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HETEROGENEOUS EXPECTATIONS AND ENDOGENOUS FLUCTUATIONS IN THE FINANCIAL ACCELERATOR FRAMEWORK

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This paper proposes a financial accelerator framework to study the effects of heterogeneous and bounded rational expectations on macroeconomic dynamics. The paper examines the fluctuations effects departing from the rational expectations hypothesis in order to understand if there are significant implications on macroeconomic volatility and policy prescriptions. The findings suggest that macroeconomic stability and inflation dynamics depend on the chosen set of forecasting rules, as well as on the monetary policy adopted. The model shows that no monetary policy is able to quickly stabilize the system, as some fluctuations persist. Central banks face a trade-off between macro-volatility and speed of convergence to the steady state. This result offers some ground for fiscal policies aiming to prompt system stability. In addition, the analysis reveals a counterintuitive result confirming the "less-is-more" effect: increasing the decision-making and computational abilities of the agents may not lead the system to converge to the preferable steady state.

Keywords: Heterogeneous Expectations, Financial Accelerator, Monetary Policy, Business Fluctuations

1. INTRODUCTION

Since the eighties the macroeconomic investigation of monetary policy has been explored by the New Keynesian Dynamic Stochastic General Equilibrium approach (NK-DSGE), for example, see Mankiw (1989). This literature investigates financial frictions[1] building a dynamic general equilibrium model with imperfections in the credit market and introducing information asymmetry, collateral constraints, costly state verifications and agency cost.[2] Using the representative agent assumption and the rational expectation hypothesis (REH), NK-DSGE models clarify the role of frictions in amplifying and propagating shocks to the macroeconomy, operating like a "financial accelerator."

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However, in the thirties, Keynes (1936) had already argued that economic fluctuations are not determined only by fundamentals: the investor's animal spirits and the market psychology (e.g., euphoria) influence financial market performance. Since the seminal work of Woodford (1990), there have been several papers that recognize the importance played by "animal spirits" in the origin and propagation of fluctuations in the economy. Consequently, during the last years behavioral economics has been developing concrete alternatives to the standard rational representative approach using bounded rationality and heterogeneous expectations, e.g., showing how heterogeneous beliefs may lead to market instability or to strange attractors using rational expectation with other types of beliefs based on past performance.[3] Moreover, a growing literature[4] has investigated the drawbacks of the rational expectations and the representative agents assumption. First of all, it is not realistic to assume perfect and wide-spread knowledge about the law of motion of the interest variables. According to the REH, agents should have extremely strong computing abilities in order to calculate the equilibrium of the model. The latter aspect becomes crucial if applied to heterogeneous agents models where rational agents are expected to have a full knowledge of the market, including the expectations of all the other agents. Conlisk (1996) also highlights the infinite regress problem: "How can we formulate an optimization problem which takes full account of the cost of its own solution?" According to Johansen (1977), the answer is as follows: "at some point a decision must be taken on intuitive grounds." In other words, deliberation costs and heuristics challenge the rational optimization as the ultimate logical basis for behavioral modeling.[5]

The financial friction framework and the heterogeneous expectations are often exploited separately to investigate the business-cycle dynamics. The goal of the model presented in this paper is to merge these two approaches in order to analyze how they jointly affect macroeconomic volatility and monetary policy effectiveness.

Besides these theoretical issues, the model presented in this paper finds its origin in the recent economic crisis that has made clearer at least two phenomena. First, borrowers face some constraints in access to credit, i.e., since they are at risk of default, there are limitations in the bank credit policy with a credit spread. This spread changes according to the net worth (or collateral) of the borrower. This spread can be modeled through the financial accelerator, i.e., the premium in the loan interest rate over the risk free interest rate. Second, both professional financial intermediaries and borrowers make mistakes in their forecast and investment decisions; hence, agents are bounded rational.[6]

According to our point of view, it becomes crucial to develop a New Keynesian framework considering the financial accelerator jointly with heterogeneous and bounded agents to better explain the evolution of the business cycle. Indeed, the financial accelerator propagates and amplifies the shocks in the system, as it happened during the last crisis. However, the macroeconomic stability also depends on the sentiment of the market, i.e., the set

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of agent's forecasting rules, and on the endogenous fluctuations produced by the agents reacting to the shocks and to the different monetary policy targets.[7]

Therefore, on the one side, this analysis fits into the growing agent-based literature, which develops complex and evolving credit networks characterized by credit relationships connecting financial soundness of the borrowers, external finance premium, and endogenous macroeconomics dynamics.[8] In this literature, the macroeconomic outcome does not coincide with the representative agent behavior but emerges from the interactions among heterogeneous agents. For example, in this model, the financial intermediaries play a crucial role in the emerging system dynamics defining the quantities and the prices of the loans extended to the heterogeneous entrepreneurs.[9] On the other side, this analysis follows the same research branch of De Grauwe (2012) and Anufriev et al. (2013) that has been investigating the effects of heterogeneous expectations on future inflation and output through the investment channel presenting a behavioral financial accelerator model where entrepreneurs update their forecasting rule switching to the best performing heuristic.

As in De Grauwe and Macchiarelli (2015), the model allows endogenous fluctuations and wave of optimism and pessimism. In line with Massaro (2013), the first policy conclusion we draw from our simulations is that the macroeconomic stability is affected simultaneously by the heuristics and by the monetary policy taken into account. Moreover, the economy is characterized by a dynamic feedback system where variables co-evolve over time and we find a trade-off between macroeconomic volatility and the speed of convergence. In the standard financial accelerator framework using rational expectations [see Bernanke et al. (1999)], the monetary authority is always able to stabilize external shocks in a very short period, usually a few quarters. Instead, in this analysis, the departures from REH propagate the fluctuations for several periods. Hence, the effectiveness of the monetary policy interestingly depends on the following: heterogeneity, bounded rationality, and heuristic updating mechanism. For example, we find that in most of the simulations, strict inflation targeting policy yields deeper fluctuations in the first periods, but, when the timing is considered, it is able to stabilize the economy in fewer quarters. However, in line with Anufriev et al. (2013), we also find that a strong monetary policy is able to send signals that help agents to correct their forecast reducing their mistakes in investment decisions. In addiction, this analysis finds a counterintuitive result confirming the "less-is-more" effect of Gigerenzer and Brighton (2009). Indeed, it is not certain that when the decision-making and computational abilities of the agents increase the system converge to the preferable steady state.

The paper is structured as follows: Section 2 presents the model with the heterogeneous expectations and the updating mechanism. Section 3 proposes some simulations results with the aim to investigate some monetary policy prescriptions, whereas the last part (Section 4) collects the relevant concluding remarks.

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2. THE MODEL

The baseline framework of the model is a simplified version of the financial accelerator introduced by Bernanke et al. model (1999) with a new behavioral assumption on the expectations formation mechanism. The basic structure of the model considers five types of agents: pessimistic and optimistic entrepreneurs, households, retailers, capital producers, and the public sector (fiscal and monetary authorities). The households' behavior is quite standard: they live forever and have to take decisions on labor supply, consumption, savings, and investment. Indeed, they can choose to hold real money or risk free assets with the aim to maximize their utility.

The two types of entrepreneurs are the core of the model. Conversely to the other agents, they have a finite horizon: each time, a portion γ of them survives to the next period and continues to produce. This assumption is meant to avoid the possibility of full self-financing by entrepreneurs.

In every period, each entrepreneur has a net worth composed by profit and wage of the previous period. With this worth the entrepreneurs purchase physical capital financing the difference through loans from the bank. There are two main variables that drive the borrowing decision that are heterogeneous among the entrepreneurs: the accumulated net worth and the expectations on the investment return. The first variable affects the cost of external finance and the agency problem, whereas the second drives the agent's behavior.

Following the extending literature[10] providing that private sector behaviors are characterized by different degrees of heterogeneity and rationality, the model assumes nonperfectly rational agents.

Solved the financial problem, the entrepreneurs hire labor to combine with the physical capital producing output in the following period. This wholesale output is then sold to the retailers that buy and resell goods to the households. By assumption, these retailers compete in a monopolistic market, so that nominal stickiness is introduced in the economy.

2.1. The Financial Intermediary Problem

At the end of time t the jth entrepreneur buys capital to be used in $t + 1$. The quantity of purchased capital is K^j_{t+1}, whereas Q_t is the price paid per unit of capital in term of the numeraire good. Assuming that capital is homogeneous, the financial constraints apply to the whole capital of the firm and not just to investment.

The entrepreneur purchases capital goods $Q_t K^j_{t+1}$ using the available net worth N^j_{t+1} and bank loans B^j_{t+1}:

$$B^j_{t+1} = Q_t K^j_{t+1} - N^j_{t+1}. \quad (1)$$

Bank loans are extended by a financial intermediary (henceforward a bank) who obtains its funds from households and faces an opportunity cost equal to the

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risk free gross rate, R_{t+1}. Entrepreneurs are risk neutral and households are risk-averse, so the entrepreneur absorbs any risk. Given the choices of the entrepreneur on K^j_{t+1}, B^j_{t+1} and given the risk free interest rate R_{t+1}; the optimal contract is characterized by a nondefault firm-specific interest rate, Z^j_{t+1}, such that

$$Z^j_{t+1} = \left(\chi + \mu \frac{B^j_{t+1}}{N^j_{t+1}}\right) R_{t+1}, \tag{2}$$

where $\mu > 0$, $\chi > 0$, and $\mu + \chi > 1$. In equation (2), in line with Delli Gatti et al. (2010b), χ represents the weight of the risk free interest rate, $\frac{B^j_{t+1}}{N^j_{t+1}}$ is the leverage ratio and μ is a "firm financial soundness" parameter. This equation shows the relationship between the external cost of funds and the financial condition of the entrepreneurs. Indeed, as in Bernanke et al. (1999), the firm-specific interest rate on loans is a mark-up over the risk free interest rate and it is increasing in $\frac{B^j_{t+1}}{N^j_{t+1}}$, i.e., it depends inversely on the financial soundness. The financial intermediary is always willing to lend, and the credit market evolves according to the entrepreneurs' demand, i.e., following their expectations on capital return and the external cost of funds.

2.2. Heterogeneous Expectations and Optimal Choices of Capital

Given the state-contingent debt contract, the expected return of the entrepreneur's investment may be written as

$$E_t\left(R^k_{t+1}\right) Q_t K^j_{t+1} - Z^j_{t+1} B^j_{t+1}, \tag{3}$$

where expectations are taken upon the return on invested capital, R^k_{t+1}, given that all the other variables are predetermined. So far, we have described how state-contingent values of Z^j_{t+1} are determined, given the ex-post realization of the market return on invested capital and the ex-ante choices of $Q_t K^j_{t+1}$ and B^j_{t+1} of the single entrepreneurs.

At this point, introducing heterogeneity in the behavior of entrepreneurs, i.e., in the heuristic forecasting rules, the expectations on the return dynamics assume a crucial role in the investment decision of the entrepreneur.

Abandoning the world of rational expectations and asserting that agents are not perfectly rational and have cognitive limitations or computational limits,[11] we assume two different forecasting rules:

$$E_{h,t}\left(R^k_{t+1}\right) = R^{k*} \pm b, \tag{4}$$

with $b > 0$ and where $h = o$, p represents optimistic and pessimistic expectations, respectively. Equation (4) describes biased behaviors of bounded rational agents, as in Brock and Hommes (1998), where b represents the bias parameter and R^{k*} is the fundamental investment return, i.e., the return prevailing when the two

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heuristics shares are equal. More precisely, these entrepreneurs have a commonly shared belief on fundamental investment return plus a type specific bias, the agents have an approximate knowledge of the correct fundamental value of the investment return but they disagree on the real current level. If the bias reduces the expected return, we are considering an agent with "pessimistic" expectation; in the opposite case, we will have an "optimistic" entrepreneur.

The optimization problem of the *j*th entrepreneur of *h*th type is defined by the following optimization problem:

$$\max_{K} E_{h,t}\left(R^{k}_{t+1}\right) Q_t K^{j}_{t+1} - Z^{j}_{t+1} B^{j}_{t+1} \text{ with } h = o,\ p,$$

where $E_{h,t}\left(R^{k}_{t+1}\right)$ represents the expected return on investment of the *j*th entrepreneur that could be optimist or pessimist.

From the maximization problem, it is possible to establish a relationship between capital expenditure and the entrepreneur's financial expectation measured by the expected discounted spread between the return of capital with the risk free rate and the entrepreneurial net worth. The capital/wealth ratio may indeed be expressed as the increasing function of the premium on external funds and it can be rewritten as

$$Q_t K^{j}_{t+1} = \psi(s_{t+1}) N^{j}_{t+1}, \tag{5}$$

where $\psi\ (s_{t+1}) = 1 + \frac{E_{h,t}\left(R^{k}_{t+1}\right) - \chi R_{t+1}}{2\mu R_{t+1}}$.

Through simple substitutions, equation (5) can be rewritten as follows:

$$B^{j}_{h,t+1} = \frac{E_{h,t}\left(R^{k}_{t+1}\right) - \chi R_{t+1}}{2\mu R_{t+1}} N^{j}_{t+1}. \tag{6}$$

Capital expenditure is proportional to the entrepreneur's net worth, with a proportionality factor increasing in the expected discounted rate of return on capital. Consequently, agents with higher wedge between expected return on capital and risk free rate (optimistic entrepreneurs) will have higher incentive to borrow from the financial intermediary.

The entrepreneurs are constrained from raising the size of their firms by the fact that while increasing the amount of borrowed capital, they also increase the leverage ratio and therefore reduce the return on investment.[12] Indeed, by increasing debt, the nonfully self-financed entrepreneurs increase the leverage ratio, and therefore the external finance costs. In this way, they reduce the return on capital that will be equal to the ratio between the investment net return and the total amount of purchased capital as will be explained in Section 2.5.

Additionally to the biased entrepreneurs' expectations on the investment return, the model will consider five different heuristics on output gap and inflation: naive, fundamentalist, weak trend-follower, strong trend-follower, and learning anchoring and adjustment rule (henceforward LAA):

$$x^{e}_{t} = x_{t-1}, \tag{7}$$

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$$x_t^e = x^*, \tag{8}$$

$$x_t^e = x_{t-1} + \varsigma\left(x_{t-1} - x_{t-2}\right), \tag{9}$$

$$x_t^e = 0.5\left(x_{t-1}^{\text{av}} + x_{t-1}\right) + \left(x_{t-1} - x_{t-2}\right), \tag{10}$$

where x is the hypothetical reference variable, x^* and x^{av} represent respectively the fundamental value and the average level and where $\varsigma > 0$.

The assumption behind this approach is that the entrepreneurs have heterogeneous expectations on the return of capital, biased around the steady-state values, but they also have expectations on the final output and the inflation of the economy. This is in line with the experimental literature,[13] according to which the agents may coordinate on different heuristic rules for different forecasting variables.

Equation (7) represents naive behaviors. This rule prescribes that the agents form their expectations on future variable rank using the last observed level.[14] Fundamentalist agents, presented in equation (8), have unbiased expectations based on fundamental levels of output gap and current inflation in the NK-IS curve and in the Phillips curve. In other words, they completely believe to the Central Bank's targets on output gap and inflation, respectively, equal to 0% and 2%. Equation (9) describes the chartists strategy deeply analysed both in laboratory experiments [e.g., Hommes (2011)] and in survey analyses on the trading rules for the financial market [e.g., Frankel and Froot (1990)]. According to Anufiev and Hommes (2012), the extrapolation coefficient measures the strength of the adjustment and it is defined in the range $\varsigma = (0.4; 1.3)$. Equation (10) describes the most sophisticated heuristic, that is the learning anchoring and adjustment heuristic[15], which requires the highest computational capabilities. This strategy exhibits an adaptive learning: the expected value is anchored to the mean between the sample average of past levels (x_{t-1}^A) and the last observation, plus the short-term variation between the two previous periods. In other words, the agents extrapolate the next expected variable level combining a reference point, $\frac{x_{t-1}^A + x_{t-1}}{2}$, with a trend component.

To summarize, the agents form their expectations considering three variables: capital return, inflation, and output. On the capital return, the entrepreneurs are biased, positively or negatively, whereas on the levels of inflation and output the agents may form their expectations according to five different rules: naive, fundamentalist, chartists (weak and strong), and LAA rule.

In Section 3, focusing on the economic fluctuations due to optimistic and pessimistic waves in investment, we begin our analysis considering fixed share in the expectations on output and inflation. Only the entrepreneurs are allowed to switch between the two biased heuristics on capital return. Therefore, we will have time-varying shares of optimistic and pessimistic entrepreneurs generating investment fluctuations but constant share of agents following a reference bounded rational heuristic: naive, fundamentalist, weak and strong chartist, and learning anchoring and adjusted rule. Particularly, the expectations will be updated for all the variables, but only the investors fractions change over times. Then, we will

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introduce more complexity in the system performing some simulations considering time varying fractions for all the variables expectations and allowing the agents to switch among all five heuristics.

2.3. Households, Retailers, and Public Sector

In this section, we will describe the features of households, retail sector, government and Central Bank. As in the standard BGG framework the households have an infinite horizon, they work, consume, hold money, and invest their savings in financial assets that pay the risk free interest rate. The household maximization problem is therefore:

$$\max_{c,h,m} \quad E_t \sum_{k=0}^{\infty} \beta^k \left[ln\left(C_{t+k}\right) + \zeta ln\left(\frac{M_{t+k}}{P_{t+k}}\right) + \xi ln\left(1 - H_{t+k}\right) \right],$$

$$s.t. \quad C_t = W_t H_t - T_t + \Pi_t + R_t D_t - D_{t+1} + \frac{(M_{t-1} - M_t)}{P_t},$$

where C_t is the consumption of the households, W_t is the household real wage, H_t is the supply of labor, T_t are lump sum taxes, Π_t are the dividends of the retail firms, D_t are the deposits held at banks and $\frac{M_t}{P_t}$ is the real money balance between periods. The first-order conditions of the problem can be written as follows:

$$C_t :\rightarrow \frac{1}{C_t} = E_t \left(\beta \frac{1}{C_{t+1}}\right) R_{t+1}, \quad (A)$$

$$H_t :\rightarrow \frac{W_t}{C_t} = \xi \frac{1}{1 - H_t}, \quad (B)$$

$$M_t :\rightarrow \frac{M_t}{P_t} = \zeta C_t \left(\frac{R^n_{t+1}}{R^n_{t+1} - 1}\right), \quad (C)$$

where R^n_{t+1} is the gross nominal interest rate. It should be remembered that in equilibrium the household deposits are equal to the total amount of loans of the bank, $D_t = B_t$. Households consumption is driven by the consumption Euler equation (Equation A). Assuming the unit coefficient on the intertemporal elasticity of substitution, the log-linearized version of the consumption Euler equation can be written as $c_t = E_t\left(c_{t+1}\right) - r_{t+1}$.

The retail sector is characterized by monopolistic competition and nominal rigidity à la Calvo (1983). The nth retailer sells the quantity of output $Y_t(n)$ at the nominal price $P_t(n)$. The total final goods and their price are therefore the combination of the individual retailer sales:

$$Y_t^f = \left[\int_0^1 Y_t(n)^{(\epsilon-1)/\epsilon} dn\right]^{\epsilon/(\epsilon-1)}, \quad \textbf{(11)}$$

with $\epsilon > 1$ and $P_t = [\int_0^1 P_t(n)^{(1-\epsilon)} dn]^{1/(1-\epsilon)}$.

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To introduce price stickiness, in each period a share of firms faces the probability $(1-\theta)$ of being able to reoptimize its price. In every phase, a retailer faces a demand curve:

$$Y_t(n) = \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t^f, \tag{12}$$

where P_t^* is the price set by retailers who are able to reoptimize and $Y_t^*(z)$ is the consequent demand given this price. The nth retailer sets the price in order to maximize his expected discounted profits:

$$\sum_{k=0}^{\infty} \theta^k E_{t-1} \left\{ \Lambda_{t,k} \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t^*(n) \left[\frac{P_t^*}{P_t} - \left(\frac{\epsilon}{\epsilon-1}\right)\frac{P_{t+k}^w}{P_{t+k}}\right]\right\}. \tag{13}$$

In equation (13), $\Lambda_{t,k} = \beta C_t / C_{t+k}$ represents the consumption based discount rate equal to the shareholders' intertemporal marginal rate of substitution, θ^k is the probability that the price is fixed for k periods and P_t^w is the nominal price of the wholesale goods.

Given that the share θ of retailers is not able to reoptimize in period t, the evolution of the price will be

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)\left(P_t^*\right)^{1-\epsilon}\right]^{1/(1-\epsilon)}. \tag{14}$$

By combining equations (13) and (14) and log-linearizing, after some calculations, it is possible to obtain the following Phillips curve:

$$\pi_t = f_1 E_{t-1}(\pi_t) + f_2 y_t,$$

where f_1 depends both on the elasticity of marginal cost with respect to output and on the sensitivity of price adjustment to marginal cost fluctuations, and f_2 is the subjective discount factor of the representative household. In line with Branch and McGough (2009) and De Grauwe (2012), both coefficients are positive.

Finally, moving to the public sector, government expenditures are financed by lump sum taxes and money creation. Hence, the government budget constrain will be $G_t = T_t + \frac{(M_{t-1}-M_t)}{P_t}$. Following the standard literature,[16] the main instrument of the monetary policy is the short-term nominal interest rate. As a consequence, the Central Bank adjusts the nominal interest rate as stated by the following Taylor Rule:

$$r_t^n = \rho r_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t\right) + \varepsilon_t,$$

the monetary authority changes the actual interest rate reacting to the current inflation, the current output, and the lagged interest rate.

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2.4. Aggregation and General Equilibrium

So far, we have described how the agents behave to solve their own maximization problem. In this subsection, individual choices will be incorporated within a general dynamic equilibrium framework.

The capital purchased by the entrepreneur is combined with labor in order to produce wholesale output through the following Cobb–Douglas production function:

$$Y_{t+1} = A_{t+1} K_{t+1}^{\alpha} L_{t+1}^{(1-\alpha)}, \tag{15}$$

where Y_{t+1} represents the aggregate production in period t+1, K_{t+1} is the aggregate amount of capital purchased by all the entrepreneurs, L_{t+1} is the labor input, and A_{t+1} is an exogenous technology parameter.

At this point, it should be noticed how the heterogeneous expectations slightly affect equation (15) through the total invested capital. Starting from the single entrepreneurs investment decisions and aggregating over the heterogeneous agents, in each period the aggregate purchased capital will be defined as follows:

$$K_{t+1} = \sum_{h=1}^{H} s_{h,t} \frac{\left(N_{h,t+1} + B_{h,t+1}\right)}{Q_t}, \tag{16}$$

where $s_{h,t}$ describes the share of entrepreneurs that uses a specific rule.

Assuming a decentralized capital market where the perfect competitive capital producing firms act as simple clearing market traders,[17] the price of capital Q_t in terms of the numeraire good will be

$$Q_t = \Phi\left(\frac{K_t}{K_{t-1}}\right), \tag{17}$$

where $\Phi(\cdot)$ is increasing and concave and $\Phi(0) = 0$.

Given that entrepreneurs sell their output to retailers which have market power, the relative price of the wholesale goods will be $1/X_t$, where $X_t = \frac{P_t}{P_t^w}$ is the gross markup of retail goods over wholesale goods. Consequently, the actual rate of capital return between two periods can be written as follows:

$$R_{t+1}^{k} = \frac{\left(\frac{1}{X_t} \frac{\alpha Y_{t+1}}{K_{t+1}}\right) + (1-\delta) Q_{t+1}}{Q_t}, \tag{18}$$

where the first numerator term is the rent paid to one unit of capital, the second is the entrepreneur return for selling the undepreciated capital.

Besides capital, technology requires also labor as input. The total labor supply of the economy is composed by households and entrepreneurial labor (H_{t+1} and H_{t+1}^{e}, respectively):

$$L_{t+1} = H_{t+1}^{\Omega} (H_{t+1}^{e})^{1-\Omega}.$$

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In a competitive market, the demand curves for labor imply that wage equates marginal product; therefore, it will be

$$(1-\alpha)\Omega\frac{Y_{t+1}}{H_{t+1}} = X_{t+1}W_{t+1},$$

$$(1-\alpha)(1-\Omega)\frac{Y_{t+1}}{H^e_{t+1}} = X_{t+1}W^e_{t+1},$$

where W_{t+1} and W^e_{t+1} are the real wage rates to the household and to the entrepreneur, respectively.

The law of motion of the aggregate entrepreneurial net worth for each type, $N_{h,t+2}$, is described as follows:

$$N_{h,t+2} = \gamma V_{h,t+1} + W^e_{t+1}, \tag{19}$$

where $V_{h,t+1}$ represents the equity held by the entrepreneurs of the hth type. Remembering that γ is the fraction of entrepreneurs, which survives in each period, $\gamma V_{h,t+1}$ are the equities held by entrepreneurs still in business in the following period. This equity is the residual part of the investment return after the repayment of the loans to the financial intermediaries.

$$V_{h,t+1} = R^k_{t+1}Q_tK_{h,t+1} - \left[\chi R_{t+1} + \mu\frac{R_{t+1}\left(Q_tK_{h,t+1} - N_{t+1}\right)}{N_{t+1}}\right](Q_tK_{h,t+1} - N_{t+1}), \tag{20}$$

where the term $[\chi R_{t+1} + \mu\frac{R_{t+1}\left(Q_tK_{h,t+1}-N_{t+1}\right)}{N_{t+1}}]$ reflects the premium for external finance that has a negative relation with the net worth and positive with the debt.

2.5. Performance Measure and Dynamic Selection Mechanism

In order to complete the specification of the model, this subsection will discuss how beliefs are updated over time and how the fractions of the different types change. According to the adaptive belief system developed by Brock and Hommes (1998), the fractions evolve according to an evolutionary fitness measure. The key intuition is that rules with higher performance will attract more followers.

The evolutionary performance measure is publicly available, but it is subject to noise and it could be expressed as follows:

$$U_{h,t+1} = U^d_{h,t+1} + \epsilon^j_{h,t+1}, \tag{21}$$

where $U^d_{h,t+1}$ is the deterministic part of the fitness measure.

As in the standard literature, e.g., Hommes (2013), $\epsilon^j_{h,t+1}$ represents an independent and identically distributed noise across individual jth entrepreneurs and the types $h = 1, 2$ and drawn from a double exponential distribution. When

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the number of entrepreneurs goes to infinity, as showed by Diks and Van der Weide (2005) and Hommes et al. (2005), the probability for agents to choose the hth forecasting rule is given by a discrete choice model with multinomial logit probabilities:

$$s_{h,t+2} = (1-\nu)\frac{e^{\vartheta U^j_{h,t+1}}}{Z_{t+1}} + \nu s_{h,t}, \qquad Z_{t+1} = \sum_{h=1}^{H} e^{\vartheta U^j_{h,t+1}}, \tag{22}$$

with $0 < \nu \leq 1$, and where ν is a persistence parameter, Z_{t+1} is a normalizing factor, so that the sum of the fractions $s_{h,t+2}$ is 1. The parameter ϑ represents the "intensity of choice," i.e., it measures how the entrepreneurs are sensitive to the selection of the optimal forecast strategy.

There are two main insights related to this dynamic mechanism: the higher is the fitness measure of the hth forecasting rule, the larger is the number of entrepreneurs who switch to this strategy; the higher is the intensity of choice, the more "rational" are the agents. The intensity of choice is inversely related to the variance of the noise terms $\epsilon^j_{h,t+1}$. One extreme case, when $\vartheta = \infty$, corresponds to the case without noise, so the deterministic part of performance measure can be observed and all the agents will switch to the optimal forecast. The other extreme case, $\vartheta = 0$, the variance of noise term is infinite; thus, the differences in the fitness measures cannot be observed and the share of entrepreneurs will be fixed and equal to $1/H$. The switching mechanism described by equation (22) allows for stickiness in the updating process. Indeed, if $\nu = 0$, the switching model assumes synchronous updating, i.e., in each period all the agents update their strategies. On the contrary, if $\nu > 0$, the switching process is more gradual. In every period, only a fraction $(1-\nu)$ of agents can revise its belief according to the new available information, whereas the remaining fraction ν maintains its current strategy introducing a realistic feature of inertia in the switching mechanism.

In this financial accelerator framework, the deterministic component of the fitness measure is the investment performance of the entrepreneur following the hth forecasting rule:

$$U^d_{h,t+1} = \frac{R^k_{t+1} Q_t K_{h,t+1} - Z_{h,t+1} B_{h,t+1}}{Q_t K_{h,t+1}}. \tag{23}$$

The investment performance defined in equation (23) is the ROI index, indeed it is the ratio between the net return of the investment and the total amount of purchased capital.[18]

Therefore, in line with Kahneman et al. (1991) and the status-quo effect, the evolution of the optimistic entrepreneur will be

$$s_{o,t+2} = (1-\nu)\frac{\exp\left[\vartheta\left(\frac{R^k_{t+1} Q_t K_{o,t+1} - Z_{o,t+1} B_{o,t+1}}{Q_t K_{o,t+1}}\right)\right]}{Z_{t+1}} + \nu s_{o,t+1},$$

with $\nu > 0$ and $s_{o,t+2} + s_{p,t+2} = 1$.

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In our model, the agents form also expectations on the output gap and the inflation level but, in most of the simulations, we will assume fixed heuristics fractions. However, in the last scenario presented in Section 3.2, we will increase the complexity in the system introducing time varying heuristic shares considering five forecasting rules: naive; fundamentalist; LAA heuristic; weak and strong trend followers, hence with coefficient ς in equation (10) equal to 0.4 and 1.3, respectively. In this case, a natural performance measure is past squared forecast errors:

$$U^d_{h,t+1} = -\left[x_{t+1} - E_{h,t}\left(x_{t+1}\right)\right]^2, \tag{24}$$

where x_t represents the reference variable (output or inflation), whereas the dynamic evolution of the heuristics fractions will be

$$s_{h,t+2} = \frac{\exp\left(\vartheta U^d_{h,t+1}\right)}{\sum_{h=1}^{H} \exp\left(\vartheta U^d_{h,t+1}\right)},$$

with $H = 5$ and without stickiness in the updating process.

3. SIMULATIONS

This section presents the results of some quantitative analyses aiming to illustrate how the heterogeneous expectations and the financial accelerator are able to endogenously modify the business cycle. Section 3.1 illustrates how, in presence of bounded rationality, heterogeneous expectations and the financial accelerator affect the business cycle in the absence of external shocks. Section 3.2 will consider some simulation exercises performing stabilization analysis considering scenarios with different monetary policies and expectation rules. In the last Section 3.3, we will change some key parameters of the model to investigate the robustness of the results.

Following Clarida et al. (1999), the choice of the parameter value for the baseline model is quite standard. Looking at the households, their quarterly discount factor (β) is 0.99, and they have a fixed labor supply elasticity (η) equal to 3. In the production function, the capital share (α) is 0.2 with a depreciation rate (δ) of 0.025, the household labor share $(1 - \alpha)$ $(1 - \Omega)$ is 0.64, whereas the correlation in the technology law of motion (ρ^a) is assumed to be 1.0. The probability θ that firms are not able to reoptimize their price within a period is equal to 0.75, and the mark-up of the retail sector with respect to the wholesale market (X) is 1.2. Looking at the financial sector, the rate of "survival" of the entrepreneurs through the periods (γ) is 0.9728, whereas the weight terms in the financial intermediary interest rate setting are $\mu = 0.7$ and $\chi = 0.6$. Entrepreneurs have a bias of 5% around the fundamental value of the investment return, and the fraction of the entrepreneurs that can revise their beliefs in the asynchronous updating mechanism will be 0.9 with intensity of choice equal to 50, in line with the parameterization of Anufriev and Hommes (2012).

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The behavior of agents is based on adapting heuristics and on the switching mechanism. The model starts from a quasi-steady-state level, thus, the updating process in the expectations formation generates endogenous fluctuations and may allow to nonlinearities in dynamics. If this happens, the model will not allow for analytical and closed-form solutions. For this reason, in this section, we run 100 Monte Carlo simulations for 1,000 periods, where each series differs for its initial condition. We introduce a shock in the economy in period $t = 300$, so that we are sure the system has reached the steady state, and we can study the dynamics and the convergence properties of the different scenarios. The negative production shock follows a normal distribution and will hit the economy from the 300th quarter presenting an autoregressive persistence of 0.95.

3.1. The Effects of Heterogeneity and Bounded Rationality

In this section, we discuss a simple but typical example comparing a bounded rational scenario (i.e., considering weak trend follower agents) with a rational framework assuming two monetary policy: strict and flexible inflation targeting (henceforward SIT and FIT).[19] In line with Anufriev et al. (2013), the rational framework is defined as the scenario where agents believe that inflation rate and output will always be at their RE level, i.e., $y_t^e = 0$ and $\pi_t^e = 0$.

By definition, fully rational entrepreneurs have full information and are able to compute the actual investment level and the investment return, R_{t+1}^k. Given that the entrepreneurial expectations on the investment return are correct, there is no reason why they should be biased. Thus, we can reduce the two entrepreneur types to the standard representative agent. Consequently, we can write the entrepreneurs expectation rule and the purchased capital as

$$E_t\left(R_{t+1}^k\right) = R^{k*};$$

$$Q_t K_{t+1}^j = N_{t+1}^j + \frac{R^{k*} - \chi R_{t+1}}{2\mu R_{t+1}} N_{t+1}^j.$$

The rational and homogeneous framework avoids the emergence of animal spirits reducing the fluctuations in the system, i.e., the system does not exhibit the optimist and pessimist waves in investment decisions. Conversely, the heterogeneous and bounded rational scenarios express these dynamics because the entrepreneurs make mistakes in the expectations formation, and therefore in the investment decisions affecting the output gap and the inflation level.

Figure 1 shows scenarios with both flexible and strict inflation targeting monetary policies. The plotted scenarios are as follows: two heterogeneous frameworks with fixed shares of chartist agents and two scenarios with homogeneous and rational agents. The baseline simulations are initialized with different levels of the model variables (production, inflation, firms' net worth, etc.). Hence, the figures represent the average dynamics of the scenarios. This is a standard approach in

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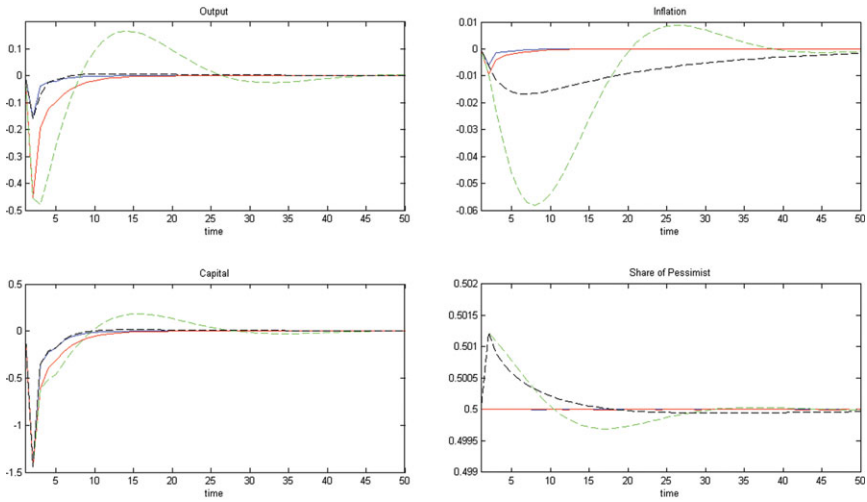


FIGURE 1. Simulation results with two inflation targeting Monetary Policies and different degrees of heterogeneity and rationality. Blue line: homogeneous and rational agents under SIT monetary policy scenario; red line: homogeneous and rational agents under FIT monetary policy scenario; black line: biased entrepreneurs and weak trend following expectations under SIT monetary policy; green line: biased entrepreneurs and weak trend following expectations under FIT monetary policy.

agent-based simulation because it allows to study the dynamics emergence, the stability of the equilibrium, and the convergence to the steady state.

The blue and red lines in each picture indicate the evolution of the model with homogeneous and rational agents with flexible and strict inflation monetary policy, respectively. Starting from initial values that differ from the steady state, the financial accelerator enlarges the oscillation of the system delaying the convergence. The initial conditions that are different from the steady-state levels affect the costs of loan, and therefore the amount of invested capital. For example, higher initial value will be translated in higher cost of loans, and therefore in a collapse of investment, as demonstrated by the negative spikes in the first period. However, as expected from the BGG model and the standard REH literature, the system converges to the steady state after few periods. Indeed, after the first period, agents are able to compute the true steady state of the variables driving the system to the equilibrium.

Conversely, in presence of heterogeneity and bounded rational expectations, we will observe the emergence of more persistent fluctuations. The combination of the financial accelerator with the switching mechanism amplifies the oscillations. In these cases, the collapse of investment is further worsened by the increase of pessimist agents. The growth of the pessimist share negatively affects the investment in capital. The convergence of capital to its steady-state level may be explained with the reduction of the inflation rate that decreases the risk free interest rate.

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Lowering this interest rate, the spread between the investment return and the interest rate on loans increases. This is translated into a long-run positive convergence path to the steady state for capital and output gap and into a reduction of pessimist entrepreneurs in the population. In the strict inflation monetary policy simulation, we register the higher fall in inflation allowing overshooting phenomena in output, capital, and forecasting share dynamics.

More precisely, the initial fall of output and inflation negatively affects the risk free interest rate. Reducing the cost of external finance, the entrepreneurs increase their demand for loans enhancing the invested capital. Indeed, until the inflation rate is low, the financial sector is willing to extend loans at relatively moderate interest rate affecting the entrepreneur's investment return. Increasing the net expected investment return, the entrepreneurs become more optimistic and rise both their demand for loans and purchased capital. In this scenario, the animal spirits are further amplified by the monetary policy that has not among its aims the closure of the output gap allowing to output and investment oscillations affecting the return on capital and the expectations of the entrepreneurs.

3.2. Stabilization Analysis with Different Monetary Policies

This subsection expands the analysis presenting several simulations on different interest rate rules when a negative production shock hits the economy.[20] This will allow to draw some policy recommendations and to show how the macroeconomic stability is affected simultaneously by the set of heuristics considered and by the different monetary policies. The comparative statics exercise on the monetary policy rules is motivated by the perspective that central banks can pursue different aims. For example, according to EU treaties, the ECB should keep stability of prices and financial system, whereas the Fed should also pursue the output stability. In order to perform comparative analyses, the hypothesized monetary policies will be

$$r_t^n = \phi_\pi \pi_t + \phi_y y_t,$$

where the inflation parameter (ϕ_π) will be equal to 1.5, whereas the coefficient on output gap (ϕ_y) may be null (strict inflation targeting monetary policy) or equal to 1 (flexible inflation targeting monetary policy).

Moreover, the subsection describes how the fluctuations of the economy due to different policies may depend on the composition of the forecasting rules on inflation and output gap. The simulations are performed assuming biased entrepreneurs in different scenarios according to the expectation rules on output gap and inflation. Then, we introduce a macroeconomic shock with the effect of an unanticipated decrease in the production, and we study the convergence to the steady state.

In the first three simulations, we consider scenarios where entrepreneurs can switch between optimist and pessimist forecasting rules but the expectations on inflation and output can only be updated according to the reference heuristic: naive,

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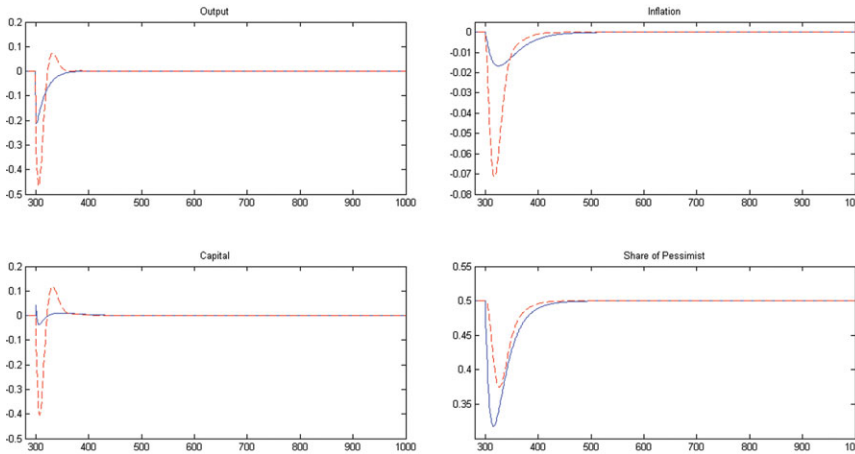


FIGURE 2. Simulation with biased entrepreneurs and naive agents; red line: strict inflation targeting monetary policy, blue line: flexible inflation targeting monetary policy.

weak chartist, and LAA rule. In other words, in these scenarios, the fractions of the biased entrepreneurs evolve over time, whereas the shares of the bounded rational expectations on inflation and output are fixed. In the fourth scenario, we increase the complexity of the system performing simulations with biased entrepreneurs and competition among five heuristics on inflation and output considering time varying fractions for all the reference expectation variables (investment return, inflation, and output).

Figure 2 shows the response of endogenous variables in a scenario with biased entrepreneurs and naive expectations on inflation and output. Looking at the scenario with flexible inflation targeting monetary policy, the shock on the total output affects negatively inflation and risk free interest rate. This reduction in the risk free interest rate has two main effects: it increases the spread between the return of capital and the interest rate on loan and it reduces the external cost of loans. Consequently, we observe a reduction in the number of pessimist entrepreneurs with a spike in the total amount of capital. This dynamics can be explained by the nature of the expectation rules. On the one side, the higher spread between safe and risky interest rates increases the relative accuracy of the optimistic entrepreneurial expectations compared to the pessimist beliefs. As a consequence, more entrepreneurs become optimist on the profitability of their investment and increase the investment causing the spike in the purchased capital. On the other side, if there is a lower external cost of funds this affects positively the investment decisions of the pessimistic agents increasing the total purchased capital. Over time, the performance of the optimist decreases, as a consequence of output levels, and the dynamics of the variables converge to their steady states. Another channel of propagation of the shock is in the fluctuation of the capital return that affects the entrepreneur's fractions. Indeed, the unanticipated

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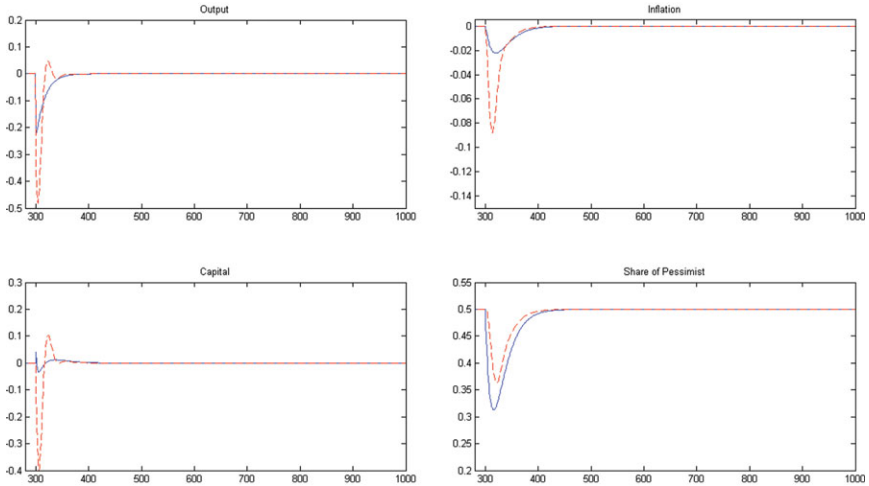


FIGURE 3. Simulation with biased entrepreneurs and weak trend following expectations; red line: strict inflation targeting monetary policy, blue line: flexible inflation targeting monetary policy.

variations in the capital return modify the spread with the interest rate on loan and consequently the net return on investment, i.e., the performance measure upon which the entrepreneurs base their expectations.

Looking at the strict inflation targeting monetary policy (henceforward SIT), the scenario exhibits comparable dynamics except for overreaction both in output and capital. Moreover, comparing the dynamics of the system of the two monetary policies, it is possible to notice how the SIT monetary policy (the red and dashed line in the graph) yields deeper fluctuations in the considered variables. These effects may be explained by the SIT monetary policy definition, which does not consider the closure of the output gap. Therefore, the output and the investment can oscillate affecting the return on capital and the expectations of the entrepreneurs. Thus, given the backward-looking nature of the heuristic, these dynamics will also affect the level of the variables in the following periods. In this scenario, as a consequence to the fall of the output, the entrepreneurs become pessimist reducing their investment in the following period more than in the flexible inflation targeting scenario. However, this chain of effects is partially mitigated by the reduction of the inflation allowing the growth in capital return and inverting the trends of capital and output around their steady-state values. Graphically, this mitigation process is depicted by the higher slope of the response curves in the convergence to the steady state.

Considering a population with weak trend following expectations, Figure 3 represents the simulations result with both monetary policies. It is possible to notice how the monetary policy deeply affects the convergence to the steady state after the shock. Indeed, the fall in the output in the flexible inflation targeting policy

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(i.e., the blue line) entails both a spike in the investment and lower fluctuations in the following period. The evolution of the purchased capital can be explained by the reductions in output and inflation, which are translated in lower risk free interest rate. As presented by the graphs on the bottom side, these variations affect the investment choice and the number of optimist entrepreneurs.

This greater spread between the investment return and the interest rate on loans brings to active investment decisions of the agents and a rise in the optimist share of entrepreneurs. However, the persistence of the shock maintaining the output level below its steady state, in combination with the nature of the heuristic, affects the purchased capital and the investment return reducing the ROI of the entrepreneur. The strict inflation targeting monetary policy implies higher undershooting phenomena due to the excess of negative feedback. The presence of bounded rational agents introduces backward-looking components in the dynamics and the pro-trend nature of the heuristic implies that, after the shock, the agents observe lower levels of output and inflation decreasing their forecasts. However, this evolution increases the expected return on investment and consequently the amount of invested capital of the entrepreneurs. This behavior origins the negative feedback system in which lower expectations may lead to higher actual level of the variables. Moreover, it should be noticed how the reduction of the number of pessimist entrepreneurs in this scenario is lower and less sudden. Again, this phenomenon may be explained by the monetary policy which, focusing only on the inflation dynamics, is less aggressive in the variations of the risk free interest rate reducing the switch consequent to the initial shock.

Concluding, it is interesting to notice how the simulation results under naive and weak trend following rule are quite similar except for wider fluctuation in the inflation level in the chartist scenarios. This finding can be explained by the common nature of these heuristics: naive agents are chartists with the null extrapolation parameter, $\varsigma = 0$. Definitely, the absence of the protrend component in the naive heuristic partially smoothes the expectations range.

Figure 4 displays simulations concerning biased entrepreneurs with all the population following the learning anchoring and adjusted forecasting rule for inflation and output. For both the policies, the model simulations exhibit very peculiar results. First, the two scenarios converge to different steady-state levels. Indeed, the two scenarios do not converge to the initial steady state. It seems that when the entire population follows the learning anchoring and adjusted rule, the monetary policy is particularly relevant in addressing the levels of output and invested capital. This result can be explained both by the initial conditions and by the reaction of the central bank to inflation and output gap.

On the one side, this environment displays persistent state dependence from the initial conditions because initial values for output gap and inflation differing from the fundamental levels affect the risk free interest rate and the external fund cost. These deviations in the cost of loans are translated into deviations from the steady-state level of investment changing the entrepreneur types fractions, $n_{h,t}$. This evolution will influence the following levels of output and purchased

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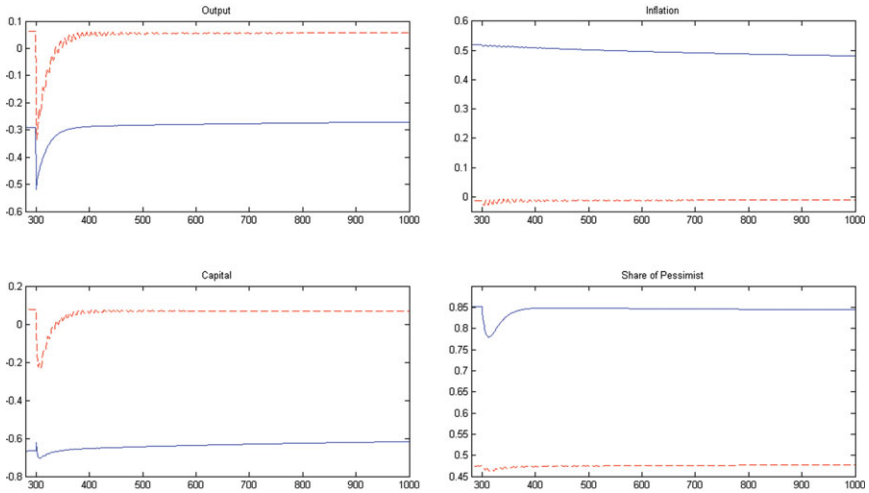


FIGURE 4. Simulation with biased entrepreneurs and LAA expectations; red line: strict inflation targeting monetary policy, blue line: flexible inflation targeting monetary policy.

capital. Moreover, the scenarios exhibit high-frequency fluctuations decreasing in amplitude and converging to new steady states in the long run. This dynamics can be explained by the adaptive learning behavior in the expectations. Indeed, in each time the agents update their information on the past realizations of variables and form their expectations adding a trend-component. Nevertheless, the nature of the LAA heuristic implies that agents anchor their forecasts to these "wrong" values conditioning the future dynamics of the variables. We define these levels "wrong" because they present deviations from the steady state characterized by null inflation and output gap and by equal fractions between optimist and pessimist entrepreneurs. With more accuracy, these two emerging levels could be defined as good and bad steady-state level. Indeed, in the first case, we observe higher output and invested capital with lower inflation rate, where the majority of the entrepreneurs is optimist. On the contrary, the bad arising steady state is characterized by lower output and purchased capital, higher inflation level, and a pessimist entrepreneurs share highly predominant.

On the other side, it should be noticed how the new steady-state levels depend on the coefficient of the interest rate rules. Indeed, the monetary policy has a crucial role in defining the level of steady state. In our simulations, when the central bank applies flexible inflation monetary policies responds more strongly to inflation than to output gap. This aggressive reaction to inflation positively affects the risk free interest rate reducing the spread between the expected return on investment and the external fund cost. As shown in the bottom hand side of Figure 4, lowering the expected return on investment the entrepreneurs will purchase inferior level of capital and the share of pessimist investors in the market will increase. On the contrary, in the SIT case the central bank is able to maintain the

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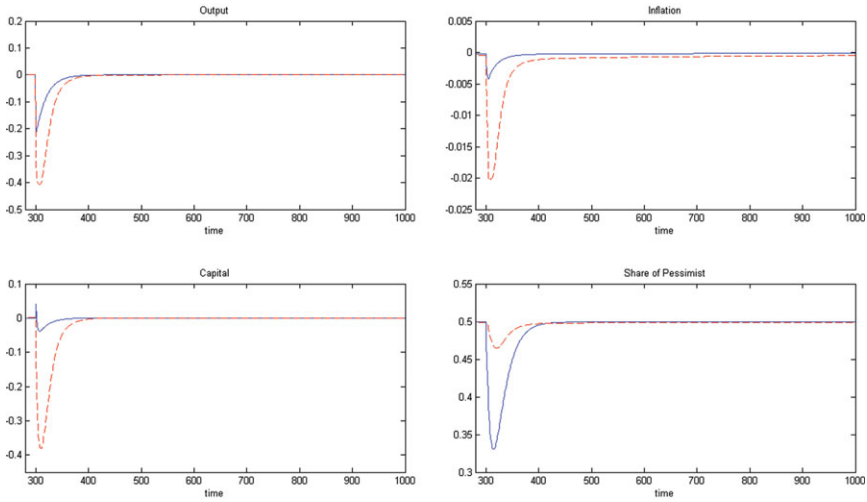


FIGURE 5. Simulation with biased entrepreneurs and time-varying expectation rules shares; red line: strict inflation targeting monetary policy, blue line: flexible inflation targeting monetary policy.

inflation close to its steady-state level. Moreover, when the monetary authority responds only to inflation we observe higher levels of investment and majority of the entrepreneurs population, which is optimist. In this case, leaving aside the stabilization of the output from its goals, the SIT monetary policy leads to lower safe interest rate, higher expected investment return, and consequently to a greater fraction of optimist agents in the economy.

Concluding, being bounded by the average in the updating process, on the one side the agents reduce the possible range of their predictions affecting also the actual variables, on the other side the anchor (i.e., the average) may constrain the convergence process to new steady-state levels. It should be noticed how there are no constrains that ensure that the reached steady state will be the optimal one. However, the central bank and its monetary policy goals can really affect the steady-state level to which the system converges.

The last scenario analyzes a more complex framework considering biased entrepreneurs and five forecasting rules for output and inflation with time varying fractions on the reference variables expectations. The result simulations are in line with the dynamics of the first two scenarios. Indeed, Figure 5 shows how the flexible inflation targeting monetary policy (henceforward FIT) is able to better stabilize the economy. In the strict inflation targeting monetary policy, the monetary authority focuses only on the inflation level without stabilizing the output gap. On the one side, this allows for wider fluctuations in output and inflation affecting the return on capital and the expectations of the entrepreneurs. On the other side, leaving aside the stabilization of the output from its goals, the SIT monetary policy

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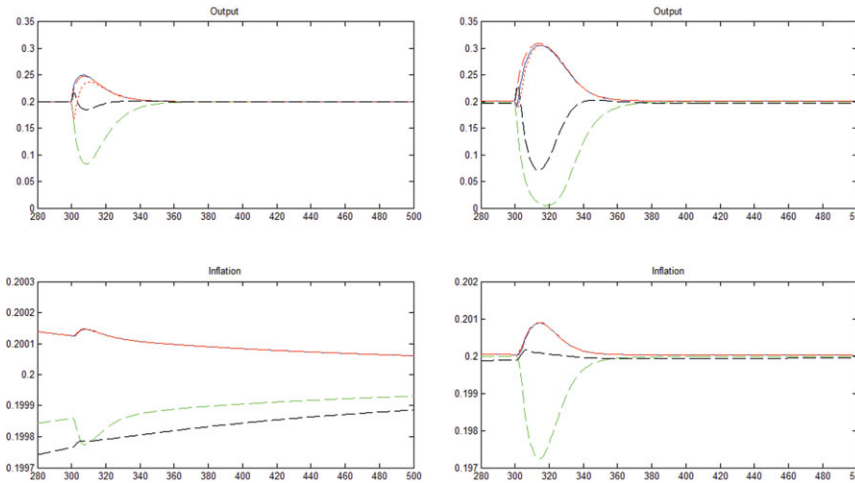


FIGURE 6. Evolutionary dynamics of the heuristic fractions; blue line: naive agent, dashed red line: weak trend follower, dotted red line: strong charitist, dashed green line: fundamentalist heuristic, black line: learning anchoring and adjustment behavior; left column: strict inflation targeting monetary policy, right column: flexible inflation targeting monetary policy.

reacts with lower strength on the risk free interest rate. This smoother revision of the rates provides lower spread between return to capital and external fund cost and a lower share of optimists in the group of entrepreneurs.

The different strength in the monetary policy reaction also affects the evolutionary dynamics of the heuristic fractions. Indeed, when the monetary authority responds weakly, focusing only on inflation, we observe lower oscillations in the shares of optimist/pessimist entrepreneurs but higher fluctuations in output, inflation, and purchased capital. As Figure 6 shows, in the FIT scenario this greater magnitude in the macroeconomic variables fluctuations generates a feedback process with the dynamic selection mechanism. For example, in the right column, the negative output shock reduces the investment return. By definition, the fundamentalist and the LAA rules present "stickiness" in their updating process. More precisely, the fundamental heuristic always forecasts the steady-state value, whereas the LAA rule is anchored at the past mean in the updating process. This "stickiness" increases the forecast error of these heuristics when there is a deviation from the steady state and consequently we observe a reduction of these fractions. On the contrary, the other three heuristics following the variables dynamic perform better. Indeed, with different strengths, both naive and chartist behaviors exploit the persistence of the shock originating more accurate forecasts. When the shock dynamic converges to zero the fundamental and the learning anchoring and adjusted rules increase their performance measures and the heuristics shares become equal.

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Summarizing, we obtain three main results from these analyses (See Table B1 in Appendix B for the main statistics of the simulations results). First, it is interesting to notice how, in the trend scenarios, the strict inflation targeting policy yields deeper fluctuations in the first periods but, when the timing is considered, it is able to stabilize the economy in fewer quarters. On the one side, these two results may be explained by the aim of the policy (i.e., to stabilize the price fluctuations) and by how this is pursued. This policy does not take into account the fluctuations in the output gap; thus, all the related graphs illustrate higher variations for this variable. On the other side, the flexible inflation targeting monetary policy brings the system to lower fluctuations even though these are more persistent. Replying strongly to the fluctuations of the variables, this policy generates greater variations on the risk free interest rate, continuously affecting the investment decisions of the entrepreneurs through the expected return on investment. With this policy, negative output gap and inflation levels imply lower safe interest rate reducing the probability for the entrepreneurs to have the "wrong attitude," i.e. the less profitable expectations. Indeed, this means higher spreads between investment return and external fund rates increasing the share of optimist agents.

Second, comparing the effects of the monetary policies in the different fixed shares scenarios an interesting insight arises: the heuristic with greater computational ability exhibits higher dependence on the objective pursued by the central bank. The LAA rule is the heuristic assuming higher agents computational capabilities; hence, it could be expected that the scenarios considering this rule of thumb should register more stable dynamics. On the contrary, according to the monetary policy goals different steady states emerge. The anchor parameter is source of a bias, which binds the forecasting range strengthening the state dependence of the dynamic from the previous period. Consequently, when the central bank reaction to output gap is positive, the system converges to the "bad" steady-state scenario characterized by higher inflation rate and lower expected return to investment. This increases the pessimist share of agents in the economy negatively affecting the invested capital and production. The dynamic propagates itself in the following period influencing positively the number of pessimist entrepreneurs and accordingly the other variables. On the contrary, when the target of the monetary policy is only the inflation rate, we observe the convergence to the "good" steady state. The existence of these two steady states implies that the LAA scenario presents higher variability in the results then the other scenarios. In these cases, even if we observe different dynamics according to the monetary policy considered, all the fluctuations are around the same steady state level with null inflation and output gap, so we do not observe the emergence of the two completely different steady states.

Third, introducing the competition among the heuristics, it is possible to notice that the spikes following the negative output shock are lower (especially in the strict inflation monetary policy scenario). However, if on the one side the last framework presents lower falls in output, inflation, and capital, on the other side, the dynamic evolution of the agents behavior increases the persistence of the oscillations reducing the convergence speed to the steady state.

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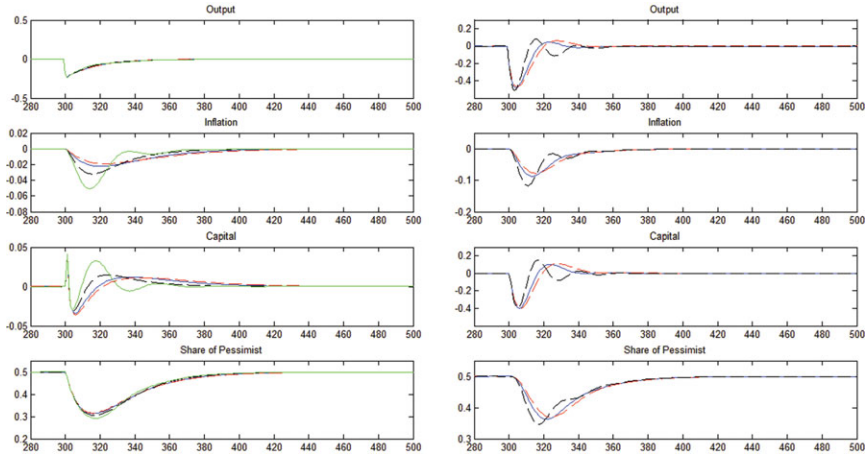


FIGURE 7. Simulation with chartist agents and different trend coefficients: $\varsigma = 0.2$ (dashed red line); 0.4 (blue line); 0.7 (dashed black line); 0.9 (green line). Left column: flexible inflation targeting monetary policy; right column: strict inflation targeting monetary policy.

Concluding, it should be underlined that the scenarios endogenously generate the economic fluctuations thanks to the heterogeneity in the entrepreneurs' expectations on the capital return. Moreover, the effectiveness of the monetary policy interestingly depends on the heterogeneity and bounded rationality, i.e., on the heuristics of the agents. In line with Anufriev et al. (2013), in most of the scenarios a strong monetary policy is able to send signals that help agents to correct their forecasts reducing their mistakes in investment decisions.

3.3. Robustness Checks

In this section, we will perform other simulations changing some key parameters in order to investigate the robustness of our results.

First, we focus the analysis on the effect of the trend component coefficient in the heuristics on the model results. Given that the majority of the heuristics presents a trend component in its definition, we will present some robustness checks considering the chartist forecasting rule. In these simulations, we expand the range of the extrapolation coefficient ($\varsigma = 0.2; 0.4; 0.7; 0.9; 1.1; 1.3$). As shown by Figure 7, the speed of convergence and the amplitude of the oscillations depend on the parameter ς. In line with Anufriev and Hommes (2012), increasing the trend coefficient the convergence to the steady state becomes oscillatory. Moreover, for weak trend extrapolations, we observe lower oscillation amplitude, the red and blue lines representing ς-values equal to 0.2 and 0.4, respectively. Increasing the trend coefficient, the dynamic of the fluctuations presents higher oscillations. It should be noticed that Figure 7 shows only four and three simulation

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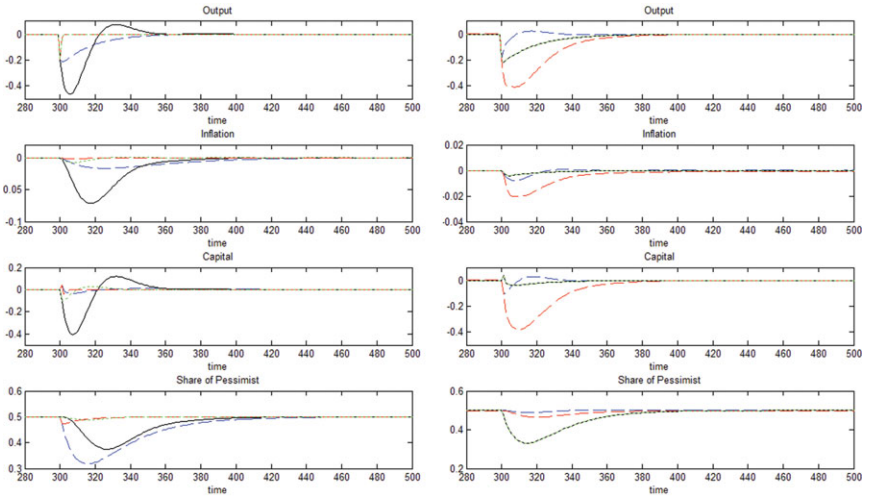


FIGURE 8. Simulation with different autoregressive shock coefficients. Left column: naive expectations; right column: time varying heuristics shares. Dashed blue line: FIT monetary policy and positive autoregressive shock persistence; dashed green line: FIT monetary policy and null autoregressive shock persistence; black line: FIT monetary policy and positive autoregressive shock persistence; dashed red line: SIT monetary policy and null autoregressive shock persistence.

results, respectively, for the flexible (left column) and the strict (right column) inflation targeting monetary policies. Indeed, for stronger trend following rules, the variable dynamics follow explosive paths diverging from the steady state through increasing amplitude oscillations. In these cases, the strong extrapolative trend coefficient causes a cumulative process of rising deviation from the steady state, which the monetary authority is not able to correct, i.e., the central bank does not respond with enough strength. Moreover, it should be underlined that the flexible inflation monetary policy is able to stabilize the divergent cumulative process emerging for high levels of the extrapolation coefficient. Conversely to the strict inflation monetary policy, the FIT monetary policy reacts with strength both to inflation and to output oscillations.

Finally, the analysis concludes investigating the robustness of the results with respect to the persistence of the negative production shock. As expected, reducing the autoregressive coefficient of the negative productivity shock increases the speed of convergence to the steady state in most of the possible scenarios. As shown in Figures 8 and 9, the scenarios with fixed heuristic share reach the convergence to the steady state in fewer quarters when the shock persistence is null. This result can be explained by the backward looking nature of the heuristics, i.e., by the fact that agents forecast future variables using past observations. Since the autoregressive coefficient affects the output gap in the following periods, it influences also the agents forecasts delaying the convergence to the steady state. On the contrary, in

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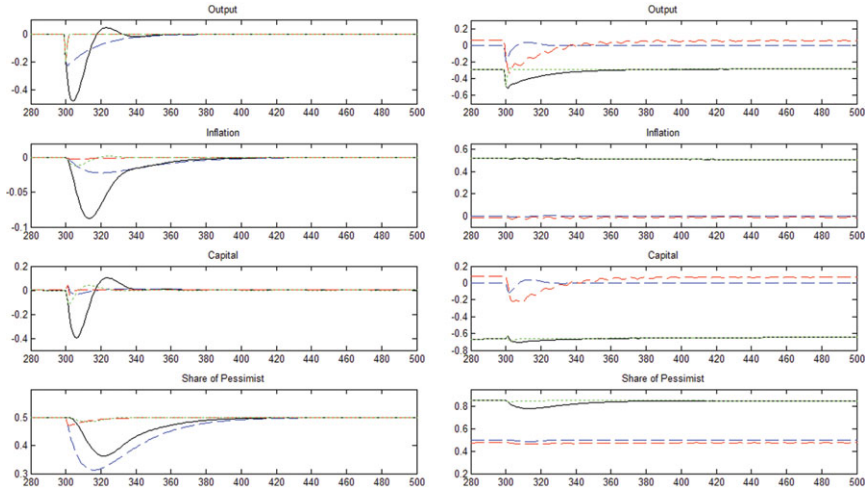


FIGURE 9. Simulation with different autoregressive shock coefficients. Left column: weak trend following expectations; right column: learning anchoring and adjustment heuristic. Dashed blue line: FIT monetary policy and positive autoregressive shock persistence; dashed green line: FIT monetary policy and null autoregressive shock persistence; black line: FIT monetary policy and positive autoregressive shock persistence; dashed red line: SIT monetary policy and null autoregressive shock persistence.

the scenario with evolutionary heuristic shares (see the right column of Figure 8), we observe slower convergence to the steady state when the shock autoregressive coefficient is null. This different result could be explained by the fact that, apart from the shock, in this scenario there is another driven force of the fluctuations: the dynamic evolution of the expectations shares. After the shock, the heuristics shares are updated amplifying and propagating their effects and producing a loop dynamic between further oscillations in the variable levels and heuristic shares updating.

4. CONCLUDING REMARKS

This paper presents a financial accelerator framework to study the effects of heterogeneous and bounded rational expectations. It uses the heterogeneous framework of Brock and Hommes (1997), where the decision to switch the expectations arises from an endogenous process aiming to the most profitable strategy. The analysis addresses the investigation of the heterogeneity and financial accelerator effects on the investment decisions, indeed the heterogeneity of the expectations on capital return brings the agents to purchase different amount of capital.

In this framework, the macroeconomic fluctuations emerge endogenously in the model thanks to the expectations updating mechanism. Indeed, this result can be explained by the bounded rationality of the entrepreneurs, i.e., they do not

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have perfect and complete information. First, they only have a common belief on fundamental investment return, but they do not know the general feeling of the market, i.e., the share of pessimist and optimist entrepreneurs in the market. Second, the different patterns of investment between the entrepreneur's types and the consequent unknown amount of total capital affect the return on investment of every single entrepreneur. This can misplace the entrepreneurs transforming an ex-ante profitable strategy into a nonprofitable one.

In this model, the financial accelerator and the asset price volatility work to amplify the fluctuations in a significant quantitative way, especially in the investment choices. These channels affect both the single investment decisions and the future fractions, amplifying the future total investment and the fluctuations of the system. Hence, the model is able to generate some important stylized facts in many financial and economical series, e.g., unpredictable returns. After having found a role for heterogeneity and bounded rationality, the paper performs some simple monetary policy simulations in order to understand if there are some general intuitions on which policies are better to stabilize the market. As in Anufriev et al. (2013) or Massaro (2013), macroeconomic stability and inflation dynamics interestingly depend on the set of forecasting strategies and on the type of the policy rule considered.

Summarizing, on the one side, the core results of these analyses seem to argue that no monetary policy manages to quickly and completely stabilize the system, actually some fluctuations persist for many quarters. The flexible inflation targeting monetary policy better stabilizes the economy yielding smaller fluctuations. In most of the cases, the lower stabilization effectiveness of the SIT monetary policy implies greater fluctuations in the return of capital bringing to higher variations in the entrepreneur's fractions. However, it is interesting to notice that even if the strict inflation targeting policy yields deeper fluctuations, these are less persistent. As explained, these results may be due to the aim of this specific monetary policy. In other words, the FIT policy replies strongly to the variables fluctuations generating greater variations on the risk free interest rate, continuously perturbing the investment decisions of the entrepreneurs through the expected return on investment and increasing the probability for them to switch to the other investment strategy. Therefore, this result offers some ground for fiscal policies—or other macroeconomic tools—with the aim to prompt system stability and to increase the speed of convergence to the steady state.

On the other side, the stabilizing effect of the monetary policy strongly depends on the nature of the forecasting rules. The scenarios assuming heuristic with greater computational ability present higher dependence to the objective pursued by the central bank. In these simulations, the system may converge to new steady state characterized by high/low inflation and to low/high levels of investment and output with wave of pessimist/optimist behaviors. Hence, this analysis seems to suggest a counterintuitive result in line with the "less-is-more" effect: it is not certain that when the decision-making and computational abilities of the agents increase the system converge to the preferable steady state. In other words, more information

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and computational capabilities may not improve accuracy in the forecasts allowing to the system to converge to less preferable equilibrium.

Moreover, introducing heuristics time varying shares both the spikes following the negative output shock and the convergence to the steady state are lower. In this case, when agents can update their belief using the new information and switch to the better strategies two contrasting dynamics emerge. Indeed, the agents are able to reduce the collapse of the economy diminishing their error through the updating and switching mechanism, and therefore moving to better performing behaviors. However, these continuous updating and switching processes produce fluctuations, which increase the persistence of the oscillations and reduce the convergence speed to the steady state.

Concluding, it seems that central banks should take seriously into account the heterogeneity and the bounded rationality of the agents when designing monetary policy. Moreover, the monetary policy alone may not be enough to stabilize the system and some other interventions may be necessary. Future work should follow two dimensions: first, it should further investigate the effect of higher degree of heterogeneity on the dynamics of aggregate output and inflation considering not only biased entrepreneurs but investors following different heuristics; second, further analyses should explore other possible tools that policy authorities may adopt to prompt the convergence to their preferred steady state and to reduce the macroeconomic fluctuations.

NOTES

1. See, for example, Greenwald and Stiglitz (1993), Kiyotaki and Moore (1997), Bernanke et al. (1999), Gertler and Karadi (2011), Curdia and Woodford (2010), or Iacoviello and Neri (2010).

2. See, for example, Bernanke and Gertler (1989), Townsend (1979), or Carlstrom and Fuerst (1997).

3. See, for example, Brock and Hommes (1997, 1998) or Branch and McGough (2009) for models with bounded rational agents.

4. See, for example, Conlisk (1980), Kirman (1992), Duffy (2006), Delli Gatti et al. (2010a), Hendry and Mizon (2010), or Coibion and Gorodnichenko (2012, 2015).

5. For supporting results in empirical analyses, see Frankel and Froot (1986, 1987a,b and 1990a,b), Shiller (1981, 1999), or Akerlof and Shiller (2009). For experimental analyses see Smith et al. (1988) or Hommes (2011).

6. Pfajfar and Zakelj (2011) or Burke and Manz (2011) support this assumption with both empirical and experimental analyses.

7. See Popoyan et al. (2017) for an interesting analysis investigating the impact on macroeconomic dynamics of alternative macroprudential regulations and monetary policy rules.

8. For an exhaustive overview, see Delli Gatti et al. (2010b), Battiston et al. (2012), Dosi et al. (2013, 2015), and Fagiolo and Roventini (2017).

9. See, for example, Ashraf et al. (2011), Kobayashi et al. (2012), and Kobayashi and Shirai (2016) for exploratory analyses on the role of banks in an evolving credit network with financial constrains.

10. See, for example, Carroll (2003), Branch (2004), or Pfajfar and Santoro (2010).

11. For some examples in the growing literature on bounded rationality and cognitive limitation, see Hommes and Zhu (2014), Hommes et al. (2005, 2007), or Branch and Evans (2005).

12. To see how the entrepreneur is constrained by increasing the invested capital, i.e., the size of the firm, it should be considered that the maximization problem imposes a trade-off between the size

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and the net investment. Indeed, increasing the size of the firm enhances the share of firm's capital investment financed by external funds negatively affecting the investment return.

13. See, for example, Assenza et al. (2013).

14. See Hommes et al. (2012) for a thorough discussion on how memory affects the dynamics of the system.

15. See Kahneman and Tversky (1973) for a thorough dissertation of this rule.

16. See Clarida et al. (2000) or Galì (2008).

17. In other words, we are assuming that at the beginning of the period these firms sell capital at price Q_t and at the end of period they purchase the undepreciated invested capital of the entrepreneurs at price Q_{t+1}, clearing the market.

18. The performance measure defined before is not the only possible measure, but it has the advantages to be clear and to avoid that the choice of switching is exclusively determined by the firm specific shock, hence, to be randomly defined. Alternative performance measures could be the difference between the firm specific investment return and the average of the market or a weighted sum between the average investment performance of the class and the just defined difference, but it adds more complexity to the system and its effect is left to future possible extensions.

19. $r_t^n = \phi_\pi \pi_t + \phi_y y_t$, with $\phi_\pi = 1.5$ and $\phi_y = (0; 1)$.

20. See Guerini et al. (2016) for a comparable analysis in a DSGE-ABM framework.

21. See Romer (2012).

REFERENCES

Akerlof, G. J. and R. J. Shiller (2009) *Animal Spirits: How Human Psychology Drives the Economy, and Why it Matters for Global Capitalism*. Princeton: Princeton University Press.

Anufriev, M. and C. Hommes (2012) Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments. *American Economic Journal: Microeconomics* 4(4), 35–64.

Anufriev, M., T. Assenza, C. Hommes, and D. Massaro (2013) Interest rate rules and macroeconomic stability under heterogeneous expectations. *Macroeconomic Dynamics* 17(8), 1574–1604.

Ashraf, Q., B. Gershman, and P. Howitt (2011) Banks, Market Organization, and Macroeconomic Performance: An Agent-based Computational Analysis. Working paper 17102, National Bureau of Economic Research.

Assenza, T., P. Heemeijer, C. Hommes, and D. Massaro (2013) Individual Expectations and Aggregate Macro Behavior. Tinbergen Institute discussion paper 13-016/II.

Battiston, S., D. Delli Gatti, M. Gallegati, B. Greenwald, and J. Stiglitz (2012) Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control* 36(8), 1121–1141.

Bernanke, B. and M. Gertler (1989) Agency costs, net worth and business fluctuations. *American Economic Review* 79(1), 14–31.

Bernanke, B., M. Gertler, and S. Gilchrist (1999) The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, vol. 1, chapter 21, pp. 1341–1393. Amsterdam: Elsevier, North-Holland.

Branch, W. A. (2004) The theory of rationally heterogeneous expectations: Evidence form survey data on inflation expectations. *Economic Journal* 114(497), 592–621.

Branch, W. A. and G. W. Evans (2005) Model uncertainty and endogenous volatility. In *Proceedings of Computing in Economics and Finance*, vol. 33, Society for Computational Economics.

Branch, W. A. and B. McGough (2009) A new Keynesian model with heterogeneous expectation. *Journal of Economic Dynamics and Control* 33(5), 1036–1051.

Brock, W. A. and C. Hommes (1997) A rational route to randomness. *Econometrica* 65(8), 1059–1095.

Brock, W. A. and C. Hommes (1998) Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control* 22(8), 1235–1274.

Burke, M. A. and M. Manz (2011) Economic Literacy and Inflation Expectations: Evidence from a Laboratory Experiment. Public Policy discussion paper 11-8, Federal Reserve Bank of Boston.

https://doi.org/10.1017/S1365100518000251 Published online by Cambridge University Press

Calvo, G. A. (1983) Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12(3), 383–398.
Carlstrom, C. T. and T. S. Fuerst (1997) Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review* 87(5), 893–910.
Carroll, C. D. (2003) Macroeconomic expectations of households and professional forecasters. *Quarterly Journal of Economics* 118(1), 269–298.
Clarida, R., J. Galì, and M. Gertler (1999) The Science of Monetary Policy: A New Keynesian Perspective. Technical report, National Bureau of Economic Research.
Clarida, R., J. Galì, and M. Gertler (2000) Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics* 115(1), 147–180.
Coibion, O. and Y. Gorodnichenko (2012) What can survey forecasts tell us about information rigidities? *Journal of Political Economy* 120(1), 116–159.
Coibion, O. and Y. Gorodnichenko (2015) Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review* 105, 2644–2678.
Conlisk, J. (1980) Costly optimizers versus cheap imitators. *Journal of Economic Behaviour & Organization* 1(3), 275–293.
Conlisk, J. (1996) Why bounded rationality? *Journal of Economic Literature* 34(2), 669–700.
Curdia, V. and M. Woodford (2010) Credit spreads and monetary policy. *Journal of Money, Credit and Banking* 42(1), 3–35.
De Grauwe, P. (2012) *Lectures on Behavioral Macroeconomics*. Princeton: Princeton University Press.
De Grauwe, P. and C. Macchiarelli (2015) Animal spirits and credit cycles. *Journal of Economic Dynamics and Control* 59, 95–117.
Delli Gatti, D., E. Gaffeo, and M. Gallegati (2010a) Complex agent-based macroeconomics: A manifesto for a new paradigm. *Journal of Economic Interaction and Coordination* 5(2), 111–35.
Delli Gatti, D., M. Gallegati, B. Greenwald, A. Russo, and J. Stiglitz (2010b) The financial accelerator in an evolving credit network. *Journal of Economic Dynamics and Control* 205(2), 459–468.
Diks, C. and R. van der Weide (2005) Herding, a-synchronous updating and heterogeneity in memory in a CBS. *Journal of Dynamics and Control* 29(4), 741–763.
Dosi, G., G. Fagiolo, M. Napoletano, and A. Roventini (2013) Income distribution, credit and fiscal policies in an agent-based Keynesian model. *Journal of Economic Dynamics and Control* 37(8), 1598–1625.
Dosi, G., G. Fagiolo, M. Napoletano, A. Roventini, and T. Treibich (2015) Fiscal and monetary policies in complex evolving economies. *Journal of Economic Dynamics and Control* 52, 166–189.
Duffy, J. (2006) Agent-based models and human subject experiments. In L. Tesfetsion and K. L. Judd (eds.), *Handbook of Computational Economics*, vol. 2, Agent-Based Computational Economics. Amsterdam: North-Holland.
Fagiolo, G. and A. Roventini (2017) Macroeconomic policy in DSGE and agent-based models redux: New developments and challenges ahead. *Journal of Artificial Societies and Social Simulation* 20(1), 1–37.
Frankel, J. and K. Froot (1986) Understanding the US dollar in the eighties: The expectations of chartists and fundamentalists. *Economic Record* (special issue), 24–38.
Frankel, J. and K. Froot (1987a) Short-term and long-term expectations of the yen/dollar exchange rate: Evidence from survey data. *Journal of the Japanese and International Economies* 1, 249–274.
Frankel, J. and K. Froot (1987b) Using survey data to test standard propositions regarding exchange rate expectations. *American Economic Review* 77, 133–153.
Frankel, J. and K. Froot (1990a) Chartist, fundamentalist and trading in the foreign exchange market. *American Economic Review* 80, 181–185.
Frankel, J. and K. Froot (1990b) Exchange Rate Forecasting Techniques, Survey Data and Implications for the Foreign Exchange Market. NBER working papers 3470, National Bureau of Economic Research.
Galì, J. (2008) *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton: Princeton University Press.

https://doi.org/10.1017/S1365100518000251 Published online by Cambridge University Press

Gertler, M. and P. Karadi (2011) A model of unconventional monetary policy. *Journal of Monetary Economics* 58(1), 17–34.
Gigerenzer, G. and H. Brighton (2009) Homo heuristicus: Why biased minds make better inferences. *Topics in Cognitive Science* 1, 107–143.
Greenwald, B. and Stiglitz, J. (1993) Financial market imperfections and business cycles. *Quarterly Journal of Economics* 108(1), 77–114.
Guerini, M., M. Napoletano, and A. Roventini (2016) No Man Is an Island: The Impact of Heterogeneity and Local Interactions on Macroeconomic Dynamics. Working paper series 2016/24, Laboratory of Economics and Management (LEM), Scuola Superiore Sant'Anna, Pisa, Italy.
Hendry, D. and G. Mizon (2010) On the Mathematical Basis of Inter-temporal Optimization. Discussion paper 497, University of Oxford Department of Economics.
Hommes, C. (2011) The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control* 35(1), 1–24.
Hommes, C. (2013) *Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems*. Cambridge: Cambridge University Press.
Hommes, C. and M. Zhu (2014) Behavioral learning equilibria. *Journal of Economic Theory* 150(C), 778–814.
Hommes, C., J. Sonnemans, J. Tuinstra, and H. van de Velden (2005) Coordination of expectations in asset pricing experiments. *Review of Financial Studies* 18(3), 955–980.
Hommes, C., J. Sonnemans, J. Tuinstra, and H. van de Velden (2007) Learning in cobweb experiments. *Macroeconomic Dynamics* 11(S1), 8–33.
Hommes, C., T. Kiseleva, Y. Kuznetsov, and M. Verbic (2012) Is more memory in evolutionary selection (de)stabilizing? *Macroeconomic Dynamics* 16(03), 335–357.
Iacoviello, M. and S. Neri (2010) Housing market spillover: evidence from an estimated DSGE model. *American Economic Journal: Macroeconomics* 2(2), 125–164.
Johansen, L. (1977) *Lectures on Macroeconomic Planning. Part 1. General Aspects*. Amsterdam: North-Holland.
Kahneman, D. and A. Tversky (1973) On the psychology of prediction. *Psychological Review* 80, 237–251.
Kahneman, D., J. L. Knetsch, and R. H. Thaler (1991) Anomalies: The endowment effect, loss aversion, and status quo bias. *Journal of Economic Perspectives* 5(1), 193–206.
Keynes, J. M. (1936) *The General Theory of Employment, Interest and Money*. London: Macmillan.
Kirman, A. (1992) Whom or what does the representative individual represent? *Journal of Economic Perspectives* 6(2), 117–136.
Kobayashi, K. and D. Shirai (2016) Heterogeneity and redistribution in financial crises. *Macroeconomic Dynamics* 20(6), 1527–1549.
Kobayashi, K., T. Nakajima, and M. Inaba (2012) Collateral constraint and news-driven cycles. *Macroeconomic Dynamics* 16(5), 752–776.
Kiyotaki, N. and J. Moore (1997) Credit cycles. *Journal of Political Economy* 105(2), 211–248.
Mankiw, N. G. (1989) Real Business Cycles: A New Keynesian Perspective. NBER working papers 2882, National Bureau of Economic Research.
Massaro, D. (2013) Heterogeneous expectations in monetary DSGE models. *Journal of Economic Dynamics and Control* 37(3), 680–692.
Pfajfar, D. and E. Santoro (2010) Heterogeneity, learning and information stickiness in inflation expectations. *Journal of Economic Behavior & Organization* 75(3), 426–444.
Pfajfar, D. and B. Zakelj (2011) Uncertainty and Disagreement in Forecasting Inflation: Evidence from the Laboratory (Replaced by CentrER DP 2012-072). Discussion paper 2011-053, Tilburg University, Center for Economic Research.
Popoyan, L., M. Napoletano, and A. Roventini (2017) Taming macroeconomic instability: Monetary and macro-prudential policy interactions in an agent-based model. *Journal of Economic Behavior & Organization* 134, 117–140.

https://doi.org/10.1017/S1365100518000251 Published online by Cambridge University Press

Romer, D. (2012) *Advanced Macroeconomics*. New York: McGraw-Hill.
Shiller, R. J. (1981) Do stock prices move too much to be justied by subsequent changes in dividends? *American Economic Review* 71(3), 421–436.
Shiller, R. J. (1999) Human behavior and the efficiency of the financial system. In J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, vol. 1(C), chapter 20, pp. 1305–1340. Amsterdam: Elsevier, North Holland.
Smith, V., G. L. Suchanek, and A. Williams (1988) Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica* 56(5), 1119–1151.
Townsend, R. M. (1979) Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory* 21(2), 265–293.
Woodford, M. (1990) Learning to believe in sunspots. *Econometrica* 58(2), 277–307.

APPENDIX A: THE COMPLETE LOG-LINEARIZED MODEL

This section presents the complete log-linearization of the model. Let lower case variables denote percent deviations from the steady state and the ratios among capital letters without time pedix denote the ratios of the respective steady state values, the log-linearization of the model is

$$b_t^h = n_{t-1}^h + \frac{1}{\frac{(RK \pm b - \chi R)}{2\mu R}} \left[\frac{Q\,(RK \pm b)}{2\mu R} \right] (q_{t-1} - r_{t-1}), \tag{A.1}$$

$$z_t^h = \frac{\chi R}{\chi R + \mu \frac{B^h R}{N^h}} r_{t-1} + \frac{\mu B^h R}{N^h \left(\chi R + \mu \frac{B^h R}{N^h} \right)} \left(b_t^h + r_{t-1} - n_{t-1}^h \right), \tag{A.2}$$

$$q_t = \varphi \left(k_t - k_{t-1} \right), \tag{A.3}$$

$$k_t^h = \frac{B^h}{\left(B^h + N^h \right)} b_t^h + \frac{N^h}{\left(B^h + N^h \right)} n_{t-1}^h - q_{t-1}, \tag{A.4}$$

$$k_t = \frac{s^o K^o}{\left(s^o K^o + s^p K^p \right)} \left(k_t^o + s_{t-1}^o \right) + \frac{s^p K^p}{\left(s^o K^o + s^p K^p \right)} \left(k_t^p + s_{t-1}^p \right), \tag{A.5}$$

$$r_t^k = \frac{1}{\frac{\alpha Y}{XKQ} + (1 - \delta)} \left[\frac{\alpha Y}{XKQ} \left(y_t - k_t - q_{t-1} \right) + (1 - \delta) \left(q_t - q_{t-1} \right) \right], \tag{A.6}$$

$$\begin{aligned} n_t^h = {} & \frac{\gamma R^k Q K^h}{\gamma \left(R^k Q K^h - Z^h B^h \right) + W^e} \left(r_t^k + q_{t-1} + k_t^h \right) \\ & - \frac{\gamma Z^h B^h}{\gamma \left(R^k Q K^h - Z^h B^h \right) + W^e} \left(z_t^h + b_t^h \right) + \cdots + \frac{W^e}{\gamma \left(R^k Q K^h - Z^h B^h \right) + W^e} w_t^e, \end{aligned} \tag{A.7}$$

$$U_{h,t}^a = \frac{R^k Q K^h}{R^k Q K^h - Z^h B^h} \left(r_t^k + q_{t-1} + k_t^h \right) - \frac{Z^h B^h}{R^k Q K^h - Z^h B^h} \left(z_t^h + b_t^h \right) - k_t^h - q_{t-1}, \tag{A.8}$$

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$$y_t = b_1 \{a_1 E_{t-1}(y_t) + a_2 [r_t - E_{t-1}(\pi_t)]\} + b_2 k_t, \quad \textbf{(A.9)}$$

$$\pi_t = f_1 E_{t-1}(\pi_t) + f_2 y_t + \varepsilon_t^{\pi}, \quad \textbf{(A.10)}$$

$$r_t^n = \rho r_{t-1}^n + (1+\rho)(\tau\pi_t + \iota y_t) + \varepsilon_t^r. \quad \textbf{(A.11)}$$

Equation (A.1) represents the log-linearized version of the solution of the entrepreneur's maximization problem. Equation (A2) is the log-linearized interest rate on loans representing the positive influence of the leverage ratio on the external finance cost and defines the financial accelerator. Equations (A.3)–(A.6) characterize the investment demand. Equations (A.3) and (A.6) are the standard relations for marginal product of capital and the link between asset price and investment. Equation (5) represents the total invested capital, which is the weighted sum of purchased capital of the two fractions, described by equation (A.5).

The evolution of the entrepreneur's net worth is described by equation (A.7), this affects the investment decision affecting the cost of loans and their required amount to the financial intermediary. Equation (A.8) is the log-linearized version of the performance measure, the return on investment. Finally equations (A.9)–(A.11) are conventional for the NK framework.[21] Equations (A.10) and (A.11) impose exogenous shocks on the inflation and interest rates. The monetary policy rule expressed by equation (A.11) is the more general as possible in order to leave some degrees of freedom for the dynamic analyses.

APPENDIX B: STATISTICS

TABLE B.1. Averages and standard deviations, in brackets, of the simulations considering different policies and heuristics (time space 280–500)

Scenario	Heuristic	Y	π	K	Share of pessimist
$\phi_\pi = 1.5,\ \phi_y = 0$	Naive	−0.0224	−0.0100	−0.0109	0.4778
		(0.1012)	(0.0194)	(0.0904)	(0.0370)
$\phi_\pi = 1.5,\ \phi_y = 1$	Naive	−0.0189	−0.0051	0.0015	0.4620
		(0.0450)	(0.0056)	(0.0091)	(0.0556)
$\phi_\pi = 1.5,\ \phi_y = 0$	Chartist	−0.0224	−0.0100	−0.0109	0.4776
		(0.0898)	(0.0210)	(0.0761)	(0.0380)
$\phi_\pi = 1.5,\ \phi_y = 1$	Chartist	−0.0189	−0.0051	0.0015	0.4621
		(0.0444)	(0.0072)	(0.0080)	(0.0577)
$\phi_\pi = 1.5,\ \phi_y = 0$	LAA rule	0.0177	−0.0139	0.0337	0.4725
		(0.0780)	(0.0038)	(0.0730)	(0.0029)
$\phi_\pi = 1.5,\ \phi_y = 1$	LAA rule	−0.3065	0.5102	−0.6587	0.8374
		(0.0400)	(0.0052)	(0.0143)	(0210)
$\phi_\pi = 1.5,\ \phi_y = 0$	Ecology	−0.0554	−0.0034	−0.0515	0.4923
		(0.1114)	(0.0053)	(0.1033)	(0.0099)
$\phi_\pi = 1.5,\ \phi_y = 1$	Ecology	−0.0206	$-6.2891e^{-04}$	−0.0041	0.4692
		(0.0442)	$(8.3391e^{-04})$	(0.0098)	(0.0510)

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