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Single-event high-compression inertial confinement fusion at low temperatures compared with two-step fast ignitor

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Abstract. Compression of plasmas with laser pulses in the 10-kJ range produced densities in the range of 1000 times that of the solid state, where however the temperatures within a few hundred eV were rather low. This induced the fast ignitor scheme for central or peripheral deposition of some 10-kJ ps laser pulses on conventional n_s-precompressed DT plasma of 3000 times solid-state density. We present results where the ps ignition is avoided and only a single-event conventional compression is used. Following our computations of volume ignition and the excellent agreement with measured highest fusion gains of volume compression, we found conditions where compression to 5000 times that of the solid state and by using laser pulses of 10 MJ produce volume ignition with temperatures between 400 and 800 eV only for high-gain volume ignition.

1. Introduction

Generation of inexhaustive, low-cost, safe and clean energy by burning deuterium–tritium mixtures to helium as nuclear fusion reactions is an important goal for avoiding the climatic catastrophe of atmospheric heating by burning fossil fuels. Within many years' attempts to reach this goal, the advancement within recent years was significant in nearly reaching the break-even condition. Based on core gain, laser fusion inertial confinement reached 30% efficiency (Soures et al. 1996) and, based on a neutral beam fusion on magnetically confined plasma, reached 67% neglecting the energy input for producing the target plasma (Guo 1998). The fact that high gains for inertial confinement fusion were measured by using 50-MJ X-radiation for compressing the fuel targets (Broad 1988) confirms the solution by this direction, where only the more sophisticated input of laser or heavy or light ion beams is expected to provide lower input energy pulses and higher gains.

The development of laser fusion arrived at a number of alternative schemes during the last 40 years of research, closely following experimental achievements.

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After the simple burn of a DT plasma of an initial density n_o (in relation to the solid-state density n_s), where energy E_o was deposited as scaled by a break-even energy E_{BE}, we arrived at an optimum (core) gain G:

$$G = \begin{cases} (E_o/E_{BE})^{1/3}(n_o/n_s)^{2/3} & \text{(Hora 1964; Hora and Pfirsch 1970)} \\ \text{const } n_o R & \text{(Kidder 1974; Fraley et al. 1974),} \end{cases} \tag{1.1}$$

where both expressions are algebraically identical (Hora et al. 1998; Hora 2000a) based on the optimum temperature of 17 keV. The break-even energy for DT is 6 MJ for adiabatic compression and expansion using the self-similarity model for the hydrodynamics (Dawson 1964; Hora 1964). The very high break-even energy and the increase of the gain the cube root of this energy were a very frustrating result though it indicated that, if the initial density n_o can be 1000 times that of the solid state, then the input energy E_o can be one million times lower for the same gain G.

To achieve this compression for better fusion physics, implosion physics (Nuckolls 2001) was studied, where the process of compression of thick layers of plasma to high densities as a recoil by the laser-driven ablation of a lower-density plasma corona was shown numerically (Mulser 1970) for plane geometry, and reached very high densities for spherical geometry (Nuckolls et al. 1972; Nuckolls 1974). For demonstrating this implosion physics, the 'exploding pusher' scheme was introduced (Nuckolls 1976) and its verification encouraged the following larger projects, though the compression was limited in the experimental cases. The scheme of spark ignition was developed (for references see Storm et al. 1988), where very high fusion gains are expected in order to overcome the rather low gains of simple fusion burn as given by (1.1). Since the thermal ablation of the plasma corona is of very low efficiency, between 5% and perhaps up to 15%, an alternative scheme was based on the direct electrodynamic conversion of the laser energy into kinetic energy of plasma motion (Hora 1976) using the nonlinear force (Hora 1969a; 2000b), where 45% or more of the laser energy is converted directly with little thermal loss into an imploding plasma shell, pushed at one interaction hit with reduced Rayleigh–Taylor instability problems.

After yields of 10^{13} DT fusion neutrons were detected (Yamanaka and Nakai 1986; Takabe et al. 1988) CD_2 targets containing some tritium were compressed to 1000 (up to 2000) times the solid-state density (Azechi et al. 1991), however with ion temperatures – concluded from the implosion velocities and from DT neutron gains – of 300 eV only. Compared with the preceding schemes of igniting the pellets with laser pulses of about a nanosecond duration, this result of high density but low temperature induced the idea (Campbell et al. 2000) of the fast ignitor (Tabak et al. 1994), where the heating of the low-temperature compressed pellet is augmented up by a 500-femtosecond laser pulse whose energy is deposited into the centre of the compressed pellet.

This paper discusses some problems (Key 2001) of this fast ignitor concept (Sec. 2) and examples of how variations of the interaction process may promise some solutions (Sec. 3). In view of the complexity of the fast ignitor process we discuss the possibility of a reconsideration of the ignition by the longer laser pulses. These longer-duration irradiation schemes need some comparison of spark ignition and volume ignition (Sec. 4). From the various discussions of why volume ignition can

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imply a strong re-absorption of the bremsstrahlung radiation, some earlier aspects are analysed of how the detailed numerical treatment arrives at relatively low-temperature ignition (Sec. 5). Based on these results, conditions are derived of how compression by larger laser pulses will lead to high densities such that the rather low temperature of the compression by nanosecond laser pulses is sufficient to ignite high-gain laser fusion in a single event, i.e. without the femtosecond additional laser that the fast ignitor needs to provide better heating (Sec. 6).

2. Problems of the fast ignitor

The fast ignitor scheme would be a most interesting solution for the stagnation-free, classical nanosecond laser pulse compression of a plasma sphere of a few thousand times solid-state density, and with the then expected low temperature of a few hundred eV would receive additional heating (Yamanaka 1983; Campbell et al. 2000) by deposition of the energy of a laser pulse of 500-fs duration into the compressed centre (Tabak et al. 1994). Producing the necessary funnel into the compressed highly superdense plasma by relativistic self-focusing (Hora 1975) covering all additional dynamics such as ponderomotive self-focusing (Hora 1969b) and double-layer effects (Hora et al. 1984; Eliezer and Hora 1989), it is clarified that the ps process is sufficiently short (Jones et al. 1982) to provide self-focusing and to avoid hole boring only (see Fig. 12.9 of Hora 2000a) without focusing. Compared to these computations with complete hydrodynamics and nonlinearities including absorption, similar results were produced by particle-in-cell (PIC) computations in a more simplified way (Pukhov and Meyer-ter-Vehn 1996) and it seems possible to expect that the second-step femtosecond laser pulse will progress to the centre of the compressed plasma. However, then the problems will appear, of how the energy is deposited (Hora et al. 1997a).

One problem can be seen clearly from the acceleration of electrons from the focus area, as was measured for a much lower laser power of 30 TW (Umstadter 1996; Umstadter et al. 1996) – compared with the 100 PW needed for the fast ignitor (Hora et al. 1997) – and analysed with quantitative agreement (Hora et al. 1997b) with the theory. The theory is based on the electron acceleration by lasers in vacuum (Hora 1988; Häuser et al. 1994), identical with the sometimes-called free-wave acceleration (Woodworth et al. 1996; Sprangle et al. 1996; Hartemann et al. 1998; Wang et al. 1998) as measured (Malka et al. 1997; Lefebvre et al. 1998). In a laser (1053-nm wavelength) plane-wave field of relativistic intensity, an electron is accelerated during a half wavelength, to an energy (Hora et al. 2000b)

$$E = (P/(1.03 \times 10^{-2}))^{1/2} \text{ eV} \quad \text{(laser power } P \text{ in W)}, \tag{2.1}$$

where a sideways motion occurs of a length

$$x = 4.74 \times 10^{-14}\ I^{1/2} \text{ cm} \quad \text{(laser intensity } I \text{ in W cm}^{-2}) \tag{2.2}$$

and the electron is taken along the laser wave direction during the half-wave by

$$z = 3.20 \times 10^{-23}\ I \text{ cm}. \tag{2.3}$$

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Nearly the same maximum energy gain is achieved if a one-wavelength laser pulse of Gaussian radial intensity profile with the optimum width of x for sufficient sideways motion hits a free electron. The interaction length in the electron system is given by z (Häuser et al. 1994). Taking into account the necessary exact longitudinal component of the laser field (Cicchitelli et al. 1990), a reduction of the maximum energy is – against simplified expectations – reduced, where from specific cases (Häuser et al. 1994) a reduction by about 37% may be applied also for other cases.

This free-electron laser acceleration can be applied to the experiments of Umstadter (1996) with a 30-TW neodymium-glass laser pulse of 400-fs duration irradiating a gas puff of atmospheric pressure, where about 10^8 electrons of 30-MeV energy were measured accelerated into the forward direction within a cone of half-angle $\alpha = 14^\circ$ aperture. It was calculated (Hora et al. 1997b) that relativistic self-focusing results in a volume in the gas containing 10^9 molecules. This number agrees roughly with the measured 10^8 electrons taking into account an intensity decay of a Gaussian beam. Based on the laser power of 30 TW, an electron energy of 50 MeV would result for the half-wavelength plane-wave case, to be reduced by 37% to 32 MeV, in rather good agreement with the measurement. For the conical angle taking $\tan(\alpha/2) = x/z$ a value of $\alpha = 21^\circ$ results, which is not too far from the experimental value (also compared with Kodama et al. 2001), since the detailed computation with Gaussian profiles, etc. will cause a correction to lower angles. It should be mentioned that there are speculations of how the experiment of Umstadter may be explained by other phenomena, e.g. by wake-field acceleration or Raman scattering; the here-mentioned numbers at least offer some reasonably convincing consistency.

In view of the just achieved model that practically all electrons (at single ionization) are taken away from the focus of the laser beam by the relativistic free-wave acceleration, the remaining ions will then be subject to a Coulomb explosion. We are aware that this is a double-layer process (Eliezer and Hora 1989; Hora 2000a; 2000b) but in such an extremely pathological case that all electrons are taken away from the (singly ionized) ions, the expression of Coulomb explosion is well justified. The electrostatic energy E_s of a charge Q within a sphere of radius r_o (all in SI units) producing a dielectric displacement $D = Q/(4\pi r^2)$ is

$$E_s = \int_{r_o}^{\infty} dr\, Q^2/(8\pi\varepsilon_o r^2) \tag{2.4}$$

or

$$E_s = 4.5 \times 10^9\, Q^2/r_o \quad (Q \text{ in Coulomb}, r_o \text{ in metre}). \tag{2.5}$$

For the 10^9 charges according to the Umstadter experiment, and the approximated radius for the volume in the focus (Hora et al. 1997b), the average ion energy is close to MeV. This process illustrated what will happen by the irradiation of the femtosecond laser pulse for heating the centre of the compressed fast ignitor: the energy will go into very energetic electrons and ions which have a long mean free path even in the highly compressed plasma, such that the goal to produce a localized spark for ignition of a fusion detonation wave is not easy to achieve. In the case with 100-PW laser pulses the energies will exceed the range of several 100 MeV compared to the case of Umstadter with 30 TW. The generation of the energetic jet of directed electrons and similarly directed ions has been observed. The electron jet, e.g. injected into a nearby lead layer, produces a large number of positrons by

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a trident process (Gahn et al. 2000), while PW laser pulses produce proton jets (Cowan et al. 1999; Roth et al. 2000) and gammas with energies up to 100 MeV, leading to nuclear photoeffects producing large numbers of new isotopes and very intense nuclear radiation (Ledingham et al. 2000; Leemans et al. 2002; Schwoerer et al. 2001). Since it may be very difficult to use spark ignition with the fast ignitor, the possibility was discussed (Hora et al. 1997a) of whether the range of the ion energy deposition may be adjusted to produce volume ignition. One alternative may be the peripheral fast ignition (Mulser and Heim 2001), where the directivity of the ion generation (Roth et al. 2001) is used in a similar way to Kodama et al. (2001) and where again the question arises of whether a volume ignition scheme may be more useful than spark ignition.

3. Interaction processes of initiating the fast ignitor

The just described difficulties for the fast ignitor (Key 2001) appear understandable in view of the fact that the laser–plasma interaction always revealed very unexpected new physical phenomena (Hora 2000a; 2000b) when opening another scale of parameters, as in the case of the petawatt pulses it includes a new dimension in nuclear and particle physics research (Cowan et al. 1999; Roth et al. 2000; Leemans et al. 2002; Ledingham et al. 2000; Gahn et al. 2000; Hora 2001; Hora et al. 2002a). We analyse here an example of the interaction for initiating the relativistic self-focusing for the funnel generation in view of more classical measurements of energetic ion generation.

When high-intensity laser pulses of longer than a few ps irradiate solid targets, it was observed from the early experiments (Linlor 1963, see also Schittenhelm et al. 1996) that, instead of the expected classical-hydrodynamic thermal ions of a few eV energy, such with up to more than 10 keV were observed, whose energy was separated in linear dependence on the ion charge number Z; therefore they are not of thermal origin. The threshold for this Linlor effect was measured to be at $P = 10^5$ to 10^6 W laser power only, as shown later (Hora 1969b) to coincide with the ponderomotive self-focusing threshold of the laser beams leading to the keV energies by nonlinear force acceleration (Hora 1969a) and Z-separation (Hora 2000a). Above the threshold of relativistic self-focusing (about 1000 times below the relativistic threshold of $I_r = 3.7 \times 10^{18}/\lambda^2$, where λ is the laser wavelength in micrometres, used only at this point of the paper for conventional scaling), the relativistic self-focusing (Hora 1975; Jones et al. 1982) can cause a shrinking of the laser beam to a diameter $\delta = 0.6$ times the wavelength. The subsequent very high laser intensities then accelerate the electrons and the double-layer attached ions to very high Z-separated energies, e.g. 0.5 GeV of 52 times ionized gold (see Fig. 2 of Begay et al. in Haseroth et al. 1996). This maximum energy fits (Hora 2000a) the relation for the fast ion energy

$$\varepsilon_{\text{trans}} = Zmc^2 Pe^2/(\pi^2 \delta^2 m^2 c^5) = Zmc^2(P/\delta^2)\ 5.7 \times 10^{-19}, \tag{3.1}$$

where m is the mass and e the charge of the electron and c the vacuum speed of light in cgs units.

It was first reported (Ehler 1975) that, apart from the fast ion group following (3.1) and apart from a slow group of thermal electrons, there is a third intermediary group of ions in between. It was found from a large number of measurements of various laboratories that the energy of this intermediary group follows a relation

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$e_{IM} = I^{1/2}$ (Gitomer 1986). It was derived that this group is of real 'hot' thermal origin, since the energy is equal to the high-energy quiver motion of the electrons in the laser field and the transfer of this quiver energy to thermalization of ions follows exactly the quantum-corrected collision process (Haseroth et al. 1996, p. 417). This process is different to the fast-electron generation by resonance absorption for oblique-incidence p-polarized lasers (Meyerhofer et al. 1993), in agreement with (Gibbon et al. 1992), though it has been known for a long time that the classical absorption may cause a drastic change of the resonance absorption (see Chap. 11.2 of Ref. 2000a).

The just mentioned good agreement of the fast ion group and of the 'hot' X-ray emission from the quivering electrons was seen in all experiments until 1990 when ps laser pulses resulted in very different observations. There was an essential difference whether any prepulse was suppressed (Zhang et al. 1998; Wolowski et al. 2002) or when a prepulse was used. With a prepulse, the usual fast ions were produced, but only very low ion energies and low X-ray emission without the prepulse. This can be directly understood from the detailed measurements of the optimum delay time τ_d between the prepulse and the main pulse (Zhang et al. 1998).

In this experiment, 5-mJ 180-fs laser pulses of 800-nm wavelength obliquely irradiated copper targets at a repetition rate of 10 Hz at a focus diameter $d_F = 0.0185$ mm corresponding to 23 wavelengths. Since the relativistic threshold intensity is 5.2×10^{18} W cm^{-2}, the focus intensity of 10^{16} W cm^{-2} relativistically shrinks to an intensity of 5.29×10^{18} W cm^{-2} if there is plasma before the target to permit the relativistic self-focusing. The maximum oscillation energy of the electrons is then $\varepsilon_{osc} = 520$ keV, while the 'hot' X-ray temperature was measured as 400 keV, which is compared with the 500 keV of the maximum oscillation energy. Averaging the oscillations in the ranges outside the Gaussian intensity profile arrives at energies near 100 keV. The expected maximum energy of the emitted protons ($Z = 1$) against the laser light

$$\varepsilon_{trans} = Z\varepsilon_{osc} \tag{3.2}$$

is then again 520 keV, which is close to the measured value of 500 keV. This measurement needed a prepulse with $\tau_p = 70$ ps. Assuming an expansion velocity of the standard value of 3×10^7 cm s^{-1}, the plasma plume just arrives at a depth of the size of the focus such that the near-critical-density plasma will provide a relativistic self-focusing within this depth (Hora 1975; 2000a) for generating the just mentioned quiver energy near 500 keV and the proton energy of 500 keV.

If there is no prepulse, no relativistic self-focusing is possible since the 10^{16} W cm^{-2} laser intensity is simply converted into oscillation energy of electrons only within the skin depth, which may be increased by the resonance absorption effects due to the oblique incidence (Meyerhofer et al. 1993). Very similar observations were reported (Hora et al. 2002b) where the laser pulses of 500-ps duration produced the expected high ion energies given by (3.1) while the same laser powers for 1.2-ps laser pulses with a carefully suppressed prepulse did not produce these energetic ions. In the petawatt experiments (Cowan et al. 1999; Roth et al. 2000), the complexity due to suppression of the prepulse was avoided by producing the very unexpected results without much differentiation with respect to the prepulses.

This example indicates that the ps laser–plasma interaction process is very complex, as it was even noted for rather low laser intensities by photoemission measurements (Malvezzi et al. 1985; Bloembergen, see Boreham et al. 1995). All these results

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may underline how much more research work has to be done to clarify all these interactions for the scheme of the fast ignitor.

4. Spark ignition and volume ignition

Being aware of the rather unexplored conditions of a number of unexpected known and yet unknown new phenomena for 100-PW laser–plasma interaction for the fast ignitor, it may be justified whether the earlier studied nanosecond laser pulses for inertal confinement fusion (ICF) may offer a way out for the solution of the inertial fusion energy production in case that the fast ignitor is too complex.

In the past, nanosecond laser fusion schemes were oriented to compression densities not far beyond 1000 times that of the solid state and for laser pulse energies not far beyond one megajoule, though conceptual discussions of the spark ignition included 10-MJ laser pulses (Storm et al. 1988) in comparison with equivalent but necessarily differing conditions of volume ignition (Hora et al. 1998). Our main attention is then the range for densities at or above 5000 times that of the solid state and laser pulse energies of up to 10 MJ and more, where we use guiding by the volume ignition scheme as a basis for the *fusion physics*.

The problems of the implosion physics as indicated by Nuckolls (2001) are well interlinked and have achieved various solutions and numerous modifications during the last 30 years of intensive research. We separate here the problems of the most complicated interaction physics between lasers and plasmas (Hora 2000b), where the nonlinear force and double-layer effects are not only causing parametric instabilities and hydrodynamic complications by Rayleigh–Taylor instabilities, plasma mixing, etc., and – perhaps as the most dominating evil – 10-ps stochastic pulsation (Hora and Aydin 1992; 1999), which may now be under control by appropriate laser-beam smoothing. To overcome these difficulties before understanding and controlling them, one way out was to use the indirect drive by embedding the fusion pellet into a high-Z capsule where laser light in the space in between is converted into Hohlraum X-radiation, which itself then smoothly compresses the pellet. The following results are indeed valid also for this indirect drive as well as for the more efficient direct drive, which may now be under control even for the more energetic longer laser wavelengths (Hora 2000a).

Implosion physics driven by laser ablation was studied numerically for spherical geometry since 1960 by Nuckolls (1974). There are several properties to be considered, whether the electrons are degenerate or not, whether the pellet is transparent (or particularly transparent for bremsstrahlung, e.g. following a Kramers spectrum), or whether the plasma is opaque with black-body radiation. What equation of state is dominating is another important question (Eliezer et al. 2002). An example is how to compress DT to 10 000 times the solid-state density n_s. Based on the Thomas–Fermi model with a solid-state density of 0.25 g cm^{-3}, a temperature of keV is reached if 3 keV per atom is transferred to the fuel at unchanged density. If, however, an additional 200 eV per atom is incorporated, corresponding to about 7% of additional energy at keV temperature, compression by a factor 20 is achieved (Eliezer and Hora 1993). The problems of keeping the initial temperature of a plasma shell at a reasonably low temperature in order to avoid too high temperatures at the desired compression was discussed also if imploding shells are driven by the high-efficiency nonlinear force ablation (Hora 1976).

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In order to regulate the implosion of shells by usual thermal ablation–compression, the selection of velocities around 4.5×10^7 cm s^{-1} is important. Using high-Z coatings (pusher) of the DT shell for the ablation, there is the possibility of sufficiently low velocity for the collapsing shell as used e.g. by Kidder, Colgate, Kirkpatrick and others for achieving volume ignition (Nuckolls 2001). Alternatively, the scheme of Nuckolls (1974) with pulse shaping can use dense DT instead of the high-Z pusher for a nearly isentropic compression; however, resulting in spark ignition. The problem of the collapsing DT consists in the ablation of the inner cold shell due to hot electrons and reaction products (3.6-MeV alphas). Nevertheless, the stagnation-free collapse and compression of a DT shell was shown to be possible in numerical studies (Velarde et al. 1986), was demonstrated experimentally for the case of highest possible fusion gains (Yamanaka and Nakai 1986; Takabe et al. 1988) with careful numerical reproduction of the extensive facts from the experimental diagnostics and was seen numerically by the appearance of nearly self-similarity ideal adiabatic compression for a high-gain heavy ion beam driven volume ignition pellet (Martinez-Val et al. 1994, see Fig. 1b in Eliezer et al. 1994). For laser pulses up to 35 kJ, the highest fusion gains were achieved (without ignition due to insufficient laser energy), where in all cases of different laboratories a perfect fit with a stagnation-free ideal self-similarity compression was confirmed (Hora et al. 1998).

All these cases confirm that both are possible: a highly uniform density compression along the self-similarity adiabatic dynamics as initial conditions for volume ignition of the whole pellet, but also the automatic generation of a hot spark in the low-density high-temperature centre of the compressed pellet, which reacts in the way of volume ignition (Nuckolls 2001) but may then produce at its surface a self-sustained fusion detonation wave in the outer low-temperature and high-density fuel for producing a high fusion gain. The complex conditions of spherical symmetry for the fusion wave, and the selection of not too high or too low temperatures in the wave front are well known, but it may be possible to control most of these parameters (Lindl 1995). The volume ignition gains of 40% to 80% lower than for spark ignition (Lackner et al. 1995) and higher gains in other cases (Hora and Ray 1978; Kirkpatrick and Wheeler 1981; Hora et al. 1991; Eliezer and Hora 1993; He and Li 1994; Martinez-Val et al. 1994; Tahir and Hoffmann 1994; Atzeni 1995, Hora et al. 1998) have the advantage of being much more robust against instabilities and asymmetries. Even fluctuations leading to stochastic initiating of the ignition within the compressed volume are compensated by radiation effects (Lackner et al. 1995). The following steps of our discussion are not so much the compression physics or the differences between volume and spark ignition, though it was shown that in detail there are clear differences (Hora et al. 1998). Otherwise, it has been shown that the volume ignition bridges conditions of spark ignition (Basko 1990; Murakami 1997; Basko and Murakami 1998; Johzaki et al. 1998) and – as mentioned (Nuckolls 2001) – the ignition of the spark is a volume ignition. The last case, however, is selecting very narrow ranges of parameters only, since the temperature for the fusion detonation wave has to be very carefully scaled as not too low and not too high. This restriction is not necessary in the following for volume ignition at large.

5. Radiation re-absorption for low-temperature volume ignition

Fusion reactions in a stationary low-density plasma with continuous emission of the bremsstrahlung need a minimum temperature of about 4.5 keV for DT to

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compensate the radiation loss by fusion-energy generation. This is no problem at the optimum reaction temperature of 17 keV for the (non-stationary) expanding and adiabatic cooling of a uniform reaction sphere (Hora 1964), from where (1.1) was derived (Hora and Pfirsch 1970; Kidder 1974). There were some attempts to achieve higher gains by lowering the optimum temperature, e.g. by preventing the emission of the bremsstrahlung using a high-Z outer shell (damper) around the pellet (Kidder 1974) or to estimate globally without any details during the adiabatic expansion how the radiation can be regained by the plasma (Caruso 1974).

A first detailed treatment of this low-temperature inertial confinement fusion was automatically possible when the main omissions of the simple burn of (1.1) were overcome by including:

- the fuel depletion;
- the alpha reheat;
- the partial re-absorption of the bremsstrahlung in the detailed computation.

For the reheat the collective model was used (Ray et al. 1976), which is basically the same as that of Gabor (1953) but where Max Born's unpublished derivation of the Fokker–Planck collision term for the Boltzmann equation was used. For the adiabatic expansion of the plasma sphere, the self-similarity model was used as before (Basov and Krokhin 1964; Dawson 1964; Hora 1964) and it was mathematically justified (Hora 1971) for using a box-like density instead of a Gaussian profile where the temperature is uniform and the expansion velocity depends linearly on the radius with temporal change of this linear profile and with adiabatic change of the temperatures as confirmed also from a Boltzmann equation (Hora and Pfirsch 1973). For each time step of the adiabatic expansion (Hora 1964; Stening et al. 1992), for each of the then present plasma temperature, density and radius the volume-integrated mean free path of the alphas was calculated and the percentage of the deposited energy determined, which for the next time step resulted in a higher temperature than that of the adiabatic cooling from the hydrodynamics of the expansion.

For the bremsstrahlung it was assumed that the conditions of transparency are fulfilled as checked before (Hora and Lochte-Holtgreven 1975) such that the Kramers spectrum for each temperature in each time step was used to calculate the spectral maximum. For this frequency the collisional absorption in the plasma given by the temperature and density of each of the steps was calculated and for the then present radius the integrated absorption length determined the percentage of the re-absorbed bremsstrahlung during this time step, causing a further increase of the temperature. These were the main ingredients in the computations (Hora and Ray 1978), which led to the discovery of volume ignition characterized by the fact that the time dependence of the temperature can increase over the initial temperature of the compressed DT sphere. The optics of the bremsstrahlung was slightly corrected (Goldsworthy 1983) but did not lead to essential changes of the results (Stening et al. 1992).

These corrections showed that the fusion gains (core gains related to the energy E_o which was deposited uniformly in the whole sphere for the subsequent adiabatic expansion) were rather similar to that of (1.1) if the gain G was below 8 for DT, Fig. 1, where the dashed parabolas (standard isochores) resulted for the varying initial temperatures with the optimum 17 keV for the highest gains as tangents to the straight lines for different compression in multiples of the solid-state density n_o.

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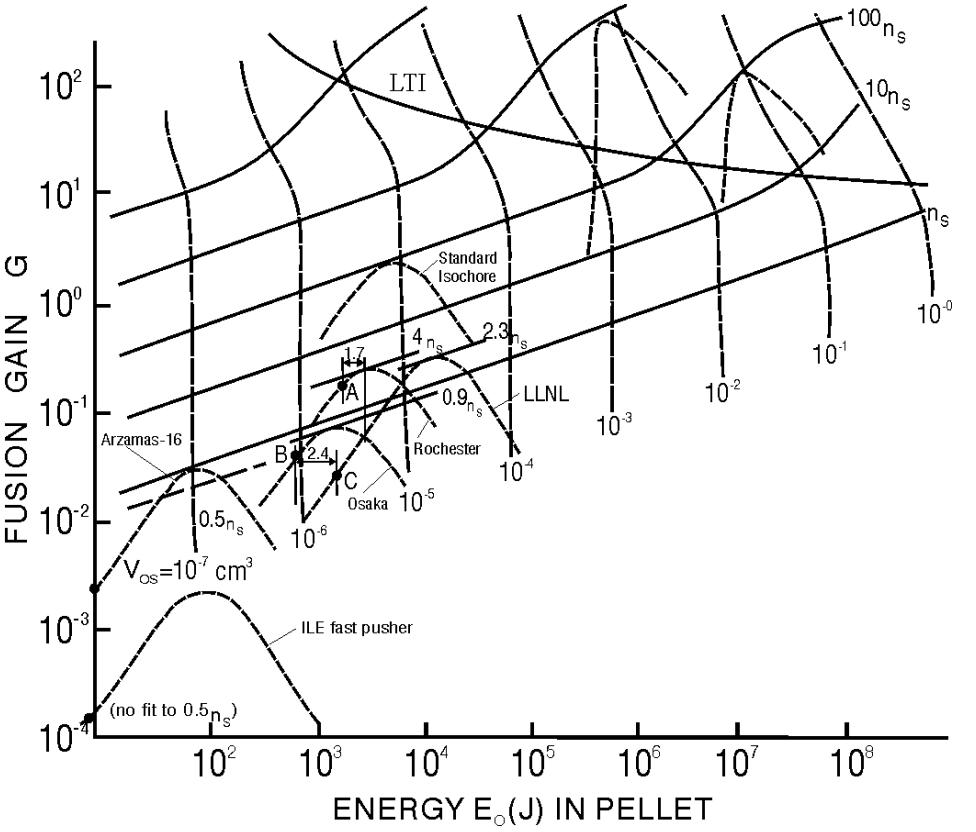


Figure 1. Optimized core fusion gains G (full lines) for the three-dimensional self-similarity hydrodynamic volume compression of simple burn ($G < 8$) (sometimes called quenching: Atzeni 1995) and volume ignition for $G > 8$ with low-temperature ignition above the LTE line. The measurements of Rochester (Soures et al. 1996, point A), Osaka (Takabe et al. 1988, point B), Livermore (Storm 1986, point C) and Arzamas-16 (Kochemasov 1996, point D) agree with the isentropic volume burn model, while the earlier fast pusher (Kitagawa 1984, point E) with strong entropy-producing shocks does not fit (Hora et al. 1998).

Above the gain 8, the volume ignition happens, as seen from the highly deformed dashed parabolas for these higher gains. The jump-up to their maximum shows how quickly the reaction from no ignition at lower initial temperature changes to ignition at very little increase of the initial temperature. The ignition leads to plasma temperatures above 100 keV and the fusion reaction profits then mostly from the alpha reheat as a kind of very much extra driver energy added to that of the laser. If the initial temperature is increased again slightly, the adiabatic expansion is so fast that the fusion gain drops remarkably. Figure 1 also shows the ideal agreement of the measured highest fusion gains with the self-similarity conditions of volume burn (Hora et al. 1998).

The vertical dashed lines are parallel for the core gain below 8, expressing the then constant optimum temperature of 17 keV. Any deviation to the left shows a decrease of the optimum temperature at volume ignition. The line LTI combines points of the optimum temperature 4.5 keV above which 'low-temperature ignition' is occurring, where indeed – suggested earlier (Kidder 1974; Caruso 1974) – the

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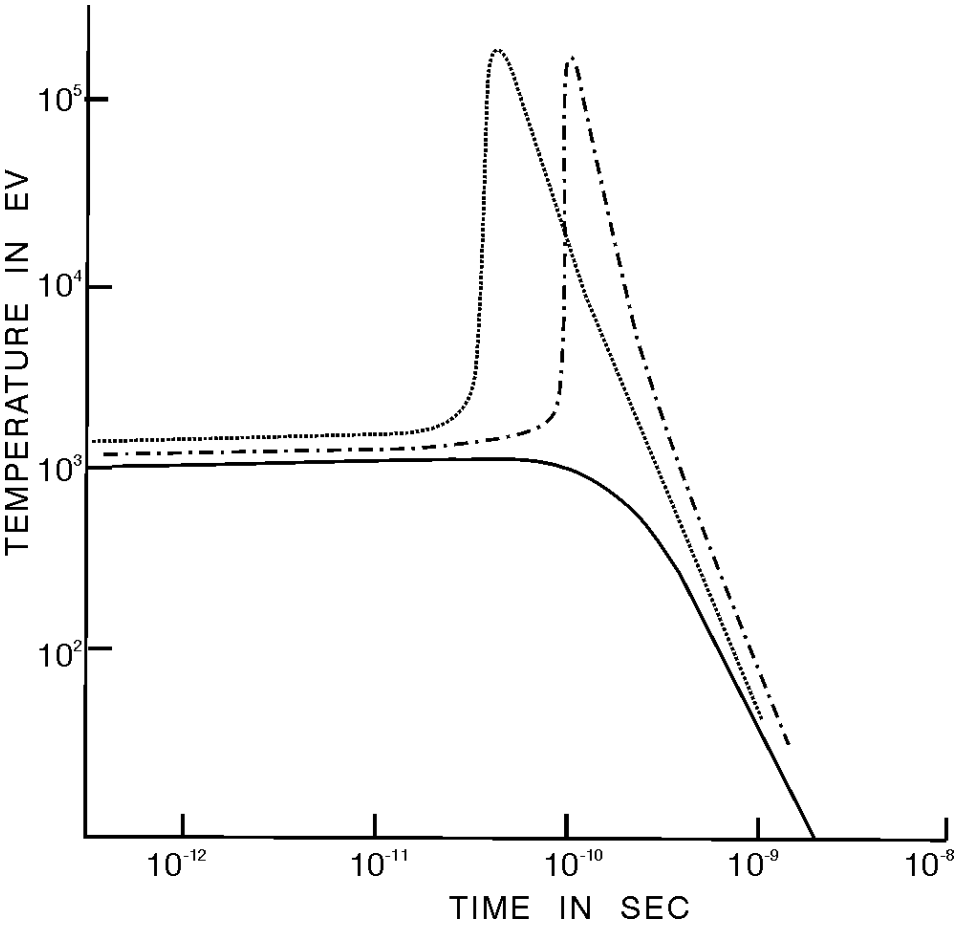


Figure 2. Time dependence of the plasma temperature T calculated (Hora et al. 1991; Eliezer and Hora 1993) for adiabatic pellet volume ignition of DT with compression to 1000 times the solid-state density n_s. At a 40-MJ input energy burn only, no ignition, occurs with a gain of 2.41 (straight line). At 41 MJ the gain is 1199 with 62.4% burnt fuel (dashed line) and at 42 MJ the gain is 1170 with 58.5% burn (remaining line).

re-absorption of the bremsstrahlung is essential. Obviously, the re-absorption is stronger for larger radius. An example is shown in Fig. 2, where the time dependence of the initial temperature can be seen drastically though the density is rather low with $1000n_s$, needing then very high energies E_o in the 40-MJ range. At an initial temperature of 1.44 keV, no ignition can be seen and the gain is low. At 1.47-keV initial temperature, the volume ignition drives the temperature up to 100 keV (dashed line) with nearly 500 (!) times higher gain, while 1.57-keV initial temperature has a faster rising temperature and faster expansion where the gain is then lower.

The rapid increase of the reaction temperature, mostly due to the extra energy input from the reheat, was the proof for the volume ignition as documented in Fig. 2 of Hora and Ray (1978). The advantage of using the self-similarity model for the hydrodynamics is the enormous reduction of the computation time, such that a large number of cases with varying parameters could lead to the general results

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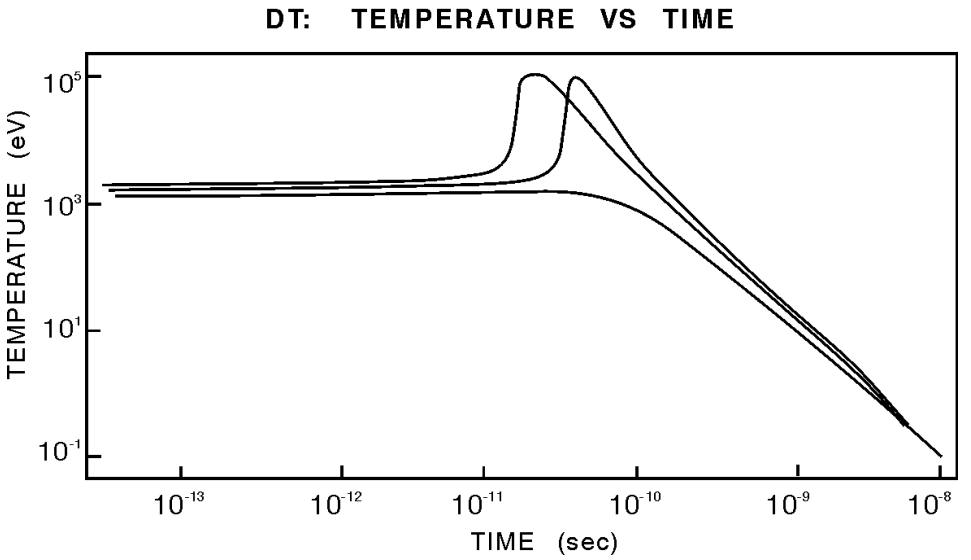


Figure 3. Same as Fig. 2 for an initial density of $5000n_s$ where the lowest curve without ignition had an energy input E_o of 200 kJ only while the initial temperature due to the strong re-absorption is 1.51 keV. Ignition occurs at the next higher energy input of 250 kJ and the highest curve is for an energy input of 300 kJ with an initial temperature of 2.25 keV.

as shown in Fig. 1. Another example of strong re-absorption of the bremsstrahlung at high density and more modest input energy E_o is shown in Fig. 3 with inclusion of recent results on fusion-cross-section theory (Clark et al. 1978; Li et al. 2000a; 2000b). After these rather crude cases were performed, the parameters for interesting selected single cases with very detailed computations can be done. A special example is that of Martinez-Val et al. (1994) (see Fig. 1b of Eliezer and Hora 1993), where the detailed ablation dynamics by 2-MJ bismuth ions of 5.25 GeV-energy with ablation in a lithium pusher layer was applied to drive a DT shell below. At the end of the heavy-ion irradiation, the whole lithium layer had gone and the stagnation-free compression led to a nearly uniform density over the whole pellet and to a uniform temperature very close to the self-similarity model for volume ignition. The very sophisticated code permitted the use of thermal non-equilibrium such that the time dependence of the ion temperature reaching more than 200 keV could be seen, different to that of the electrons (being below 100 keV) and producing the black-body radiation temperature with a maximum of less than 8 keV. This all led to the very favourable yield of 140-MJ fusion energy, presenting a total gain of 70.

6. Single-event solution

Returning to the initial question of whether a single-event laser fusion with pulses in the nanosecond range may lead to a solution of inertial fusion energy instead of the two-step fast ignitor, the low-temperature ignition is important in the range below keV. In order to look to higher densities, the question of the Fermi energy has to be taken into account. The Fermi energy is given by

$$E_F = (h^2/2m)(3n_e/8\pi)^{2/3} \tag{6.1}$$

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and the limit where the Fermi energy is less than the plasma temperature is shown in Fig. 4 as the Fermi limit.

Since a laser compression to $1000n_s$ was measured (Azechi et al. 1991) in CD_2, assuming full ionization the Fermi energy is 1.12 keV, corresponding to a pressure of about one tera-atmosphere. Nevertheless, the ion temperature was only about 300 eV or less. For lower ionization of the carbon the values of the pressure are at least of the same order of magnitude. It is very remarkable that this compression by lasers – or even to $2000n_s$ – was experimentally verified, very probably stagnation-free, such that these conditions could be considered later (Campbell et al. 2000) for the scheme of the fast ignitor for increasing the temperature in the compressed centre with sufficient energy deposition from the sub-picosecond laser pulse.

The computation results in Fig. 1 (Hora et al. 1998) were all performed for non-degenerate plasma, therefore the Fermi energy of the electrons had to be less than the temperature of the plasma. Since the compression and the expansion of the plasma occur adiabatically, the once determined property of non-degeneracy is preserved over the whole compression and expansion process. Figure 4 shows the results of the core gains of DT fusion for the here interesting range of high input energies E_o and high densities. The crosshatched area is that of degenerate electrons. The computations Fig. 4 were done as if the electrons are non-degenerate; therefore certain corrections of the results are there necessary in the crosshatched area. For simplification, we may use here for a first comparison the results of Fig. 4 and are aware that some not essentially different values are valid for the completely degenerate calculation.

We now compare cases similar to the compression measurements where the ion temperatures were below 300 eV and the electron Fermi energy is above keV. Taking under the same conditions the case of a laser pulse energy of 10 MJ with a 10% hydrodynamic efficiency for the compression for the 50:50 DT fuel, the optimum temperature for volume ignition is 560 eV, arriving at a core gain of $G = 1300$. This corresponds to a total fusion gain related to the incident laser energy of $G_{tot} = 130$. Slightly higher total gains of 140 are achieved with 20-MJ laser pulse energy, where the optimum temperature decreases to 425 eV for the same compression to 10 000 times the solid-state density.

The just mentioned result of very low ignition temperature, in continuation of Johzaki et al. (1998) – mostly due to the self-absorption of the bremsstrahlung but highly favoured by the very high extra self-generated driver energy from the reheat – may well be an orientation for the conditions of how the rather 'robust' (Lackner et al. 1995) volume ignition may work within the here mentioned extrapolated parameters. Since the measurements of high compression and low temperature (Azechi et al. 1991) were performed with laser pulses in the 10-kJ range, the experiments with 10 MJ pulses may well differ and may arrive at higher temperatures than that required for the optimum ignition temperature concluded from the computations of Fig. 4. Any higher temperature will then relax the requirements of the high compression and the high laser pulse energy, and not the contrary. The more optimistic conditions can be expected due to the here not included reheating by neutrons and if the thermal non-equilibrium decoupling of the electrons, ions and the black-body radiation are included, as e.g. in the specific computations by Martinez-Val et al. (1994).

We conclude that, from the various options of laser fusion with the experimentally confirmed goal of reasonably low driver energies known from the driving with

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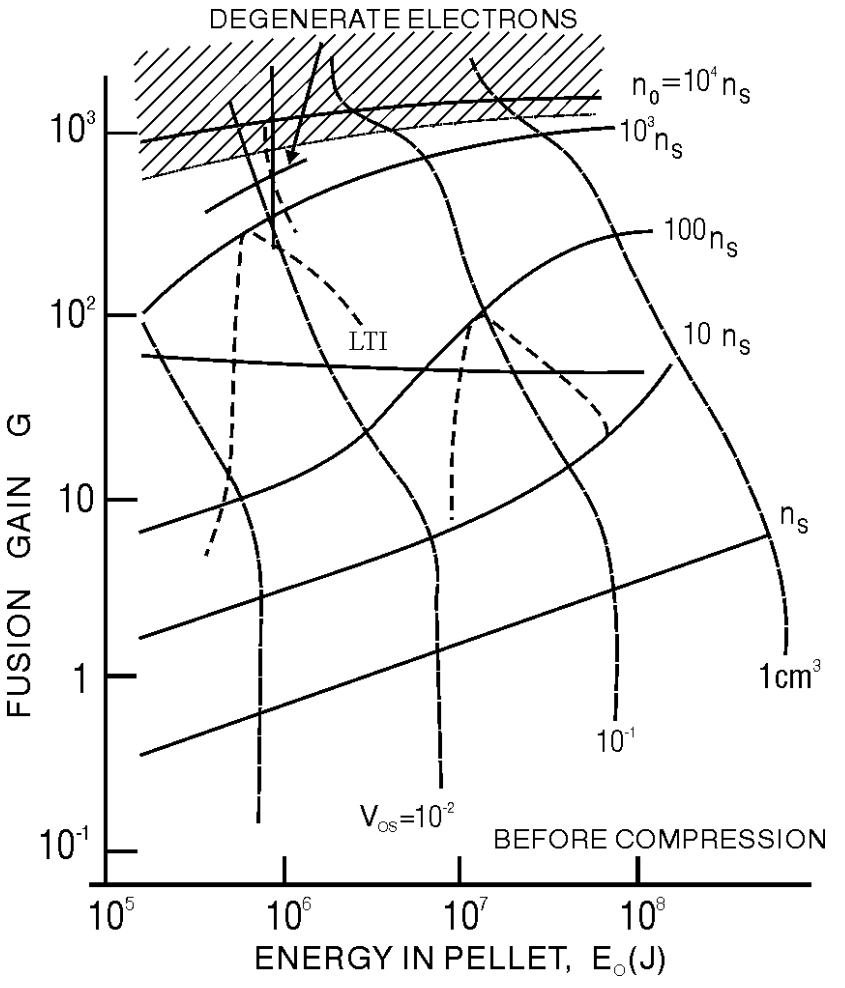


Figure 4. Optimized core fusion gains G as in Fig. 1 with the crosshatched area for Fermi degenerate electrons for evaluation of optimized volume ignition at temperatures in the 500-eV range.

intense X-rays of underground nuclear explosions (Broad 1988), there is a realistic possibility for the use of a one-step robust volume ignition with conventional nanosecond laser pulses in the 5- to 10-MJ range. These laser pulses are available with present-day technology if red light is used by saving the expensive higher-harmonic generation if the difficulties of direct drive (which have not been understood for many years) are overcome by appropriate laser-beam smoothing (Hora and Aydin 1992; 1999; Hora 2000b) for suppression of the stochastic 10-ps pulsation. The problems with the optical properties due to unavoidable defects in the crystals for harmonic generation are well known – see Bodner in Dawson (2001). A step back to the second harmonics will not help much since the generation of wide-band laser radiation for the smoothing process needs to work with the fundamental frequency only (Peng 2001). One should at least consider the single-event option for a first basis of a power station while other more elegant options with hopefully higher

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gains by spark ignition, indirect drive, or – despite their unsolved problems (Key 2001) – fast ignitors (Kodama et al. 2001), may lead to further improvements.

One example of how a single-event nanosecond compression to 6000 times the solid-state density of DT with a small amount of tritium can offer a solution is given by Miley and Hora (2001). Though the ρR value is higher, the results are comparable to that using the fast ignitor as shown by Latkowski et al. (2001).

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