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CORRIGENDUM

The L^2 $\bar{\partial}$-problem on manifolds with piecewise strictly pseudoconvex boundaries

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This note is to correct [**2**], where the proofs of the theorems stated require extra conditions on the manifolds. This is because it is assumed tacitly in [**2**] that a locally square integrable $\bar{\partial}$-closed (p,q)-form on a manifold Ω with a piecewise strictly pseudoconvex boundary is $\bar{\partial}$-exact. This is equivalent to $H^q(\Omega, D_p) = 0$, $q \geqslant 1$, where D_p is the sheaf of germs of holomorphic p-forms on Ω. This is not true in general, but true if Ω is Stein; but to assume Ω is Stein is too restrictive a condition.

We will show that the results of [**2**] are true provided that all manifolds with piecewise strictly pseudoconvex boundaries considered there are assumed to have C^2 strictly plurisubharmonic functions defined on them. We do this by showing that on manifolds Ω with piecewise strictly pseudoconvex boundaries, having C^2 strictly plurisubharmonic functions defined on them, the cohomology groups $H^q(\Omega, D_p)$ vanish for $q \geqslant 1$.

We prove as our main result in this note the following

Theorem 1. *Let X be a complex manifold and Ω a relatively compact subdomain with a piecewise strictly pseudoconvex boundary. Let there be a C^2 strictly plurisubharmonic function defined on Ω. Then $H^q(\Omega, D_p) = 0$ for $q \geqslant 1$.*

This theorem depends heavily on two theorems. The first which is folk-lore follows from arguments in [**1**; section 2·2] or [**5**; sections 3·2 and 3·3] and is:

Theorem 2. *Let X be a complex manifold and Ω a relatively compact subdomain with C^∞ strictly pseudoconvex boundary. Let there be a C^2 strictly plurisubharmonic function defined in a neighbourhood of the closure in X of Ω. Then, if $f \in L^2_{(p,q)}(\Omega)$ is $\bar{\partial}$-closed, there is $u \in L^2_{(p,q-1)}(\Omega)$ such that $\bar{\partial} u = f$ and*

$$\|u\|_{L^2_{(p,q-1)}(\Omega)} \leqslant c \|f\|_{L^2_{(p,q)}(\Omega)}, \quad q \geqslant 1,$$

where c depends on Ω.

The proof of the next theorem is completely parallel to that of [**3**: theorem IX; C.4]

Theorem 3. *Let Ω be a manifold with a piecewise strictly pseudoconvex boundary. Then there are a Stein manifold Y and a proper holomorphic mapping $\pi : \Omega \to Y$ with the following properties*:

(i) $\pi : O_Y \cong O_\Omega$,

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(ii) *there are a finitely many points* $x_1, \ldots, x_t$ *in* Y *such that* $\pi^{-1}(x_j)$ *is a compact subvariety of* Ω *of positive dimension, and* $\pi: \Omega \backslash \bigcup \pi^{-1}(x_j) \cong Y \backslash \{x_1, \ldots, x_t\}$.

The first statement means that the rings of holomorphic functions O_Y and O_Ω are isomorphic and the last part of the second means that $\Omega \backslash \bigcup \pi^{-1}(x_j)$ is biholomorphic to $Y \backslash \{x_1, \ldots, x_t\}$.

To prove Theorem 1, it is enough, by Dolbeault's Theorem, to prove that if f is a C^∞ $\bar{\partial}$-closed (p, q)-form on Ω then there is a $C^\infty(p, q-1)$-form u on Ω such that $\bar{\partial} u = f$. The proof of this is modelled on an argument of Hörmander [**4**; theorem 2·7·8]. First of all, since a Stein manifold can be exhausted by a sequence of strictly pseudoconvex domains, each with a C^∞ boundary, we use Theorem 3 to lift to an exhaustion of Ω by a sequence $\Omega_1 \subset\subset \Omega_2 \subset\subset \ldots$ of strictly pseudoconvex domains, each with a C^∞ boundary. Secondly, using Theorem 3 again, by a lifting argument Oka–Weil approximation theorem is possible in neighbourhoods of large holomorphically convex compact subsets of Ω. These compact sets are large in the sense that they contain $\bigcup \pi^{-1}(x_j) \subset \Omega$ in their interiors, where π and the x_j are as in Theorem 3. Thirdly, since Ω is holomorphically convex, there is an exhaustion of Ω by holomorphically convex compact subsets of Ω, $K_1, K_2, \ldots$, such that $K_1 \subset \Omega_1 \subset K_2^0 \subset K_2 \subset \Omega_2 \subset K_3^0 \subset K_3 \subset \Omega_3 \subset \ldots$, where K_j^0 is the interior of K_j. Lastly, we apply Theorem 2 to each Ω_j of the exhaustion of Ω, noting that if f is a $C^\infty \bar{\partial}$-closed (p, q)-form on Ω_j, the u such that $\bar{\partial} u = f$ on Ω_j given by Theorem 2, can be chosen to be C^∞.

Now we have the same situation as in [**4**; theorem 2·7·8] and we can apply the arguments there almost word for word.

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