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BOOK REVIEWS

NEW INTRODUCTION TO MULTIPLE TIME SERIES ANALYSIS

by Helmut Lütkepohl
Springer, 2005

REVIEWED BY
LUTZ KILIAN
University of Michigan

1. INTRODUCTION

Vector time series analysis has become a standard tool in macroeconometrics and in empirical finance since the early 1980s. Over the last two decades, Helmut Lütkepohl has published extensively on virtually all topics covered in this book, which puts him in an excellent position for writing an authoritative textbook on this subject.

New Introduction to Multiple Time Series Analysis is a substantially revised and extended version of an earlier book by the same author that first appeared in 1991 under the title *Introduction to Multiple Time Series Analysis* and was reprinted in 1993. Unlike that second edition of *Introduction to Multiple Time Series Analysis*, this new book has undergone substantial changes. The discussion of old material and the reference section have been updated in light of more recent developments in the literature, and several new chapters have been added. The choice of topics reflects to some extent the author's research interests but includes all topics that are relevant to the analysis of vector autoregressive (VAR) and vector error correction (VEC) models. The book focuses on parametric time series models throughout. There is no discussion of frequency domain methods. The book strikes the right balance in assembling a broad range of topics. Only rarely would I have shifted the emphasis somewhat.

2. TARGET AUDIENCE

The book is designed for graduate students and for practitioners. It may be used in teaching a topics course at the M.A. level or a field course at the Ph.D. level. The book focuses on statistical tools. It is assumed that the reader is familiar

Address correspondence to Lutz Kilian, Department of Economics, University of Michigan, 611 Tappan Street, Ann Arbor, MI 48019-1220, USA; e-mail: lkilian@umich.edu.

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with matrix and vector notation. Some knowledge of univariate time series analysis at the level of Hayashi (2000) will also be helpful. Although the book contains many examples, it is not concerned with applications. In teaching a Ph.D. course on macroeconometrics, the book should therefore be supplemented by selected readings drawn from journal articles that illustrate the practical importance of the tools developed in this book and put them in historical perspective. In teaching a Ph.D. course on time series analysis, the book should be supplemented by a general reference such as Hamilton (1994). Alternatively, a topics course on VAR and VEC models could be organized around this book alone.

The book bridges the gap between introductory time series textbooks such as Enders (2004) and the more difficult technical literature found in journals. It contains a comprehensive collection of theoretical results for estimators and test statistics used in the analysis of VAR and VEC models. The text provides precise statements of the estimators and test statistics in question, it motivates and illustrates their use, and it derives their asymptotic properties in the form of propositions and proofs. It also provides an extensive and up-to-date list of references for further reading. As the title states, the book is intended as an introduction. In practice, this means that proofs are sometimes presented in greater detail than would be the case in journal articles. It also means that technical arguments are not always presented in their greatest generality. For example, some proofs assume normality of the regression error when that assumption could be relaxed. The emphasis is on conveying the idea underlying the proof, not on presenting the most general case. If students grasp the basic idea underlying the proof, they are free to consult the original papers cited in the reference section for a more rigorous treatment. The proofs are accessible to second-year Ph.D. students who have taken two semesters of statistics and econometrics at the level of Casella and Berger (2002) and Hayashi (2000). Several proofs constitute nice applications of material typically covered in the first year of a Ph.D. program. For example, the derivation of the Jarque–Bera test of the Gaussianity of the VAR regression innovations makes for a great illustration of the Lagrange multiplier (LM) approach to testing. Although the proofs will be an important part of any Ph.D. course based on this book, the casual reader may skip most proofs without loss of continuity.

The book's treatment of the derivation of estimators and test statistics and their asymptotic distributions is more complete than alternative sources, reflecting in part its more selective coverage. Often alternative estimators of the same object are discussed. This parallel development of alternative approaches may seem pedestrian or even redundant at first, but it actually helps students appreciate the differences between and commonalities of these estimators. For example, the book contrasts the least-squares (LS) estimator, the conditional maximum likelihood (ML) estimator, and the Yule–Walker estimator of the parameters of the unrestricted VAR model. The book further distinguishes the equation-by-equation LS estimator and multivariate LS estimator and demonstrates why they are the same (setting the stage for the subsequent discussion of estimators of

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models subject to parameter constraints). It also distinguishes between estimation based on demeaned data and estimation of models including an intercept.

Given the proliferation of alternative estimators and approaches, the book does a good job of maintaining consistent and intuitive notation throughout. The initial investment in notation pays off when students have to implement these procedures on a computer. The book is unusually explicit in that every procedure is laid out in matrix notation step by step, making it easy for students to follow. The same feature also makes the book uniquely suited for a course that involves computer programming. To take full advantage of this book, it is recommended that students have access to matrix algebra software such as MATLAB or GAUSS that allows them to replicate the numerical results in the book and the additional computational exercises at the end of each chapter. The data sets for the empirical exercises are no longer provided in the book but can be downloaded as ASCII files free of charge from the Web site www.jmulti.de. Virtually all techniques discussed in the book are illustrated by concrete empirical examples. In many cases, full numerical answers are provided. The great advantage of this approach is that students can trace any discrepancies between their own empirical results and the results shown in the book to their source, thereby detecting and eliminating coding errors. Students not only learn to be precise, but over time they become proficient programmers who are able to write and debug their own code.

In addition, the programming experience provides a good check on how well students have absorbed the material. A good example is the section on lag order selection for VAR models. In my experience, most students believe that they have understood the material on information criteria after I cover this section in class, yet often the programming exercises reveal their inability at first to replicate the empirical results in the book, forcing them to review the material. Thus, the programming aspect of teaching from this book becomes an integral part of the learning experience. It also is possible to do the same empirical exercises using canned packages such as Eviews, RATS, PCGive, MicroFit, or JMulTi. The latter option may be more suitable for M.A. level courses. The JMulTi software can be downloaded free of charge from www.jmulti.de, which may be attractive for instructors whose students do not have access to commercial software.

3. CONTENTS

The book is organized as follows. Following a nontechnical introduction to the topic of multivariate time series analysis, Part I (chapters 2–5) of the book deals with the reduced form analysis of stationary finite-order VAR models. Chapter 2 introduces notation under the assumption that all model parameters are known.

Chapter 3 discusses in depth the implementation and asymptotic properties of the least-squares, Yule–Walker, and conditional maximum likelihood estima-

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tors of the parameters in the unrestricted finite-order stationary VAR model. It also addresses the question of what uses we can put the estimated VAR model to. Of particular interest are the construction of point forecasts and of interval forecasts, tests of Granger noncausality, tests of instantaneous and of multistep noncausality, reduced form impulse response analysis, and the construction of forecast error variance decompositions. The book pays particular attention to the asymptotic properties of each procedure and differentiates between models with an intercept and models for demeaned data. A natural extension would have been a brief discussion of models with deterministic time trends. A useful tool that is missing from this chapter is historical decompositions that allow the researcher to isolate the cumulative impact of any one shock on each variable in the VAR system at a given point in time (as discussed, e.g., in the RATS manual).

Chapter 4 deals with model selection. It covers all commonly used methods of lag order selection for VAR models including sequential likelihood ratio (LR) tests for lag order selection, lag order selection based on information criteria (FPE, AIC, SIC, HQC), portmanteau and LM tests for the absence of serial correlation in the model residuals. It also includes other model diagnostics such as the Jarque–Bera tests for the normality of the model innovations and tests of the structural stability of the model parameters.

Chapter 5 deals with the estimation of VAR models subject to parameter constraints. The first part of this chapter focuses on restricted feasible generalized least-squares (GLS) and restricted maximum likelihood estimation of VAR processes subject to linear parameter restrictions. Estimation subject to nonlinear parameter restrictions is discussed briefly. There also is extensive discussion of the estimation and specification of subset VAR models. The second part of chapter 5 discusses estimation and inference in VAR models from a Bayesian point of view. The focus is on Gaussian priors and the use of hyperparameters in stationary models. The introduction of the independent random walk prior (also known as the Litterman or Minnesota prior) is deferred until chapter 7. Although Bayesian VAR analysis has come a long way since the 1980s, the discussion in this section continues to be practically relevant and remains the most accessible introduction to Bayesian VAR modeling that I am aware of.

Part II (chapters 6–8) includes three chapters on the reduced form analysis of cointegrated finite-order VAR models. Chapter 6 introduces the concepts of integrated processes and of cointegrated time series. As in chapter 2, the discussion assumes that the VAR model parameters are known. Chapter 7 discusses estimation of cointegrated VAR (or VEC) models by least-squares, by feasible GLS, and by the method of maximum likelihood. The chapter also covers two-stage estimation of the VEC model given a superconsistent estimate of the cointegrating vector. The latter estimator includes the Engle and Granger (1987) approach as a special case. Following a discussion of the asymptotic properties of each procedure this chapter concludes with a discussion of forecasting, impulse response analysis, and Granger noncausality tests and of how

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parameter restrictions may be imposed in the estimation of VEC models. It is unfortunate that the new book omits any discussion of the pitfalls of interpreting the estimated coefficient of a variable in the (normalized) cointegrating vector as the long-run effect of a unit increase in that variable (see section 11.3.3.a of *Introduction to Multiple Time Series Analysis*). This point always struck me as important in teaching this material because it is frequently ignored in applied work. Chapter 8 deals with issues of model selection in cointegrated VAR models ranging from the VAR lag order selection problem and the rank selection problem to general model diagnostics, the specification of subset models, and the treatment of deterministic terms.

Part III (chapters 9 and 10) of the book deals with structural VAR models and with dynamic simultaneous equation models in which some variables are treated as exogenous with respect to the system of equations. The latter models are also referred to as conditional models, if the data generating process of the exogenous variables is left unspecified. Chapter 9 covers structural VAR models based on short-run identifying restrictions, long-run identifying restrictions, or both. It also covers structural VEC models. This chapter is a most welcome addition. Among existing time series textbooks only Amisano and Giannini (1997) contains any formal discussion of this important topic. Chapter 9 discusses estimation of the structural parameters of VAR and VEC models in the context of maximum likelihood estimation. Here some additional discussion of the generalized method of moments approach of Bernanke (1986) and of the instrumental variable approach to structural VAR models in Shapiro and Watson (1988) would have been useful. Chapter 10 provides a concise overview of the specification and uses of dynamic simultaneous equation models from a modern point of view, including the estimation of the model parameters and some distributional results.

Part IV (chapters 11–15) drops the assumption that the data generating process is a finite-order VAR process. Chapter 11 extends the analysis to stationary finite-order vector autoregressive–moving average (VARMA) models. Chapter 12 is concerned with the problem of estimating VARMA problems by the method of maximum likelihood and deriving the asymptotic properties of that estimator. The chapter also discusses impulse response analysis and forecasting from VARMA models. Chapter 13 covers model selection tools for VARMA models. Chapter 14 extends the analysis to cointegrated VARMA models.

An obvious question is how the interpretation and statistical properties of finite-order VAR models are affected when the dynamics of the data generating process are misspecified. A leading example is the use of finite-order VAR approximations to infinite-order VAR processes. In that situation an alternative asymptotic thought experiment in which the VAR lag order increases at a suitable rate with the sample size may be appealed to. Chapter 15 highlights the important changes in the asymptotic properties of VAR estimators in the latter case.

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Part V covers special topics in multivariate time series analysis. Much of the book is concerned with modeling the conditional mean of a vector time series, but especially in finance one is often interested in modeling the conditional variance. Chapter 16 deals with the latter case. It focuses on the multivariate generalized autoregressive conditionally heteroskedastic (GARCH) model. The chapter addresses quasi-maximum likelihood estimation, asymptotic inference, specification testing, and the interpretation of the GARCH model estimates.

In chapter 17, nonstationary VAR processes with time-varying parameters are considered. The chapter distinguishes two cases: In intervention models a one-time event such as a natural disaster or legislative change causes a structural break in one or more model parameters. In periodic VAR processes, in contrast, parameters vary over time according to some recurring seasonal pattern. The discussion in this chapter perhaps does not do full justice to the wide range of econometric work on seasonality and structural change, but it is true that much of that work has focused on the univariate case, and the discussion in chapter 17 does suffice as an introduction. In particular, maximum likelihood estimation of various types of intervention models and periodic models is discussed in addition to LR and other tests for time-varying parameters. A useful extension of this chapter would have been a section on multivariate regime-switching models.

Chapter 18 introduces state-space representations of VAR models. State-space models provide a unifying framework for the time-invariant VAR and VARMA models in addition to the time-varying VAR models discussed in earlier chapters. They also allow estimation of an even more general class of time-varying parameter VAR models with randomly evolving coefficients. Chapter 18 discusses in detail how various models may be cast in state-space form and estimated by maximum likelihood methods using the Kalman filter.

A valuable addition to the text is provided by the appendixes, which review many of the technical tools used in the book, including some that are not commonly taught in first-year courses. These additions help make the book largely self-contained. Appendix A provides a concise and comprehensive review of matrix and vector algebra, including some topics that not all beginning graduate students will be familiar with such as vector and matrix differentiation and the optimization of vector functions. Appendixes B and C contain a review of basic distributional results, of the concept of stochastic convergence, and of the derivation of the asymptotic distributions of estimators and test statistics. Appendix C has been expanded to include some material on Brownian motions and the functional central limit theorem. Appendix D contains a very brief introduction to the mechanics of simulating draws from multivariate Gaussian distributions and of using residual-based bootstrap methods for inference. Throughout the book, the author makes a laudable effort to illustrate the use of bootstrap methods of inference in VAR and VEC models as an alternative to conventional large-sample approximations. One suggestion for future editions would

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be to expand the list of references for the asymptotic validity of the bootstrap in this context.

4. CONCLUSION

Helmut Lütkepohl has provided an excellent review of the state of the art of VAR and VEC modeling. The book offers enough material for a two-semester course on vector time series analysis. In practice, users will have to set priorities. I have found chapters 1–9 particularly useful in teaching a Ph.D. course on macroeconometrics. The selection of additional chapters largely depends on the priorities of the instructor.

The book is well written, comprehensive, and appropriate for the target audience. It will undoubtedly become a standard textbook for applied time series courses and a useful reference book for users of VAR and VEC models. There is no other book on the market that is as comprehensive and up to date or as accessible to students. The book can also be recommended for an independent study course or for self-study. The one point that deserves criticism is that the publisher demands US$199 for the hardcover version of this book. This price will be prohibitive for most students, no matter how good the book is. One can only hope for a more reasonably priced paperback edition in the near future.

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