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Asymmetric Information, Debt Capacity, and Capital Structure

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Abstract

Capital structure choice based on costs associated with asymmetric information is examined in order to present a new perspective on the standard pecking order and trade-off theories. In the model, both the face value of debt and the restrictiveness of the associated debt covenants are chosen as part of the financial structure, allowing a more complete characterization of this decision. Debt structure choice balances ex ante adverse selection against ex post moral hazard, providing a natural integration of the pecking order and trade-off theories and the development of interesting empirical implications.

I. Introduction

This paper develops a model of capital structure choice in which asymmetric information between the firm and external investors is the sole friction. In the model, both the face value of debt and the restrictiveness of the associated debt covenants are chosen as part of the financial structure, allowing a more complete characterization of this decision. Debt structure choice balances ex ante adverse selection against ex post moral hazard, providing a natural integration of the pecking order and trade-off theories and the development of interesting empirical implications.

Despite its trade-off theory nature, the model can be most easily thought of as an extension of Myers and Majluf (1984) and Myers (1984). A firm with uncertain cash flow, facing asymmetric information between itself and investors, is examined. To establish the firm, a capital-constrained entrepreneur seeks financing from an inferiorly informed capital market. Myers and Majluf (1984) and Myers (1984) examine asymmetric information at the time of financing and show

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that existing equity holders benefit from the sale of securities with low information sensitivity (debt). These models, however, do not consider the effect of the chosen financing scheme on decision-making subsequent to the financing event nor the impact these ex post incentives have on the ex ante financing choice.

Extending the traditional framework, we assume that subsequent to the initial financing, the information asymmetry remains; however, a signal concerning the strength of the economy is publicly observed. Based, in part, on this signal, the firm may decide to either continue, generating an uncertain future cash flow, or to liquidate, for an immediate and certain value. The liquidation decision is a simple representation of ex post investment decision-making. The use of debt in the initial financing creates a bias toward continuation. The cost of this ex post moral hazard offsets the entrepreneur's initial motivation to issue debt at the financing stage in response to the adverse selection faced at that time.[1]

The incentive problem implies that debt covenants, which allow the lender to demand immediate repayment of the debt on a state-contingent basis, may be valuable features of debt contracts. In the model, the debt covenant effectively allocates control of the liquidation decision to the firm or the lender based on the realization of the public signal. The lender's inferior information implies allocation of control to the lender may be costly; however, this cost may be mitigated by renegotiation of the covenant. The balance of this net expected ex post cost and the benefit of issuing debt ex ante represents the trade-off in the capital structure choice. Consideration of the subsequent incentive effects of financing decisions (a natural addition to the pecking order) places the model within the realm of the trade-off theory. The greater the ex ante asymmetric information and the lower the cost of renegotiation, the more likely it is that firms will elect to use high leverage and restrictive debt covenants. A measure of debt capacity arises endogenously within the model; however, the optimal debt level may lie significantly below this benchmark.

Covenants are standard features of debt contracts. They take a variety of forms and may restrict firms from taking certain actions (engaging in mergers, paying dividends, issuing additional debt) that transfer wealth from lenders to shareholders or may proscribe conditions that must be maintained (minimum levels of net worth or interest coverage) to avoid technical default. The covenants modeled here are "proscriptive" covenants, which act as an "early warning" for deterioration of the firm's financial health.

Debt covenants have been shown to have a significant ex post impact on firm behavior. Asquith, Gertner, and Scharfstein (1994) found technical violation of covenants to be the leading cause of default (slightly ahead of failure to make scheduled payments) in a sample of junk-bond issuers. Chava and Roberts (2008) demonstrate that capital investment declines following violation of a covenant, as creditors use the threat of default to intervene in managerial decision-making. Nini, Smith, and Sufi (2012) further show that violations of covenants are followed by reductions in acquisitions, capital expenditures, leverage, and payouts

[1]The examination of the implications of the initial financing choice for subsequent decision-making is the fundamental departure of our model from traditional pecking order models (Myers and Majluf (1984) or Viswanath (1993) in a dynamic context). Davis (2016) makes a related point.

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to shareholders, as well as an increase in chief executive officer (CEO) turnover. They attribute these policy changes to creditors' influence on the firm's decision-making. The absence of control right allocations in the form of debt covenants from the ex ante capital structure discussion, therefore, may be an important omission. By presenting a model in which the amount of debt and the structure of the associated control rights are jointly considered in the leverage decision, we hope to further understand the role of debt covenants and to shed light on the broader capital structure question.

Gârleanu and Zwiebel (2009) present a model that is closely related to our work. They point out that debt covenants provide an interesting and important example of the broader property rights literature developed in Grossman and Hart (1986) and Hart and Moore (1990). Gârleanu and Zwiebel develop a model of incomplete contracting based on asymmetric information that illustrates one way (the motivation of costly information acquisition) in which restrictive covenants that allocate control of decisions to a party with *inferior* information may enhance value. In our model, the contractual incompleteness also derives from asymmetric information; however, our study differs from theirs primarily in the nature and role of the covenants. The focus of their paper is on the nature of the covenants themselves, and they take the leverage decision and the managerial incentive problem as exogenous. The leverage decision is our focus, and leverage and the induced managerial incentives are endogenous in our model.

Our model provides a complementary explanation to that in Gârleanu and Zwiebel (2009) for the use of restrictive rather than unrestrictive covenants. Further, we show that an important consideration in determining the restrictiveness of debt covenants is a firm's ability to renegotiate the covenants when they inefficiently bind. Thus, the use of restrictive covenants and the fact that they are frequently renegotiated are closely tied.[2]

Dynamic models of optimal financial contracting (e.g., DeMarzo and Fishman (2007)) have been developed in which the history-dependent dynamic contracts can be interpreted as the result of the renegotiation of violated covenants. For example, in DeMarzo and Fishman, the fundamental contracting friction is the agent's ability to appropriate a portion of the realized cash flows. The optimal dynamic contract can be implemented using equity, long-term debt, and a line of credit. The agent holds a portion of the equity, while the long-term debt, the line of credit, and the ability of the external investors to terminate the project provide the agent with incentives to truthfully reveal the amount of cash flow generated each period (or instant in time). Interpreting the dynamically changing overall debt level as the result of covenants that are violated and renegotiated, these models suggest debt levels that are renegotiated upward and downward in exchange for covenants that are made less and more restrictive, respectively.

The main theoretical results of the model can be summarized as follows. Adverse selection at the financing stage introduces a standard pecking order in the model. Firms prefer to first issue riskless debt, then risky debt, and finally

[2]Leland (1994) is another important paper considering the impact of debt covenants on the capital structure decision. It is a model of debt value and capital structure choice whose focus is very different from ours.

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external equity. The model identifies liquidation value as a measure of the firm's debt capacity, the point at which there is no longer any motivation for the firm to issue debt in response to the adverse selection problem. However, the model also indicates that the optimal level of debt may be significantly below the firm's debt capacity. If the renegotiation of covenants is not possible, it is optimal for firms to use low amounts of debt combined with unrestrictive covenants, and it is never optimal to use a restrictive covenant. However, when renegotiation is costless, the use of high levels of debt combined with restrictive covenants represents equilibrium initial financial structures; these results establish the connection between the use of restrictive covenants and the ability to renegotiate them. In the most natural case, when renegotiation is possible but costly, depending on parameter values, firms find either a "high" level of debt combined with a restrictive covenant or a "low" level of debt and an unrestrictive covenant to represent the equilibrium financial structure.

Empirically, the main capital structure implications in the model may be summarized as follows. The firm is more likely to choose higher leverage and more restrictive covenants the greater is the value added (NPV) of the investment opportunity, the greater is the ex ante adverse selection problem, the greater is the firm's debt capacity, the stronger is the economic outlook, and the lower is the cost of renegotiation. Additionally, when firms attempt to renegotiate violated covenants, the concession the firm must make to waive the violation will be larger when the economic outlook is stronger.

Consideration of the standard trade-off theory costs and benefits of debt would be accomplished in this model very differently than in the pecking order. The pecking order theory assumes that trade-off theory frictions (taxes and bankruptcy costs) are second-order effects, except at extreme debt levels at which point they influence financing choices in the usual way. Here, trade-off theory frictions, were they to be included, would affect leverage choice within an intermediate range of leverage. Therefore, if the tensions examined in the model and its notion of debt capacity are important considerations in capital structure choice, the model suggests a new approach to empirical examinations of capital structure.

The remainder of this article proceeds as follows: Section II introduces the model. Section III presents the equilibrium outcome when renegotiation is not possible, and Section IV considers the case of costless renegotiation. Section V provides the results when renegotiation has a finite cost. Section VI discusses the empirical implications of the model, and Section VII concludes.

II. The Model

In the model, an entrepreneur/manager seeks funding in order to establish a firm. The entrepreneur's type or quality (t) is assumed to be known privately by the entrepreneur (we will interchangeably refer to the entrepreneur's type or the firm's type; i.e., good entrepreneurs run good firms). Managers are assumed to be observationally equivalent from the perspective of the external investors/market. Ex ante, external investors know only that type is drawn from the set $\{B, G\}$, where $0 < B < G$, the prior probability of a good type (G) is θ, and the mean type

is denoted $\bar{t}_\theta$. Market participants do observe the financing structure chosen by the firm and update their beliefs based on this choice. The model is, therefore, a signaling model in which managers choose a financial structure in order to establish the firm and, based on beliefs about the type of firm, market participants price the associated securities. Asymmetric information between the manager and the market is the fundamental friction in the model. We denote the market's posterior probability that a manager is of type G by μ and the resulting expected type by $\bar{t}_\mu$.

The level of initial capital, I, required to initiate a project and establish a firm is assumed to be common knowledge. The investment's time 2 payoff is assumed to depend upon the entrepreneur's type and the value of a signal, w. This signal is publicly observable and verifiable at time 1, where w is drawn from the set $\{w_2, w_1\}$, where $0 < w_2 < w_1$ and the probability $w = w_1$ is equal to p. The signal is assumed to be independent of type. For expositional convenience, the signal will be discussed as an indication of the strength of the overall economy or the industry, as it will affect the fortunes of all observationally equivalent firms. However, we could equivalently consider the signal to be an industry- or firm-specific release of information. As long as each possible realization of the signal may be reported by all firm types, the implications of the model remain the same. We refer to the realization $w = w_1$ as a strong market and $w = w_2$ as a weak market.

If the project is initiated and allowed to continue until time 2, it generates a cash flow of H or L, where $H > L > 0$. The cash flow H (success) is generated at time 2 with a probability equal to the product of the entrepreneur's type and the realization of the public signal, tw, and the cash flow L (failure) is realized with the complementary probability $(1 - tw)$. For internal consistency, we assume $1 > Gw_1$. Note also that $Gw_1 > Bw_2 > 0$.

An alternative to continuation of the investment project, available at time 1, is that it may be liquidated (or "quit"). Liquidation of a firm generates a time 1 cash flow of Q with certainty. The timing of the model is such that the liquidation decision is made conditional on the realization of the public signal w. The liquidation decision introduces the possibility of ex post moral hazard and serves as a simple representation of ex post decision-making that may be influenced by the incentives established by the initial financing decision. We assume all agents are risk neutral and that the risk-free rate is 0.

The entrepreneur/manager owns the rights to the project but has no capital. The required capital, I, must be raised by issuing a combination of equity and debt.[3] We consider that the entrepreneur chooses the face value of debt, F, and the restrictiveness of the debt's covenant. The debt covenant effectively allocates the

[3] As in Gârleanu and Zwiebel (2009), we assume the use of debt contracts is optimal based on standard arguments from the security design literature and that it is impossible for the face value of debt to be state contingent. Strictly speaking, we cannot make the standard assumption that the future states are not sufficiently describable ex ante to allow contingent contracting, as that would rule out covenants. Altering the model to consider many possible future states that the public signal separates into subsets would allow the use of the standard assumption. This, however, complicates the model for no other purpose. Instead, we assume that the cost of renegotiation is associated with identifying the appropriate face value of debt for each realized state, in which case a fully state-contingent face value would be unnecessarily costly.

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right to make the liquidation decision on a state-contingent basis. We indicate the restrictiveness of the debt covenant by a level of the signal w', *below* which control of the liquidation decision is allocated to the lender (the lender has the right to call the loan). The parties renegotiate this allocation of control rights at time 1 at a cost c. The choice of debt structure (the face value and the level of the covenant) and the level of the required capital determine the proportion of the firm's equity to be sold externally, α. Entrepreneurs make decisions considering their informed valuation of their retained equity. Note that this is equivalent to assuming that entrepreneurs (given their superior information) act in the interest of shareholders. Intuitively, this structure generates an agency problem that is increasing in the amount of debt financing. Asymmetric information motivates the ex ante use of debt and the resulting agency problem provides an ex post cost to the use of debt financing.

A. First-Best Liquidation/Continuation

We assume that in a strong market ($w = w_1$), it is efficient for both firm types to continue operations. In contrast, in a weak market ($w = w_2$), it is efficient for a good firm to continue but a bad firm to liquidate. Thus, we restrict the parameter values so that

$$
\begin{aligned}
(1) \qquad & Gw_1H + (1 - Gw_1)L = Gw_1(H - L) + L > Q, \\
& Bw_1(H - L) + L > Q, \\
& Gw_2(H - L) + L > Q, \\
& Bw_2(H - L) + L < Q.
\end{aligned}
$$

These restrictions allow us to examine the interesting aspects of the financing choice in a relatively simple environment. We also assume that the required investment, I, is such that there is value to both good and bad firms being established.

B. The Agency Problem

The time 1 unconstrained liquidation decision of an entrepreneur of type t with debt outstanding follows standard intuition. Entrepreneurs act to maximize the informed ex post value of their retained shares for a given face value of debt. Assuming $L \leq F \leq Q$ (restrictions we address subsequently), the informed conditional expected value of levered equity is either $Q - F$ when the firm is liquidated or $tw(H - F)$ when the firm is allowed to continue. The convexity of their claim in the final cash flow implies that equity holders have a bias toward continuation, a bias that increases with leverage. (Similarly, debt holders prefer liquidation.)

Given this bias, managers of good firms will always make efficient liquidation decisions (it is always efficient for good firms to continue). Also, both types of firms will make efficient decisions in a strong market. However, managers of bad firms will make inefficient liquidation decisions in a weak market if the face value of debt is sufficiently high. The assumption that $Q - L > Bw_2(H - L)$ indicates that, for face values of debt that are risk free ($F \leq L$), entrepreneurs in bad firms will liquidate efficiently. When the face value of debt increases sufficiently, $Q - F < Bw_2(H - F)$. In particular, there exists a level of debt, F^L, defined

implicitly by $Q - F^L = Bw_2(H - F^L)$, at which the manager of a bad firm is indifferent between continuation and liquidation in a weak market. The lender has the opposite bias and, when $L < F \leq Q$, will strictly prefer liquidation to continuation. Thus, the model presents a simple representation of standard intuition.

The nature of the managerial incentives and the structure of the model imply that it is sufficient to consider two levels of covenant: a restrictive covenant and an unrestrictive covenant. We denote an unrestrictive covenant as $w' = w_2$, where the covenant conveys the right to call the loan to the lender if $w < w'$. With an unrestrictive covenant, the lender is never allocated the right to call the loan and the manager makes the liquidation decision in all states. Under a restrictive covenant ($w' = w_1$), the lender is assigned the right to call the loan (and so force liquidation) when $w = w_2$. Note that it is never optimal to assign control of the liquidation decision to the lender when $w = w_1$ because the lender strictly prefers liquidation and both types of manager make efficient decisions.

III. The Capital Structure Problem: No Renegotiation

In the absence of renegotiation, equity, debt, and firm value, as well as the value of the manager's retained shares, are defined as follows. Assuming $L \leq F$, a generic representation of time 0 equity value for a given firm type, t, is

$$
\begin{aligned}
(2) \qquad S^t(F, w') \;=\; & p\,[(1 - \ell(F, w_1, w'))tw_1 \max(H - F, 0) \\
& + \ell(F, w_1, w') \max(Q - F, 0)] \\
& + (1 - p)\,[(1 - \ell(F, w_2, w'))tw_2 \max(H - F, 0) \\
& + \ell(F, w_2, w') \max(Q - F, 0)],
\end{aligned}
$$

where $\ell(F, w, w')$ is an indicator function that takes the value 1 in the event the firm will be liquidated given the signal w, debt covenant w', and face value of debt F. Because of the cumbersome notation, we will commonly suppress the indicator function and the max operator. Assuming efficient liquidation, the value of the good firm's equity, for example, is more simply written as

$$
(3) \qquad S^G(F, w') \;=\; pGw_1(H - F) + (1 - p)Gw_2(H - F)
$$

and the value of the bad firm's equity is

$$
(4) \qquad S^B(F, w') \;=\; pBw_1(H - F) + (1 - p)(Q - F).
$$

The value of the good and bad firms' debt and firm values (the sum of the equity plus the debt) are defined analogously (see the Appendix). Uninformed firm value, given market beliefs about the probability a firm is good (μ), is written as

$$
\begin{aligned}
(5) \qquad & V^U(F, w') \;= \\
& p\bar{t}_\mu w_1(H - L) + (1 - p)[\mu Gw_2(H - L) + (1 - \mu)(Q - L)] + L.
\end{aligned}
$$

Uninformed values of equity ($S^U(F, w')$) and debt ($D^U(F, w')$) are defined analogously.

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Managers of firms of type t will choose the initial financial structure (F, w', α) (where α represents the fraction of a firm's equity retained by the manager) of the firm in order to optimize the informed value of their retained shares subject to raising capital, I, by selling external claims.[4] The capital constraint can be used to solve for α, and the objective function of a manager of type t is written as, dependent upon the choice of the face value of debt and the restrictiveness of the covenant,

$$\max_{F,w'} \frac{S^t(F,w')}{S^U(F,w')}\left(V^U(F,w') - I\right). \tag{6}$$

The relation between our model and that of Myers and Majluf (1984) can be illustrated using equation (6). While not strictly true, one can think of the ratio as capturing the impact of asymmetric information on the manger's choice while the uninformed firm value captures the impact of any inefficiencies in decision-making. It is also true that, for a given covenant, this ratio is strictly increasing (decreasing) in the face value of debt for a good (bad) firm for F in the interval $[L, Q]$. This is the essence of the pecking order developed by Myers and Majluf.

Proposition 1. The pecking order and debt capacity: Asymmetric information at the time of financing (time 0), for a given covenant, implies there is a pecking order for external financing. The manager of a good firm prefers to issue riskless debt to the extent possible $(F = L)$ and then risky debt to the point at which its informational sensitivity is equal to that of external equity $(F = Q)$. At this level of debt financing, the entrepreneur is indifferent between issuing additional risky debt or external equity.

Proof. Immediate from the preceding discussion.

Proposition 1 illustrates that the model captures the standard pecking order preference of the good manager given ex ante asymmetric information. However, this is a characterization of financing choice only if we ignore ex post costs associated with the induced incentives. Proposition 1 further demonstrates that by including a liquidation decision in the model, a notion of "debt capacity" arises endogenously. The structure of the cash flow implies that when $F = Q$, there is no further motivation to issue debt rather than external equity.[5] This result, and noting that $F = L$ is the maximal amount of risk-free debt, implies there is no loss of generality in restricting attention to $L \leq F \leq Q$.

The innovation in our model is that we also consider how subsequent decision-making is influenced by the firm's response to the ex ante informational asymmetry. There are two issues to consider. The first issue is that the use of risky debt in the initial financing of the firm may distort the incentives of the manager for making the time 1 liquidation decision. The second one is that debt covenants

[4] For simplicity, we assume that firms raise exactly I in the external markets. This can be derived from the fact that good firms pay a premium for external capital given the asymmetric information, but the complications from doing so detract from the main focus of the paper.

[5] Williamson (1988), Hart and Moore (1994), and Shleifer and Vishny (1992) have all, for varied reasons, identified liquidation value as a measure of debt capacity. This simple version of debt capacity is a "soft constraint" in that, while there is no positive motivation for the further use of debt, there is also no cost.

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can help to limit the cost of the distorted incentives. The cost of the ex post distortions in the incentives of the decision-maker induced by the use of debt financing is balanced against the adverse selection benefits of the ex ante sale of debt in determining the firm's capital structure.

We examine the pure strategy perfect Bayesian (PSPB) equilibria of the firm's choice over debt structure. An equilibrium entails the following: i) a debt structure choice (F and w'); ii) a renegotiation strategy for both types of manager, when allowed; iii) a set of beliefs held by market participants (posterior probabilities) regarding the type of firm issuing a given set of securities, such that neither type of manager can increase the ex ante informed value of their retained equity by choosing an alternative strategy; and iv) market pricing, which is an unbiased expectation of the value of the issued securities given the market's posterior probabilities concerning firm type, where the market's posterior probabilities are formed following Bayes law (when possible) given the equilibrium behavior of each type of firm.

In order to focus attention on the most interesting region, we restrict attention to the set of parameter values that admit only pooling equilibria.[6] The capital structure decision, therefore, consists of identifying ex ante capital structures that represent pooling equilibria in which good and bad firms choose the same initial financing structure.

Assumption 1. Parameter Restriction

In order to focus on pooling equilibria in the model, we assume the following:

$$\left(1-\frac{B}{G}\right)(I-L) \quad > \quad (1-p)[(Q-L)-Bw_2(H-L)]. \tag{7}$$

The right-hand side of equation (7) represents the expected change in value for a bad firm from efficient versus inefficient liquidation in a weak market. In order to rule out separating equilibria, this value must be relatively small since the most effective way for the good firm to separate from the bad is to induce inefficient decision-making for the bad firm when it mimics the good firm's financial structure. The left-hand side of the inequality is larger the greater the difference between the good and bad firm types. The larger this difference, the greater the benefit to the bad firm from mimicking the good. Similarly, the greater the amount of risky capital raised, the more valuable is mimicking for the bad firm.

Lemma 1. In the absence of renegotiation, with an unrestrictive covenant ($w' = w_2$), the range for the face value of debt $[L, Q]$ may be usefully separated into low debt $[L, F^L]$ and high debt $(F^L, Q]$. $F^L = (Q - Bw_2H)/(1 - Bw_2) > L$. For low debt levels, both firm types have incentives to make efficient liquidation decisions in weak and strong markets. For debt levels above F^L, the liquidation decision-making of bad firms is inefficient.

[6] For some parameter values, there exist separating equilibria in the model based on the two-dimensional signal provided by choice over face value and the restrictiveness of the covenant. Constantinides and Grundy (1989) discuss this issue. Assumption 1 is sufficient to rule out separating equilibria.

Proof. See the Appendix.

The level F^L is the debt level for which the bad firm is indifferent to liquidation or continuation in a weak market. As will be shown, for most of the results, it suffices to discuss low or high debt levels rather than specific choices within these intervals.

A final issue that must be considered before we can establish the PSPB equilibria of the signaling game is the off-the-equilibrium-path beliefs of the market participants. Given that bad firms have an incentive to mimic choices of good firms to improve the pricing of their securities, one possible set of beliefs is that, observing *any* deviation from the equilibrium financial structure, the market believes the firm is bad ($\mu = 0$). It is straightforward to show that these beliefs support a relatively large set of pooling equilibria. For example, for some parameter values, all low levels of debt ($L \leq F \leq F^L$) combined with an unrestrictive covenant would be PSPB equilibria of the signaling game.

However, consider one such equilibrium, $F = L$ (risk-free debt), and an unrestrictive covenant. For some parameter values, this is an equilibrium financial structure under the suggested beliefs.[7] For the good firm, this is the least desirable of all the equilibria; for the bad firm, however, it is the most desirable. This equilibrium is supported because the market is assumed to view any deviation as coming from a bad firm. However, the good firm benefits most from a deviation to higher debt. Bad firms benefit from low debt levels and are not interested in deviating to higher debt unless they can convince the market that they are good rather than average firms by doing so. In this case, the suggested off-the-equilibrium-path beliefs are not reasonable.

We will assume, therefore, that for a given deviation from any candidate equilibrium, if one type of firm would strictly benefit from this deviation under the assumption that market beliefs are unchanged from the equilibrium beliefs (i.e., $\mu = \theta$), then the market will assign a probability equal to 1 that the deviation is from that type of firm.[8] In the event neither type of firm or both types of firms strictly benefit from a given deviation under the beliefs $\mu = \theta$, the market will assign "passive" beliefs to the firm type that has deviated, $\mu = \theta$. For example, consider these off-the-equilibrium-path beliefs for a candidate equilibrium with debt F such that $L < F < F^L$ and an unrestrictive covenant. Under the assumption that market beliefs are $\mu = \theta$ following any deviation, good firms have an interest in increasing the level of debt and bad firms have an interest in decreasing the debt level. The market will then assign beliefs so that any deviation to a higher debt level sets $\mu = 1$ and deviations to lower debt sets $\mu = 0$,[9] implying that only $F = F^L$ is a possible equilibrium strategy from the range $L < F < F^L$.

[7] For all parameter values, it is easily shown that for these off-the-equilibrium-path beliefs, there is some level of debt $F < F^L$ combined with an unrestrictive covenant that serves as a pooling equilibrium for which the same discussion applies. The choice here is simply for illustration.

[8] We assign a probability of 1 (or 0) for simplicity. All results hold, assuming off-the-equilibrium-path beliefs of some $\mu > \theta$ when a good firm benefits from a deviation under equilibrium beliefs.

[9] While this set of beliefs has the "flavor" of the intuitive criterion introduced by Cho and Kreps (1987), their refinement does not generate this set of beliefs. This is because these beliefs do not satisfy the equilibrium dominance requirement in Cho and Kreps. If a bad firm deviates from a pooling equilibrium by increasing the debt level and this convinces the market it is a good firm, the bad firm

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The initial financial structure in the absence of renegotiation may now be examined.

Proposition 2. When renegotiation of debt covenants is prohibited (or is infinitely costly) and off-the-equilibrium-path beliefs are as described previously, only the debt level F^L combined with an unrestrictive covenant represents a PSPB equilibrium.

Proof. See the Appendix.

Proposition 2 establishes that when the initial financing choice considers both ex ante adverse selection and ex post moral hazard induced by the initial financing, absent renegotiation, neither high debt nor restrictive covenants are equilibrium choices.

IV. Costless Renegotiation of Covenants

For simplicity, we will assume that in the renegotiation process, the firm makes a take-it-or-leave-it offer to the lender (and is responsible for any associated costs). This assumption gives all the bargaining power in the renegotiation to the firm and may be justified by the availability of alternative financing after a covenant violation. This assumption does not affect our conclusions; we highlight below the circumstances in which the assignment of bargaining power has a significant impact on outcomes.

Suppose that the initial financial structure consisted of a debt level F^R with a restrictive covenant (where $L \leq F^R < Q$ and R denotes an initial debt level associated with a restrictive covenant). A restrictive covenant allows the lender to make the liquidation decision in a weak market. The lender will always prefer to liquidate, introducing a potential inefficiency for a good firm. However, because it is efficient for a good firm to continue in a weak market and a good firm's continuation cash flow distribution stochastically dominates that of a bad firm, there is, in a weak market, a separating renegotiation offer a good firm is willing to make, $F^N > F^R$, that a bad firm will not mimic. The lender, believing a good firm has made this offer, will accept it in exchange for waiving the covenant violation. Note, however, that conditional on the realization of a weak market, good firms desire the lowest acceptable renegotiation offer. This may not imply separation from bad firms ex post.

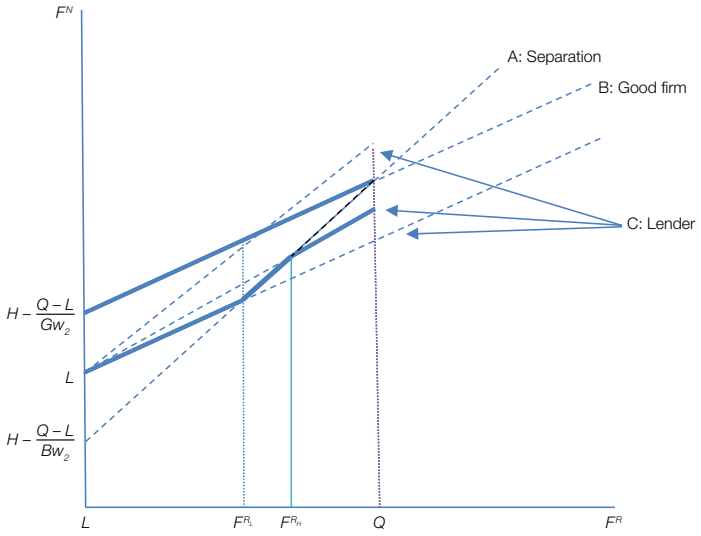
Renegotiation offers made by firms will be those that are acceptable to the lender in exchange for waiving the covenant violation. What is acceptable to the lender will depend on the type or types of firms the lender believes have made the offer. For any set of parameter values, there are two versions of the lender's participation constraint in a renegotiation: one if the offer separates good from bad firms and another if it does not.

Figure 1 illustrates the possible combinations of an initial debt level with a restrictive covenant (F^R) and an acceptable renegotiation offer (F^N) in a weak market. All allowable combinations of F^R and F^N lie within the outlined area.

may have an incentive to do so. We note that the proposed beliefs do not support any additional equilibria; rather, they serve only to eliminate those that are not "reasonable," as described previously.

FIGURE 1
Costless Renegotiation of Restrictive Covenants

Line A in Figure 1 represents the "separation" constraint and identifies the necessary renegotiation offer for each initial debt level with a restrictive covenant, F^R, for which the bad firm is indifferent between mimicking the offer and accepting liquidation with a debt level F^R. Line B represents the maximum offer, F^N, of an increase in debt burden that the good firm is willing to make for each initial debt level F^R. The three lines C represent the minimum increase in debt level the lender is willing to accept in exchange for waiving the covenant when the lender believes it is i) a good firm making the offer, ii) an average firm making the offer and it is efficient for an average firm to continue, and iii) an average firm making the offer and it is inefficient for an average firm to continue. The area inside the heavy lines represents the renegotiation region, and its bottom edge represents the set of optimal offers from the good firm's perspective.



Combinations on the lower edge of this area are those that extract the good firm's greatest benefit. The combinations on the upper edge of this area are those for which the good firm's participation constraint binds, representing renegotiation offers for which the good firm is indifferent between renegotiating the covenant and liquidating the firm in a weak market.

Lemma 2. When a restrictive covenant is violated and there is costless renegotiation, the initial debt level can be separated into low and high debt regions.
(*i*) For low debt,

$$L \quad \leq \quad F^R \quad \leq \quad \frac{Gw_2((Q-L)-Bw_2(H-L))}{Gw_2-Bw_2}+L \quad = \quad F^{R_L}.$$

The renegotiation offer is chosen so that the lender's separating participation constraint is binding, $F^N(F^R) \geq (F^R-L)/Gw_2+L$, which also satisfies the separation constraint.
(*ii*) For high debt, $F^{R_L} < F^R \leq Q$.
(*iia*) If parameter values are such that $\bar{t}w_2(H-L)+L \leq Q$, then the renegotiation offer will be chosen so that the separation constraint, $F^N(F^R) \geq H-(Q-F^R)/Bw_2$, is binding.
(*iib*) If $\bar{t}w_2(H-L)+L > Q$, the renegotiation offer will be such that the lender's pooling participation constraint, $F^N(F^R) \geq (F^R-L)/\bar{t}w_2+L$, binds.

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For cases in which the separation constraint is satisfied, (*i*) and (*iia*), bad firms liquidate and the lender, believing a good firm has made the offer, accepts and waives the covenant violation. When the separation constraint is not satisfied, (*iib*), both firm types make the same renegotiation offer and the lender accepts, believing the firm is of average type.

Proof. See the Appendix.

An interesting conclusion of Lemma 2 is that for high debt levels, the offer good firms must make to separate from bad firms increases in w_2. Given that it is efficient for good firms to continue in a weak market and for bad firms to liquidate, all else equal, the "stronger" the "weak" state is, the more difficult it is for good firms to separate from bad.

If the initial financial structure includes a debt level greater than F^L and an unrestrictive covenant, there is an induced inefficiency for bad firms. Renegotiation offers then exist in which firms offer to liquidate in a weak market (an increased restrictiveness of the covenant) in exchange for a reduced debt burden.[10] Figure 2 illustrates the offers bad firms may make to renegotiate unrestrictive covenants in a weak market.

Lemma 3. Assume renegotiation is costless, that a debt level F^U in combination with an unrestrictive covenant was chosen at $t=0$, and that a weak market has been realized.
(*i*) For very low levels of initial debt F^U such that $L \le F^U \le F^L$, a bad firm has an incentive to liquidate the firm and there is no renegotiation offer the lender will accept.
(*ii*) For low levels of debt,[11]

$$F^L \quad < \quad F^U \quad \le \quad \frac{Gw_2H - Bw_2L - (Q-L)}{Gw_2 - Bw_2} \quad = \quad F^{U_L},$$

the bad firm offers to liquidate in exchange for a reduction in the face value of debt such that the lender's participation constraint, $F^N(F^U) = Bw_2(F^U - L) + L$, binds.
(*iiia*) For high debt levels, $F^{U_L} < F^U < H$, if $\bar{t}w_2(H-L)+L > Q$, the bad firm's offer will satisfy the separation constraint $F^N(F^U) = Q - Gw_2(H - F^U)$.
(*iiib*) If $\bar{t}w_2(H-L)+L < Q$, then for debt levels

$$F^{U_L} \quad < \quad F^U \quad \le \quad F^{U_H} \quad = \quad \frac{Gw_2H - \bar{t}w_2L - (Q-L)}{Gw_2 - \bar{t}w_2},$$

the offer satisfies the separation constraint $F^N(F^U) = Q - Gw_2(H - F^U)$.
(*iiic*) If $\bar{t}w_2(H-L)+L < Q$ and the required capital, I, is large enough that the initial debt level is $F^{U_H} < F^U < H$, then a renegotiation offer $F^N(F^U) = \bar{t}w_2(F^U - L) + L$ to liquidate in exchange for a reduction in the debt's face value is made by both firms.

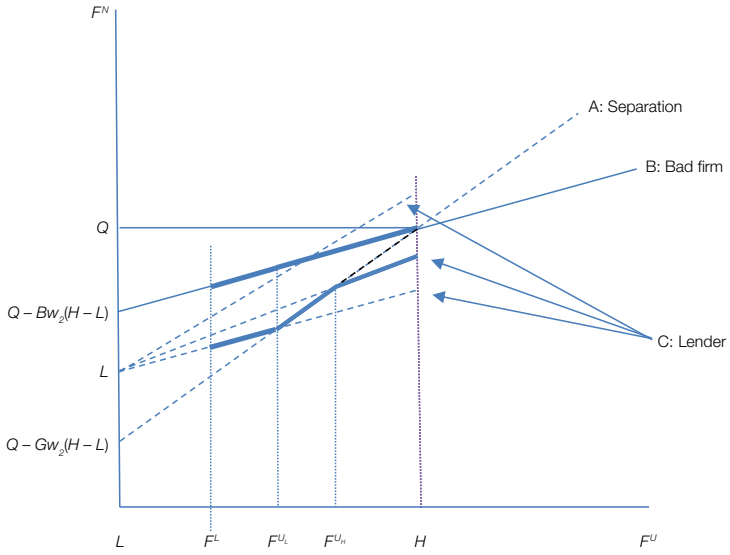
[10] This is equivalent to an endogenous violation of absolute priority, an insight provided by the referee.

[11] The statement of Lemma 4 assumes that $F^L < F^{U_L}$. It is possible that $F^{U_L} < F^L$, in which case F^L simply becomes a lower bound on the initial debt levels that permit renegotiation of an unrestrictive covenant rather than F^{U_L}.

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FIGURE 2
Costless Renegotiation of Unrestrictive Covenants

Line A in Figure 2 represents the separation constraint and identifies the necessary renegotiation offer of a new lower face value the bad firm may make, for each initial level of debt, F^U, with an unrestrictive covenant which the good firm will not mimic. Line B represents the minimum offer, F^N, of a decrease in debt burden that the bad firm is willing to make in exchange for a voluntary liquidation for each initial debt level. The three lines C represent the minimum increase in debt the lender is willing to accept in exchange for waiving the covenant when the lender believes it is i) a good firm making the offer, ii) an average firm making the offer and it is efficient for an average firm to continue, or iii) an average firm making the offer and it is inefficient for an average firm to continue. The area inside the heavy lines represents the renegotiation region, and its top (bottom) edge represents the set of optimal choices from the good (bad) firm's perspective.



In cases (*i*), (*ii*), (*iiia*), and (*iiib*), the lender, believing the offer has been made by a bad firm, accepts and good firms continue under the initial contract. In case (*iiic*), the lender, believing that the offer is from an average type firm, accepts.

Proof. See the Appendix.

We now examine the ex ante financial structure choice with costless renegotiation.

Proposition 3. When renegotiation of covenants is costless, the set of possible PSPB equilibria of the signaling game include initial financial structures with any level of "high debt" in the range $[F^{R_L}, Q]$, where

$$F^L \quad < \quad F^{R_L} \quad = \quad \frac{Gw_2((Q-L) - Bw_2(H-L))}{Gw_2 - Bw_2} + L \quad < \quad Q$$

combines with a restrictive covenant and an initial financial structure with low debt, $F = F^L$, combines with an unrestrictive covenant. Following the realization of a weak market, in the high debt restrictive covenant equilibria depending on the initial level of debt and parameter values, separating or pooling renegotiation may occur.

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There are no equilibrium financial structures that use an unrestrictive covenant other than at the low initial debt level, $F = F^L$.

Proof. See the Appendix.

When renegotiation is costless, the good firm is indifferent between low debt with an unrestrictive covenant and high debt with a restrictive covenant. Within the range of high initial debt levels in Proposition 3, increasing the level of debt ex ante (increasing the good firm's benefit from the use of debt) increases the concession required to renegotiate the covenant. This increased cost of implementing efficient decision-making ex post balances the increase in the ex ante benefit derived from the use of high initial debt.

Proposition 3 not only illustrates the value of bond covenants for controlling the agency costs of debt but also the importance of a firm's ability to renegotiate these covenants. If renegotiation is impossible, restrictive covenants are never employed in equilibrium. When renegotiation is allowed, restrictive covenants may be used in combination with high levels of debt financing. High debt combined with an unrestrictive covenant, even when the covenant may be renegotiated, is never part of an equilibrium financial structure.

Proposition 3, again, identifies liquidation value as the firm's debt capacity. There never exists a positive incentive for the firm to choose an initial debt level $F^R > Q$. Further, when parameters are such that it is efficient for an average type firm to liquidate, if the initial debt level is greater than the debt capacity, there is no possibility of a successful renegotiation, implying a positive expected cost for violating debt capacity.

Figure 1 is useful for considering how a change in the allocation of the bargaining power between the firm and the lender affects the results. When the firm has all the bargaining power, in renegotiation, either the separation constraint or the lender's participation constraint is binding. Separating renegotiation offers share the efficiency gains between the lender and the good firm. If the lender were assumed to have some (or all) of the bargaining power in renegotiation, accepted offers would, for each initial F^R, lie on the interior (the upper edge) of the shaded area in Figure 1. Ex ante, such outcomes would be inferior from the perspective of the good firm's manager and the set of equilibria would decrease to the low debt/unrestrictive covenant equilibrium. It is, however, true that the use of high debt and restrictive covenants is superior to the use of high debt and unrestrictive covenants for any division of the bargaining power between the firm and the lender.

Proposition 3 illustrates that an important aspect of the decision to use restrictive or unrestrictive covenants is the identity of the party at the bargaining table in renegotiation. When an unrestrictive covenant is renegotiated, the bad firm bargains with the lender. If the bad firm extracts any of the gains from trade in the renegotiation, the value of the good manager's ex ante value will be reduced; that is, the lender accounts for these gains in the initial pricing and the good firm does not receive the ex post gains. This result offers an explanation, complementary to that in Gârleanu and Zwiebel (2009), for the observed use of restrictive covenants and their renegotiation to be less restrictive rather than the use of initially less restrictive covenants that are renegotiated to be more restrictive.

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V. Costly Renegotiation of Covenants

We now consider the model, including a dissipative cost $c > 0$ of renegotiating debt, a cost paid by the firm at time 1. The equivalence of the low debt, unrestrictive covenant and the high debt, restrictive covenant financial structures under costless renegotiation would seem to imply that high debt must be strictly inferior under costly renegotiation. However, the cost of renegotiation changes the separation constraint faced by the good firm in the renegotiation of a restrictive covenant (bad firms also bear the renegotiation cost).[12] This change implies the equilibrium financial structure is parameter dependent. For some parameter values, the high debt, restrictive covenant financial structure represents an equilibrium; for others, the low debt, unrestrictive covenant combination is an equilibrium financial structure.[13]

With a cost c for renegotiating a violated restrictive covenant, the constraints on a separating renegotiation offer are as follows. The lender's constraint remains as

$$F^R \quad \leq \quad Gw_2(F^N - L) + L.$$

The offer required for separation from a bad firm is now written as

$$Q - F^R \quad \geq \quad Bw_2(H - F^N) - c.$$

Finally, any offer the good firm makes must be such that it will prefer renegotiating the covenant to liquidating the firm under the original contract:

$$Gw_2(H - F^N) - c \quad \geq \quad Q - F^R.$$

Limited liability of the equity implies that $Gw_2(H - F^N) \geq c$.

These constraints identify the feasible combinations of F^R and F^N in Figure 3. Note that the separation constraint and the good firm's participation constraint are shifted downward relative to these constraints in a costless renegotiation. Limited liability bounds the renegotiated face value, which, using the good firm's participation constraint, limits the initial debt to $F^R \leq Q$, again identifying liquidation value as debt capacity. We first examine the renegotiation and then the ex ante financial structure choice.

Lemma 4. Assume that the cost of renegotiation is such that there are gains to a good firm renegotiating a restrictive covenant, $Gw_2(H - L) + L - Q > c$. Consider a good firm faced with the violation of a restrictive covenant in a weak market.

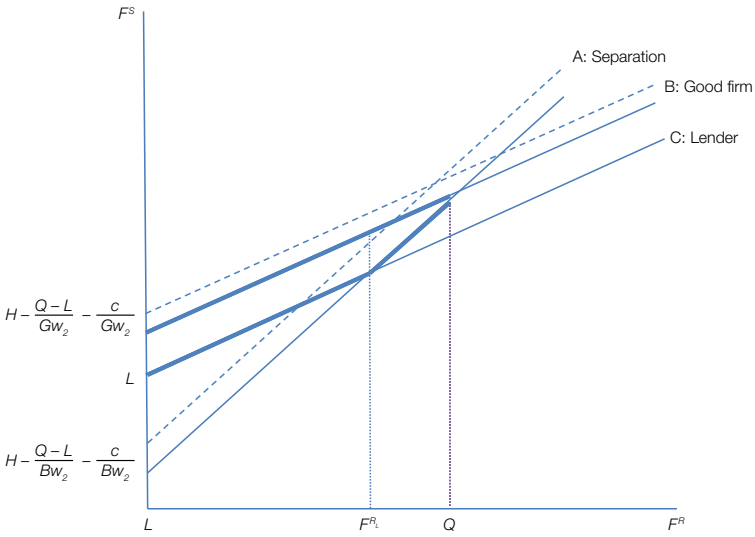
[12] Costly renegotiation must be *ex post* efficient; therefore, we restrict parameters so that $Gw_2(H - L) + L - Q > c$ (for restrictive covenants) and $Q - Bw_2(H - L) - L > c$ (for unrestrictive covenants). These restrictions simply place upper bounds on the cost of renegotiation.

[13] For simplicity, we ignore the pooling renegotiation strategies identified in Section III. Their consideration complicates the presentation and contributes no additional understanding. Formally, we can assume that $\bar{t}w_2(H - L) + L < Q$, in which case there are no pooling renegotiation strategies for restrictive covenants, and while the strategy exists for the case of unrestrictive covenants, it never appears in equilibrium.

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FIGURE 3
Costless Renegotiation of Unrestrictive Covenants

Figure 3 presents a comparison of the renegotiation of restrictive covenants with costly renegotiation to the case of costless renegotiation. The constraints are as defined in Lemmas 4 and 5. The figure shows that the separation constraint is now shifted downward, allowing the good firm to capture more of the gains from any separating renegotiation than was possible in a costless renegotiation. This increase in the good firm's benefit to renegotiation allows that, for some parameter values, it is superior ex ante for the good firm to select a high debt level and a restrictive covenant rather than a low debt level and an unrestrictive covenant.



(*i*) For low initial debt,

$$L \leq F^R \leq F^{R_L} = \frac{Gw_2((Q-L) - Bw_2(H-L))}{Gw_2 - Bw_2} + L + \frac{Gw_2 c}{Gw_2 - Bw_2},$$

the renegotiation offer made by a good firm satisfies the lender's participation constraint

$$F^N(F^R) = \frac{F^R - L}{Gw_2} + L.$$

(*ii*) For high initial debt, $F^{R_L} \leq F^R \leq Q$, the renegotiation offer made by a good firm satisfies the separation constraint

$$F^N(F^R) = H - \frac{(Q - F^R)}{Bw_2} - \frac{c}{Bw_2}.$$

In both cases, bad firms liquidate and the lender (believing a good firm has made the offer) accepts in exchange for waiving the violation of the covenant.

Proof. See the Appendix.

Note that for high initial debt, the separating renegotiation offer that is necessary to waive a restrictive covenant is increasing in w_2. This, again, indicates that renegotiation requires a larger increase to the debt burden the stronger the outlook is in a weak market. Renegotiation strategies following the realization of a weak

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market when the initial financial structure includes an unrestrictive covenant are identified in a similar way.

Lemma 5. Assume a weak market is realized and that there are gains to a bad firm renegotiating an unrestrictive covenant, $Q-(Bw_2(H-L)+L)>c$.
(*i*) For low initial debt,

$$F^L \quad < \quad F^U \quad \leq \quad F^{U_L} \quad = \quad \frac{Gw_2H - Bw_2L - (Q-L)+c}{Gw_2 - Bw_2},$$

the renegotiation offer the bad firm makes is governed by the lender's participation constraint

$$F^N(F^U) \quad = \quad Bw_2(F^U - L)+L.$$

(*ii*) For an initial debt level, F^U, such that $F^{U_L} < F^U < H$, the renegotiation offer a bad firm will make in a weak market is governed by the separation constraint

$$F^N(F^U) \quad = \quad Q - Gw_2(H - F^U) - c.$$

In both cases, good firms continue and the lender (believing the offer was made by a bad firm) accepts in exchange for liquidation (making the covenant more restrictive).

Proof. See the Appendix.

The equilibrium initial financial structure with costly renegotiation may now be identified.

Proposition 4. Under costly renegotiation, two initial financial structures may be PSPB equilibria of the signaling game. If

$$(8) \qquad \left(V^U(F^L, w_2) - I\right) - \left(\frac{\theta \bar{t}}{(1-\theta)(G-B)}\right) \times \left(pGw_1(H-Q)+(1-p)\left(\frac{G-B}{B}\right)c\right) \quad > \quad 0,$$

$F^R = Q$ (debt equal to debt capacity) combined with a restrictive covenant is the equilibrium financial structure. If the reverse is true, the equilibrium financial structure is debt with an initial face value of F^L and an unrestrictive covenant. In the razor's edge case of an equality, both are equilibria of the signaling game.

Proof. See the Appendix.

Proposition 4 shows there are only two equilibria. The first, which is more likely when c is high, mirrors the unique equilibrium in the case of no renegotiation. Firms choose low debt with an unrestrictive covenant. This equilibrium is quite different from the result derived by Myers and Majluf (1984) and is driven by our consideration of the impact of the initial financial structure on subsequent decision-making. The second equilibrium, when c is low, is similar in spirit to that in Myers and Majluf: debt equal to the firm's debt capacity combined with a restrictive covenant (renegotiated by good firms in a weak market). The difference is that our definition of debt capacity and the use of the restrictive covenant

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explicitly recognize the ex post moral hazard induced by the firm's response to the ex ante asymmetric information.

Interestingly, the simple structure of the model is not responsible for this stark result. This is true in the sense that if we enrich the model, the set of equilibrium financial structures may not change. Consider a model including a third "very weak" state and a fourth "disastrous" state, $w_4 < w_3 < w_2 < w_1$, along with the associated public signals. Further, include a "medium" quality firm, with $G > M > B$. Assume that bad firms should liquidate in all but the best state, medium firms should liquidate in states w_4 and w_3, and good firms should continue in all but the worst state, w_4.

For low face values of initial debt, depending on parameter values, the first possible equilibrium debt level is the minimum of the point at which the medium type firm is indifferent between continuing in state w_3 or liquidating and the level, $F = F^L$, for which the bad firm is just indifferent between continuing in state w_2 or liquidating. Assuming that for some debt level $F < F^L$ the medium type firm will make inefficient liquidation decisions, in the case of costly renegotiation, this allows a third possible equilibrium initial financial structure. This structure includes a very low initial debt level, $F < F^L$, associated with a minimally restrictive covenant that transfers control to the lender if state w_4 is realized (i.e., the least restrictive efficient covenant).

If parameter values are such that the medium firm makes inefficient liquidation decisions when $F > F^L$, the only low debt equilibrium is as given in Proposition 4. The face value of debt is equal to F^L and the covenant will transfer control in state w_3 and w_4 in order to restrict the bad firm's decision-making. Depending on the structure of incentives, with many firm types, there may be only two possible equilibria.

Regardless of the parameter values, the only high debt equilibrium (with $F > F^L$) has debt equal to debt capacity and the associated covenant will be "very restrictive" in the sense that control over the liquidation decision will be transferred to the lender in all states except for those in which it is efficient for *all* types to continue. Enriching the model, as described previously, however, quickly complicates the statement of the model and the equilibrium analysis. Other than more fully illustrating the notion that more restrictive covenants are associated with higher debt levels, there appear to be no additional insights generated by the more complex model.

A comparative static analysis of equation (8) identifies when it is likely that the equilibrium will include a low or high debt level. This is accomplished by taking partial derivatives of the left-hand side of equation (8) with respect to the relevant exogenous parameters. For example, all else equal, the larger the firm's debt capacity (Q), the greater the available ex ante benefit from the use of debt and the more likely it is that the equilibrium will include high debt and a restrictive covenant.

The greater the initial capital required, I (the lower is the NPV of the project, $V^U(F^L, w_2) - I$), the less likely it is that the equilibrium will be the high initial debt financial structure. The required capital represents the market's valuation of the securities sold externally, and all else equal, the greater this value (the lower is the value of the retained equity), the lower is the incentive to use high initial debt.

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As the cost (c) of renegotiation rises (representing the net cost of moral hazard), it is less likely that firms will use high debt and a restrictive covenant. Raising the cost of renegotiation affects the cost but not the benefit of initial debt use. For a high cost of renegotiation, the equilibrium financial structure corresponds with the equilibrium in the absence of renegotiation, and with a low renegotiation cost, the equilibrium financial structure corresponds to a subset of the equilibrium with costless renegotiation. The results concerning debt capacity and the renegotiation cost reflect the basic trade-off in the model, ex ante adverse selection balanced against ex post moral hazard.

An increase in the probability of a strong market, p (improvement in the ex ante economic outlook), makes it more likely the equilibrium initial financial structure is high debt and a restrictive covenant. The model, therefore, reflects the idea that, with costly renegotiation, there is an incentive for the initial contract to restrict the likelihood of renegotiation. It also suggests that more debt will be used in economic expansions.

As the quality of the pool of firms is stronger (the higher is the probability of a good firm, θ), the high debt initial financial structure becomes less likely. The stronger the pool of observationally equivalent firms, the lower the adverse selection discount faced by a good firm; therefore, the benefit from issuing a large amount of debt is lower. Increasing θ also increases the expected cost of renegotiation, increasing the cost of high initial debt. However, increasing θ increases the NPV of the investment, generating an opposite effect. The net effect is to reduce the likelihood of the high debt equilibrium.

VI. Empirical Implications and Discussion

Empirical work examining debt covenants provides a context in which we may evaluate some of the predictions of our model. Summarizing the preceding discussion, high debt and restrictive covenants are jointly determined. Restrictive covenants, when violated, are renegotiated to be less restrictive in exchange for an increased debt burden. Further, the smaller the cost of renegotiation, the greater the informational asymmetry, the stronger the economic outlook, or the larger the firm's debt capacity, the more likely we are to observe high debt and restrictive covenants. In drawing links to the empirical evidence, it is important to note that our model is quite simple and the predictions are "all else equal"; thus, we focus on qualitative connections to the empirical results.

There are at least three broad findings in the literature consistent with our model's predictions. First, Bradley and Roberts (2015) find a positive correlation between their covenant index (a measure of the number of restrictive provisions) and firm leverage. They also find that protective covenants are more likely to be observed in firms that are smaller and have higher growth opportunities (typical proxies for the degree of information asymmetry). Second, Chava and Roberts (2008) find that covenants are initially written to be remarkably tight. They report that for covenants written on the firm's current ratio, the difference between the value of the firm's actual current ratio and the level at which the covenant is violated, relative to the firm-specific standard deviation of the current ratio, has an average (median) value of 1.09 (0.84). For covenants written based on the firm's

net worth, the average (median) value of this measure of covenant tightness is 0.68 (0.56). Third, Roberts and Sufi (2009b) find that renegotiation is common. They show that over 90% of long-term debt contracts are renegotiated prior to their stated maturity. More importantly, renegotiations result in large changes to the amount, maturity, and pricing of the contract and are largely driven by the accrual of new information concerning the credit quality, investment opportunities, and collateral of the borrower, as well as macroeconomic fluctuations related to credit and equity markets. These results are broadly consistent with our finding that firms with significant amounts of debt will have very restrictive covenants that are frequently renegotiated after a violation.

Another set of studies, including Roberts and Sufi (2009a) and Nini et al. (2012), examine what happens after a debt covenant violation. Some of the findings presented in these studies are also broadly consistent with results developed here. Nini et al. find that violation of debt covenants leads to revisions in decisions that are generally consistent with a greater weight put on debt-holder incentives. Most interestingly, they find that these changes increase firm performance and value. These findings may suggest that given the conflict of interest between debt holders and equity holders and the state of the firm/market indicated by a covenant violation, creditor influence on the decision-making process limits (or prevents) equity holders' ability to engage in value-destroying activities, thus increasing performance and value. In our model, violation of restrictive covenants and their renegotiation separates good and bad firms ex post, implementing efficient decision-making and resulting in an upward revision in value. The model is, however, not rich enough to capture the various changes in behavior examined by Nini et al.

A direct relation between existing empirical results and the predictions of this model is represented by results reported in Roberts and Sufi (2009a). Roberts and Sufi ((2009a), Table VIII) show that covenant violations result in a greater reduction in post-violation debt issuance for firms that have high pre-violation leverage, low equity valuations (market-to-book ratios), and lack an S&P credit rating. They interpret this as indicating that when the firm has few available alternate sources of financing given a covenant violation (the lender has more bargaining power), the lender will be able to extract more concessions from the firm in exchange for a waiver of the covenant violation. This result is consistent with the predictions of our model concerning renegotiation following a covenant violation. All else equal, the greater the bargaining power of the lender, the greater the concessions a good firm provides to renegotiate the covenant.

This empirical result also allows for an interesting ex ante interpretation. The model presented in the paper suggests that ex ante firms are indifferent to the extent of a lender's ex post bargaining power. This stems from the assumption that the debt market is competitive when the firm is initially financed. However, if we alter the model to allow for market power for lenders in the debt market at time 0 (as may be plausible for firm/lender pairs for which the firm will have few alternate sources of financing post-violation), we see that the impact of greater ex-post bargaining for the lender on the renegotiation of a violated covenant will not be completely priced into the initial debt contract. This would translate to costlier debt and, therefore, the use of less debt ex ante.

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Lemmon and Zender (2001) and Strebulaev and Yang (2013) examine very low leveraged firms. They find that firms with very low or zero leverage make up a significant portion of the set of large public firms and that this policy choice is persistent. Both studies, however, find this choice to be inconsistent with the standard trade-off theory forces. The low debt/unrestrictive covenant equilibrium identified here offers a possible equilibrium explanation of their empirical findings.

As a final note on empirical implications, we consider what happens when standard trade-off theory forces are considered in the model. Such tensions would help determine the firm's leverage within a range between the firm's riskless level of debt and the firm's debt capacity. This is very different from the idea, introduced by Myers (1984), that trade-off theory forces affect financing choice only at extreme debt levels. In our model, debt in excess of capacity prevents renegotiation of restrictive debt covenants and, consequently, results in the possibility of inefficient investment/liquidation decision-making. Standard trade-off frictions, therefore, would impact the firm's choice between the low versus the high debt equilibrium. If the forces determining ex ante leverage in this model are important for the broad cross section of firms, then the model suggests an adjustment to standard empirical modeling of the trade-off theory that recognizes an important role for debt capacity.

VII. Conclusion

The capital structure decision is examined in a setting where asymmetric information is the sole friction. A parsimonious model is developed that allows consideration of the level of debt and the associated covenants as components of the trade-off between debt and equity financing. The model can be thought of as a simple extension of Myers and Majluf (1984), where the implications for subsequent decision-making of having debt in the capital structure when there is asymmetric information are considered. Incentive problems created by debt financing and asymmetric information are controlled by the use of debt covenants, which transfer control of the relevant decision-making to the lender in some states of nature.

The model considers the renegotiation of debt covenants. It has been shown that debt covenants are commonly written to be very restrictive and are often renegotiated when violated (Chava and Roberts (2008), Nini et al. (2012)). Our results indicate that the use of restrictive debt covenants is value enhancing only when they may be renegotiated to remove the induced ex-post inefficiencies. We examine the initial capital structure choice and the renegotiation of restrictive covenants that have been violated and highlight a number of testable implications from the model.

Continuing research examines the extent to which the model's conclusions depend on its relatively simple structure. While this model may be simple and transparent and the results intuitive, it is important to understand whether the results are robust to the examination of a more complex environment. Because the ultimate goal is to inform empirical tests of capital structure choice, this is also an important avenue of continuing research.

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Appendix. Proofs of the Lemmas and Propositions

Some initial definitions will ease the presentation of the proofs of the results. The time 0 value of the equity for an initial debt level is F^i, an initial covenant is w', a final debt level is F^N, the probability the firm is liquidated in a weak market is denoted $\ell \in [0,1]$, belief is given as $\mu \in [0,1]$ (which is 1 if the firm is known to be a good firm, 0 if it is known to be bad, and μ in a pooling strategy where $\mu = \theta$ is in equilibrium), and renegotiation cost $c \in \{0, c, \infty\}$ is written $S(F^i, w', F^N, \ell, \mu, c)$. Similarly, for the time 0, the value of debt is $D(...)$ and that of the firm is $V(...)$. For example, for initial debt, $F \in [L, F^L]$, and an unrestrictive covenant, ($S^G(F, w_2)$ in the text), the informed value of a good firm's equity is given as

$$S(F, w_2, F, \ell = 0, \mu = 1, c) \quad = \quad pGw_1(H - F) + (1 - p)Gw_2(H - F).$$

In this case, there is no value to renegotiation so initial and final debt levels are the same. With an unrestrictive covenant, the manager of the good firm will not liquidate in a weak market and c may take any value. Under the same conditions, we write the informed value of the bad firm's equity as

$$S^B(F, w_2) \quad = \quad S(F, w_2, F, \ell = 1, \mu = 0, c) \quad = \quad pBw_1(H - F) + (1 - p)(Q - F).$$

The initial and final debt levels are the same and the bad firm efficiently liquidates. The uninformed value of equity is a probability weighted average ($\mu = \theta$ in equilibrium) of these values.

Proof of Lemma 1. Because $Q > Bw_2(H - L) + L$ and $Bw_2 < 1$, the quantity $(Q - F) - Bw_2(H - F)$ is monotonically decreasing in F for $F \in [L, Q]$, is strictly positive for $F = L$, and is strictly negative for $F = Q$. Define F^L implicitly by $(Q - F^L) = Bw_2(H - F^L)$. This equality indicates the debt level at which the bad manager is indifferent between continuation and liquidation conditional in a weak market. Solving gives $F^L = (Q - w_2BH)/(1 - w_2B) < Q$, the level of debt above which the manager of a bad firm will continue in a weak market. It is efficient for a good manager to continue in both weak and strong markets, and for any $F \in [L, Q]$, they have an incentive to do so. □

Proof of Proposition 2. No Renegotiation

The structure of the problem implies that the strategy space for managers can be reduced to four options: low debt ($F \in [L, F^L]$) and an unrestrictive covenant, high debt ($F \in (F^L, Q]$) and an unrestrictive covenant, low debt and a restrictive covenant, and high debt and a restrictive covenant. The value to the manager of a type G firm choosing low debt and an unrestrictive covenant is given by

$$\text{(A-1)} \qquad \frac{S(F, w_2, F, \ell = 0, \mu = 1, c = \infty)}{S(F, w_2, F, \ell = 1 - \mu, \mu, c = \infty)} \times (V(F, w_2, F, \ell = 1 - \mu, \mu, c = \infty) - I),$$

where μ represents the market's beliefs about the probability the firm choosing the associated financial structure is a good firm. Similarly, the value of a financial structure with low debt and an unrestrictive covenant for the manager of a bad firm is given by

$$\text{(A-2)} \qquad \frac{S(F, w_2, F, \ell = 1, \mu = 0, c = \infty)}{S(F, w_2, F, \ell = 1 - \mu, \mu, c = \infty)} \times (V(F, w_2, F, \ell = 1 - \mu, \mu, c = \infty) - I).$$

In any equilibrium, it must be that $\mu = \theta$. Note that for any market belief, μ, equation (A-1) is strictly increasing in the face value of debt for $F \in [L, F^L]$ while equation (A-2) is strictly decreasing F. Therefore, the market's off-the-equilibrium-path beliefs indicate that for any

candidate equilibrium with an unrestrictive covenant and debt $F \in [L, F^L]$, if the market observes a deviation to a lower debt unrestrictive covenant strategy, the market will believe the deviation was done by a bad firm, whereas if the deviation represents an increase in the initial debt level relative to the candidate equilibrium, the market will believe the deviation was done by a good firm. Only $F = F^L$ can represent a low debt, unrestrictive covenant equilibrium strategy.

For any financial structure with high debt ($F \in (F^L, Q]$) and an unrestrictive covenant, the value to the manager of a good firm is

$$\text{(A-3)} \qquad \frac{S(F, w_2, F, \ell = 0, \mu = 1, c = \infty)}{S(F, w_2, F, \ell = 0, \mu, c = \infty)} \times (V(F, w_2, F, \ell = 0, \mu, c = \infty) - I),$$

which has the same value for *all* $F \in (F^L, Q]$. Similarly, for a bad firm choosing a high debt unrestrictive covenant financial structure, the value to the manager is given by

$$\text{(A-4)} \qquad \frac{S(F, w_2, F, \ell = 0, \mu = 0, c = \infty)}{S(F, w_2, F, \ell = 0, \mu, c = \infty)} \times (V(F, w_2, F, \ell = 0, \mu, c = \infty) - I),$$

which is also independent of the initial debt level in the range $F \in (F^L, Q]$.

If a firm selects a financial structure with a restrictive covenant (with either high or low debt), the value to the manager of a good firm is

$$\text{(A-5)} \qquad \frac{S(F, w_1, F, \ell = 1, \mu = 1, c = \infty)}{S(F, w_1, F, \ell = 1, \mu, c = \infty)} \times (V(F, w_1, F, \ell = 1, \mu, c = \infty) - I)$$

and for a bad firm's manager, the value is

$$\text{(A-6)} \qquad \frac{S(F, w_1, F, \ell = 1, \mu = 0, c = \infty)}{S(F, w_1, F, \ell = 1, \mu, c = \infty)} \times (V(F, w_1, F, \ell = 1, \mu, c = \infty) - I).$$

Equation (A-5) is strictly increasing in F for debt levels in the interval $[L, Q]$, while equation (A-6) is strictly decreasing in F. This implies only $F = Q$ may be an equilibrium.

The set of PSPB equilibria for the signaling game without renegotiation can then be established by considering equations (A-1) and (A-2) evaluated at $F = F^L$ and equations (A-3)–(A-6) evaluated at $F = Q$. Direct valuation of the retained equity for good and bad firms in the candidate equilibria versus the value offered from a deviation to any other strategy shows that only the strategy with $F = F^L$ and an unrestrictive covenant survives as an equilibrium in the signaling game without renegotiation. □

Proof of Lemma 2. Costless Renegotiation of a Restrictive Covenant

When the initial financial structure includes a restrictive covenant following the realization of a weak market, $w = w_2$, it is inefficient to allow the lender to make the liquidation decision. The lender does not know the firm's type and prefers to liquidate regardless of type. Renegotiation can take place in a weak market if there are gains to having the firms the lender believes are attempting to renegotiate continue rather than liquidate. For the first case, assume that $\bar{t} w_2(H - L) + L \leq Q$. It is only efficient to renegotiate a restrictive covenant in a weak market with good firms. In equilibrium, therefore, only separating renegotiation offers will be accepted. For an initial debt level F^R, the final debt level offered must satisfy the lender's participation constraint $G w_2(F^N - L) + L \geq F^R$, which ensures the lender, believing a good firm has made the offer, will accept the new debt level in exchange for waiving the covenant violation. The good firm's participation constraint $G w_2(H - F^N) \geq Q - F^R$ ensures good firms are willing to increase the debt level rather than liquidate under the initial debt. The separation constraint ensures bad firms prefer to liquidate under the initial debt level rather than continue under the new debt $Q - F^R \geq B w_2(H - F^N)$.

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When the firm makes a take-it-or-leave-it offer to the lender, renegotiation offers are those that satisfy all three constraints, with the lowest new debt level for each initial debt level. Solving the separation constraint and the lender's participation constraint for F^N and setting them equal identifies

$$F^{R_L} = \frac{Gw_2((Q-L)-Bw_2(H-L))}{Gw_2-Bw_2}+L,$$

the separation between low and high initial debt levels.

For low initial debt levels $F^R \le F^{R_L}$, the offer a good firm will make is given by the lender's participation constraint written as an equality:

$$F^N(F^R) = \frac{F^R-L}{Gw_2}+L. \tag{A-7}$$

For initial debt levels $F^{R_L} < F^R \le H$, the offer is determined by the separation constraint

$$F^N(F^R) = H-\frac{Q-F^R}{Bw_2}. \tag{A-8}$$

These offers will be accepted by the lender, and the bad firm will not renegotiate.

If, instead, $\bar{t}w_2(H-L)+L > Q$, then there is scope for renegotiation of the initial debt level by good firms or good and bad firms (a pooling renegotiation). The change in parameter values introduces one change in the preceding discussion. For initial debt levels $F^R \le F^{R_L}$, separating renegotiation offers are given by equation (A-7). For initial debt levels $F^{R_L} < F^R \le F^{R_H}$, where

$$F^{R_H} = \frac{\bar{t}w_2((Q-L)-Bw_2(H-L))}{\bar{t}w_2-Bw_2}+L,$$

separating renegotiation offers are given by equation (A-8). For initial debt levels $F^{R_H} < F^R \le Q$, renegotiation offers are pooling offers. The lender, believing that both types renegotiate, accepts offers given by

$$F^N(F^R) = \frac{F^R-L}{\bar{t}w_2}+L \tag{A-9}$$

and both good and bad type firms make offers to renegotiate the violated covenant. □

Sketch of Proof of Lemma 3. Costless Renegotiation of an Unrestrictive Covenant

The proof of Lemma 4 closely follows that of Lemma 3, so we sketch the proof. One difference is that for any initial debt level $F^U \le F^L$, the lender will not accept any offer to renegotiate an unrestrictive covenant as both types of firms already have an incentive to make efficient decisions.

For parameter values such that it is efficient for the average firm to continue, $\bar{t}w_2(H-L)+L \ge Q$, there is only scope for bad firms to renegotiate an unrestrictive covenant. In this case, the separating renegotiation offers made by a bad firm in a weak market entail an offer to lower the debt level in exchange for a voluntary liquidation. For initial debt F^U such that $F^L < F^U \le F^{U_L}$, separating renegotiation offers will be derived from the lender's participation constraint. If the initial debt level is such that $F^{U_L} \le F^U \le H$ ($F^U = H$ only if the NPV of the investment is 0), then separating renegotiation offers are determined by the separation constraint. The lender, believing the offer is made by a bad firm, accepts and good firms continue with the initial debt.

For parameter values such that it is inefficient for the average firm to continue, $\bar{t}w_2(H-L)+L < Q$, there is scope for pooling renegotiation of an unrestrictive covenant.

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In this case, for initial debt in the range $F^L < F^U \leq F^{U_L}$, renegotiation offers will be separating and again derived from the lenders' participation constraint. For initial debt such that $F^{U_L} \leq F^U \leq F^{U_H}$, acceptable renegotiation offers will be given by the separation constraint. For initial levels of debt such that $F^{U_H} \leq F^U \leq H$, renegotiation offers will be pooling offers where both firm types renegotiate. Renegotiation offers are

$$F^N(F^U) \quad = \quad \bar{t} w_2(F^U - L) + L.$$

The lender, believing that both types will attempt to renegotiate, accepts. □

Sketch of Proof of Proposition 3. Costless Renegotiation

This follows the proof of Proposition 2, so we again sketch the proof. For Proposition 3, we first specify the ex ante values of the debt, equity, and the firm, considering the expectations for ex post renegotiation and the combinations of initial and final debt level for the given covenants. The value to the manager of good and bad firms is then identified, and the equilibrium initial financial structure can be derived.

For low levels of debt, $L \leq F^R \leq F^{R_L}$, combined with a restrictive covenant, the informed values of equity, debt, and the firm are written (using Lemma 2) as

$$\begin{aligned}
S(F^R, w_1, F^N, \ell = 0, \mu = 1, c = 0) &= pGw_1(H - F^R) + (1-p)Gw_2(H - L) \\
&\quad -(1-p)(F^R - L), \\
D(F^R, w_1, F^N, \ell = 0, \mu = 1, c = 0) &= pGw_1(F^R - L) + (1-p)(F^R - L) + L, \\
V(F^R, w_1, F^N, \ell = 0, \mu = 1, c = 0) &= pGw_1(H - L) + (1-p)Gw_2(H - L) + L, \\
S(F^R, w_1, F^N, \ell = 1, \mu = 0, c = 0) &= pBw_1(H - F^R) + (1-p)(Q - F^R), \\
D(F^R, w_1, F^N, \ell = 1, \mu = 0, c = 0) &= pBw_1(F^R - L) + (1-p)(F^R - L) + L, \\
V(F^R, w_1, F^N, \ell = 1, \mu = 0, c = 0) &= pBw_1(H - L) + (1-p)(Q - L) + L.
\end{aligned}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad ($1-\mu$).

The value of a good type manager's retained equity for this financial structure is

$$\frac{S(F^R, w_1, F^N, \ell = 0, \mu = 1, c = 0)}{S(F^R, w_1, F^N, \ell = \mu, \mu, c = 0)} \left(V(F^R, w_1, F^N, \ell = \mu, \mu, c = 0) - I\right)$$

and the value of the bad type manager is written similarly. The good manager's value is strictly increasing in the initial debt level F^R and the bad firm manager's value is strictly decreasing in F^Ron the interval $L \leq F^R \leq F^{R_L}$, so the only financial structure with low debt and a restrictive covenant that may be an equilibrium is when $F^R = F^{R_L}$.

For high debt $F^{R_L} < F^R \leq Q$, if it is inefficient for an average firm to continue (or for debt levels $F^{R_L} < F^R \leq F^{R_H}$when it is efficient for an average firm to continue), the value of retained equity for both the good and bad type manager is independent of F^R.

For debt levels $F^{R_H} < F^R \leq Q$, when it is efficient for an average firm to continue (using Lemma 3), the value of retained equity for both managers is independent of F^R.

For low levels of debt, $F^L < F^U \leq F^{U_L}$, combined with an unrestrictive covenant, the value of the retained equity for the good (bad) firm manager is strictly decreasing (increasing) in F^U. The only candidate equilibrium with a level of debt in the interval $F^L < F^U \leq F^{U_L}$ combined with an unrestricted covenant is an initial financial structure with $F^U = F^L$ combined with an unrestrictive covenant.

For a level of debt $F^{U_L} < F^U \leq F^{U_H}$ (if it is inefficient for an average firm to continue) or $F^{U_L} < F^U \leq H$ (if it is efficient for an average firm to continue), combined with an unrestrictive covenant, the value of the retained equity for both good and bad managers is

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independent of F^U. Therefore, all of the relevant financial structures or none of them will be equilibria in the signaling game for the initial choice of financial structure.

For levels of debt $F^{U_H} < F^U \leq H$ (when it is inefficient for an average firm to continue) combined with an unrestrictive covenant, the value of the retained equity for a good (bad) firm manager's value is decreasing (increasing) in F^U. Therefore, only an initial debt level equal to F^{U_H} is a candidate equilibrium for this range of initial debt levels combined with an unrestrictive covenant.

Establishing the set of equilibria with costless renegotiation is now straightforward. For each of the candidate equilibria (low debt ($F^U = F^L$) and an unrestrictive covenant, high debt ($F^{U_L} \leq F^U \leq H$) and an unrestrictive covenant, and high debt ($F^{R_L} \leq F^R \leq Q$) combined with a restrictive covenant), compare the equilibrium value to the manager of a good or a bad firm to the value from deviation to an alternate initial financial structure. Applying off-the-equilibrium-path beliefs, only the low debt, unrestrictive covenant or the set of high debt, restrictive covenant financial structures survive as equilibria of the signaling game. In each of the other cases, deviation by the good firm to the low debt unrestrictive covenant strategy results in a strict gain, implying these candidates cannot be equilibria of the signaling game. For the two equilibria identified, deviations are either strictly improving for bad firms (implying a market belief $\mu = 0$) or neutral for both firm types (implying a market belief $\mu = \theta$) and neither firm has an incentive to deviate. □

Sketch of Proof of Lemma 4. Costly Renegotiation of a Restrictive Covenant

For low initial debt levels, $L \leq F^R \leq F^{R_L}$, combined with a restrictive covenant, renegotiation offers will be determined by the lender's participation constraint. The separation constraint is satisfied, so when a weak market is realized, good firms make renegotiation offers so that the participation constraint binds, bad firms do not attempt to renegotiate preferring liquidation, and the lender, believing a good firm has made the renegotiation offer, accepts the offer to waive the violation of the covenant.

For high initial debt levels, $F^{R_L} \leq F^R \leq Q$, combined with a restrictive covenant, renegotiation offers will be determined by the separation constraint. The lender's participation constraint is satisfied, so again, bad firms do not attempt to renegotiate following the realization of a weak market and the lender, believing a good firm has made the offer, accepts the offer to waive the violation of the covenant. □

Sketch of Proof of Lemma 5. Costly Renegotiation of an Unrestrictive Covenant

For low initial debt levels, $F^L < F^U \leq F^{U_L}$, combined with an unrestrictive covenant, renegotiation offers are such that the lender's participation constraint is binding. The separation constraint is satisfied so good firms will continue under the original debt contract and the lender, believing a bad firm has made the offer to liquidate the firm in exchange for a reduced debt level, accepts.

For high initial debt levels, $F^{U_L} < F^U \leq H$, combined with an unrestrictive covenant, renegotiation offers are such that the separation constraint is binding. The lender's participation constraint is satisfied so good firms continue under the original debt contract and the lender, believing that a bad firm has made the offer to liquidate the firm in exchange for a reduced debt level, accepts. □

Sketch of Proof of Proposition 4. Costly Renegotiation

For the initial financial structure with $F^U = F^L$ combined with an unrestrictive covenant, there is no renegotiation.

Using Lemma 4, for low initial values of debt $L \leq F^R \leq F^{R_L}$ combined with a restrictive covenant, the value of the retained equity is strictly increasing (decreasing) in the level of initial debt for good (bad) firms over the interval $L \leq F^R \leq F^{R_L}$, which implies that the only possible equilibria has $F^R = F^{R_L}$.

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Using Lemma 4, for high initial values of debt $F^{R_L} \leq F^R \leq Q$ combined with a restrictive covenant, the value of the retained equity is strictly increasing (decreasing) in the level of initial debt for good (bad) firms so that the only candidate equilibrium is at the firm's debt capacity, $F^R = Q$.

Using Lemma 5, for low initial values of debt combined with an unrestrictive covenant, the value of the manager's retained equity is strictly decreasing (increasing) in the initial debt level for good (bad) firms over the interval $F^L \leq F^U \leq F^{U_L}$, so that only an initial debt level $F^U = F^L$ may be an equilibrium.

Using Lemma 5, for high initial values of debt, $F^{U_L} \leq F^R < H$, combined with an unrestrictive covenant, the value of the retained equity for the good (bad) manager is decreasing (increasing) in the initial debt level. Only $F^U = F^{U_L}$ may be an equilibrium.

There are then three candidate equilibria in the case of costly renegotiation: low initial debt and an unrestrictive covenant that will not be renegotiated ($F^U = F^L$), low initial debt and an unrestrictive covenant that may be renegotiated ($F^U = F^{U_L}$), and high initial debt ($F^R = Q$) combined with a restrictive covenant. Consider the financial structure with low initial debt and an unrestrictive covenant that may be renegotiated. Holding beliefs at $\mu = \theta$, good firms strictly benefit from a deviation to the financial structure with low debt and an unrestrictive covenant that will not be renegotiated. Such a deviation will result in the off-the-equilibrium-path belief that a good firm is deviating, implying this cannot be an equilibrium.

For the other two candidate equilibria, depending on parameter values, each may represent a PSPB equilibrium of the signaling game. Comparing the value of the good manager's retained equity for initial debt $F^R = Q$ with a restrictive covenant to that for initial debt $F^U = F^L$ with an unrestrictive covenant results in the inequality given in Proposition 4. Consider the high debt/restrictive covenant candidate. If the inequality in Proposition 4 holds, then, assuming beliefs are represented by $\mu = \theta$, the good firm will be strictly worse off from any deviation, implying that any deviation results in the belief that the deviating firm is a bad type and no type will deviate from the candidate equilibrium. If the stated inequality is violated, then, assuming beliefs are represented by $\mu = \theta$, good firms strictly benefit from deviating to the low debt, unrestrictive covenant financial structure, and this deviation will be believed to have been made by a good firm. The result is that both types will wish to deviate. If the inequality is reversed, a similar argument shows that the low debt, unrestrictive covenant (which will never be renegotiated) financial structure is the equilibrium of the signaling game. □

Comparative Statics

The comparative static analysis is done by taking the partial derivative of the expression in equation (8) with respect to the parameters I, Q, c, and p. For the comparative static analysis with respect to θ, it is helpful to rewrite the expression as

$$\frac{G(V^U(F^L, w_2) - I)}{\theta \bar{t}} - \frac{(V^U(F^L, w_2) - I)}{\theta} \quad > \quad pGw_1(H - Q) + (1 - p)\frac{G - B}{B}c.$$

The partial derivative of the left-hand side of this expression with respect to θ can be shown to be negative, while the right-hand side is not dependent upon θ.

References

Asquith, P.; R. Gertner; and D. Scharfstein. "Anatomy of Financial Distress: An Examination of Junk-Bond Issuers." *Quarterly Journal of Economics*, 109 (1994), 625–658.

Bradley, M., and M. R. Roberts. "The Structure and Pricing of Bond Covenants." *Quarterly Journal of Finance*, 5 (2015), 1–37.

Chava, S., and M. R. Roberts. "How Does Financing Impact Investment? The Role of Debt Covenants." *Journal of Finance*, 63 (2008), 2085–2121.

Cho, I. K., and D. Kreps. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics*, 102 (1987), 179–222.
Constantinides, G. M., and B. D. Grundy. "Optimal Investment with Stock Repurchase and Financing as Signals." *Review of Financial Studies*, 2 (1989), 445–465.
Davis, J. E. "Optimal Issuance across Markets and over Time." Working Paper, University of North Carolina (2016).
DeMarzo, P. M., and M. J. Fishman. "Agency and Optimal Investment Dynamics." *Review of Financial Studies*, 20 (2007), 151–188.
Gârleanu, N., and J. Zwiebel. "Design and Renegotiation of Debt Covenants." *Review of Financial Studies*, 22 (2009), 749–781.
Grossman, S. J., and O. D. Hart. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy*, 94 (1986), 691–719.
Hart, O. D., and J. Moore. "Property Rights and the Nature of the Firm." *Journal of Political Economy*, 98 (1990), 19–58.
Hart, O. D., and J. Moore. "A Theory of Debt Based on the Inalienability of Human Capital." *Quarterly Journal of Economics*, 109 (1994), 841–879.
Leland, H. E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *Journal of Finance*, 49 (1994), 1213–1252.
Lemmon, M. L., and J. F. Zender. "Looking under the Lamppost: An Empirical Examination of the Determinants of Capital Structure." Working Paper, University of Colorado (2001).
Myers, S. C. "The Capital Structure Puzzle." *Journal of Finance*, 39 (1984), 575–592.
Myers, S. C., and N. S. Majluf. "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have." *Journal of Financial Economics*, 13 (1984), 187–221.
Nini, G.; D. C. Smith; and A. Sufi. "Creditor Control Rights, Corporate Governance, and Firm Value." *Review of Financial Studies*, 25 (2012), 1713–1761.
Roberts, M., and A. Sufi. "Control Rights and Capital Structure: An Empirical Investigation." *Journal of Finance*, 64 (2009a), 1657–1695.
Roberts, M., and A. Sufi. "Renegotiation of Financial Contracts: Evidence from Private Credit Agreements." *Journal of Financial Economics*, 93 (2009b), 159–184.
Shleifer, A., and R. W. Vishny. "Liquidation Values and Debt Capacity: A Market Equilibrium Approach." *Journal of Finance*, 47 (1992), 1343–1366.
Strebulaev, I. A., and B. Yang. "The Mystery of Zero-Leverage Firms." *Journal of Financial Economics*, 109 (2013), 1–23.
Viswanath , P. V. "Strategic Considerations, the Pecking Order Hypothesis, and Market Reactions to Equity Financing." *Journal of Financial and Quantitative Analysis*, 28 (1993), 213–234.

https://doi.org/10.1017/S0022109018000443 Published online by Cambridge University Press