SUPERCONDUCTING CYLINDERS

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1. Introduction

It has been shown experimentally* that there is no hysteresis in the magnetization curve of a superconductor provided it is pure and has an ellipsoidal shape (e.g. a sphere or very long cylinder). In these circumstances the transition from super to normal conductivity can be explained† in terms of an intermediate state for which $H = H_c$ and $0 < B < H_c$ (H_c being the critical field), and this transition takes place reversibly. It has not however been possible to calculate theoretically how the transition should take place for a non-ellipsoidal shape, and it seemed of interest, therefore, to investigate experimentally the magnetization curves of some simple non-ellipsoidal shapes such as circular cylinders of finite lengths. The results of such an investigation are described in the present paper, and it is shown how the magnetization curve and, in particular the hysteresis which it displays, is affected by the shape, the temperature and the purity of the specimen. The paper concludes with some remarks about the time lags reported to occur between changes of field and the corresponding changes of magnetization‡.

2. Experimental details

The magnetization was measured by the method previously used by the author (*loc. cit.*) for the sphere and ring, namely by measuring the force on the specimen in a slightly inhomogeneous field. Experiments were made with cylinders of various length to diameter ratios, but mostly with a 1 : 1 ratio, in order to have a shape as different as possible from an ellipsoid (if the ratio is greater than one, the specimen approaches an infinite cylinder, the limiting case of a long spheroid, while if the ratio is less than one it approaches a thin disc, the limiting case of a flat spheroid). The cylinders (usually 3 or 4 mm. in diameter) were prepared either by turning or by pressing in a mould (in the case of lead), and

* Shoenberg, *Proc. Roy. Soc.* A, 155 (1936), 712. This contains references to earlier work.

† Peierls, *Proc. Roy. Soc.* A, 155 (1936), 613; London, *Physica*, 3 (1936), 450.

‡ Rjabinin and Schubnikow, *Phys. Zeit. d. Sow. Union*, 6 (1934), 557; Mendelssohn and Pontius, *Physica*, 3 (1936), 327; *Nature*, 138 (1936), 29.

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the metals investigated were lead (Hilger HS 8334), tantalum (from Tungsten Mfg. Co. Ltd.) and tin (Hilger HS 10,000). We shall deal first with the results obtained with tin, since this appeared to be the purest superconductor and showed no complicating features due to impurities.

3. Magnetization curve of 1 : 1 tin cylinder

It is convenient to plot our results in a way rather different from that adopted in the earlier paper. Since a non-ellipsoidal specimen is not in general uniformly magnetized, the observed magnetic moment* of the specimen, divided by its volume, gives only an average value, which we shall denote by σ, of the magnetization I. Instead of plotting σ against the external field H, we plot $-4\pi\sigma/H_c$ against H/H_c, where H_c is the critical field. This has two advantages, first that the results on the same specimen at different temperatures (and consequently, different critical fields), or on similar specimens of different superconductors, may be directly compared and secondly because, as we shall see directly, it gives an immediate indication how far the specimen behaves according to the theory for an ellipsoid.

According to the present description of superconductivity, the value of $-I$ at any point in a superconducting metal can never exceed $H_c/4\pi$. For, if the metal is in the pure superconducting state, it has $B = 0$, and so $I = (B-H)/4\pi = -H_i/4\pi$ (H_i meaning here the "local" field), while as soon as H_i reaches H_c at any place, the metal goes into the intermediate state at that place; for further increase of the external field, the local field remains equal to H_c, but $0 < B < H_c$, so that $-I < H_c/4\pi$. Finally the metal returns to the normal state, and I is then zero. Consequently for any external field the average value $-\sigma$ of $-I$ also cannot exceed $H_c/4\pi$, and the maximum value of $-4\pi\sigma/H_c$ must be less than or equal to one.

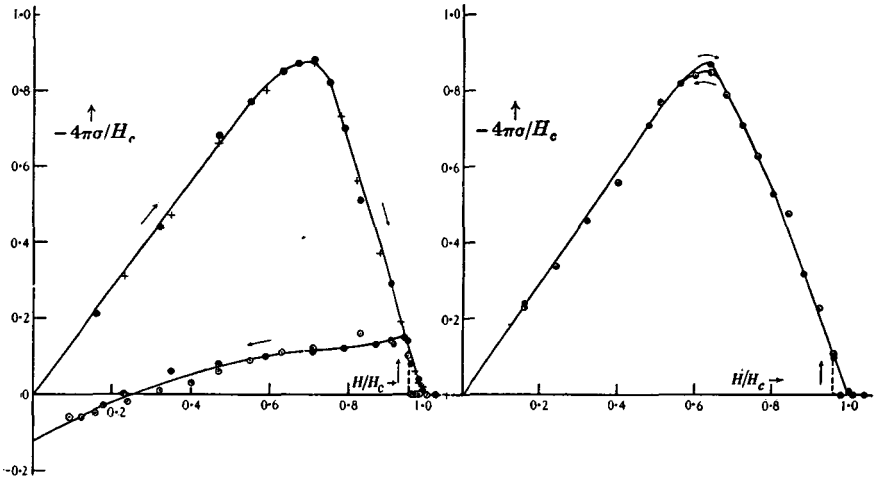
For an ellipsoid, the local field is always uniform inside and parallel to the external field, so that the whole of the specimen reaches the intermediate state for the same value of the external field, and the maximum value of $-4\pi\sigma/H_c$ is just unity. For any other shape this is no longer true and the local field in some parts of the body will reach H_c before it does so in others, so that the maximum value of $-4\pi\sigma/H_c$ should be less than one. The amount by which it is less than one gives some idea of how far the behaviour of the body differs from that of an ellipsoid, for instance how much penetration of the external field into the body has taken place before the turning point in the magnetization curve is reached. Also it is easy to show that for an ellipsoid the slope of the initial linear

* Strictly speaking, what we measure is not exactly the magnetic dipole moment, owing to the contribution to the observed force of higher order moments produced in a non-ellipsoidal specimen. No correction has been made for the influence of these higher order moments, but the correction is probably always small and does not invalidate any of our conclusions.

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relation between $-4\pi\sigma/H_c$ and H/H_c is just the reciprocal of the value of H/H_c at which the magnetization curve turns over (i.e. at which the metal goes into the intermediate state); the extent to which this relation holds for a non-ellipsoidal shape provides another comparison with the behaviour of an ellipsoid.

The magnetization curves of the 1 : 1 tin cylinder are shown in Figs. 1 and 2 plotted in this way, Fig. 1 being for the field parallel to the axis of the cylinder (longitudinal case)* and Fig. 2 for the field perpendicular to the axis of the cylinder (transverse case). Both curves were obtained in a single experiment by rotating the solenoid through 90° about an axis perpendicular to that of the cylinder, and both are virgin curves, since superconductivity was destroyed in between the two series of measurements by warming the specimen above its transition temperature.



● Increasing field } T=2·04° K.
⊙ Decreasing field } H_c=193 gauss.
+ Increasing field } T=2·70° K.
⊕ Decreasing field } H_c=129 gauss.

Fig. 1. Tin cylinder (1 : 1), sharp rims, longitudinal.†

● Increasing field } T=2·08° K.
⊙ Decreasing field } H_c=191 gauss.

Fig. 2. Tin cylinder (1 : 1), sharp rims, transverse.

Several interesting features may at once be noted:

(1) The magnetization curves in both cases are initially linear, showing that, in spite of the sharp rims of the cylinder, the whole specimen or a very large proportion of it remains superconducting over a fairly large range of fields. The significance of this is that although the field at the rims must reach H_c for very low

* Mendelssohn and Pontius (*Nature*, 138 (1936), 29) have published a B-H curve for a short tin cylinder, and, apart from the time-lag effects, its general form is confirmed by our Fig. 1.

† No great accuracy is claimed either for the temperature or the absolute field values given in this and other diagrams.

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fields, this has no great influence on the rest of the body, i.e. only a very small proportion of the volume ceases to be superconducting. The slopes of the initial linear portions are approximately equal in the transverse and longitudinal cases and not greatly different from the value ($\frac{3}{2}$) for a sphere (see § 8).

(2) In both curves the maximum value of $-4\pi\sigma/H_c$ is only slightly less than one, so that in this respect the specimen behaves not very differently from an ellipsoid. Correspondingly the value of H/H_c at which the curves turn over is rather less than the reciprocal of the initial slope.

(3) The magnetization curves do not turn over sharply as in the case of an ellipsoid, but the falling part of the curve is approximately linear over a fairly large range, suggesting that most of the specimen is then in the intermediate state.

(4) The most striking feature is the difference in the hysteresis between the longitudinal and transverse cases. In the latter there is practically no hysteresis at all, just as for a sphere for instance, while in the former the curve for decreasing fields follows the virgin curve for a very short range of fields (see the next paragraph), then abruptly departs, becoming very flat and finally leaving the specimen with a small frozen-in moment in zero external field. We shall discuss this hysteresis after dealing with some more experimental evidence.

(5) When the field is reduced below H_c, in both longitudinal and transverse cases, the magnetization at first remains zero until a value rather below H_c and then quite suddenly jumps to a value on the virgin curve to continue as described in the preceding paragraph*. The field at which the jump occurs is not exactly reproducible, and the phenomenon seems to resemble supercooling†; we have not observed this sudden jump with our lead and tantalum (both of which, as we shall see, contain impurities), so this "supercooling" is probably not an impurity effect. Nor is it a shape effect or due to the inhomogeneous field, since we have observed it also in some other experiments (unpublished) with a tin sphere in a uniform field.

Finally we should mention another relevant feature not shown in the diagrams. This is that if, after the cylinder has acquired a frozen-in moment in the longitudinal direction, the field is again increased from zero, but now perpendicular to the axis of the cylinder, and without in the meantime destroying superconductivity by warming the specimen, the transverse magnetization curve obtained is almost identical with the virgin curve (as in Fig. 2). In other words a frozen-in moment perpendicular to the field has no great effect on the magnetization curve. A similar result was found in the case of a sphere of lead-bismuth alloy‡, but there the frozen-in moment was much larger, and the magnetization curve with the

* We verified that the magnetization remains zero for several minutes in this region, so the phenomenon has not the character of a time lag between magnetization and field.

† A similar effect has been reported by Mendelssohn and Pontius (*loc. cit.*).

‡ See Shoenberg (*loc. cit.*), p. 721, Fig. 3.

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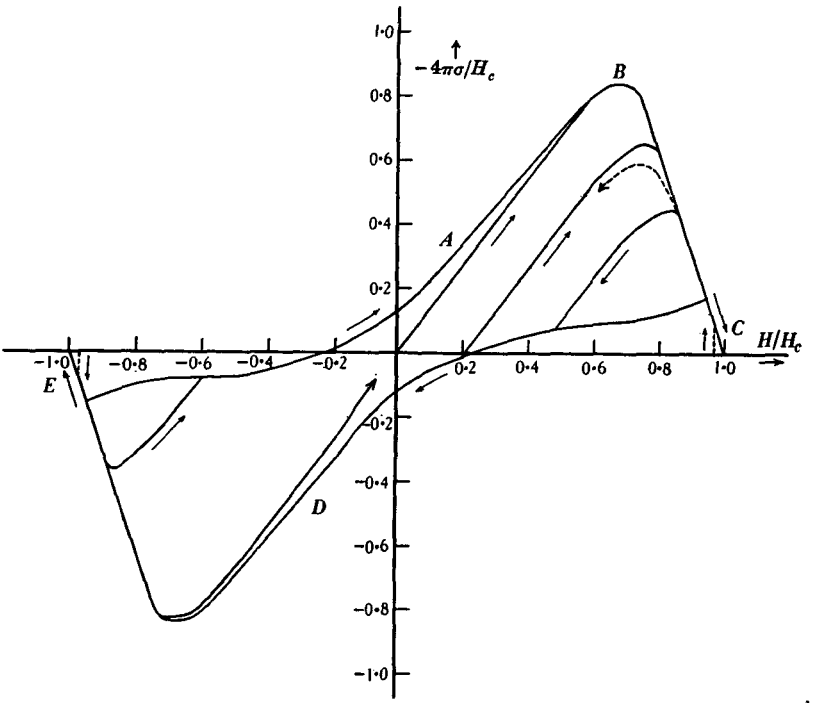
field perpendicular to the frozen-in moment did show a slight difference from the virgin curve (it is possible however that this may have been due to a slight play in the suspension of the specimen, so that the latter may have turned slightly when the magnet was rotated through 90°, i.e. that the frozen-in moment may not have been accurately perpendicular to the field).

4. SOME EXPERIMENTS ON THE LONGITUDINAL HYSTERESIS

In order to understand better the mechanism of the hysteresis in the longitudinal case it is important to know how the magnetization varies

(*a*) if the field is increased again from any point on the return curve,

(*b*) if the field is reduced from a point on the descending branch of the outward curve.



---- Beginning of return curve for split cylinder.

Fig. 3. Tin cylinder (1 : 1), sharp rims, longitudinal, $H_c = 190$ gauss.

This was investigated in a special experiment and the results are shown* in Fig. 3. It appears that the curve $ABCDE$ is a kind of boundary curve and only

* It will be noticed that the maximum of the curve and the form of the hysteresis are not exactly the same as in Fig. 1, although the same specimen was used. Probably these features are very sensitive to the exact position of the specimen in the field, which cannot be exactly reproduced after removing and replacing the specimen.

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states represented by points inside this boundary can be physically realized. If the field is increased or decreased from any point within this boundary, the magnetization curve is a straight line approximately parallel to the ascending branch of the virgin curve, until when the straight line approaches the boundary it turns rather suddenly and continues along the boundary. This suggests that points inside the boundary correspond to the whole or a great proportion of the surface of the cylinder superconducting, but with lines of force frozen-in. If the external field is varied, the flux of frozen-in lines remains constant, until the magnetization curve reaches the boundary, when further change of field causes a variation of the flux through the specimen. Exactly similar results were found earlier* in experiments with a sphere of impure lead, where the hysteresis was due to the impurity (5 % bismuth) instead of to shape, and also in some new experiments with a ring (see § 9). This suggests an important analogy between the mechanisms of hysteresis due to shape and to impurities (in spite of the difference pointed out below, § 7), and we shall discuss this question further in § 9.

We also investigated the effect on the hysteresis of splitting the cylinder along a diametral plane and rejoining the two halves with an insulating layer in between. The outward curve in the longitudinal case was much the same as before, but the return curve did not depart appreciably from the outward curve until much lower field (the beginning of the return curve for the split cylinder is shown by a broken line in Fig. 3); in other words the hysteresis was much less marked than before splitting.

In the transverse position (with the plane of splitting containing the field direction) the magnetization curve was quite similar to Fig. 2.

5. Influence of sharpness of the rims of the cylinder

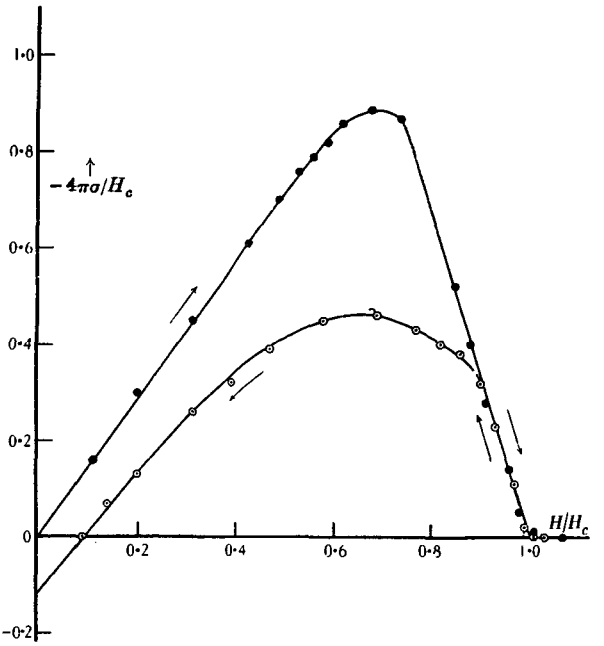
In the earlier experiments with lead (see § 7) some of the cylinders were made by pressing in a mould, and consequently had less sharp rims than cylinders prepared by turning. It was found that, apart from features which could be ascribed to impurities in the lead, the hysteresis in the longitudinal magnetization curve was considerably less marked than in Fig. 1. This suggested that the hysteresis might be greatly influenced by the sharpness of the rims and we therefore made a direct comparison between the magnetization curves of two similar tin cylinders, in one of which however the rims had been rounded off † (giving a radius of curvature something like 10 % of the radius of the cylinder). As will be seen from Fig. 4 (which should be compared with Fig. 1), the rounding off has very little influence on the curve for increasing field, but quite changes the character of the return

* See Shoenberg (*loc. cit.*), footnote p. 720; in the experiment mentioned there we obtained results exactly like those described here for a pure cylinder.

† This was done by filing, and care was taken to avoid filing any regions other than those at the rims, i.e. the end faces were left flat over nearly their whole areas.

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curve. There is now much less hysteresis and the return curve departs gradually from the outward curve instead of abruptly as in Fig. 1. There may be some significance in the fact that the frozen-in moments are almost exactly equal, but this is probably a coincidence, since in the limiting case of "complete" rounding off, i.e. for a sphere, there is no frozen-in moment at all. We may mention also that the rounding off had no significant effect on the transverse magnetization curve.



● Increasing fields } $T=2{\cdot}04°$ K.
⊙ Decreasing fields } $H_c=193$ gauss.

Fig. 4. Tin cylinder, rounded rims, longitudinal.

6. INFLUENCE OF TEMPERATURE

It seemed possible that the hysteresis in the longitudinal case might depend on temperature. This would be the case, for instance, if the hysteresis was connected with the magneto-caloric effect, which might temporarily warm up the inner part of the specimen more than the outer and so leave superconducting rings as the field was reduced. Since the magneto-caloric effect (and also the thermal conductivity and specific heat of the specimen) depend quite strongly on temperature, such a mechanism would lead to a different hysteresis at different temperatures (i.e. with the magnetization curves plotted as explained above; in terms of ordinary magnetization curves we should say that it would lead to the curves at different temperatures not being scale models of each other). Actually, as we see from Fig. 1, where the points marked + are for a higher temperature (with

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H_c about two-thirds of its value for the lower temperature), the temperature has no appreciable influence on the hysteresis. The slight differences can be ascribed to two experimental errors: first that the temperature was not kept sufficiently constant (which means that the critical field was not exactly the same throughout a cycle) and secondly that for the lower critical field the forces measured become rather small, and the errors in measurement are of course exaggerated by plotting the curve on the same scale as for the higher critical field.

We can therefore say that the hysteresis phenomenon for a pure specimen depends only on the shape of the specimen.

7. Influence of impurities

The experiments already mentioned with lead cylinders showed that our lead was not sufficiently pure for studying the shape effects in their simplest form, and we were forced to use tin with some loss of convenience and accuracy*, but with a corresponding gain in clarity. The lead results are however interesting as showing how impurities affect the magnetization curves, and in particular how the hysteresis due to impurities differs from that due to shape.

A characteristic influence of the impurities is that they produce a "tail" at the end of the magnetization curve (as in Fig. 5, which is, however, for tantalum), so that there is no longer a single sharply defined critical field. Moreover, the critical field obtained by extrapolating the linear portion of the descending curve is slightly lower in the transverse than in the longitudinal case (by about 2 % at 4·2° K.). In the transverse position the magnetization curve of the lead cylinder shows a hysteresis a good deal more pronounced than for the sphere, and it is possible that this may be because the sensitivity of the magnetization curve to impurities depends on the shape†. Just as in the alloy sphere (*loc. cit.*), there is a rather sharp drop of magnetization when the field is slightly reduced from a point in the "tail"; after this the magnetization again increases and follows a curve similar to that described for a tin cylinder.

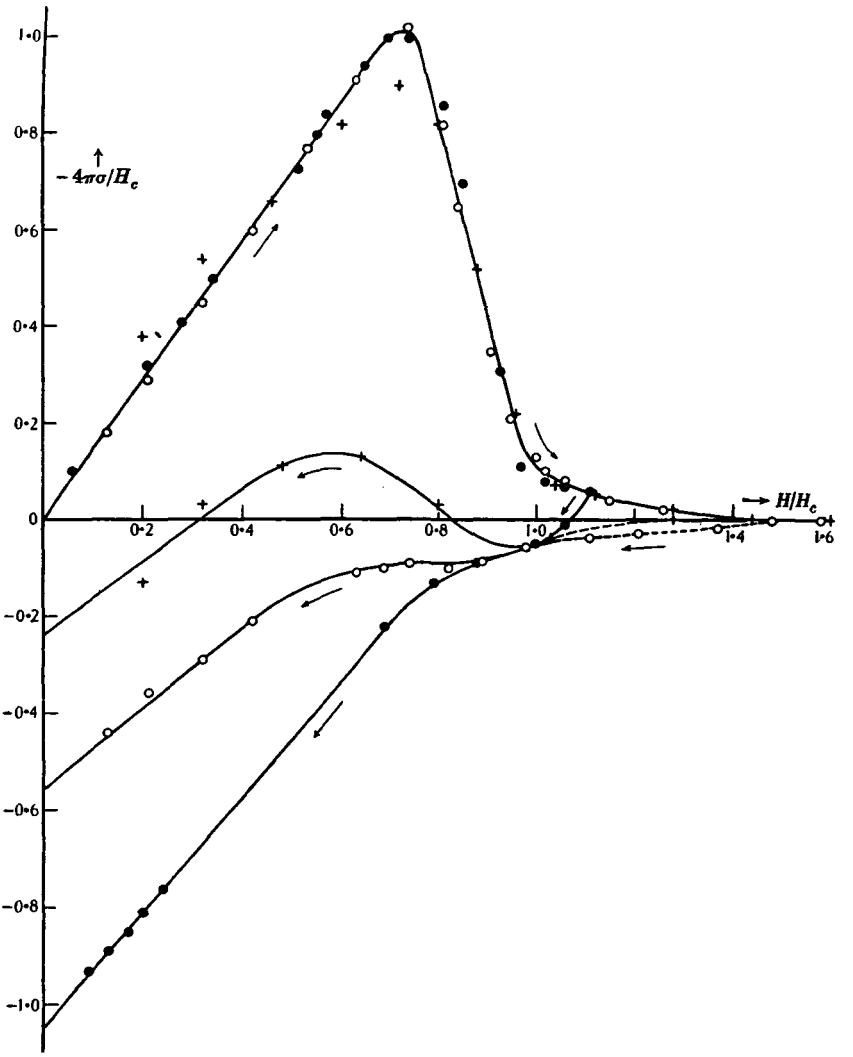
The most important difference between the hysteresis caused by impurities and that due to shape is that the former, unlike the latter, is influenced by temperature. This could be observed with our lead cylinder, but is much more clearly shown for a greater amount of impurity and for a case where there is no "shape" hysteresis. Fig. 5 shows the transverse curve for a 1 : 1 tantalum cylinder at three

* Lead has a large critical field at 4·2° K., and so gives easily measurable forces, while the use of tin involves pumping (introducing difficulties of temperature control), and the critical fields are less than half those of lead. The previous experiments with lead spheres had suggested that the lead was rather pure.

† The absence of a tail for our lead sphere, and the comparatively slight hysteresis found in our lead sphere, may of course be because different samples of the same batch of lead have different degrees of purity, so that the lead sphere was accidently much purer than all the cylinders.

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different temperatures, and since there is no hysteresis for a similar pure tin cylinder we can conclude from the hysteresis here that the tantalum is very impure (from a superconducting point of view at least)*. It will be seen that the return curves and frozen-in moments, when plotted relative to the critical field



\+ T=3·64° K. H_c=237 gauss.
○ T=3·27° K. H_c=360 gauss.
● T=1·7° K. H_c=820 gauss.

Fig. 5. Tantalum cylinder (1 : 1), sharp rims, transverse.

* The word impurity is used in a general sense, i.e. the hysteresis may be caused by the mechanical state of the tantalum rather than chemical impurities (cf. Mendelssohn and Moore, *Phil. Mag.* 21 (1936), 532).

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(defined somewhat arbitrarily* by the intercept on the axis of the linear portion of the descending part of the virgin curve), are no longer the same at different temperatures, and the hysteresis becomes more pronounced as the temperature is lowered.

This temperature effect can be understood in a general way by supposing that the critical fields for the less pure parts of the specimen increase more rapidly with decrease of temperature than does the critical field for the purer parts. Consequently the spread of critical fields becomes greater at lower temperatures and the "sponge" mechanism proposed by Mendelssohn† becomes more effective and is able to freeze-in more lines of force. Thus, roughly speaking, the effect of lowering temperature is similar to that of increasing the impurity content.

The absence of all these characteristic effects of impurities in the case of tin makes it very plausible that the tin really is pure as far as superconductivity is concerned, and that the phenomena discussed earlier are entirely due to shape. At present very little is known about which impurities are important in affecting superconductivity, and indeed the analyses supplied by Hilger's with the lead and tin used in these experiments assign a higher overall spectrographic purity to the lead than to the tin, so that the spectrographic purity is no sure indication in itself. It is possible that dissolved gases (not revealed by spectrographic analysis but present in both the lead and the tin) may be rather important, and that the same amount of impurity may have quite different influences on different superconductors. These questions and the influence of mechanical state will be important for the development of a more precise theory of superconducting alloys.

8. Cylinders of different dimensions

Since it seemed possible that the absence of hysteresis in the tin cylinder transverse to the field might be due to the particular ratio of length to diameter, we measured the magnetization curves of two other cylinders with length to diameter ratio 1 : 2 and 2 : 1 respectively. For both of these there was again practically no hysteresis in the transverse case, so that the absence of hysteresis in the transverse position must be a general property of cylinders. The hysteresis in the longitudinal position was a good deal less marked for the 2 : 1 than for the 1 : 2 cylinder, which showed a hysteresis about as marked as that of the 1 : 1 cylinder (Fig. 1). Probably this is partly connected with the ratio of the radius of curvature of rounding-off of the rims‡ to the radius of the cylinders, since all three cylinders were made in the same way, i.e. with equal radii of curvature at the rims,

* It is no doubt because of this method of definition that the maximum value of $-4\pi\sigma/H_c$ is about 1 in this case, although it is only 0·88 for the similar tin cylinder. Mendelssohn and Moore (*loc. cit.*) have shown that the critical fields for reappearance of resistance are much higher, and so the "average" value of critical field may be rather higher than our extrapolated values.

† *Proc. Roy. Soc.* 152 (1935), 34.

‡ The rounding-off referred to here is that unavoidably produced in the making of the cylinder, and should not be confused with the rounding-off deliberately introduced in the experiments of § 5.

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but the radius of the 2 : 1 cylinder was about half that of the other two cylinders, so that the radius of curvature of rounding-off was relatively twice as large for the 2 : 1 cylinder, and consequently it shows less hysteresis for the same reason that

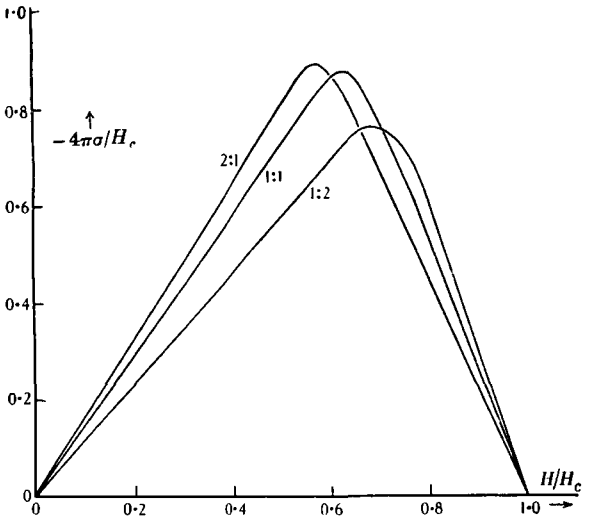


Fig. 6. Longitudinal tin cylinders.

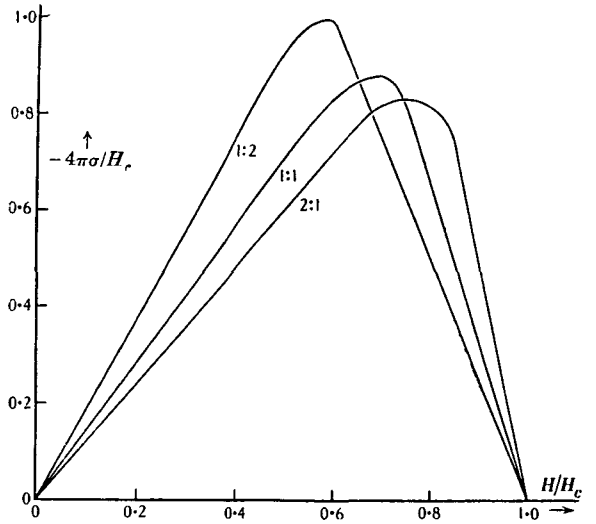


Fig. 7. Transverse tin cylinders.

Fig. 4 differs from Fig. 1. Also if the form of the hysteresis is connected with the relative importance of the rims, we should in any case expect less hysteresis for a longer and thinner cylinder.

Figs. 6 and 7 show the progressive changes in the magnetization curves as the length to diameter ratio increases, both for the longitudinal and transverse cases

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(to avoid obscuring the diagrams we have omitted the experimental points and the curves for decreasing fields). The slopes of the ascending and descending branches of these curves are given in the table, compared with the values for the corresponding rotation ellipsoids* (i.e. whose principal axes are in the ratio $l : d : d$, where l and d are the length and diameter of the cylinder). The experimental values, especially for the descending branches (owing to their steepness), are not very accurate, but it will be seen that they are usually slightly smaller than the ellipsoid values†. We could express this result in another way by saying that the effective demagnetizing coefficients for cylinders of zero and of infinite permeability are not very different from those of the corresponding spheroids.

Table of Slopes

Length to diameter ratio	Longitudinal		Transverse	
	Ascending	Descending	Ascending	Descending
1 : 2	1·86 (2·13)	2·50 (1·89)	1·14 (1·31)	3·00 (4·25)
1 : 1	1·41 (1·50)	3·30 (3·00)	1·47 (1·50)	2·63 (3·00)
2 : 1	1·20 (1·22)	4·94 (5·65)	1·64 (1·69)	2·22 (2·47)

The values in brackets are for the corresponding rotation ellipsoids.

9. The hysteresis mechanism

Peierls (*loc. cit.* p. 624) has shown that for a body of not too complicated shape there is a field H_0 between which and the critical field H_c the whole of the body can be in the intermediate state. This field H_0 depends on the shape, and would probably be smaller for a cylinder with rounded rims than for one with sharp rims, so it is natural in the longitudinal case to identify it as the field at which the return curve departs abruptly from the outward curve (see Fig. 3). The question of what happens for fields lower than H_0 is rather a difficult one; one possibility is that macroscopic superconducting rings are formed (which freeze in lines of force for further reduction of field), but then the difficulty is to explain why an increase of

* If a substance has volume susceptibility χ, then the slope κ of the magnetization curve of an ellipsoid of demagnetizing coefficient N is given by $1/\kappa = N + 1/\chi$. For the ascending branch of the curve $\chi = -1/4\pi$, while for the slope of the descending branch $\chi = \infty$ (see Peierls, *loc. cit.*, p. 624), hence we can obtain $-4\pi\kappa$. The value of N is determined from formulae given in Clerk Maxwell's *Electricity and Magnetism*, 2 (Oxford, 1873), 64–5.

† Probably the ascending and descending branches are not exactly linear, so that the slopes of the assumed straight lines are smaller than the actual slopes of the curve near the axis of abscissae. The experimental accuracy is not sufficient, however, to show this slight curvature of the magnetization curves.

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field from any point on the return curve causes an increase of magnetization parallel to the virgin curve. As pointed out in § 4, this implies that the whole or nearly the whole of the surface of the cylinder is superconducting, i.e. that the superconducting rings are formed over nearly the whole surface; and there seems no obvious reason why this should happen, i.e. why the local field should fall below H_c over nearly the whole surface. The analogy with the behaviour of an alloy pointed out in § 4 suggests the alternative possibility that the hysteresis mechanism is similar to that of the Mendelssohn sponge, i.e. that when the external field is smaller than H_0 the body breaks up into some sort of sponge structure with superconducting meshes. For an increase of field this sponge would indeed behave as if only the outside surface of the cylinder were superconducting, but it is difficult to understand at first sight why the same is not true along the return curve, i.e. why the return curve does not run parallel to the virgin curve.

A similar difficulty arises in the interpretation of the magnetization curve of the ordinary thin ring, and there the explanation has been given by Schubnikow and Chotkewitsch* who point out that on the return curve the current in the ring cannot reach its full value (i.e. corresponding to a return curve parallel to the virgin curve), since then the local field at the *inside* periphery of the ring would exceed the critical value. In fact the return curve for a ring is determined by the condition that the local field at the inside periphery of the ring is just H_c. If however the external field is increased again, the current in the ring (and therefore its magnetic moment) can increase according to the ordinary law of induction, without the field anywhere on the surface of the ring reaching H_c, and similarly any change of field inside the "boundary" curve (see § 4) will cause a change of magnetic moment in accordance with the ordinary law of induction. The last point has not previously been proved experimentally (except for a rise from zero field; see Shoenberg, *loc. cit.*), so we made a special experiment on the magnetization curve of a thin lead ring (diametral ratio 13·7 : 1), the results of which are shown in Fig. 8, and clearly illustrate the point†.

It is difficult to see how this theory can be applied exactly to the superconducting parts of the cylinder, but roughly speaking we can say that the return curve is determined by the condition that the local field at some surface inside the cylinder should not exceed the critical value (as soon as the external field is less than H_0, the field of the currents induced in the superconducting circuits is, on

* *Phys. Zeit. d. Sow. Union*, 10 (1936), 231. The question, however, of how the current in the ring adjusts itself to values on the boundary curve (different from those given by the law of induction) is still an open one; the explanation suggested by the authors is not necessarily the correct one.

† Owing to the thinness of the lead wire (0·35 mm. diameter) from which the ring was made, the ring is thinner in some places than others and the slopes of the various linear portions do not agree well with theory. The discrepancies are however such as can be attributed to lack of uniformity of the ring.

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an inside surface, in the same direction as the external field and so the total field is greatest on an inside surface)*.

If the sponge idea is true, then, as in the alloy, it is not yet possible to describe the nature of the sponge in detail; for an alloy the sponge is connected with the inhomogeneity of critical field (and consequently the form of the hysteresis depends on temperature), while for the cylinder the sponge is presumably produced by the inhomogeneity of the local field when the external field is less than

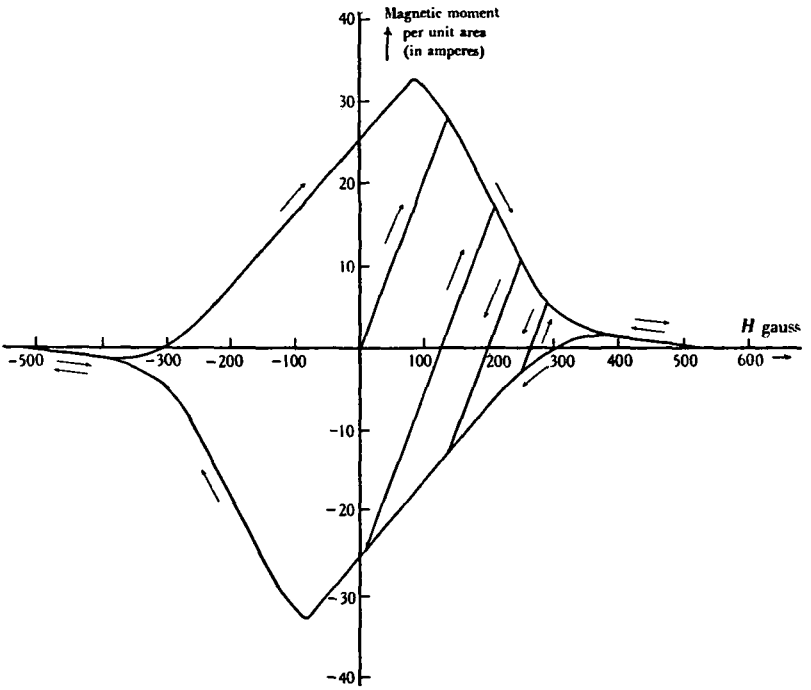


Fig. 8. Lead ring (diametral ratio 13·7 : 1), 4·24° K.

H_0. The experiment with the split cylinder (§ 4) shows that, if currents are prevented from crossing a diametral plane, hysteresis begins only at lower fields. In terms of the sponge mechanism this may be interpreted as meaning that at first (i.e. just below H_c) the sponge is formed only near the surface, so that the superconducting paths right round the cylinder are broken by splitting and no freezing-in occurs; as the field is reduced, however, the sponge spreads to the interior, and superconducting paths enclosing appreciable areas are formed in each half separately, and lines of force can be frozen-in.

Just as on London's views the intermediate state is an intimate micro-

* For a uniform ring the theory requires the return curve to be linear, and the fact that this is not so for the cylinder (see Fig. 3) suggests that the inside surface at which the field is just H_c is different for different external fields, or in other words that the distribution of the superconducting regions changes with reduction of field.

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scopic mixture of normal and superconducting states, so the state of the metal below H_0 may be some different microscopic mixture of states, which cannot be described by a division into macroscopic regions of intermediate, superconducting and normal states. This view is, however, speculative, and requires a sounder theoretical basis before it can be accepted.

The absence of hysteresis in the cylinder in the transverse position could be explained on either of two hypotheses: either that H_0 is so low that the cylinder can be wholly in the intermediate state until the turning point, so that the transition is practically reversible, or that H_0 is about as large as in the longitudinal case, but that the state of the metal below H_0 does not form a sponge or superconducting rings and so does not freeze-in the field. Of these hypotheses, the first seems less likely because the maximum value of $-4\pi\sigma/H_c$ is definitely less than one, so that at the turning point there must be both superconducting and intermediate regions (see § 3). The second hypothesis might be true on account of the different configuration of the cylinder relative to the field in the longitudinal and transverse positions; thus in the transverse position the cross-section of the cylinder normal to the field direction is a square and the local field will certainly not be constant round this cross-section, thus preventing formation of plane superconducting rings for instance. Although the second hypothesis seems more likely, it must again be regarded as speculative until a more detailed theory is made.

10. TIME LAGS

When these experiments were begun, one of the aims in view was to obtain more information about the time lag between magnetization and field which has been reported* to occur especially when the specimen is non-ellipsoidal. The present evidence about this effect is rather conflicting, particularly as regards the order of magnitude of the "relaxation time" and the conditions for its occurrence. Thus Rjabinin and Schubnikow find the effect in long lead cylinders only if they are pure single crystals and describe the effect by a rapid fall in a time of order 1 min., followed by a slow change for about 5 min., while Mendelssohn and Pontius find the effect in pure tin, both polycrystalline and single crystal, and find a rapid fall in a time of order 2 or 3 min., followed by a slow fall for about 20 min. Since the latter authors have pointed out that the effect is most marked if the shape of the specimen is non-ellipsoidal, we hoped to have the most favourable conditions for the occurrence of the effect in our short cylinders.

In our experiments with lead (polycrystalline) cylinders† we found marked time effects only around the top of the magnetization curve‡. Thus if the field was suddenly increased in this region, the magnetization momentarily rose above its

* References in § 1.

† Similar effects were found with the lead ring mentioned in § 9.

‡ Less marked time effects were found also at the beginning of the return curve.

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final value and then fell off rapidly, usually in about 5 or 10 sec. Our accuracy was not sufficient to decide definitely whether the fall was exponential, but it seemed there was rather a longer tail than would be the case with an exponential fall; however, assuming the decay to be as $e^{-t/\tau}$, we estimated τ to be 2 or 3 sec. Owing to slight "zero" drift of the balance, and slight changes of the current in the solenoid, we were unable to say definitely whether there was any further slow change of magnetization after 10 sec.; i.e. the slight subsequent changes of deflection could always be attributed to these stray effects. We tried also to see if the relaxation time was affected by temperature, but the results were very uncertain owing to the additional difficulty of temperature control below 4·2° K.; it did seem, however, that the relaxation time became rather longer at lower temperatures. We also investigated a lead single crystal of approximately cylindrical shape (it had rounded corners and some small holes) but found its behaviour as regards time effects no different from that of the polycrystalline lead.

In the experiments with tin we also noticed short relaxation times (of the same order of magnitude as in lead), but (as mentioned in footnote * on p. 267) the experimental accuracy with tin is less than with lead owing to the smaller forces to be measured, and it was impossible to draw any conclusions about the phenomenon beyond this observation.

The time effects may perhaps be caused by the dying away of currents due to local reappearance of resistance (which might occur near the top of the magnetization curve for non-ellipsoidal specimens), and in this case we should expect the relaxation time to depend on the size of the regions in which resistance appeared. Thus our short relaxation times may be due to the small size of our specimens compared with those of other workers. It is also possible that the shortness of relaxation time is in some way connected with the inhomogeneity of the field (which is of course an essential feature of our method), since all previously reported experiments have been carried out in uniform fields. In any case the general irreproducibility of the phenomenon and its dependence on the particular experimental arrangement suggest that it is not a primary phenomenon.

Summary

The magnetization curves of short tin, lead and tantalum cylinders were measured by the force method, and it was shown that although for tin there was no appreciable hysteresis in the transverse position, there was a very marked hysteresis in the longitudinal position, of the same form at different temperatures but depending on the sharpness of the cylinder rims. Lead and tantalum showed additional temperature dependent features which could be attributed to impurities.

The hysteresis due to shape in the longitudinal cylinder is analogous to the hysteresis of alloy ellipsoids, in that the magnetization for a given field always

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lies within a certain boundary curve and varies "classically" within this boundary. This analogy suggests that the mechanism of the shape hysteresis for a pure superconductor may be similar to the sponge mechanism proposed by Mendelssohn for alloys, but it is also possible that the hysteresis is due to the formation of macroscopic superconducting rings. The considerations which fix the boundary curve are probably similar to those in the superconducting ring, and in this connection some new measurements on a lead ring are reported. The paper concludes with a discussion of the time lag between magnetization and field, and it is suggested that this is not a primary phenomenon.

In conclusion I wish to thank Prof. Lord Rutherford and Dr Cockcroft for their interest in this work, and Dr Peierls for many valuable discussions and suggestions concerning the theoretical interpretation of the results.

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