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On waves in incompressible Hall magnetohydrodynamics

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Abstract. Hall magnetohydrodynamics (HMHD) is a mono-fluid approximation extending the validity domain of the ordinary MHD system to spatial scales down to a fraction of the ion skin depth or frequencies comparable to the ion gyrofrequency. In the paper by Galtier (2006 *J. Plasma Physics*), an incompressible limit of the HMHD system is used for developing a wave turbulence theory. Nevertheless, the possibility and the consequences of such an approximation are different in HMHD and in MHD. Here, we analyse these differences by investigating the properties of the HMHD equations in the incompressible limit: the existence of linear modes, their dispersion relations and polarizations. We discuss the possibility of replacing the fluid closure equation of a complete HMHD system by an incompressibility hypothesis and determine the validity range.

1. Introduction

The bi-fluid plasma model and its various reductions have been extensively used to describe various phenomena in laboratory and astrophysical plasmas. In its most general form, it describes electrons and ions as two fluids coupled by Maxwell's equations. This complete bi-fluid system has six eigenmodes, three at low frequency (magnetohydrodynamics (MHD) modes and their continuation toward high frequency) and three at high frequency (the two vacuum electromagnetic modes, and one electrostatic plasma mode). If one considers the non-relativistic regime and neglects the effects due to the ion–electron charge separation (ensured by neglecting, respectively, the transverse and longitudinal components of the displacement current), a reduced fluid model having three eigenmodes can be obtained (e.g. Sahraoui et al. 2003). These modes are the three low frequency modes of the complete bi-fluid system but only for frequencies up to the electron gyrofrequency ω_{ce} and spatial scales down to the electron skin depth (smaller scales, where the electrostatic effects can become important, may not be correctly described in this approximation). A further consistent approximation is possible by fully neglecting the electron inertia ($m_e/m_i \sim 0$). One obtains then the Hall magnetohydrodynamics (HMHD) model. The last limit, i.e. the MHD model, is reached if one considers

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scales larger than the ion Larmor radius and times larger than the ion gyroperiod ($\omega \ll \omega_{ci}$). (The upper part of the HMHD range $\omega_{ci} \ll \omega \ll \omega_{ce}$, called the 'electron MHD' or 'whistler' range, also deserves specific simplifications but we will not refer to it here.)

Astrophysical plasmas are often described in the MHD approximation. However, recent works have reported the importance of small scales (not covered by the MHD theory) for phenomena such as turbulence or magnetic reconnection (e.g. Huba 1995; Ghosh and Goldstein 1997; Bhattacharjee 2004; Sahraoui et al. 2004; Vaivads et al. 2004). In collisionless plasmas particularly, these small scales are believed to play a key role, both for energy dissipation and for breaking the frozenness of the magnetic field in MHD flows. To model all these physical phenomena, there is therefore a clear need to include non-MHD effects, or at least to include the Hall effect. This last effect has indeed been shown to control the global reconnection scales and to allow fast reconnection rates (Fitzpatrik 2004; Vaivads et al. 2004; Birn et al. 2005). It is also believed to be very important for turbulence and dynamo effect in astrophysics (see, e.g., Balbus and Terquem 2001; Mininni et al. 2003; Cumming et al. 2004). For weak/wave turbulence studies, most previous works have been limited to the MHD approximation allowing descriptions of the weak nonlinear coupling of Alfvén waves (see Galtier et al. (2000) for a complete review). A similar theory has been developed for the electron MHD regime to describe whistler wave turbulence (Galtier and Bhattacharjee 2003). The next step should be the building of a HMHD wave turbulence theory for the intermediate scale range. Nevertheless, owing to the intrinsic complexity of the HMHD system for such studies, a first step consists of reducing HMHD to a more tractable 'incompressible HMHD' system. The aim of this short paper is to take stock of the situation concerning the incompressible HMHD system and its use: conditions for the existence of incompressible linear modes, dispersion, polarization and the relation to pure MHD modes.

2. Incompressible HMHD modes

We consider the inviscid HMHD equations in the incompressible limit (n is constant):

$$\partial_t \mathbf{V} + \mathbf{V}\cdot\nabla\mathbf{V} = \mathbf{B}\cdot\nabla\mathbf{B} - \nabla\left(\frac{\mathbf{B}^2}{2} + \frac{P}{mn}\right), \tag{2.1}$$

$$\partial_t \mathbf{B} = \nabla\times(\mathbf{V}\times\mathbf{B}) - d_i\nabla\times[(\nabla\times\mathbf{B})\times\mathbf{B}], \tag{2.2}$$

$$\nabla\cdot\mathbf{V} = 0, \tag{2.3}$$

$$\nabla\cdot\mathbf{B} = 0. \tag{2.4}$$

Here the magnetic field is normalized to a velocity ($\mathbf{B} \longrightarrow \sqrt{\mu_0 nm}\,\mathbf{B}$) and $d_i = B_0/\omega_{ci}$ is the ion skin depth (B_0 being the Alfvén speed). Equation (2.2) is Faraday's equation, coupled with the generalized Ohm's law in the HMHD approximation. One can indeed recognize the curl of the Hall term in (2.2) which is proportional to d_i. We note that no fluid closure equation, in the usual sense (i.e. coupling, for instance, the pressure P to the density n), is introduced here. Below we will discuss this point further.

To obtain the linear solutions, we linearize the previous equations around the homogeneous equilibrium state, by introducing the first-order variables $\mathbf{b}$, $\mathbf{v}$, p such that $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $\mathbf{V} = \mathbf{v}$, $P = P_0 + p$, where $\mathbf{B}_0$ and P_0 are assumed to be constant. A

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zeroth equilibrium flow is considered. We notice that, under the incompressibility hypothesis, the pressure perturbation p is implicitely assumed to be the result of temperature fluctuations. However, since no closure equation is used so far, there is no need to specify the form of the pressure. We will come back later to this issue. Now, we decompose the previous equations into longitudinal and transverse contributions with respect to the gradient operator (labelled hereafter, respectively, by the subscripts L and T). The longitudinal projections of (2.3) and (2.4) give vanishing components of the velocity and magnetic perturbations

$$\mathbf{v}_{\mathrm{L}} = \mathbf{b}_{\mathrm{L}} = \mathbf{0}. \tag{2.5}$$

Replacing these two equations into (2.1) leads to

$$\nabla\left(\mathbf{B}_0 \cdot \mathbf{b} + \frac{p}{mn}\right) = \mathbf{0}. \tag{2.6}$$

Equation (2.6) couples the pressure and the magnetic field perturbations. Similarly, replacing (2.5) in (2.2) provides only a trivial equality (i.e. all terms cancel). On the other hand, the projection on the transverse direction provides

$$\partial_t \mathbf{v}_{\mathrm{T}} = \mathbf{B}_0 \cdot \nabla \mathbf{b}_{\mathrm{T}}, \tag{2.7}$$

$$\partial_t \mathbf{b}_{\mathrm{T}} = \mathbf{B}_0 \cdot \nabla \mathbf{v}_{\mathrm{T}} - d_{\mathrm{i}} \nabla \times [\mathbf{B}_0 \cdot \nabla \mathbf{b} - \nabla(\mathbf{b}) \cdot \mathbf{B}_0]. \tag{2.8}$$

Owing to the curl operator, the Hall term above is purely transverse. Using $\mathbf{b}_{\mathrm{L}} = 0$ from (2.5) in (2.8) gives

$$\partial_t \mathbf{b}_{\mathrm{T}} = \mathbf{B}_0 \cdot \nabla(\mathbf{v}_{\mathrm{T}} - d_{\mathrm{i}} \nabla \times \mathbf{b}_{\mathrm{T}}). \tag{2.9}$$

Equations (2.7)–(2.9) form a set of first-order linear differential equations coupling self-consistently the transverse variables $\mathbf{v}_{\mathrm{T}}$ and $\mathbf{b}_{\mathrm{T}}$. In Fourier space ($\nabla \longrightarrow i\mathbf{k}$, $\partial_t \longrightarrow -i\omega$, and $i^2 = -1$) these equations become

$$-i\omega \mathbf{v}_{\mathrm{T}\mathbf{k}} = i\mathbf{B}_0 \cdot \mathbf{k} \mathbf{b}_{\mathrm{T}\mathbf{k}} \tag{2.10}$$

$$-\omega \mathbf{b}_{\mathrm{T}\mathbf{k}} = \mathbf{B}_0 \cdot \mathbf{k}(\mathbf{v}_{\mathrm{T}\mathbf{k}} - i d_{\mathrm{i}} \mathbf{k} \times \mathbf{b}_{\mathrm{T}\mathbf{k}}). \tag{2.11}$$

Equations (2.10) and (2.11) yield finally (for either $\mathbf{b}_{\mathrm{T}\mathbf{k}}$ or $\mathbf{v}_{\mathrm{T}\mathbf{k}}$)

$$\mathbf{e}_{\mathbf{k}} \times \mathbf{b}_{\mathrm{T}\mathbf{k}} = -i\Lambda \mathbf{b}_{\mathrm{T}\mathbf{k}}, \tag{2.12}$$

where $\mathbf{e}_{\mathbf{k}} = \mathbf{k}/k$ and

$$\Lambda = \frac{\omega^2 - {k_\parallel}^2 {B_0}^2}{\omega d_{\mathrm{i}} B_0 k_\parallel k}. \tag{2.13}$$

The subscript $\parallel$ denotes the parallel component to the Alfvén speed $\mathbf{B}_0$.

Let us now solve (2.12) with respect to Λ. The projection of (2.12) on $\mathbf{e}_{\mathbf{k}}$ yields the trivial relation

$$\mathbf{e}_{\mathbf{k}} \cdot \mathbf{b}_{\mathrm{T}\mathbf{k}} = 0. \tag{2.14}$$

Similarly, using (2.12) and its cross product by $\mathbf{e}_{\mathbf{k}}$ provides

$$\mathbf{e}_{\mathbf{k}} \times (\mathbf{e}_{\mathbf{k}} \times \mathbf{b}_{\mathrm{T}\mathbf{k}}) = -\Lambda^2 \mathbf{b}_{\mathrm{T}\mathbf{k}}. \tag{2.15}$$

By using relation (2.14), one obtains the following solution:

$$\Lambda = \pm 1. \tag{2.16}$$

Combining (2.13) and (2.16) yields the dispersion equation

$$\omega^2 - {k_\parallel}^2 {B_0}^2 = \Lambda \omega d_{\mathrm{i}} B_0 k_\parallel k. \tag{2.17}$$

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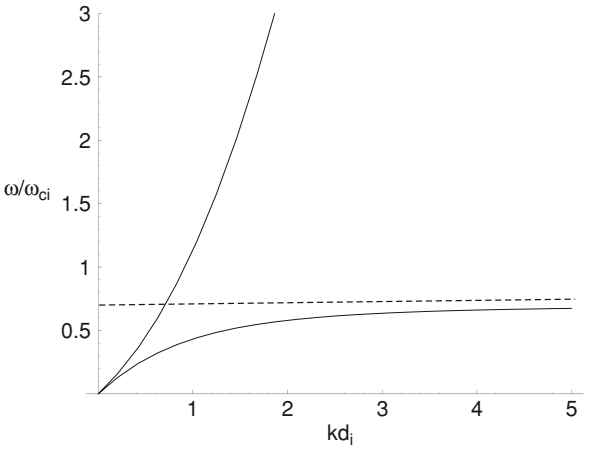


Figure 1. The dispersion relations of the incompressible HMHD modes calculated for a given propagation angle $\theta = (\mathbf{k}, \mathbf{B}_0) = \cos^{-1}(0.7)$. The horizontal dashed line is the resonance frequency $\omega_{\rm ci}\cos\theta$ of the Alfvén mode. Note the degeneracy of the two modes in the large scale limit $kd_{\rm i} \longrightarrow 0$.

The right-hand side term is the correction to the MHD dispersion as a result of the Hall effect. Solving (2.17) with respect to ω provides the dispersion relations of the incompressible HMHD modes,

$$\omega_\Lambda^s = \frac{k_\parallel k d_{\rm i} B_0}{2}\left(\Lambda + s\sqrt{1+\frac{4}{{d_{\rm i}}^2 k^2}}\right), \quad s = \pm 1. \tag{2.18}$$

Note that these solutions have been obtained previously by Krishan and Mahajan (2005; equation (29)). An illustration of the dispersion relation is given by Fig. 1. Equation (2.18) provides the dispersion relations of two modes: $\Lambda s > 0$ and $\Lambda s < 0$, which are, respectively, the circularly polarized right and left modes. We will call them, respectively, whistler and cyclotron modes. These names can also be justified by calculating the asymptotes of the corresponding dispersions.

Two limits are found.

(a) The small scale limit:

$$kd_{\rm i} \gg 1 \longrightarrow \omega_\Lambda^s \sim s k_\parallel B_0 k d_{\rm i}\left(\frac{\Lambda s+1}{2} + \frac{1}{k^2 d_{\rm i}^2}\right),$$

which has two values depending on the signs of Λ and s:

- $\Lambda s > 0 \longrightarrow \omega_\Lambda^s \sim s k_\parallel B_0 k d_{\rm i}$;
- $\Lambda s < 0 \longrightarrow \omega_\Lambda^s \sim s k_\parallel B_0 / k d_{\rm i} = s\omega_{\rm ci}\cos\theta$.

The first dispersion relation is the classical whistler mode, known to have a right-handed polarization. The second one is close to the Alfvén ion cyclotron asymptote. However, the resonance frequency here is corrected by the propagation angle with respect to the local magnetic field.

(b) The large scale limit:

$$kd_{\rm i} \ll 1 \longrightarrow \omega_\Lambda^s \sim s k_\parallel B_0,$$

which is the well-known degenerated Alfvén mode in the MHD theory.

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3. Wave polarizations

To point out a subtle difference between the HMHD and MHD approximations, we compute the polarization vectors of the previous waves, which can be obtained by solving (2.12) with respect to $\mathbf{b}_{\mathbf{Tk}}$. For this purpose we assume that $\mathbf{e}_{\mathbf{k}}$ lies in the (x, z) plane and forms an angle θ with respect to $\mathbf{B}_0$, which is assumed to be along the z-axis. Thus (2.12) yields the following equations (we omit the subscripts T and k for the field components):

$$i\Lambda b_x - \cos\theta b_y = 0, \tag{3.1}$$

$$\cos\theta b_x + i\Lambda b_y - \sin\theta b_\parallel = 0, \tag{3.2}$$

$$\sin\theta b_y + i\Lambda b_\parallel = 0, \tag{3.3}$$

which provide

$$\mathbf{b}_{\mathbf{Tk}} = \begin{pmatrix} -\cot\theta \\ -i\Lambda/\sin\theta \\ 1 \end{pmatrix} b_\parallel, \quad \Lambda = \pm 1. \tag{3.4}$$

We see that the three components are coupled linearly: they derive from one single free parameter $b_\parallel$. The vector $\mathbf{b}_{\mathbf{Tk}}$ corresponds to two waves polarized oppositely ($\Lambda = \pm 1$) whose dispersions are those calculated above.

It is worth noticing here that the previous solutions, i.e. the dispersion relations and polarizations, are obtained from only the transverse components of the dynamical equations. Therefore, we may ask what constraints are brought by the longitudinal equations. In Fourier space, (2.6) becomes

$$B_0 b_\parallel + \frac{p_k}{mn} = 0. \tag{3.5}$$

Equation (3.5) corresponds to the usual balance between thermal and magnetic pressures and consequently couples the pressure with the parallel component of the magnetic field perturbations. Taking into account this longitudinal constraint, the solution to (3.4) can be written as

$$\mathbf{b}_{\mathbf{Tk}} = \begin{pmatrix} \cot\theta \\ i\Lambda/\sin\theta \\ -1 \end{pmatrix} \frac{p_k}{mnB_0}. \tag{3.6}$$

As long as the initial system has not been closed by any 'equation of state', (3.5) plainly determines the new variable p_k as a function of the magnetic field solution found above from only the transverse system. If an equation of state is added to close the initial system, as is done in the usual MHD method, this new constraint dramatically changes the preceding result since it generally forbids any solution of this form but the trivial solution. Let us suppose for instance a polytropic closure such as

$$\frac{p_k}{P_0} = \gamma \frac{n_k}{n_0} = \gamma \frac{k}{\omega} v_{\mathrm{L}} = 0. \tag{3.7}$$

The polytropic assumption, together with the incompressible assumption therefore lead in general to $p_k = 0$, which yields the trivial solution $\mathbf{b}_{\mathbf{Tk}} = 0$ (3.6). This situation contrasts with the MHD model. Indeed, in the MHD limit ($kd_{\mathrm{i}} \longrightarrow 0$), (2.12) reduces to

$$(\omega^2 - k_\parallel^2 B_0^2)\mathbf{b}_{\mathbf{Tk}} = \mathbf{0}, \tag{3.8}$$

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which has non-trivial solutions only if

$$\omega = sk_{\parallel}B_0, \quad s = \pm 1.$$

This is the Alfvén dispersion relation. In this case, the polarization vector $\mathbf{b}_{\mathrm{Tk}}$ has the general form

$$\mathbf{b}_{\mathrm{Tk}} = \begin{pmatrix} -\cot\theta b_{\parallel} \\ b_y \\ b_{\parallel} \end{pmatrix}, \tag{3.9}$$

which obviously satisfies the transverse condition $\mathbf{e}_{\mathbf{k}} \cdot \mathbf{b}_{\mathrm{Tk}} = 0$. However, as one can see, only two components of the magnetic field are linearly coupled; the b_y component is independent from the two others. By using the longitudinal equation (3.5), the solution (3.9) becomes

$$\mathbf{b}_{\mathrm{Tk}} = \begin{pmatrix} p_k \cot\theta / nmB_0 \\ b_y \\ -p_k / nmB_0 \end{pmatrix}, \tag{3.10}$$

which does not contradict the polytropic closure (3.7): letting $p_k = 0$ in (3.10) simply leads to the usual polarization of the magnetic field along the y-axis for the MHD Alfvén wave.

We have therefore shown that, contrary to the ordinary MHD situation, there is no exact non-trivial incompressible solution in HMHD when a given equation of state of the plasma is used (coming from the underlying microphysics). We will nevertheless show now that approximate incompressible solutions can be found as limits of the compressible modes when the beta plasma parameter becomes very large.

4. From compressible to incompressible HMHD

Solving the complete compressible HMHD system, closed with a polytropic equation of state, leads to the following dispersion relation (e.g. Sahraoui et al. 2003; Ohsaki and Mahajan 2004):

$$(\lambda - \cos^2\theta)[\lambda^2 - \lambda(1+\beta) + \beta\cos^2\theta] = \frac{d_{\mathrm{i}}^2}{B_0^2}\omega^2(\lambda - \beta)\cos^2\theta, \tag{4.1}$$

where $\lambda = \omega^2/k^2B_0^2$, $\beta = C_{\mathrm{s}}^2/B_0^2$ and $C_{\mathrm{s}} = \sqrt{\gamma P_0/mn_0}$ is the sound speed. In the limit $\beta \longrightarrow \infty$, two solutions may be derived.

- Waves with phase velocities close to the sound speed: $\lambda \sim \beta$.

Equation (4.1) becomes

$$\lambda[\lambda^2 - \lambda\beta] \sim 0 \longrightarrow \lambda \sim \beta, \tag{4.2}$$

which provides the expected dispersion relation of the magnetosonic mode $\omega_{\mathrm{s}} \sim skC_{\mathrm{s}}(s = \pm)$.

- Waves with phase velocities much slower than the sound speed: $\lambda \ll \beta$.

Equation (4.1) yields

$$(\lambda - \cos^2\theta)[-\lambda\beta + \beta\cos^2\theta] = -\frac{d_{\mathrm{i}}^2}{B_0^2}\omega^2\beta\cos^2\theta, \tag{4.3}$$

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which gives

$$\lambda - \cos^2\theta = \Lambda \frac{d_i}{B_0}\omega \cos\theta, \tag{4.4}$$

or equivalently

$$\omega^2 - {k_\parallel}^2 {B_0}^2 = \Lambda d_i \omega B_0 k_\parallel k. \tag{4.5}$$

This dispersion equation is identical to (2.17), which has been obtained in the preceding section, without any expansion in beta, but using the incompressible limit. Calculating the polarization of this solution as above indeed confirms that it tends toward incompressibility when $\beta \longrightarrow \infty$, as could be expected for any subsonic solutions.

5. Conclusion

The incompressibility hypothesis, when justified, allows for a noticeable simplification in the calculations, not only in HMHD but also in MHD. In particular, this approach allows the analytical treatment of weak turbulence. Nevertheless, the exact range of validity of this hypothesis in both cases has to be known for its appropriate use and the main goal of this paper has been to address this problem. In ordinary MHD, a pure incompressible mode, the Alfvén mode, exists, and the use of a reduced incompressible MHD system allows the study of its properties whatever the value of the plasma beta. On the other hand, HMHD does not include any strictly incompressible mode. This means that the use of an incompressible hypothesis is meaningless for any low beta plasma. The main result of this paper that should be kept in mind is that incompressible HMHD can only be used in the limit of very large betas, where two of the three eigenmodes tend to incompressibility. A first study of weakly nonlinear interactions between HMHD waves is addressed in the paper by Galtier (2006). A more general model of weak/wave turbulence including compressibility may be applicable to a wider range of problems. Such an approach requires a powerful formalism such as the Hamiltonian field theory (Sahraoui et al. 2003). This general model is now in progress and will be the subject of a forthcoming paper.

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