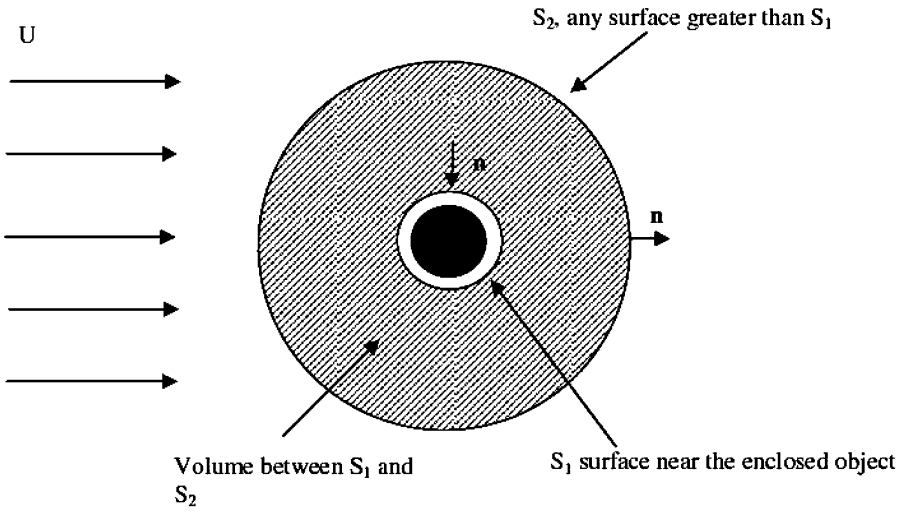


Figure 2. A diagram to illustrate the momentum–stress calculation.

However, the first integral is equal to the force on the enclosed object so that

$$\mathbf{D} = -\iint_{S_2} [\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p_e\mathbf{n} - \mathbf{T}\cdot\mathbf{n}]\, dS. \tag{12}$$

Evaluation of the integral over the surface S_2 can therefore be used to determine the force on the immersed object. If S_2 is at a large distance from the object the term associated with Maxwell's stress tensor may be neglected, but near the immersed object this is *not* the case. The force on the object is the result of the direct impact of ions, the electron pressure (sometimes negligible), and an electrostatic force. In the case where the immersed object is a conductor, the electric force acts like a 'negative pressure', but one that varies with position on the surface. In the case of an insulator, (12) would be applicable in a complete theory of the ion flow to the object.

4. Application to the case of weak flows

We shall apply our basic theorem by determining the surface integral of (12) over the surface of a large sphere centred on the immersed object (Fig. 3). At large distances from the object the (normalized) velocity potential for weak flows, i.e. for ion-acoustic Mach numbers much less than one, is given by

$$\Phi = \nu r\cos\theta + \frac{J}{r}, \tag{13}$$

where $4\pi J$ is the normalized flux to the object [11]. The ion-acoustic Mach number is given by $\nu = U/c_s$, where U is the flow velocity, and c_s is the ion-acoustic speed $(kT_e/M)^{1/2}$; the distances have been normalized in terms of the electron Debye distance, and consequently the unit of flux is $\lambda_d^2 c_s$ (or $\lambda_d^3\omega_{pi}$). The ion velocity is given by the gradient of the velocity potential, i.e.

$$\nabla\Phi = \left[\nu\cos\theta - \frac{J}{r^2}\right]\mathbf{u}_r - \nu\sin\theta\,\mathbf{u}_\theta. \tag{14}$$

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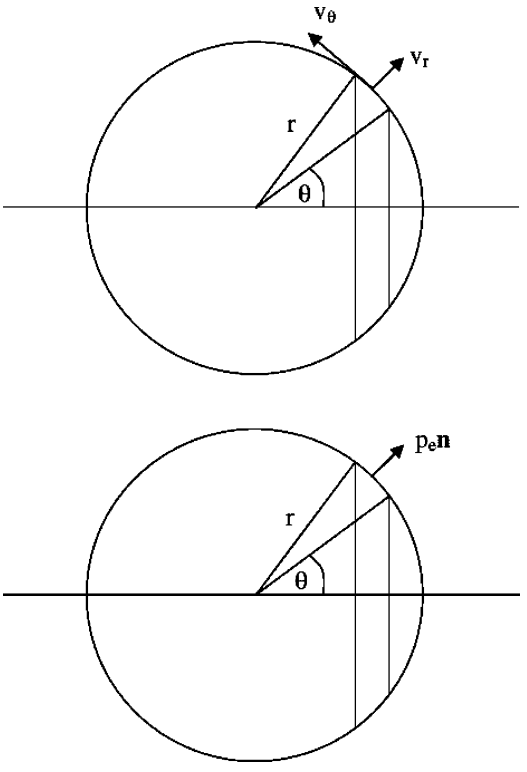


Figure 3. A diagram to illustrate the momentum–stress calculation for weak flows.

The rate of flow of momentum leaving the spherical surface is given by

$$M_{\mathrm{S}} = \int_0^{\pi} 2\pi \hat{R}^2 \hat{\rho} v_r \sin\theta (v_r \cos\theta - v_\theta \sin\theta)\, d\theta, \tag{15}$$

where the hat refers to normalized quantities. The integral can be readily evaluated to give

$$M_{\mathrm{S}} = -\frac{16\pi\nu J}{3}. \tag{16}$$

Converting to physical values we have

$$\iint \rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v}\, dS = -\frac{4}{3} MI\mathbf{U}, \tag{17}$$

where $\mathbf{U}$ is the stream velocity and I is the flux of ions (number of ions per second) to the immersed object, dust particle, or otherwise. We have now determined the first term in the required surface integral in our basic theorem of (12). The effects of the electron pressure and the Maxwell stresses remain to be examined.

The integral of the electron pressure over the surface of the large spherical surface is given by the following expression, referring again to Fig. 3:

$$\iint p_{\mathrm{e}}\mathbf{n}\, dS = \int_0^{\pi} p_{\mathrm{e}} 2\pi R^2 \sin\theta\cos\theta\, d\theta \mathbf{u}_z. \tag{18}$$

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The (normalized) electric potential at large distances, for weak flows only, is a simple dipole field [11]

$$\phi = \frac{J\nu\cos\theta}{r^2}. \tag{19}$$

This result can be readily obtained from the conservation of energy if the simple flow pattern described by (13) is assumed to be valid, which is the case for weak flows.

The variation of electron density on the spherical surface is obtained from the Boltzmann relation, (4), which can be linearized at large distances. The integral of (18) can then be written, after normalization,

$$\mathbf{P} = \int_0^\pi 2\pi J\nu\cos^2\theta\sin\theta\,d\theta\mathbf{u}_z, \tag{20}$$

which can be readily evaluated to give

$$\mathbf{P} = \frac{4\pi J\nu}{3}\mathbf{u}_z. \tag{21}$$

Reverting to physical quantities we have

$$\iint p_e\mathbf{n}\,dS = \frac{MUI}{3}\mathbf{u}_z. \tag{22}$$

We have now determined the first two terms of the surface integral of (12); it remains to consider the last term, the contribution of the Maxwell stress. The electric field at large radii is given by taking the gradient of (19), i.e. it is the dipole field

$$\mathbf{E} = \frac{2J\nu\cos\theta}{r^3}\mathbf{u}_r + \frac{J\nu\sin\theta}{r^3}\mathbf{u}_\theta. \tag{23}$$

The Maxwell stress can be described in terms of a tension of $\varepsilon_0 E^2$, together with an isotropic pressure of $\frac{1}{2}\varepsilon_0 E^2$. The stress could be integrated over the surface of a large spherical surface, but since $E^2 \sim O(1/r^6)$ and the area of the surface is $4\pi r^2$ the integral tends to zero as r tends to infinity.

It is interesting to note that there is a flux of ions across a large spherical surface, but no flux of the electric field **E**.

To determine the force on the immersed object, dust grain, or other, we employ our basic momentum–stress theorem, (11), which we recall for reference,

$$\iint_{S_1}[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p_e\mathbf{n} - \mathbf{T}\cdot\mathbf{n}]\,dS + \iint_{S_2}[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p_e\mathbf{n} - \mathbf{T}\cdot\mathbf{n}]\,dS = 0.$$

Substituting for the first two terms in the surface integral S_2, having demonstrated that the third term is negligible, we find that

$$\iint_{S_1}[\rho(\mathbf{v}\cdot\mathbf{n})\mathbf{v} + p_e\mathbf{n} - \mathbf{T}\cdot\mathbf{n}]\,dS = MUI\mathbf{u}_z. \tag{24}$$

Thus the force on the immersed object is equal to the initial momentum associated with those ions which are destined to be captured by the object.

5. Application to the case of strong subsonic flows

At large distances from the dust particle, or other object, the quasi-neutral electron–ion mixture will behave like a gas with constant sound speed [2]. We can then

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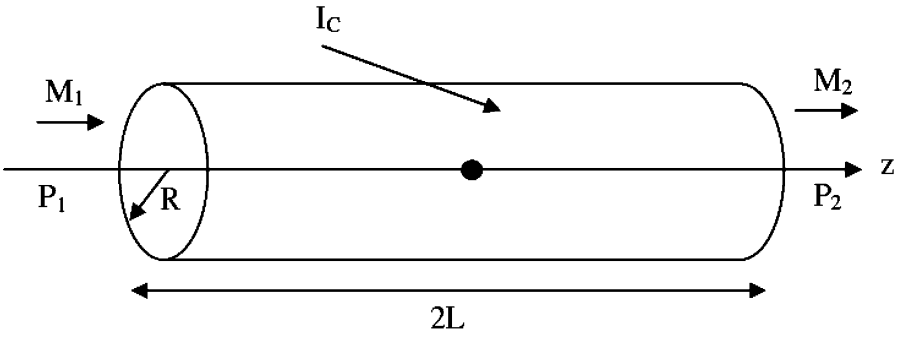


Figure 4. A diagram to illustrate the flow of momentum and the pressure force for strong subsonic flows.

apply results that that are known from fluid mechanics, in particular those results obtained for the linearized theory of subsonic flow with a sink [12]. The velocity potential at large distances from the sink can be written, in cylindrical coordinates, as

$$\Phi = Uz + \frac{k}{4\pi(z^2 + \beta^2 r^2)^{1/2}}, \tag{25}$$

where k is a measure of the flow to the sink in $\mathrm{m}^3\ \mathrm{s}^{-1}$ and $\beta = (1 - \nu^2)^{1/2}$ where $\nu = U/c_\mathrm{s}$ is the Mach number.

The velocity components given by taking the gradient of the velocity potential are

$$v_z = U - \frac{kz}{4\pi(z^2 + \beta^2 r^2)^{3/2}} \tag{26}$$

and

$$v_r = -\frac{k\beta^2 r}{4\pi(z^2 + \beta^2 r^2)^{3/2}}. \tag{27}$$

Another result that can be adopted from the linearized equations of fluid mechanics is an expression for the gas pressure [12]

$$p = p_\infty - \rho_\infty U \frac{\partial}{\partial z}(\Phi - Uz) \tag{28}$$

so that

$$p = p_\infty + \frac{\rho_\infty U k z}{4\pi(z^2 + \beta^2 r^2)^{3/2}}. \tag{29}$$

In the present case, the electron gas is assumed to be isothermal so that the local density follows from the last equation and can be written in the form

$$\rho = \rho_\infty \left[1 + \frac{Ukz}{4\pi(z^2 + \beta^2 r^2)^{3/2} c_\mathrm{s}^2}\right]. \tag{30}$$

Instead of the spherical surface considered previously, we shall now consider a large cylindrical control surface of length $2L$ and radius R, as shown in Fig. 4.

The z-directed momentum per unit area entering the cylindrical surface is given by

$$\rho v_z v_r = \frac{\rho_\infty U^2 k \beta^2 R}{4\pi(z^2 + \beta^2 R^2)^{3/2}}, \tag{31}$$

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where higher-order terms have been neglected. Hence the required flow of momentum is given by

$$I_C = \int_{-L}^{L} \frac{\rho_\infty U^2 k \beta^2 R^2}{2(z^2 + \beta^2 R^2)^{3/2}}\, dz, \tag{32}$$

which is readily evaluated to give

$$I_C = \frac{\rho_\infty U k L}{(L^2 + \beta^2 R^2)^{1/2}}. \tag{33}$$

The pressure forces at the plane ends of the cylindrical control volume will now be calculated using the expression for the pressure given above, (29). Integration over the plane surfaces gives

$$P_1 - P_2 = \int_0^R - \frac{\rho_\infty U k L r}{(L^2 + \beta^2 r^2)^{3/2}}\, dr, \tag{34}$$

which can be integrated to give the result

$$P_1 - P_2 = - \frac{\rho_\infty U k L}{\beta^2} \left[\frac{1}{L} - \frac{1}{(L^2 + \beta^2 R^2)^{1/2}} \right]. \tag{35}$$

The flow of z-directed momentum at the plane ends remains to be calculated. Using the expression for v_z, (26), we have

$$v_z^2 = U^2 - \frac{kUL}{2\pi(L^2 + \beta^2 r^2)^{3/2}} + \cdots. \tag{36}$$

Recalling the expression for density (30), the flow of z-directed momentum per unit area can now be written as

$$\rho v_z^2 = \rho_\infty \left[1 + \frac{UkL}{4\pi(z^2 + \beta^2 r^2)^{3/2} c_s^2} \right] \left[U^2 - \frac{kUL}{2\pi(L^2 + \beta^2 r^2)^{3/2}} \right], \tag{37}$$

which gives

$$\rho v_z^2 = \rho_\infty U^2 \left[1 - \frac{kz(1 + \beta^2)}{4\pi(L^2 + \beta^2 r^2)^{3/2} c_s \nu} \right] + \cdots. \tag{38}$$

The difference between the flow of momentum in and that flowing out is

$$M_1 - M_2 = \rho_\infty U^2 \int_0^R \frac{kL(1 + \beta^2) r}{(L^2 + \beta^2 r^2)^{3/2} c_s \nu}\, dr, \tag{39}$$

which can be evaluated to yield

$$M_1 - M_2 = \frac{\rho_\infty U k L (1 + \beta^2)}{\beta^2} \left[\frac{1}{L} - \frac{1}{(L^2 + \beta^2 R^2)^{1/2}} \right]. \tag{40}$$

Application of the principle of the conservation of linear momentum, using the fact that the Maxwell stress is unimportant at large radii, gives the drag force on the dust particle or other object as

$$D = (M_1 - M_2) + (P_1 - P_2) + I. \tag{41}$$

Collecting together our results, (33), (35), and (40), the resulting drag is found to be

$$D = \rho_\infty U k. \tag{42}$$

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We can note that the units of k are $\mathrm{m}^3\ \mathrm{s}^{-1}$, but we can write the expression for the drag in the same form as (24). We have now demonstrated that the same result is obtained for strong subsonic flows, i.e. for ion-acoustic Mach numbers between zero and one:

$$D = MUI. \tag{43}$$

The electric potential does not appear in the last calculation, although it is the electric field that acts upon the positive ions in the guise of a pressure gradient. It is of interest, therefore, to obtain an expression for the electric potential. The Boltzmann relation (4) can be linearized and compared with (30) to give

$$\frac{eV}{kT_\mathrm{e}} = \frac{k\nu z}{4\pi(z^2+\beta^2r^2)^{3/2}c_\mathrm{s}}. \tag{44}$$

Expressed in dimensionless units, for easier comparison with the previous results for weak flows, this becomes

$$\phi = \frac{k\nu z}{4\pi(z^2+\beta^2r^2)^{3/2}}, \tag{45}$$

where subscripts have been discarded. The flow to the dust particle or other object is now in normalized units of $\lambda_\mathrm{D}^3\omega_{pi}$. It is seen that the potential approaches that of a dipole as $\nu \to 0$ and $\beta \to 1$. (Note that cylindrical coordinates have been used here, whereas spherical coordinates were employed in the calculation for weak flows.)

6. Application to experimental work on dusty plasmas

The set of basic equations (3), (4), and (6), together with the continuity equation for the positive ions, remains to be solved [11]. Thus our principal result, (43), gives the total drag force on a dust particle in terms of the ion current, which is an unknown quantity. This paragraph indicates what can be done at present.

The charge Q on a dust particle can be determined in various ways, for example by observing the period of oscillation in the background electric field [13]. An estimate of the floating potential acquired by the dust particle can then be made using the 'vacuum capacitance' formula

$$Q \simeq 4\pi\varepsilon_0 r_d V_f. \tag{46}$$

where r_d is the radius of the dust particle and V_f is the floating potential.

This approximation should be accurate for small dust particles. Hence the electron flux to the dust particle can be calculated using

$$I_\mathrm{e} = 4\pi r_d^2 n_\infty e^{eV_f/kT_\mathrm{e}}\sqrt{\frac{kT_\mathrm{e}}{2\pi m}} \tag{47}$$

This flux is equal to the positive ion flux to the dust particle, if secondary processes are negligible, so using our principal result, (43), we obtain

$$D = 4\pi r_d^2 n_\infty e^{eV_f/kT_\mathrm{e}} MU\sqrt{\frac{kT_\mathrm{e}}{2\pi m}}. \tag{48}$$

The force on the dust particle is the result of ion impact, a (reduced) electron pressure, and an electric force. The theory presented in this paper does not give these individual components, but only their sum, the total force.

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7. Conclusions

The theory presented here assumes that the directed velocity of the positive ions is more important than their random velocities, which is the case in many laboratory plasmas. It is clear that in some situations collisions [14] will become important and secondary emission can also play a role, sometimes leading to the immersed object acquiring a positive charge [15]. It is hoped, however, that the present model will help to clarify the basic physics concerning the momentum transfer from the flowing ions to the immersed object, dust particle, or otherwise.

The complete set of equations [11] remains to be solved, as already stated above, and this represents a considerable challenge; the problem appears to be intractable analytically, so it is likely that a numerical approach will be necessary.

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