Robotica (2001) volume 19, pp. 641–648. Printed in the United Kingdom © 2001 Cambridge University Press

Sliding mode control with perturbation estimation: application on DD robot mechanism

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(Received in Final Form: May 30, 2001)

SUMMARY
In this work another perturbation estimation sliding mode based control algorithm is introduced for a class of robotic systems in the presence of structured and unstructured uncertainties and external disturbances. The effects of these uncertainties are combined into a single quantity. A full order device with the actuator voltages as control inputs is assumed in control design. The decentralized control scheme with only a partial state feedback is applied. A modification of the switching functions with perturbation estimation is introduced. The salient features of this approach is that the perturbations are effectively treated by a computationally straightforward procedure. The proposed controller is applied to a minimal configuration direct drive robot mechanism.

KEYWORDS: Sliding mode control; Perturbation estimation; Uncertainties; External disturbances; Control scheme.

1. INTRODUCTION

Trajectory tracking with a certain accuracy is a main task of robotic control. The control is often based on a mathematical model of the system. This model is never an exact representation of reality, since modeling errors are inevitable. Moreover, one can use a simplified model for control. In this paper, the structured and unstructured uncertainties are of primary interest, i.e. modeling errors due to parameter variations and unmodeled modes, especially the friction and sensor dynamics, neglected time delays, etc.

The faulty model and the demand for high performance require the controller to be robust. Sliding mode controllers (SMC) based on variable structure control can be used if the inaccuracies in the model structure are bounded with known bounds. However, a SMC has some disadvantages, related to chattering of the control input signal. Often this phenomenon is undesirable, since it causes excessive control action leading to increased wear of the actuators and to excitation of unmodeled dynamics.

The attempts to attenuate this undesirable effect result in the deterioration of the robustness characteristics. This is a well-known problem and widely treated in the literature. In order to obtain smoothing in the bang-bang typed discontinuities of the sliding mode controller, different schemes have been suggested. Slotine and Sastry[1] introduced a boundary layer around the sliding surface and used a continuous control within the boundary layer. Burton and Zinober[2] also considered the implications of a continuous approximation of the discontinuous VSS control law within a predefined subspace. Elmali and Olgac[3] and Jezernik and Curk[4] developed perturbation estimation schemes, Kaynak and Denker[5] suggested to supplement the control input with a predictive correction term. Recently, fuzzy logic smoothing algorithms have received increased popularity.[6]

Another important issue limiting the practical applicability of SMC is the overconservative control law due to the upper bounds of the uncertainties. In practice, most often the worse case implemented in control law does not take place and the resulting large control inputs become unnecessary and uneconomical.

In this paper we suggest an approach to the design of decentralized motion controllers for mechatronics systems besides the sliding mode motion controller structure and disturbance torque estimation. The accuracy of the estimation is the critical parameter for robustness in this scheme, as opposed to the upper bounds of the perturbations themselves. Consequently, the driving terms of the error dynamics are reduced from the uncertainties (as in the conventional SMC) to the accuracy in their estimates. The result is a much better tracking accuracy without being over conservative in control.

2. PRELIMINARIES

First, the design of the control system will be demonstrated for an uncertain nonlinear system presentable in the form:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x}) + (\mathbf{B}(\mathbf{x}) + \Delta\mathbf{B}(\mathbf{x}))\mathbf{u} + \mathbf{d}(\mathbf{x}, t), \tag{1}$$

where the state vector $\mathbf{x} \in R^n$, the control vector $\mathbf{u} \in R^m$, $\mathbf{f}(\mathbf{x})$ and $\Delta\mathbf{f}(\mathbf{x}) \in R^n$, $\mathbf{B}(\mathbf{x})$ and $\Delta\mathbf{B}(\mathbf{x}) \in R^{n\times m}$. $\mathbf{f}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ represent known nominal models, $\Delta\mathbf{f}(\mathbf{x})$ and $\Delta\mathbf{B}(\mathbf{x})$ are uncertain perturbations. $\mathbf{d}(\mathbf{x}, t)$ represents the effect of unmodeled dynamics and external disturbances. Assuming the satisfaction of the matching conditions:[7]

$$\begin{aligned} \Delta\mathbf{f}(\mathbf{x}) &= \mathbf{B}(\mathbf{x})\Delta\mathbf{f}'(\mathbf{x}) \\ \Delta\mathbf{B}(\mathbf{x}) &= \mathbf{B}(\mathbf{x})\Delta\mathbf{B}'(\mathbf{x}) \\ \mathbf{d}(\mathbf{x}, t) &= \mathbf{B}(\mathbf{x})\mathbf{d}'(\mathbf{x}, t) \end{aligned} \tag{2}$$

for certain $\Delta\mathbf{f}'(\mathbf{x})$, $\Delta\mathbf{B}'(\mathbf{x})$ and $\mathbf{d}'(\mathbf{x}, t)$ it is possible to lump the total plant uncertainty into a single vector $\mathbf{w}(\mathbf{x}, t)$ and represent the uncertain system in form:

https://doi.org/10.1017/S0263574701003629 Published online by Cambridge University Press

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} + \mathbf{B}(\mathbf{x})\mathbf{w}(\mathbf{x}, t). \tag{3}$$

The goal of the sliding mode control synthesis is to find the control $\boldsymbol{u}$ such that the motion of the system (3) is restricted to belong to the sliding mode manifold $\boldsymbol{S}$.

$$\mathbf{S} = \{\mathbf{x}: \sigma(\mathbf{x}, t) = \mathbf{0}\}, \quad \sigma = (\sigma_1, \sigma_2, \ldots, \rho_m)^T, \tag{4}$$

where $\sigma_i(\mathbf{x}, t)$, $i = 1, \ldots, m$ are continuous switching functions also continuously differentiable with respect to time. Equations $\sigma_i(\mathbf{x}, t) = 0$, $i = 1, \ldots, m$ are so-called switching surfaces. In the theory of the VSS with sliding modes it is proven that the closed-loop behavior of the system (3) in the manifold (4) is determined by the selection of the manifold.[8]

The condition under which the state will move toward and reach the manifold $\boldsymbol{S}$ is known as the reaching condition. By choosing the Lyapunov function candidate:

$$V(\mathbf{x}, t) = \sigma^T \sigma/2 \tag{5}$$

a global reaching condition is given by $\dot{V}(\mathbf{x}, t) < 0$ when $\sigma \neq \mathbf{0}$ or equivalent for (5): $\sigma^T \dot{\sigma} < 0$. Various choices of time derivative of the Lyapunov function candidate $\dot{V}$ lead to different rates of change for σ. For example, $\dot{V}(\mathbf{x}, t) = -\sigma^T \mathbf{K}\,\mathbf{sig}(\sigma)$, $\mathbf{sig}(\sigma) = (\mathbf{sig}\ \sigma_1, \mathbf{sig}\ \sigma_2, \ldots, \mathbf{sig}\ \sigma_m)^T$ leads to a constant rate reaching law

$$\dot{\sigma} = -\mathbf{K}\,\mathbf{sig}\,(\sigma). \tag{6}$$

With choosing $\dot{V}(\mathbf{x}, t) = -\sigma^T \mathbf{D}\sigma$ the proportional rate is selected

$$\dot{\sigma} = -\mathbf{D}\sigma \tag{7}$$

provided that $\boldsymbol{K}$ and $\boldsymbol{D}$ are positive definite matrices.

The control law can be calculated directly from the required form of $\dot{V}$. For the system (3) the control law is given by (8). It is assumed that the product $\partial\sigma/\partial\mathbf{x}\ B(\mathbf{x})$ is nonsingular for all x

$$\mathbf{u} = \mathbf{u}_{eq} + \Delta\mathbf{u}. \tag{8}$$

The equivalent control $\boldsymbol{u}_{eq}$ assures a tangential motion of the system according to sliding manifold. The reaching part $\Delta\mathbf{u}$ assures a reaching of the sliding mode in finite time. Equivalent control $\boldsymbol{u}_{eq}$ can be decomposed in two parts: nominal part $\mathbf{u}_{nom}$ and disturbance compensation part $\mathbf{u}_{cmp}$.

$$\mathbf{u}_{eq} = \mathbf{u}_{nom} + \mathbf{u}_{cmp}. \tag{9}$$

The nominal part of equivalent control depends on known nominal part of system (3) and from selected switching functions:

$$\mathbf{u}_{nom} = -\left(\frac{\partial\sigma}{\partial\mathbf{x}}\mathbf{B}(\mathbf{x})\right)^{-1}\left(\frac{\partial\sigma}{\partial\mathbf{x}}\mathbf{f}(\mathbf{x}) + \frac{\partial\sigma}{\partial t}\right). \tag{10}$$

The disturbance compensation part $\boldsymbol{u}_{cmp}$ of equivalent control depends on total plant uncertainties:

$$\mathbf{u}_{cmp} = -\mathbf{w}(\mathbf{x}, t). \tag{11}$$

The reaching part of control vector $\Delta\mathbf{u}$ depends on the chosen reaching law. For a constant rate reaching law (7) this control part is discontinuous:

$$\Delta\mathbf{u} = -\left(\frac{\partial\sigma}{\partial\mathbf{x}}\mathbf{B}(\mathbf{x})\right)^{-1}\mathbf{K}\,\mathbf{sig}(\sigma), \tag{12}$$

or for a proportional rate reaching law (7) this control part is continuous:

$$\Delta\mathbf{u} = -\left(\frac{\partial\sigma}{\partial\mathbf{x}}\mathbf{B}(\mathbf{x})\right)^{-1}\mathbf{D}\sigma. \tag{13}$$

Obviously, the disturbance compensation part $\boldsymbol{u}_{cmp}$ of equivalent control cannot be applied in the form (11) because the perturbation vector $\boldsymbol{w}(\boldsymbol{x}, t)$ is unknown by assumption. If system perturbation is measurable, then a suitable $\boldsymbol{u}_{cmp}$ can be determined to provide a proper corrective action. However, direct measurements of the perturbation in any system are difficult to obtain.

Traditionally, robustness to the uncertainty is achieved by using of discontinuous reaching part sufficiently large regarding the bounds of $\|\boldsymbol{w}(\boldsymbol{x}, t)\|$ to ensure that the dynamics of the sliding mode converges to zero. The control law (8) with (12) is discontinuous across the switching surfaces, and can be written in the classical form:[8]

$$\mathbf{u} = \begin{cases} \mathbf{u}^+(\mathbf{x}, t) & \text{if} \quad \sigma(\mathbf{x}, t) > \mathbf{0} \\ \mathbf{u}^-(\mathbf{x}, t) & \text{if} \quad \sigma(\mathbf{x}, t) < \mathbf{0}. \end{cases} \tag{14}$$

The sliding mode exists if $\boldsymbol{u}_{eq}$ is unique and if reaching conditions (15) are satisfied.

$$\mathbf{u}^+(\mathbf{x}, t) < \mathbf{u}_{eq} < \mathbf{u}^-(\mathbf{x}, t). \tag{15}$$

The applied switching control is merely dependent on the application device. In some applications, for example, switching converters related systems for the on-off operation is a way of life. On the other hand, some systems require smooth control signals to avoid an excitation of chattering.

$$\mathbf{u}_{cmp} = -\hat{\mathbf{w}}(\mathbf{x}, t). \tag{16}$$

Using an estimation of the perturbation vector (16) in the discontinuous control law, the amplitude of the discontinuous part of the control signal can be reduced according to the reduction of the upper bounds of the disturbance estimation error $\|\Delta\mathbf{w}(\mathbf{x}, t)\| = \|\mathbf{w}(\mathbf{x}, t) - \hat{\mathbf{w}}(\mathbf{x}, t)\| < \|\mathbf{w}(\mathbf{x}, t)\|$.

In this case the reaching conditions for the system (3) with control (8), (12) and (16) can be expressed in the form (17). For Lyapunov function candidate (5) the time derivative is:

$$\dot{V}(\mathbf{x}, t) = -\sigma^T \mathbf{K}\,\mathbf{sig}\,(\sigma) + \sigma^T \frac{\partial\sigma}{\partial\mathbf{x}}\mathbf{B}(\mathbf{x})\Delta\mathbf{w}(\mathbf{x}, t). \tag{17}$$

Stability of the sliding mode on $\boldsymbol{S}$ is guaranteed if $\boldsymbol{K}$ covers the contribution of the disturbance estimation error $\partial\sigma/\partial\mathbf{x}\ \mathbf{B}(\mathbf{x})\Delta\mathbf{w}(\mathbf{x}, t)$. Note that $\boldsymbol{K}$ can be time and/or state dependent.

On the other hand, if the switching law cannot be implemented, as in the case of continuous control law (13),

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then the reaching condition for the system (3) with control (8), (13) and (16) can be expressed in the form (18). In this case for a Lyapunov function candidate (5) the time derivative is:

$$\dot{V}(\mathbf{x}, t) = -\sigma^T \mathbf{D}\sigma + \sigma^T \frac{\partial \sigma}{\partial \mathbf{x}} \mathbf{B}(\mathbf{x}) \Delta \mathbf{w}(\mathbf{x}, t). \quad (18)$$

There are evidently stability problems in the vicinity of $\sigma = \mathbf{0}$. Consequently, the disturbance estimation algorithm in the continuous sliding mode control must satisfy a much rigorous condition: convergence of disturbance estimation error $\Delta\mathbf{w}(\mathbf{x}, t)$ to zero before the system reaches a sliding mode manifold. This condition is hard to fulfil. Simplification is given by extension of the term sliding mode. Similar to the definition adopted by Sabanovic[9] any motion of the system (1) that occurs in the ε-vicinity of the manifold (4) on which the sliding mode motion can exist with control (8), (13), will be referred as the quasi sliding mode motion. This reflects the essence of the continuous sliding mode design method. The control is selected to ensure that the reaching time from any initial condition to the ε-vicinity of the manifold $\boldsymbol{S}$ is finite. In this framework the design of the control system should ensure stability of the sliding mode manifold $\boldsymbol{S}$, while the selection of the manifold will provide the means to determine the equations of motion.

Above considerations have been applicable to continuous time systems. Transition to discrete-time systems is not so obvious as in other design procedures. Contrary to continuous time systems in discrete-time systems the sliding mode existence of perturbed systems does not require discontinuity of control.[10] That is an important fact, which allows chattering free sliding mode design.

The disturbance estimation algorithms can be classified into three groups:

- Filtering of the implemented discontinuous control signal. It is based on the proposition that in the sliding mode the average value of the control is the same as the equivalent control. The averaging is made by dynamic filtering.[11] The disturbance vector can then be calculated from equivalent control. From (9)

$$\hat{\mathbf{w}} = \mathbf{u}_{nom} - \hat{\mathbf{u}}_{eq}, \quad (19)$$

  where $\boldsymbol{u}_{nom}$ is a known nominal control and $\hat{\mathbf{u}}_{eq}$ is an estimated equivalent control. The disadvantage of this approach is in the reaching phase when the average control differs from the equivalent control.

- Calculation of the equivalent controls from the time derivative of switching functions and from the applied control vector can be used for continuous control. The time derivative of the switching function is given by:

$$\dot{\sigma} = \frac{\partial \sigma}{\partial \mathbf{x}} \dot{\mathbf{x}} + \frac{\partial \sigma}{\partial t}. \quad (20).$$

Putting the system equation (3) in (20), leads to the relation between equivalent control term, applied control vectors and time derivative of switching function.

$$-\left(\frac{\partial \sigma}{\partial \mathbf{x}} \mathbf{B}(\mathbf{x})\right)^{-1} \dot{\sigma} = \mathbf{u}_{eq} - \mathbf{u}. \quad (21)$$

This algebraic relation is valid in any time and does not depend on the system mode. Suppose the control law:

$$\mathbf{u}_c = \mathbf{u}_{eq} + \Delta\mathbf{u}, \quad (22)$$

where $\mathbf{u}^c$ represents the necessary control which satisfies a chosen reaching condition. Insertion of (21) in (22) leads to:

$$\mathbf{u}_c = \mathbf{u} + \Delta\mathbf{u} - \left(\frac{\partial \sigma}{\partial \mathbf{x}} \mathbf{B}(\mathbf{x})\right)^{-1} \dot{\sigma}. \quad (23)$$

This relation is of recursive form and can be implemented in discrete time control systems:[9]

$$\mathbf{u}(kT+T) = \mathbf{u}(kT) + \Delta\mathbf{u}(kT) - \left(\frac{\partial \sigma}{\partial \mathbf{x}} \mathbf{B}(\mathbf{x})\right)^{-1} \dot{\sigma}(kT). \quad (24)$$

This powerful approach often requires an additional approximation in the calculation of time derivation of the switching function and the stability of a closed loop system is hard to prove.

- Observer based disturbance estimation algorithms are often used for model reference control systems. Various nonlinear and linear techniques are also used as well as the sliding mode technique.[12]

  In this paper another approach for disturbance estimation is now proposed. In this approach; the disturbance estimator is incorporated into the modified switching function calculation and control task, decay into two steps. First, reaching the sliding mode on a modified sliding surface; and, second, asymptotic convergence to the original sliding manifold, where the system has given dynamics.

3. CONTROL OBJECT

An *n*-DOF robotic manipulator fully actuated by electrical machines, is assumed as a control plant. A mechanical system can always be modeled in the form (1) and it always satisfies the matching condition (2). Thus the dynamic model of the robot mechanism in the joint space can be put into the form

$$\bar{\mathbf{J}}(\mathbf{q})\ddot{\mathbf{q}} = \tau + \mathbf{w}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t), \quad (25)$$

where $\mathbf{q} = [q_1, \ldots, q_n]^T$ is vector of joint generalized coordinates, $\tau = [\tau_1, \ldots, \tau_n]^T$ is drive torque vector, $\bar{\mathbf{J}}(\mathbf{q})$ is $[nxn]$ positive definite matrix with meaning of equivalent inertia. $\mathbf{w}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t) = [w_1(.), \ldots, w_n(.)]^T$ is the sum of the centrifugal, Coriolis, gravitational, friction forces and all perturbations and disturbances. Equation (25) is the "*reduce order*" dynamic model with the torque input.

The electrical machines are in most cases taken as the actuator in the robotic system. For all electrical machines the change of the torque generated by machine can be

https://doi.org/10.1017/S0263574701003629 Published online by Cambridge University Press

expressed as a first order differential equation of the following form for *i*-th actuator

$$\dot{\tau}_i = \beta_i(u_i - e_i), \tag{26}$$

where β_i is the gain that depends on the machine parameters as the air gap flux, u_i is the voltage input and e_i is the disturbance which represent the EMS and all voltage drops. The gain β_i could be in most cases accepted as a constant. Taking into account the above equation the plant description may be arranged to make 3*n* order dynamical system with voltages as the control inputs. This description will be referred as "*full order*" and may be given by

$$\frac{d}{dt}\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\\ \tau\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}_{n\times n} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{J}}^{-1}(\mathbf{q})\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\\ \tau\end{bmatrix}$$

$$+\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \beta\end{bmatrix}\mathbf{u} + \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \bar{\mathbf{J}}^{-1}(\mathbf{q}) & \mathbf{0}\\ \mathbf{0} & -\beta\end{bmatrix}\begin{bmatrix}\mathbf{w}\\ \mathbf{e}\end{bmatrix}, \tag{27}$$

where β is *nxn* diagonal matrix $\beta = \text{diag}\,(\beta_i)$, $\mathbf{u} = [u_1, \ldots, u_n]^T$ is control input and $\mathbf{e} = [e_1, \ldots, e_n]^T$ is voltage disturbance vector. If the effective inertia matrix $\bar{\mathbf{J}}(\mathbf{q})$ is chosen as a diagonal matrix $\bar{\mathbf{J}}(\mathbf{q}) = \text{diag}\,(\bar{J}_i(\mathbf{q}))$, thus all interconnected terms are included in $\mathbf{w}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, t)$, then the system (27) may be presented as a block structured of all blocks being of the third order. Each block represents the dynamics of the *i*-th subsystem corresponding to the *i*-th DOF of the whole system (27).

The control plant can be represented in decentralized structure

$$\dot{\mathbf{x}}_i = \mathbf{A}_i(\mathbf{x})\mathbf{x}_i + \mathbf{b}_i u_i + \mathbf{d}_i(\mathbf{x}, t), \tag{28}$$

where $\mathbf{x} = [\mathbf{x}_1, \ldots, \mathbf{x}_n]^T$ is the whole state vector of dimension 3*n* and $\mathbf{x}_i = [q_i, \dot{q}_i, \tau_i]$ is the state vector of *i*-th subsystem, u_i is the scalar input voltage and $\mathbf{A}_i$, $\mathbf{b}_i$ and $\mathbf{d}_i$ are

$$\mathbf{A}_i(\mathbf{x}) = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & \bar{J}_i^{-1}(\mathbf{q})\\ 0 & 0 & 0\end{bmatrix}, \mathbf{b}_i = \begin{bmatrix}0\\ 0\\ \beta_i\end{bmatrix}, \mathbf{d}_i = \begin{bmatrix}0\\ \bar{J}_i^{-1}(\mathbf{q})w_i\\ -\beta_i e_i\end{bmatrix},$$

where $\bar{J}_i(\mathbf{q})$ is a scalar positive definite function corresponding to *i*-th diagonal element of matrix $\bar{\mathbf{J}}(\mathbf{q})$ and $\bar{J}_i^{-1}(\mathbf{q})$ is its inverse. Note that the full order model with voltages of electrical machines as plant inputs (in most cases generated by switching converters) requires a discontinuous control approach as distinguished from the reduce order model with torque inputs, where the control must be continuous.

4. DECENTRALIZED CONTROL DESIGN

In order to obtain smooth mechanical motion of the mechanism continuous desired trajectories with values q_i^d, $\dot{q}_i^d$ and $\ddot{q}_i^d$ are considered.

The goal of the control design is to find a discontinuous control:

$$u_i = \begin{cases} u_i^+(\mathbf{x}, t) & \text{if} \quad \sigma_i(\mathbf{x}, t) > 0\\ u_i^-(\mathbf{x}, t) & \text{if} \quad \sigma_i(\mathbf{x}, t) < 0\end{cases} \quad i = 1, \ldots, n. \tag{29}$$

The switching function, which determines the mechanical motion, is chosen to be of the second order and a function of angular position, velocity and acceleration errors:

$$\sigma_i = \ddot{q}_i^c - \ddot{q}_i, \quad i = 1, \ldots, n, \tag{30}$$

where $\ddot{q}_i^c$ is a calculated acceleration composed of desired and measured signals:

$$\ddot{q}_i^c = \ddot{q}_i^d + K_{vi}(\dot{q}_i^d - \dot{q}_i) + K_{pi}(q_i^d - q_i), \quad i = 1, \ldots, n. \tag{31}$$

K_{vi} and K_{pi} are control design parameters that in the sliding mode ($\sigma_i = 0$) determine the damping and the maximum frequency of the decentralized prescribed dynamics of second order and can be chosen directly, or may be assigned, using Ackermann's formula.[13]

The sliding mode exists if $\boldsymbol{u}_{eq}$ is unique and if applied control functions u_i^+ and u_i^- satisfy reaching conditions

$$u_i^+(\mathbf{x}, t) < u_{ieq} < u_i^{-1}(\mathbf{x}, t), \quad i = 1, \ldots, n. \tag{32}$$

According to the so-called equivalent control method[8] a fictitious continuous control u_{ieq} is calculated as a solution of the system of the algebraic equation $d(\sigma(\mathbf{x}, \mathbf{u} = \mathbf{u}_{eq}))/dt = \mathbf{0}$ and denoted as the equivalent control.

In the case of the system (28) with switching functions (30), the corresponding equivalent controls can be represented in the form

$$u_{ieq} = e_i + \beta_i^{-1}\left(\frac{d}{dt}(\bar{J}_i\ddot{q}_i^c) + \dot{w}_i\right), \quad i = 1, \ldots, n. \tag{33}$$

The fulfilling of the reaching condition (32) depends on the bounds of the voltage disturbance e_i, desired trajectory and tracking error and on the time derivative of the torque disturbance $\dot{w}_i$ which can be large. Note that for the switching function calculation (30) a full state feedback is required.

5. PERTURBATION ESTIMATION

For the practical control implementation the measured quantities are the state variables q_i. The acceleration signal $\ddot{q}_i$ is not measurable and can be obtained by double differentiation of the angular position q_i, but it is contaminated by the measurement noise to such a degree that it can no longer be used.

Consequently, the acceleration signal $\ddot{q}_i$ needs to be replaced by an estimated value $\hat{\ddot{q}}_i$, which is obtained simply from the differential equation of motion.

$$\bar{J}_i(\mathbf{q})\hat{\ddot{q}}_i = \tau_i - \hat{w}_i, \tag{34}$$

where τ_i is the active measurable drive torque developed by the actuator and $\hat{w}_i$ is the estimate value of the load torque. The expression (34) is inserted into the control scheme (30) by replacing the real acceleration $\ddot{q}_i$ with an estimated value $\hat{\ddot{q}}_i$.

The consequence of this replacement is that the equivalent dynamics in the sliding mode is described by a modified switching function:

$$\sigma_i^e = \bar{J}_i\sigma_i + \Delta w_i, \tag{35}$$

where $\Delta w_i = w_i - \hat{w}_i$ is the perturbation estimation error. In the realization this modified switching function can be

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expressed as a difference between the reference and measured torque.

$$\sigma_i^e = \tau_i^c - \tau_i, \tag{36}$$

$$\tau_i^c = \bar{J}_i \ddot{q}_i^c + \hat{w}_i. \tag{37}$$

The reference torque (37) consists of a motion controller and perturbation estimation outputs.

The stability of the sliding mode is guaranteed if reaching conditions (32) with the equivalent control (38) are satisfied.

$$u_{ieq} = e_i + \beta_i^{-1}\left(\frac{d}{dt}(\bar{J}_i \ddot{q}_i^c) + \hat{w}_i\right), \quad i = 1, \ldots, n. \tag{38}$$

Obviously, time derivatives of the estimated perturbances $\hat{w}_i$ requires a much smaller amplitude of control signal than in the case of a full state feedback (33) and conditions (32) can be easier satisfied. On the other hand, in the case of bounded control inputs, faster reference trajectories and higher sliding mode dynamics can be applied without stability problems.

It is easy to see that $\sigma_i^e = 0$ and $\sigma_i = 0$ define the same sliding surface if $\Delta w_i = 0$. However, if Δw_i converges to 0, then the manifold $\mathbf{S}^e = \{(\mathbf{q}, \dot{\mathbf{q}}, \tau)$: $\sigma^e = \mathbf{0}\}$ reaches the manifold $\mathbf{S} = \{(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$: $\sigma = \mathbf{0}\}$. For $\sigma_i = 0$ the error dynamics can be expressed as:

$$\Delta\ddot{q}_i + K_{vi}\Delta\dot{q}_i + K_{pi}\Delta q_i = -\frac{1}{\bar{J}_i}\Delta w_i. \tag{39}$$

The equation (35) shows how important a good disturbance estimation algorithm is for closed-loop dynamics (39). There are many algorithms to estimate the disturbance w_i.

The first choice is a conventional Luenberger reduced order observer:

$$\dot{\hat{z}}_i = -\frac{h_i}{\bar{J}_i}(\hat{z}_i - h_i\dot{q}_i - \tau_i), \tag{40}$$

$$\hat{w}_i = -\hat{z}_i + h_i\dot{q}_i. \tag{41}$$

An improvement, called PI estimator, is proposed[4] to overcome the noisy signal τ_i in the estimator algorithm:

$$\hat{w}_i = -h_i\int_0^t \sigma_i dv, \tag{42}$$

where h_i is the design parameter.

Both algorithms (41) and (42) work well if the disturbances vary slowly ($dw_i/dt \approx 0$).[14] The dynamics of the disturbance error is given by:

$$\Delta\dot{w}_i = -\frac{h_i}{\bar{J}_i}\Delta w_i + \dot{w}_i \tag{43}$$

for the Luenberger observer and for the PI estimator if the sliding mode is guaranteed in σ_i^e.

In this paper we propose an improved estimator in the form:

$$\dot{\hat{w}}_i = -\frac{h_i}{K_i}(\sigma_i + \Gamma_i\dot{\sigma}_i), \tag{44}$$

where K_i is the *disturbance rejection factor* and can be chosen as the positive number and $\Gamma_i = (1 - K_i)/h_i$. The disturbance error dynamics of the estimator (44) is:

$$\Delta\dot{w}_i = -\frac{h_i}{\bar{J}_i}\Delta w_i + K_i\dot{w}_i. \tag{45}$$

The advantage of the proposed estimator is its ability to estimate the high frequency modes of w_i. This ability depends on the design coefficient K_i. In the Laplace domain (45) becomes:

$$\Delta w_i(p) = K_i\frac{p}{p + p_{0i}}w_i(p) \tag{46}$$

with $p_{0i} = h_i/\bar{J}_i$ and p is the Laplace variable. For $K = 1$ (46) is a high-pass filter with input w and output Δw. For $K < 1$ the estimator attenuates the high frequency contents too. In this case the error dynamics of i-th subsystem on $\sigma^e = 0$ can be expressed from (39) and (46) in the Laplace domain as:

$$\Delta q_i(p) = \frac{K_i p}{(p^2 + K_{vi}p + K_{pi})(p + p_{0i})}w_i(p). \tag{47}$$

The influence of disturbances is reduced through a disturbance rejection factor K_i at all frequencies. Figure 1 illustrates these facts for different choices of the parameter K_i.

Additionally, the control law linearizes the closed-loop behavior of position error dynamics. The position error behavior is described by the 3rd order linear relation (39) of which the transfer function is expressed by (47). The controller parameters K_i, K_{vi}, K_{pi} and p_{0i} are arbitrarily chosen and define the robust properties of the position error $\Delta q(p)$ considering perturbation $w(p)$ in the frequency domain. The resulted magnitude of $\Delta q(p)$ in the frequency domain consists of the disturbance estimator part and the

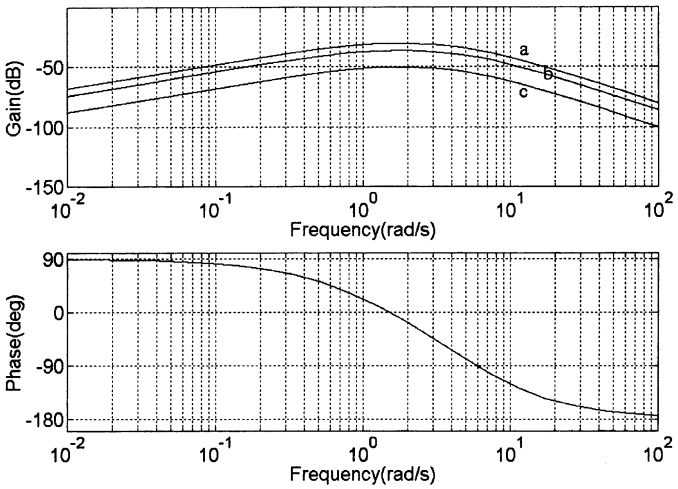


Fig. 1. Frequency characteristic of closed-loop system. (a) $K = 1$, (b) $K = 0.5$, (c) $K = 0.1$.

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feedback position controller part (47). The estimator suppresses perturbation as a high-pass filter in the desired low-pass frequency domain of the position variation with 10 dB/dec. High-pass frequency disturbances (noise, etc.) are rejected with 20 dB/dec by the position controller working as a low-pass filter. Consequently, excellent robust properties due to the system perturbations could be expected in the low-pass frequency domain, where with the choice for $K < 1$, the estimation effectively suppresses the influence of disturbances on the position error.

6. SIMULATION RESULTS

To illustrate previous results the SISO system described by model (48) is simulated for different values of the disturbance rejection factor K. The results are presented in Figures 2 and 3.

$$\ddot{x} = B(u - d)$$

$$B = \bar{B} + \Delta B = 3 + (4 + 2\cos(128.28t)) \tag{48}$$

$$d = \begin{cases} 5 & \text{for} \quad t \le 0.1 \\ 5 + 7.5(\cos(25.6t) + \cos(76.8t)) & \text{for} \quad t > 0.1 \end{cases}$$

System (48) is forced to track the desired trajectory $\ddot{x}^d$, $\dot{x}^d$, x^d with the prescribed dynamics $\sigma^e = \bar{B}\ddot{x}^c + \hat{w} = 0$ with $\ddot{x}^c = \ddot{x}^d + 10(\dot{x}^d - \dot{x}) + 25(x^d - x)$ and $\hat{w}$ is calculated by (44) and $p_0 = 1$. There is a large scale variation of the input parameter B around the nominal value $\bar{B} = 3$. The disturbance rejection factor K is chosen as $K = 1$, $K = 0.5$ and $K = 0.1$. Figure 2 shows the perturbation estimation error between perturbation $w(t) = d(t) + \Delta B\, u$ and estimated value $\hat{w}$ for different Ks. Figure 3 presents the dependence of the tracking error on the value of the disturbance rejection factor. It is evident that the tracking error rejection occurs for $K < 1$.

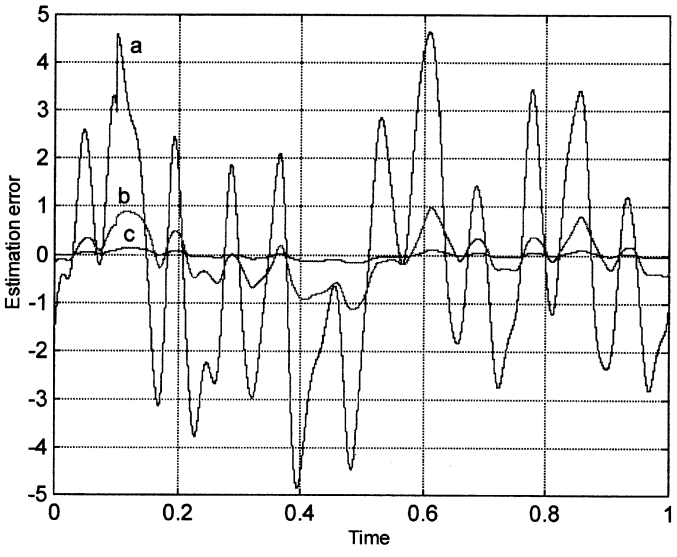


Fig. 2. Perturbation estimation error Δw, (a) $K = 1$, (b) $K = 0.5$, (c) $K = 0.1$.

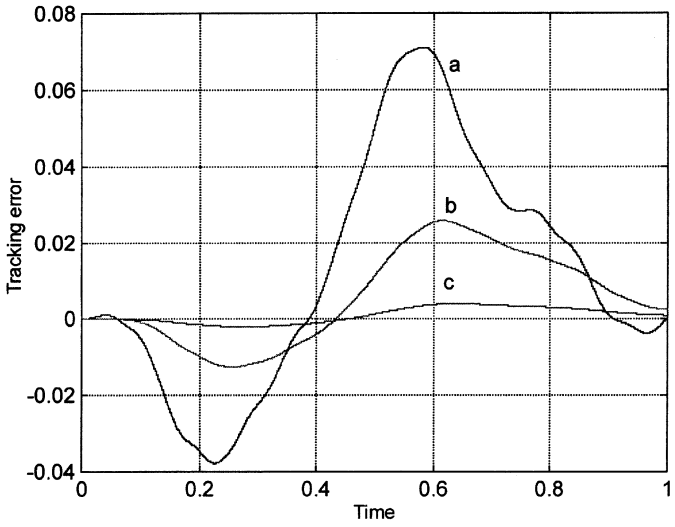


Fig. 3. Tracking error $x^d - x$ of system (48). (a) $K = 1$, (b) $K = 0.5$, (c) $K = 0.1$.

7. EXPERIMENTAL RESULTS

For the experiment we used a 3-DOF Puma like configuration direct drive robot[15], built at our institute (Figure 4). The robot is equipped with Dynaserv AC-motors produced by the Yokogawa company. The controller is based on eight transputer cards connected by high speed serial links in a multiprocessor transputer network. Three cards are used as joint controllers, one for the communication with user and the rest for interpolating and control algorithms. Communication with the user, programming and storing of the data is taking place on the personal computer also connected in the multiprocessor network. The sampling time of the whole system is 1 ms.

The placement of the design parameters (K, $h/\bar{J}$, K_p, K_v) is defined by the physical limitations of the system. The main limitation is maximal available torque, but also sampling frequency; measurement delay, quantized signals and nonlinear friction play a significant role. The test trajectory is PTP movement with $\sin^2$ acceleration profiles that is the same for all three joints. The desired trajectory with an end position of 0.5 rad in joint space is represented by Figure 5. In the control algorithm constant nominal values of inertia matrix were used. Parameters of the matrix were calculated from the dynamic model of the robot, analytically derived with the use of the Lagrangian equations. Differences between nominal inertia matrix and actual inertia matrix represent structured uncertainties. Unmodeled manipulator dynamics or unstructured uncertainties are gravitational forces, centrifugal forces, Coriolis forces, friction and coupling effects. Both structured and unstructured uncertainties are contained in the disturbance torque w. The disturbance torque for the second axis reaches the value that is seven times bigger than the estimated part of the model. For the first and third axis the disturbance torque mostly consists of friction force. The chosen equivalent system dynamics in the sliding mode is determined by the coefficients $K_{vi} = 28.28$, $K_{pi} = 200$, $i = 1, 2, 3$, and $\bar{J}_1 = 5$, $\bar{J}_2 = 2.4$, $\bar{J}_3 = 0.2$, $h_1 = 40$, $h_2 = 60$, $h_3 = 100$.

Figure 6 shows the robot tip error in the work space for $K = 1$ and $K = 0.5$. The improvement in the accuracy of the

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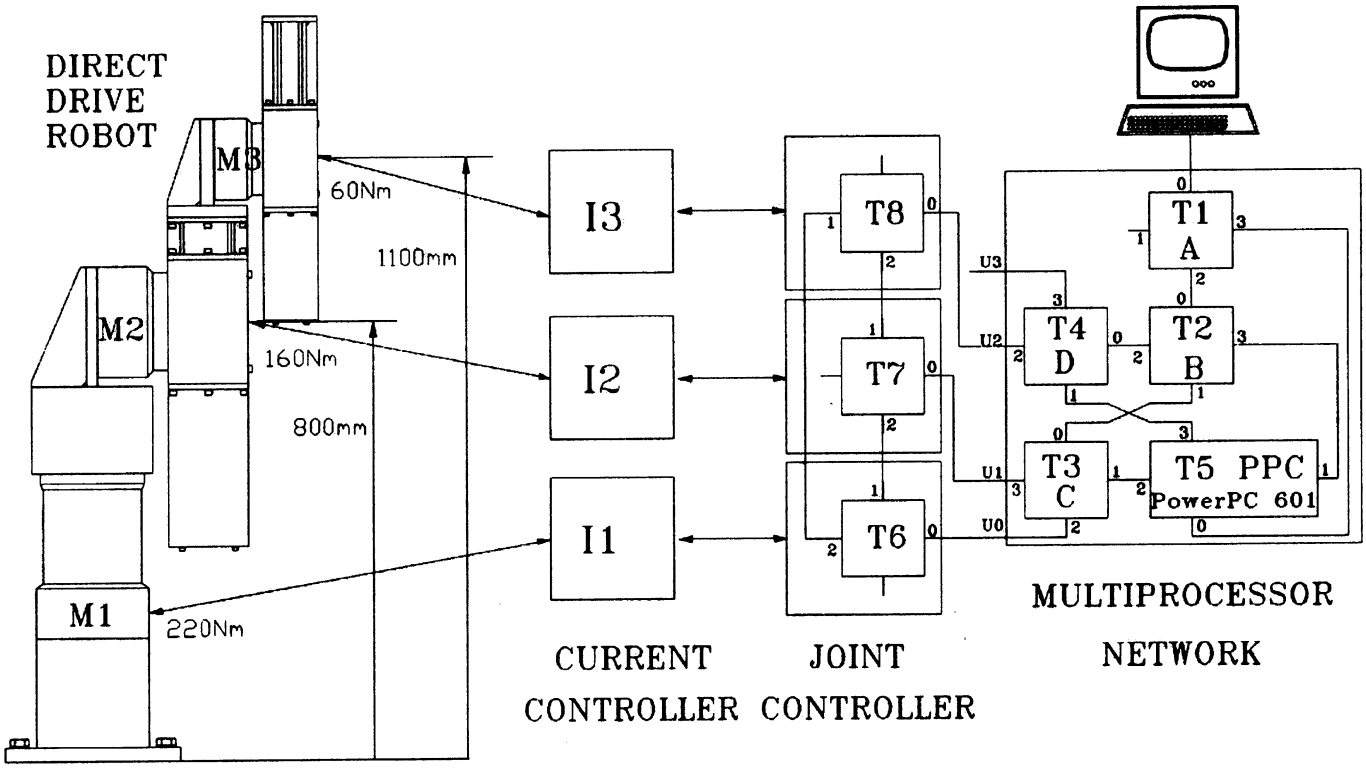


Fig. 4. The robot system.

trajectory tracking PTP movement with a new controller is obvious. At that point the main restriction for lowering the parameter K and consequently reaching better trajectory tracking accuracy is the quality of the measured signals. The construction of the motion controller results in zero steady state error and in a dynamic error that is asymptotically stable. The high-speed trajectory tracking test and the test of sudden load changes show improved performance too.

8. CONCLUSION

This paper introduces a new robust tracking control algorithm for a class of second order nonlinear motion control systems. The consolidation of the perturbation estimator into the sliding mode controller demonstrates significant performance improvement over the full state feedback form of the sliding mode control. The stability analysis shows that as long as the estimator stability is guaranteed, the controller can also be stable. A design procedure for a systematic formulation of the new routine is presented utilizing frequency domain techniques. Experiments on a 3-DOF Puma like DD robot confirm the superior performance of proposed sliding mode motion controller. The salient feature of the proposed approach is a computa-

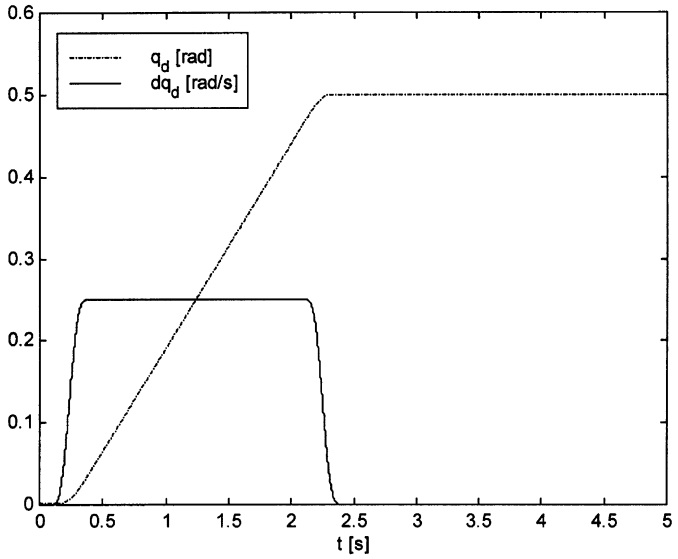


Fig. 5. Desired joint trajectories q_i^d and $\dot{q}_i^d$, $i=1, \ldots, 3$.

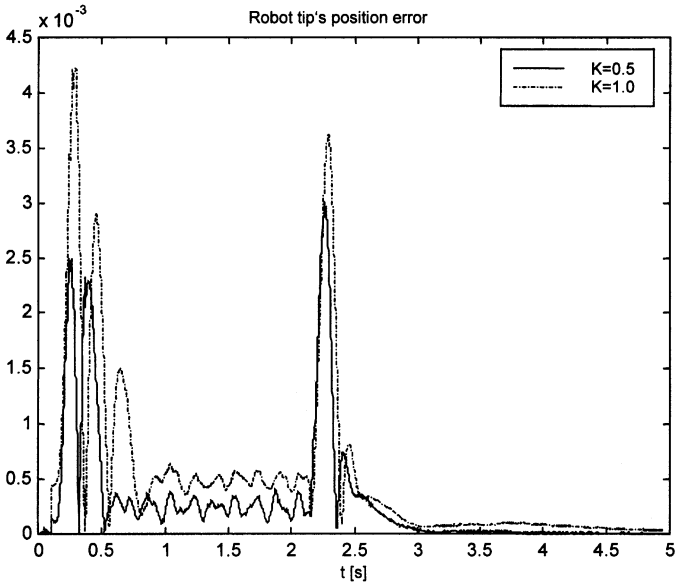


Fig. 6. Tracking error of robot tip in work space for $K=1.0$ and $K=0.5$.

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tionally straightforward procedure, which is easy to implement.

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