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# Measuring Bias and Uncertainty in DW-NOMINATE Ideal Point Estimates via the Parametric Bootstrap

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DW-NOMINATE scores for the U.S. Congress are widely used measures of legislators' ideological locations over time. These scores have been used in a large number of studies in political science and closely related fields. In this paper, we extend the work of Lewis and Poole (2004) on the parametric bootstrap to DW-NOMINATE and obtain standard errors for the legislator ideal points. These standard errors are in the range of 1%–4% of the range of DW-NOMINATE coordinates.

## 1 Introduction

This paper presents a method for obtaining standard errors for the legislator parameters estimated by DW-NOMINATE. We extend the method developed by Lewis and Poole (2004) in which they use the parametric bootstrap (Efron 1979; Efron and Tibshirani 1993) to obtain standard errors and other measures of estimation uncertainty for W-NOMINATE (Poole and Rosenthal 1985, 1991, 1997) and the quadratic normal (QN) scaling procedure (Poole 2001).

Because of the sheer size of the matrices being analyzed by DW-NOMINATE, it is not practical to invert the information matrix or the analytical Hessian matrix directly. Consequently, traditional DW-NOMINATE computed only conditional standard errors. For example, the standard errors for a legislator were computed with the roll call parameters held fixed. This is possible to do because each legislator's parameters are independent of every other legislator's parameters (conditional on the roll call parameters). Although it is

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straightforward to compute these conditional standard errors, it is likely that they substantially understate the full unconditional sampling variability of the estimates. Indeed, we find that the conditional standard errors are 25% smaller on average than the unconditional standard errors that we derive here.

In Section 2, we outline the DW-NOMINATE model and briefly discuss the history of its development. In Section 3, we discuss the application of the parametric bootstrap to DW-NOMINATE. In Section 4, we show basic descriptive statistics of our results and we conclude in Section 5.

## 2 A Brief History of NOMINATE

The acronym DW-NOMINATE stands for dynamic, weighted, nominal three-step estimation. It is a *FORTRAN* program that estimates a probabilistic model of binary choice of legislators in a parliamentary setting over time. The basic choice model is the random utility model developed by McFadden (1976). In the random utility model, a legislator's overall utility for voting Yea is the sum of a deterministic utility and a random error. Specifically, legislator $i$'s utility for the Yea outcome on roll call $j$ in legislative session $t$ is

$$U_{ijyt} = u_{ijyt} + \epsilon_{ijyt}, \tag{1}$$

where $u_{ijyt}$ is the deterministic portion of the utility function and $\epsilon_{ijyt}$ is the stochastic or random portion of the utility function. It is standard to work with the difference between the utilities of Yea and Nay,

$$U_{ijyt} - U_{ijnt} = u_{ijyt} - u_{ijnt} + \epsilon_{ijyt} - \epsilon_{ijnt}.$$

Given the probability density for function $\epsilon_{ijyt} - \epsilon_{ijnt}$, the probability of a Yea vote by legislator $i$ is

$$\Pr_i(\text{Vote} = \text{Yea}) = \Pr(U_{ijyt} - U_{ijnt} > 0) = \Pr(\epsilon_{ijnt} - \epsilon_{ijyt} < u_{ijyt} - u_{ijnt}). \tag{2}$$

The "NOMINATE model," as it has come to be known, uses the normal density function as the deterministic utility function. The original one-dimensional NOMINATE (Poole and Rosenthal 1983, 1985, 1987a, 1987b) used "logit" error for the stochastic portion of the utility function. Technically, $\epsilon_{ijyt}$ and $\epsilon_{ijnt}$ are a random sample of size 2 from the log of the inverse exponential distribution, that is,

$$f(\epsilon) = \mathrm{e}^{-\epsilon}\mathrm{e}^{-\mathrm{e}^{-\epsilon}},$$

where $-\infty < \epsilon < \infty$.

Dhrymes (1978) shows that the probability distribution of $\epsilon_{ijnt} - \epsilon_{ijyt}$ is

$$f(z) = \frac{\mathrm{e}^{-z}}{(1+\mathrm{e}^{-z})^2},$$

where $z = \epsilon_{ijnt} - \epsilon_{ijyt}$ and $-\infty < z < \infty$. This is the logit distribution. The probability the legislator votes Yea is

$$\Pr_i(\text{Vote} = \text{Yea}) = \int_{-\infty}^{u_{ijyt} - u_{ijnt}} \frac{\mathrm{e}^{-z}}{(1+\mathrm{e}^{-z})^2}\mathrm{d}z = \frac{\mathrm{e}^{u_{ijyt}}}{\mathrm{e}^{u_{ijyt}} + \mathrm{e}^{u_{ijnt}}}. \tag{3}$$

The log of the inverse exponential distribution and the logit distribution are unimodal but not symmetric. However, neither distribution is badly skewed, and the logit distribution

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function is reasonably close to the normal distribution function. The advantage of the logit approach is that the probabilities are formulas. In contrast, with a normal error model the probabilities must be computed with some sort of series approximation. Given the computing resources of the early 1980s, the logit model was far easier to estimate.

This early approach was extended to D-NOMINATE, which was developed from 1986 to 1989. D-NOMINATE was multidimensional and could analyze more than one legislature at a time. Two modifications to D-NOMINATE were made when McCarty, Poole, and Rosenthal (1997) developed DW-NOMINATE. DW-NOMINATE is based on normally distributed errors rather than logit errors, and it uses the W-NOMINATE weighted distance model.

Although DW-NOMINATE, like D-NOMINATE, was designed to deal with the U.S. House and Senate, it can be used to analyze any voting body that meets in a series of legislative terms over time, for example, U.S. state legislatures, the United Nations, or the European Parliament. For our purposes, a "Legislative Term" is any distinct meeting of a voting body. In the U.S. context, we define a term to be one "Congress," generally a 2-year period. DW-NOMINATE as structured requires that terms be indexed by integers. Future work could adapt the program to measure time by date, etc.

Let $T$ be the number of legislative terms (Congresses, Parliaments, etc.) that are indexed by $t = 1, \ldots, T$; $s$ denote the number of policy dimensions ($k = 1, \ldots, s$); $p_t$ denote the number of legislators in legislative session $t$ ($i = 1, \ldots, p_t$); $q_t$ denote the number of roll call votes in legislative session $t$ ($j = 1, \ldots, q_t$); and $T_i$ denote the number of Legislative Terms in which legislator $i$ served ($t = 1, \ldots, T_i$). To allow for spatial movement over time, the legislator ideal points are treated as being polynomial functions of time, namely, legislator $i$'s coordinate on dimension $k$ at time $t$ is given by

$$X_{ikt} = \chi_{ik0} + \chi_{ik1}T_{t1} + \chi_{ik2}T_{t2} + \cdots + \chi_{ikv}T_{tv}, \tag{4}$$

where $v$ is the degree of the polynomial, the $\chi$'s are the coefficients of the polynomial and the time-specific terms—the $T$'s—are Legendre polynomials. Specifically, the first three terms of a Legendre polynomial representation of time are

$$T_{t1} = -1 + (t-1)\frac{2}{T-1}, \tag{5}$$

$$T_{t2} = \frac{3T_{t1}^2 - 1}{2}, \tag{6}$$

and

$$T_{t3} = \frac{5T_{t1}^3 - 3T_{t1}}{2}. \tag{7}$$

Legendre polynomials are orthogonal on the interval $[-1, +1]$. This is convenient because DW-NOMINATE scales the legislators and roll call midpoints to be in the unit hypersphere (more on this below). The orthogonality of the Legendre polynomials is a continuous property, but even with discrete data the linear and quadratic terms will be orthogonal. If ordinary powers of time were used instead, that is, $t$, $t^2$, $t^3$, and so on, these would be correlated.

The two roll call outcome points associated with Yea and Nay on the $k$th dimension at time $t$ can be written in terms of their midpoint and the distance between them, namely,

$$O_{jkyt} = Z_{jkyt} - \delta_{jkt}$$

and

$$O_{jknt} = Z_{jknt} + \delta_{jkt},$$

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where the midpoint is

$$Z_{jkt} = \frac{O_{jkyt} + O_{jknt}}{2}$$

and $\delta_{jkt}$ is half the signed distance ($\delta_{jkt}$ can be negative) between the Yea and Nay points on the $k$th dimension, that is,

$$\delta_{jkt} = \frac{O_{jkyt} - O_{jknt}}{2}.$$

Legislator $i$'s utility for the Yea outcome on roll call $j$ in legislative session $t$ is

$$U_{ijyt} = u_{ijyt} + \epsilon_{ijyt} = \beta e^{-\frac{1}{2}\sum_{k=1}^{s} w_k^2 d_{ijkyt}^2} + \epsilon_{ijyt}, \tag{8}$$

where

$$d_{ijkyt}^2 = \left(X_{ikt} - O_{jkyt}\right)^2$$

and the $w_k$ are the salience weights. Because the stochastic portion of the utility function is normally distributed with constant variance, $\beta$ is proportional to $\frac{1}{\sigma^2}$, where

$$\epsilon_{ijnt} - \epsilon_{ijyt} \sim N\left(0, \sigma^2\right).$$

Hence, the probability that legislator $i$ votes Yea on the $j$th roll call in legislative session $t$ is similar to that shown for the logit approach except that the standard normal cumulative distribution function ($\Phi$) is substituted for the logistic:

$$\Pr_i(\text{Vote} = \text{Yea}) = \Phi(u_{ijyt} - u_{ijnt}) = \Phi\left(\beta\left\{e^{-\frac{1}{2}\sum_{k=1}^{s} w_k^2 d_{ijkyt}^2} - e^{-\frac{1}{2}\sum_{k=1}^{s} w_k^2 d_{ijknt}^2}\right\}\right). \tag{9}$$

The natural log of the likelihood function is

$$\mathcal{L} = \sum_{t=1}^{T}\sum_{i=1}^{p_t}\sum_{j=1}^{q_t}\sum_{\tau=1}^{2} C_{ij\tau t} \ln P_{ij\tau t}, \tag{10}$$

where $\tau$ is the index for Yea and Nay, $P_{ij\tau t}$ is the probability of voting for choice $\tau$ (Yea or Nay) as given by equation (9), and $C_{ij\tau t} = 1$ if the legislator's actual choice is $\tau$ and 0 otherwise. Note that if a legislator did not vote on a roll call, then $C_{ijyt} = C_{ijnt} = 0$.

The total number of parameters is at most

$$2s\sum_{t=1}^{T} q_t + sp(v+1) + s,$$

where $2s\sum_{t=1}^{T} q_t$ is the number of roll call parameters, $sp(v+1)$ is the maximum possible number of legislator parameters—that is, all $p$ legislators serve in at least $v+1$ legislative terms so that the $v+1$ $\chi_{ik}$'s can be estimated for each member—and the last term represents $\beta$ and $w_2$ through $w_s$ that must be estimated (the weight on the first dimension can be set equal to 1). In practice, $v$ is usually set to 0—the constant model where the legislator's ideal point is the same in every legislative session in which she serves—or $v$ is set to 1—the linear model in which legislators are allowed to follow a straight line trajectory through the space over time. In their original D-NOMINATE work on the U.S. House and Senate,

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Poole and Rosenthal (1991, 1997, 2001, 2007) found that the linear model in two dimensions was the best combination of explanatory power and parsimony.

The spatial model outlined above has no natural metric. The metric used by DW-NOMINATE is established by requiring the constant point of the legislator's ideal point polynomial as well as the midpoints of the roll call outcomes ($Z_{ji}$'s) to lie within a unit hypersphere. These constraints are similar to those used in D-NOMINATE (Poole and Rosenthal 1997).

DW-NOMINATE like its predecessor D-NOMINATE does not generate its own parameter starting values because of the sheer size of the roll call data set being analyzed. Good starting values are crucial to reliably performing this complex nonlinear estimation. In their development of D-NOMINATE, Poole and Rosenthal experimented for over 2 years until they had a satisfactory solution to the problem. Essentially, they began with what they thought was a sensible set of starting values and studied the estimated legislator coordinates by turning them into computer animations that could be played on a VHS video recorder. In these animations, letter tokens and colors for the estimated legislator coordinates were used to identify parties and regions of the United States.[1] Poole and Rosenthal then relied upon their knowledge of American political history to see if the output made visual sense.[2] If they saw anomalies, they adjusted the output coordinates to correct for the anomalies and then used these adjusted coordinates as new starting values. At each step of this process, the fit of the model increased but it was a relatively arduous task because the supercomputers of the 1980s were limited in their capabilities and the animations took about a week to make on the equipment at the Pittsburgh Supercomputer Center.

The upshot of what one of us has described as ''Poole and Rosenthal as the outer loop of the estimation'' is that the final coordinates from D-NOMINATE that were released to the larger research community in 1989–90 were about as close to the global maximum of the (constrained) likelihood function shown in equation (10) as was practicable given the computer resources of the time.

Consequently, when DW-NOMINATE was developed in 1996 it simply used for starts the D-NOMINATE coordinates along with patched-in W-NOMINATE coordinates for Congresses 100–105. The converged coordinates that maximized equation (10) were then checked against the D-NOMINATE coordinates both in terms of simple Pearson correlations along with the overall fit of the model for the common period of Congresses 1–99. DW-NOMINATE has a slightly higher overall fit (Poole and Rosenthal 2007), and the Pearson correlations are all over 0.95 for dimensions one and two.

## 3 Applying the Parametric Bootstrap to DW-NOMINATE

The parametric bootstrap is based on classical statistics. The parameters are treated as fixed constants to be estimated, and consequently, they are not given probability distributions as in the Bayesian approach. Rather, the underlying probability model, with its parameters set equal to the maximum likelihood estimates, is used to generate repeated simulated samples. For each of these repeated samples, the model is refit, and the empirical distribution

[1] Because DW-NOMINATE is a dynamic estimation, animations remain our preferred means for understanding the results and obtaining a rapid overview of the evolution of ideology in the political history of Congress. The most current animations, programmed in 2005, are on the Web at http://voteworld.Berkeley.edu and http://voteview.ucsd.edu.

[2] For a short history of this, see the discussion in Poole and Rosenthal (1997).

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of these estimates across the pseudo-samples is used to estimate the relevant sampling distributions.

The parametric bootstrap is conceptually simple. In a maximum likelihood framework, the first step is to compute the likelihood function of the sample. The second step is to draw, for example, 1000 samples from the likelihood density and compute for each sample the maximum likelihood estimates of the parameters of interest. The sample variances computed from these 1000 values are the estimators of the variances of the parameters (Efron and Tibshirani 1993).[3]

When the parametric bootstrap is applied to a scaling method such as DW-NOMINATE, the first step is to run the program to convergence and then calculate the probabilities for the observed choices. This produces a legislator by roll call matrix containing the estimated probabilities for the corresponding actual roll call choices of the legislators. Note that the product of these probabilities is the likelihood, that is,

$$\mathcal{L}(\hat{\theta}\Big| Y) = \prod_{t=1}^{T}\prod_{i=1}^{p_t}\prod_{j=1}^{q_t}\prod_{\tau=1}^{2}\hat{P}_{ij\tau t}^{C_{ij\tau t}}, \tag{11}$$

where $\hat{\theta}$ is the set of all the estimated ideal point, roll call, and utility function parameters, $Y$ is the $p \times \sum_{t=1}^{T} q_t$ matrix of roll calls for all legislative sessions with the number of nonmissing entries equal to $\sum_{t=1}^{T}\sum_{i=1}^{p_t}\sum_{j=1}^{q_t}\sum_{\tau=1}^{2} C_{ij\tau t}$, $\hat{P}_{ij\tau t}$ is the estimated probability that legislator $i$ votes for choice $t$ on the $j$th roll call in legislative session $t$ as given by equation (9), and $C_{ij\tau t} = 1$ if the legislator's actual choice is $t$ and 0 otherwise.

To draw a random sample, we treat each estimated probability as a weighted coin and "flip" the coin. To do the "flip," we draw a number from a uniform distribution over zero to one—$U(0, 1)$—and if the estimated probability is greater than or equal to the random draw, the sampled value is the observed choice. If the random draw is greater than the estimated probability, then the sampled value is the opposite of the observed choice, that is, if the observed choice is Yea, then the sampled value is Nay. Note that this is equivalent to drawing a random value from $\sum_{t=1}^{T}\sum_{i=1}^{p_t}\sum_{j=1}^{q_t}\sum_{\tau=1}^{2} C_{ij\tau t}$ separate Bernoulli distributions with corresponding parameters $\hat{P}_{ij\tau t}$. In the context of a Bernoulli distribution, the two outcomes are either "success" or "failure." Here a "success" is the observed choice and "failure" is the opposite choice.

The sample roll call matrix is then analyzed by DW-NOMINATE. This process is repeated $n$ times, and the variances of the legislator ideal points are calculated from the $n$ bootstrap configurations. Technically, let $\pi$ be a random draw from $U(0,1)$. The sample rule is such that if the observed choice is Yea (Nay):

If $\pi \leqslant \hat{P}_{ij\tau t}$, then the sample value, $\hat{C}_{ij\tau t}$, is Yea (Nay). If $\pi \geqslant \hat{P}_{ij\tau t}$, then the sample value, $\hat{C}_{ij\tau t}$, is Nay (Yea).

This technique allows the underlying uncertainty to propagate through to all the estimated parameters. To see this, note that as $\hat{P}_{ij\tau t} \to 1$, then $\hat{C}_{ij\tau t} \to C_{ij\tau t}$, that is, sample choices become the observed choices so that the bootstrapped variances for the parameters of the model go to zero. If the fit of the model is poor, for example, if $\hat{P}_{ij\tau t}$ are between 0.5 and 0.7, then the bootstrapped variances for the parameters will be large.

[3]The parametric bootstrap assumes that the model has been correctly specified so that its assumptions are stronger than the nonparametric bootstrap. The nonparametric bootstrap is not readily applicable in this case because the data (roll call votes) are dependent across both members and roll calls (unconditionally).

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Applying the parametric bootstrap to DW-NOMINATE is not as easy as the application to W-NOMINATE and QN discussed in Lewis and Poole (2004). W-NOMINATE and QN are completely self-contained programs that generate their own starting values for the parameters. They read the sample roll call matrix and generate the estimated parameters for that bootstrap trial. Consequently, for each bootstrap run, new starting values can be computed; from the new starts, W-NOMINATE or QN can be run to convergence. This procedure leads to bootstrapped sets of parameters and the corresponding bootstrapped variances. In contrast, as we discussed above, DW-NOMINATE reads starting values for the parameters. Specifically, for the two-dimensional model, let $\tilde{X}$ be the $\sum_{t=1}^{T} p_t \times 2$ matrix of starting values for the legislators, let $\tilde{O}$ be the $\sum_{t=1}^{T} q_t \times 4$ matrix of starting values for the roll calls, $\tilde{w}_2$ be the starting value of the second-dimension weight (recall that $w_1$ is set equal to 1), and $\tilde{\beta}$ be the starting value for the signal-to-noise parameter $\beta$. The steps in DW-NOMINATE are the following:

1. Read $Y$, $\tilde{X}$, $\tilde{O}$, $\tilde{w}_2$, and $\tilde{\beta}$.
2. Estimate $\tilde{w}_2$, given $\tilde{X}$, $\tilde{O}$, and $\tilde{\beta}$.
3. Estimate $\beta$, given $\tilde{X}$, $\tilde{O}$, and $\tilde{w}_2$.
4. Estimate $O$ given $\tilde{X}$, $\tilde{\beta}$, and $\tilde{w}_2$.
5. Estimate $X$ given $O$, $\beta$, and $w_2$.
6. Go to step 2.

Steps 1–6 are repeated until the improvement in geometric mean probability is less than 0.001. Typically, this is only about three to five iterations through the six steps.

Let $h = 1, \ldots, m$ be the number of bootstrap trials and let $Y_h$ be the $p \times \sum_{t=1}^{T} q_t$ matrix of roll calls for all legislative terms produced by the bootstrap draw procedure shown in equation (11). Let $\tilde{X}$, $\tilde{O}$, $\tilde{w}_2$, and $\tilde{\beta}$ be the actual starting values used to produce $\hat{\theta}$. Given that the Poole-Rosenthal ''outer loop'' cannot be programmed, one possible approach is for step 1 to be

1. Read $Y_h$, $\tilde{X}$, $\tilde{O}$, $\tilde{w}_2$, and $\tilde{\beta}$.

That is, we could use the same starting coordinates for each bootstrap trial. However, because the DW-NOMINATE algorithm is not run to full convergence, the estimates from the bootstrap samples might depend in important ways on the starting values. In particular, the variability in the estimates obtained from the bootstrap samples might be attenuated by using a fixed set of starting values. To avoid this potential source of attenuation, we created new starting values for each bootstrap iteration by randomly perturbing the maximum likelihood estimates. Specifically, we set

$$\tilde{X}_h = \tilde{X} + \Psi$$

and

$$\tilde{Z}_h = \tilde{Z} + \Xi,$$

where the $s \sum_{t=1}^{T} p_t$ elements of $\Psi$ and the $s \sum_{t=1}^{T} q_t$ elements of $\Xi$ were randomly drawn from a normal distribution with zero mean and constant variance, that is, $N(0, \sigma^2)$. Note that we perturbed the midpoints of the outcomes. This was easier to implement computationally and is sufficient to produce dramatically different starting values for the roll calls depending upon how large $\sigma^2$ is. Consequently, at the $h$th iteration step 1 is

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1. Read $Y_h$, $\tilde{X}_h$, $\tilde{O}_h$, $\tilde{w}_2$, and $\tilde{\beta}$.

As it turns out, the magnitude of the estimated standard errors that we obtained using this starting value method does not substantially depend upon the value chosen for $\sigma^2$. Because the estimated standard errors remained stable as we increased $\sigma^2$ from 0 to 0.4, we are reasonably confident that the starting values are in no way driving the uncertainly estimates presented below.

One issue that we had to consider was that the legislator and roll call coordinates are only identified up to an arbitrary rotation in the *s*-dimensional space. DW-NOMINATE pins down the rotation, setting the first dimension to the one that explains the greatest amount of variation in the voting, the second dimension to dimension orthogonal to the first that explains the second most variation, and so forth. The polarity of these dimensions is also arbitrary. That is, each of the dimensions can be reversed without changing the fit or meaning of the estimates. We avoid this problem through the use of starting coordinates that lead the estimator to consistently find the same polarity of the space in each bootstrap trial.

A more important issue is the interaction of the second-dimension weight, $w_2$, with the linear terms of the legislator polynomials, the $\chi_{ik1}$'s. The only constraints used by DW-NOMINATE are that the constant point of the legislator—that is, the $s \times 1$ vector $\chi_{i0}$ from the $s$ polynomials defined by equation (4)—and the midpoints of the roll call outcomes—the $Z_{jt}$'s—are within a unit hypersphere. Consequently, there is some interaction between $w_2$ and the linear terms for the legislators, the $s \times 1$ vectors $\chi_{i1}$. Namely, the legislators' time trends on the first dimension can all increase, the time trends on the second dimension can all increase a little, and the effect can be canceled in terms of overall fit by increasing the size of $w_2$ to compensate. Let $\hat{X}$ be the $\sum_{t=1}^{T} p_t \times s$ matrix of legislator coordinates estimated by DW-NOMINATE, and let $X_h$ be the $\sum_{t=1}^{T} p_t \times s$ matrix of legislator coordinates estimated on the $h$th bootstrap trial. We removed this effect by computing the regression

$$\hat{X} = X_h \Delta + \epsilon, \tag{12}$$

where $\Delta$ is an $s \times s$ diagonal matrix. Because DW-NOMINATE always produces legislator coordinates with mean zero, we do not show a vector of intercepts in equation (12). To reduce notational clutter, we will simply use $X_h$ to denote the $h$th bootstrap trial with this correction taken into account.

The mean legislator ideal point on the $k$th dimension at time $t$ is

$$\tilde{X}_{ikt} = \frac{\sum_{h=1}^{M} X_{hikt}}{m},$$

where $X_{hikt}$ is the estimated coordinate on the $h$th trial. The corresponding standard deviation is

$$\sigma_{X_{ikt}} = \sqrt{\frac{\sum_{h=1}^{M} \left(X_{hikt} - \hat{X}_{ikt}\right)^2}{m-1}}.$$

We take a conservative approach and use the estimated coordinate, $\hat{X}_{ikt}$, rather than the mean of the bootstrap trials, $\tilde{X}_{ikt}$, as our "sample mean" in our calculation of the standard deviation. This inflates the standard deviations somewhat, but we feel it is better to err on the safe side and not underreport the standard deviations.

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## 4 DW-NOMINATE Standard Errors and Auxiliary Quantities of Interest for the U.S. House and Senate, 1789 to 2006

We applied the bootstrap procedure developed above to roll call votes taken in the U.S. House and Senate between the First Congress and the 109th Congress. The selection criterion for which members and roll calls to include in the analysis was the same as was used in Poole and Rosenthal (2007) and includes every roll call vote with at least 2.5% in the minority and every legislator casting more than 20 votes in any Congress. In total, for 10,419 unique members of the House and 1828 unique Senators, 35,739 locations are estimated for Representatives and 8644 locations for Senators across all Congresses in which they served. These estimates are based on 42,029 scalable roll call votes in the House with a total of 11,866,698 individual vote choices and 43,188 scalable roll call votes in the Senate with a total of 2,479,057 individual vote choices.

Each member's ideal point in each Congress is estimated along two dimensions. As described above, the constraints imposed on how each member's location can vary over time allow the estimated locations to be compared across Congresses. Previous research has demonstrated that the first dimension locations reveal standard left-right or economic cleavages, whereas the second-dimension locations reflect social and sectional divisions. The importance of the second dimensions has waxed and waned throughout American history. Readers are encouraged to consult Poole and Rosenthal (2007) for a complete discussion of the DW-NOMINATE scores in the U.S. Congress. In what follows, we will assume a basic familiarity with NOMINATE scores and how they are commonly applied to the study of the U.S. Congress.

For the Senate, estimated first-dimension locations ranged from −1.252 (John Langdon, Jeffersonian, of New Hampshire in the sixth Congress) to 1.392 (James Ross, Federalist, of Pennsylvania in the third Congress) with 90% of the estimated locations falling between −0.511 and 0.516. Second-dimension locations ranged from −1.482 (John McCain, Republican, of Arizona in the 109th Congress) to 1.475 (Richard Russell, Democrat, of Georgia, well known as a segregationist, in the 91st Congress) with 90% falling between −0.764 and 0.760. Estimates of first-dimension locations in the House ranged from −1.273 (Adam Clayton Powell, Democrat, of New York, a controversial African-American from Harlem, in the 79th Congress) to 1.586 (Harold Gross, Republican, of Iowa, well known as an isolationist conservative, in the 93rd Congress). Ninety percent of the first-dimension locations fall between −0.545 and 0.587. Second-dimension locations range from −1.691 (Frank Hook, Democrat, of Michigan in the 79th Congress) to 1.505 (John Shafer, Republican, of Wisconsin in the 68th Congress). Ninety percent of the second-dimension locations fall between −0.833 and 0.928.

In order to obtain standard errors for the estimated legislator locations and to calculate standard errors and confidence intervals for other quantities of interest based on these estimates, we produced 1000 bootstrap trials using the procedure described in Section 3. For the purposes of calculating the standard error of each estimated legislator location, 1000 trials are certainly more than we need. Indeed, subsamples of as few as 50 of the 1000 total samples produced estimated standard errors that were very similar to those based on the full set of bootstrap trials. Treating the standard errors obtained after 1000 trials as reflecting the true sampling uncertainty, Fig. 1 shows the root mean squared error in the standard error estimates associated with U.S. House members' locations resulting from a range of bootstrap trial sizes. For each bootstrap sample size considered, 10 different bootstrap samples were drawn and plotted. Note that the difference between the standard errors estimated from 1000 trials and those estimated from smaller numbers

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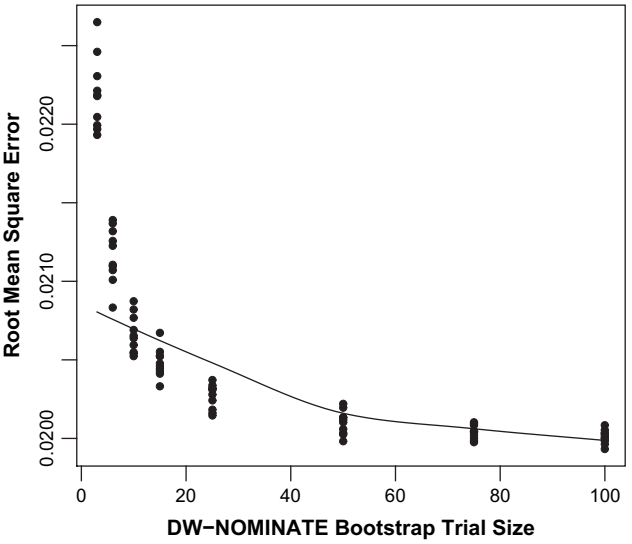



**Fig. 1** The root mean squared error of the standard error estimates of U.S. House member locations for bootstrap sample sizes between 3 and 100, along with lowess smoother. These average errors are relative to the standard errors obtained after 1000 trials.

of trials drops off quickly as the smaller number of trials is increased and that estimated standard errors based on as few as 50 bootstrap trials are quite similar to those based on 1000 trials. However, for other quantities of interest, such as confidence intervals for overlap partisan intervals (described below), more samples are useful. For researchers interested in calculating confidence intervals for any function of the DW-NOMINATE scores from the U.S. Congress, we have made the results of all 1000 of these bootstrap trials, as well as estimated standard errors for legislator locations, available on for download from the Internet.[4]

In the Senate, the average estimated standard error for first-dimension locations was 0.05 and 90% of the first-dimension standard errors fall between 0.02 and 0.10. The largest first-dimension standard error was 0.41 and is associated with James Ross (Federalist) of Pennsylvania in the third Congress. This large standard error is not unexpected given that Ross cast only 361 votes in his career, served at a time when the Senate had fewer than 35 members, and was located far from the middle of the dimension. The average second-dimension standard error is 0.094. The largest second-dimension standard error was 0.541 and is associated with Thaddeus Betts (Whig) of Connecticut in the 26th Congress. Betts cast only 44 votes in his career and served in a Senate with only 52 members. Overall, 90% of the second-dimension standard errors fall between 0.042 and 0.166.

In the House, the average first-dimension standard error is 0.047. Ninety percent of the first-dimension standard errors fall between 0.014 and 0.101. The largest first-dimension standard error of 0.496 is associated with the estimated location of Harold Gross (R) of Iowa in the 93rd Congress. This large standard error results from the large (but uncertain) estimated linear trend in his estimated first-dimension location that moves him from 0.396 (with a standard error of 0.054) in the 81st Congress to 1.586 (0.496) in the 93rd Congress.

[4]The full set of bootstrap trials can be downloaded from http://www.voteview.com.

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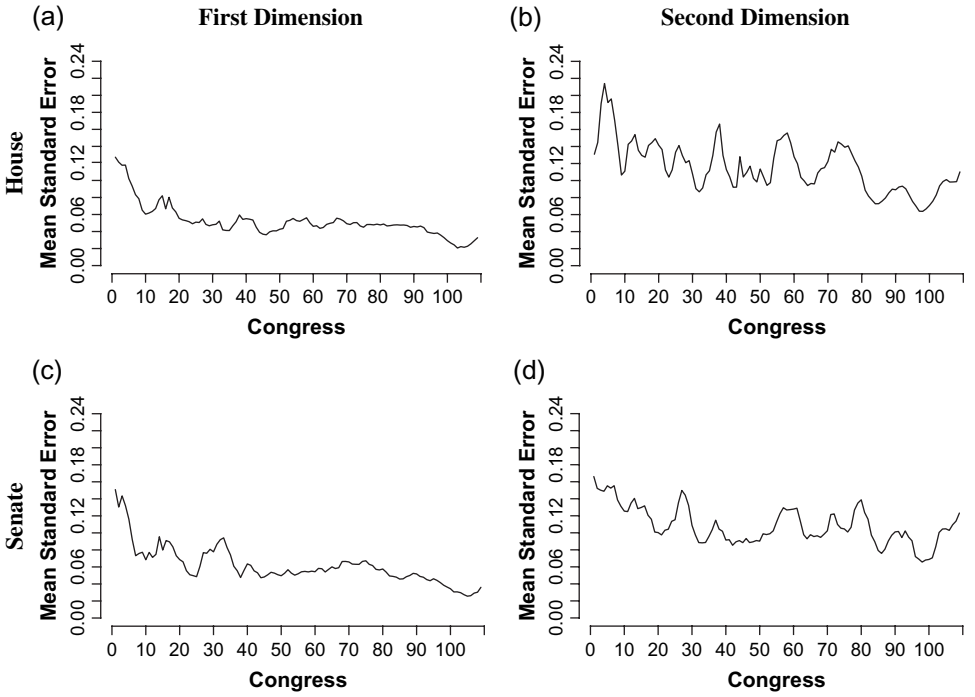



**Fig. 2** Mean DW-NOMINATE standard errors by chamber and dimension.

The average House second-dimension standard error is 0.11 with 90% of the estimated standard errors falling between 0.037 and 0.235. The largest second-dimension standard error, 0.738, is associated with the estimated second-dimension position of John Reid (D) of Missouri in the 37th Congress (Reid only cast 31 votes and only served in the 37th—the first 2 years of the Civil War).

Because of the substantially greater predictive power of the first dimension, it is unsurprising that first-dimension locations are typically estimated with roughly twice the precision as second-dimension locations. Figure 2a and 2b plot the average standard errors for the first- and second-dimension locations, respectively, for the House, and Fig. 2c and 2d show the same information for the Senate. A scatter plot of all first-dimension standard errors for the House is shown in Fig. 3. Note that there is considerable variability over time. We are still investigating all the factors that drive this variability. However, the historically small uncertainty associated with the first-dimension locations of members of recent Congresses results at least in part from the relatively large size of the modern House and Senate, the large number of roll calls taken in recent Congresses in both chambers, and the large number of long-serving members. In contrast, second-dimension locations have been increasingly uncertain over past few decades. This lack of precision results from the declining salience of the second dimension (Poole and Rosenthal 2007). That is, relatively few votes taken during the last few decades have cutting lines that divide Members of Congress mainly and strongly along the second dimension.

Figure 4a and 4c plot the locations of the floor medians for the House and Senate, respectively, and Fig. 4b and 4d show the Democrat and Republican Party medians since the end of Reconstruction (46th to 109th Congresses). One of the useful features of the bootstrap procedure is that it allows for easy calculation of standard errors and confidence

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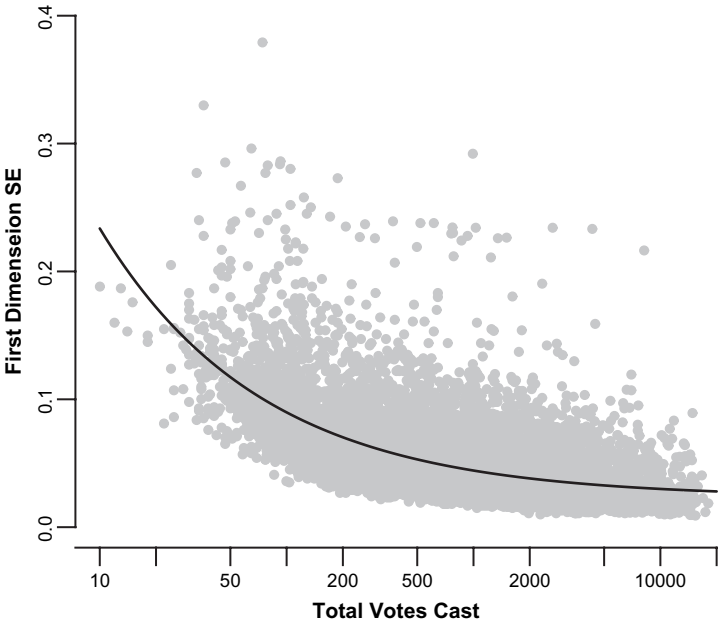



**Fig. 3** First-dimension standard errors of legislators versus number of votes for U.S. House of Representatives. Standard errors shown are mean standard errors of legislators across all years of service. Fitted curve represents the best-fit $\frac{1}{\sqrt{N}}$ line through the points, with $R^2 = 0.36$.

intervals not only of individual members' locations but also for any function of those locations. The vertical line through each of the points plotted in Fig. 4a to 4d represent the 95% confidence interval for the corresponding estimated median. Not surprisingly, the medians are estimated with greater precisions than individual member locations and indeed are quite tightly estimated. Given that a good deal of scholarship turns on how the ideological composition of the House and Senate floor and party caucuses has varied over time as measured by DW-NOMINATE scores, it is comforting that that variation is not driven by estimation imprecision to any significant degree.

Another function of the ideal point estimates is the extent of ideological overlap between the two major parties. This measure has been used to study political polarization by Poole and Rosenthal (1997, 2001, 2007) and McCarty, Poole, and Rosenthal (2006). As Fig. 5 shows, overlap is estimated with great precision.

Figure 6a and 6b show confidence ellipses for several prominent members of the 109th Congress. Figure 6a shows that then minority leader Nancy Pelosi (D-CA) is to the left of the Democratic caucus and her rival for party leader Steny Hoyer (D-MD) is to the right of the caucus. Figure 5b reveals that Democrat Senate leader (Reid of NV) was significantly more moderate along the first dimension than the caucus that he leads and that he held a distinct position from the dimension-by-dimension Democratic median. Both Senator Hillary Clinton (D-NY) and Senator Barak Obama (D-IL) have confidence ellipses that overlap the center of the Democratic caucus (and each of these Senators has relatively large confidence ellipses due in part to their short tenure in the Senate). Interestingly, John McCain (R-AZ) despite his long service in the Senate is nevertheless quite imprecisely located in the 109th Congress.

McCain's move away from the Republican caucus is shown in Fig. 7. Note that as he has moved farther away from his caucus, his position has become increasingly uncertain. The

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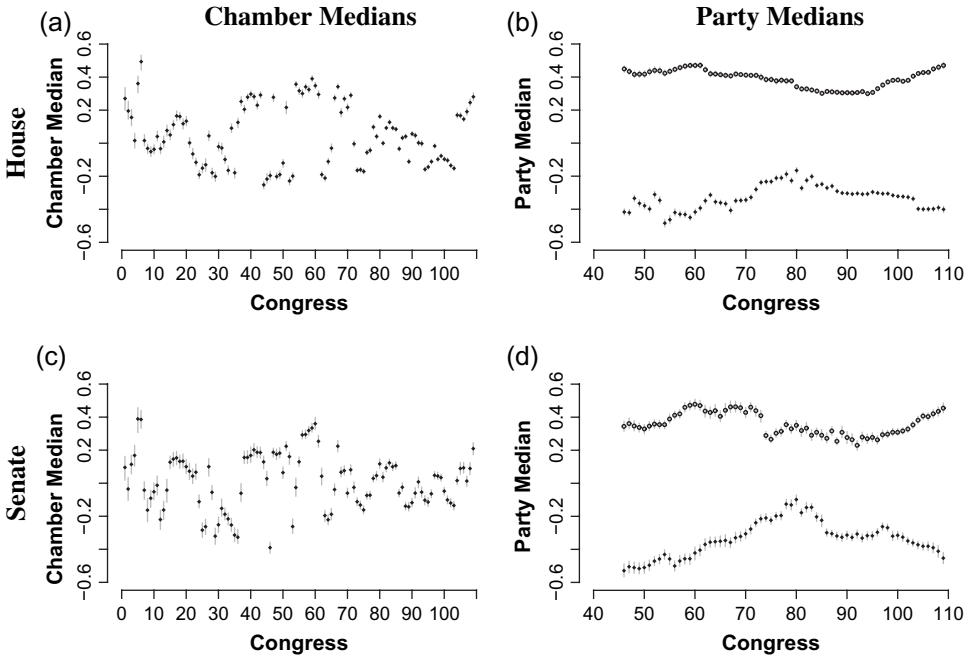



**Fig. 4** Chamber and party median DW-NOMINATE scores by chamber. Chamber medians begin from the first Congress, whereas party medians begin from the 46th Congress dating back only to the joint existence of the modern Democratic (lower band) and Republican (upper band) parties. Lines from each point represent the empirical 95% confidence intervals of the estimates.

movement of the Senate median to the right in part reflects increased Republican membership following the elections of 1994 and 2002. The movement of the Republican median to the right reflects the increasing polarization of American politics. McCain's position has been very stable on the first dimension. The roll call voting behavior associated with McCain's ''maverick'' reputation may be reflected in his movement along this second dimension. This has been picked up by his sharp movement on the second dimension, which, in recent years, has served mainly to pick up ''odd'' votes that fail to fit the liberal

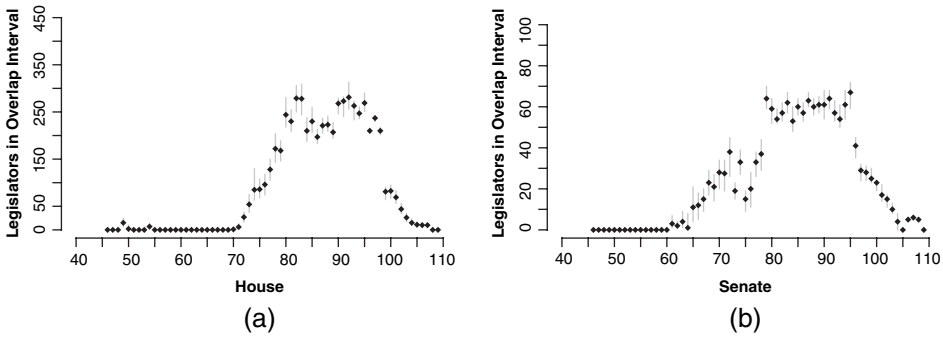



**Fig. 5** Overlap intervals by chamber from the 46th to the 109th Congresses. Overlap intervals are computed as the number of legislators with ideal points lying between the most conservative Democrat and the most liberal Republican. Lines show 95% confidence intervals of the estimated overlap intervals.

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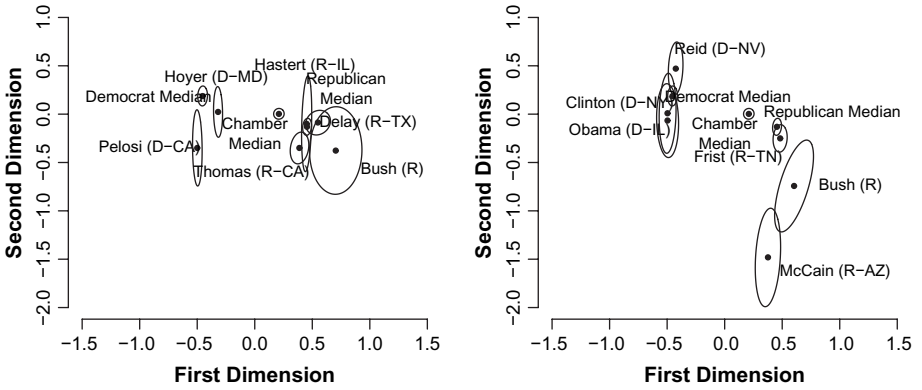



**Fig. 6** DW-NOMINATE scores in two dimensions of key members of the 109th Congress. House results are shown on the left, and Senate results are shown on the right. Ellipses are 95% confidence ellipsoids generated using R's ellipse package. The chamber and party medians have also been plotted. The estimates for President Bush are based on the votes selected by Congressional Quarterly to compute Bush's presidential support score.

conservative dimension. Finally, note that McCain's ellipses are smaller in the middle of his Senate career. This reflects the typical result in this setting that the estimates are more accurate near the center of a range of data.

## 5 Conclusions

Our results show that the parametric bootstrap is an effective method for obtaining standard errors for DW-NOMINATE. These standard errors are relatively small in

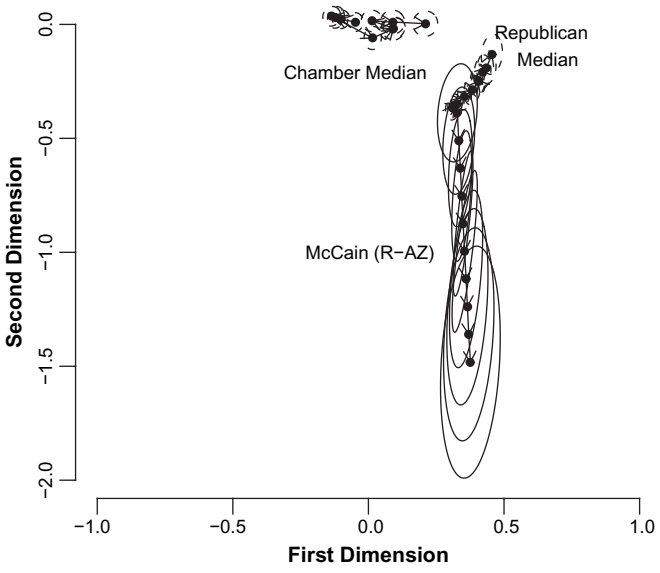



**Fig. 7** DW-NOMINATE shifts of McCain (R-AZ), Senate median, and Republican Senate median from the 100th to 109th Congress. Ellipses are 95% confidence ellipsoids. Arrows point to a shift from one Congress to the next.

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magnitude, and in the aggregate they reflect the well-known periods of Congressional history when roll call voting did not fit the spatial model (Silbey 1967; Poole and Rosenthal 1997, 2007).

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