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Parametric study of repeatable gait for a planar five-link biped

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SUMMARY

Parametric formulation is a systematic method that ties the constraint functions to the resulting joint profiles for bipedal robots. Acceptable joint profiles for continuous and steady walking must satisfy the additional condition of being repeatable. This extra condition imposes restrictions on the selection of constraint functions, gait parameters and the initial posture of each step, and can make the synthesis of joint profiles extremely challenging. This paper presents a correlation between the gait parameters used in a class of specific constraint functions and the initial posture that will create repeatable gait. Furthermore, a general necessary and sufficient condition for repeatable gait is proposed. This condition provides a guideline for selecting constraint functions and their associated gait parameters in the context of producing repeatable gait. This research can not only provide valuable insights into bipedal walking, but also serve as a stepping stone for employing parametric formulation with various constraint functions.

KEYWORDS: Bipedal model; Repeatable gait; Parametric formulation; Joint analysis

1. INTRODUCTION

The design of joint profiles is a crucial step in the development of bipedal robots. However, there is a lack of systematic methods for synthesizing the joint profiles and most of the previous methods have been based on trial and error. Tzafestas *et al.*[1] developed a sliding mode control for the motion regulation of a five-link bipedal model walking on flat ground and climbing stairs. The joint profiles were synthesized based on the effects of gravity on the gait patterns and the requirement that the gait must be repeatable. Furusho and Masubuchi[2,3] developed numerical and experimental models for five-link bipeds. Their approach is based on linearizing the equations of motion around the vertical equilibrium point and applying proportional plus derivative feedback at the individual joints to track the prescribed joint profiles. However, a coherent method for specifying the joint profiles was never mentioned. Vukobratovic *et al.*[4] has studied locomotion directly. The fundamental aspect of their approach is the adaptation of human walking data to prescribe the motion of the lower limbs. Two immediate problems arise when using human walking data directly. First, a complex dynamic model is required. Secondly, the designer has no freedom to synthesize the joint profiles based on tangible gait characteristics, such as walking speed, step length or step elevation. None of the aforementioned approaches give a well-defined method for the design of joint profiles. Hurmuzlu[5,6] developed a parametric formulation that ties together the constraint functions and joint profiles. The constraint functions are cast in terms of coherent physical characteristics of gait. In light of prevailing locomotion literature, Humuzlu's method[5,6] fills the gap in the design of walking machines regarding the specification of constraint functions. He further found a correlation between the parameters used in the constraint functions and the outcome of the contact of the limb with the walking surface.

Hurmuzlu has applied his method to the design of joint profiles for a five-link bipedal model.[5,6] His five constraint functions lead to four algebraic equations and one first-order ordinary differential equation. Thus, once the initial angles of every step are selected, the joint profiles will be uniquely determined. However, to have continuous and repeatable gait on flat ground, the postures at the beginning and end of each step have to be identical. This requires the selection of specific initial joint angles, constraint functions and their associated gait parameters. This selection can be extremely challenging, if not impossible. This issue has not been addressed previously since the special constraint functions used by Hurmuzlu[5] automatically produced repeatable gait. The restriction of creating repeatable gait and the lack of rules for selecting proper initial angles, constraint functions and their gait parameters have severely limited the applications of Hurmuzlu's method.[5]

In the course of our research on the design of joint profiles for a 5-link biped using different constraint functions,[7] we noticed that the initial angles and certain gait parameters, such as step length and step period, are interdependent. Finding this relationship is important for specifying the compatible initial joint angles when gait parameters are pre-selected. Thus, the objective of this work is: (i) to investigate the correlation between the gait parameters used in a specific class of constraint functions and initial conditions in the context of producing repeatable gait, and (ii) to develop a general repeatability condition, which can provide a guideline for the selection of constraint functions and their associated gait parameters. This research can provide valuable information about bipedal gait. It is a stepping stone for employing Hurmuzlu's parametric formulation[5,6] with various constraint functions.

The paper is organised as follows: The five-link bipedal system is first described and five constraint equations, as used by Hurmuzlu,[5,6] are then outlined in the next section. Explicit relationships among the initial angles and the gait parameters, based on Hurmuzlu's constraint functions, will

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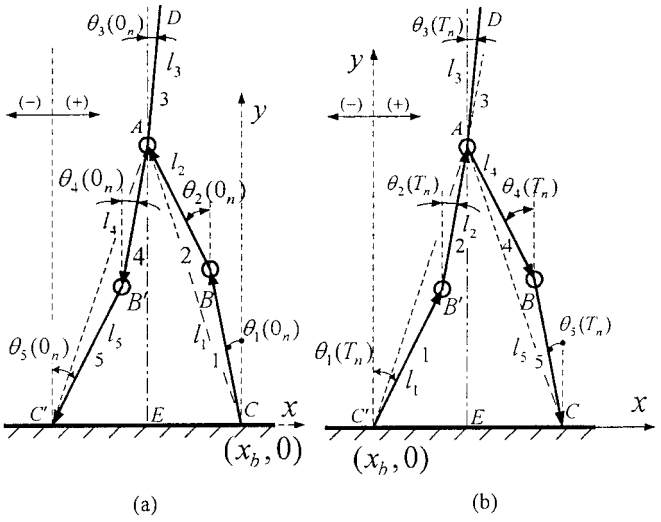


Fig. 1. Posture of a 5-link biped (a) at start-of-step and (b) at end-of-step.

be developed in Section 3. These relationships will be generalised to a certain class of constraint functions. A general sufficient and necessary condition for repeatable gait will be established and proven in Section 4. The application of this repeatability condition for selecting constraint functions will be discussed in the same section, followed by conclusions.

2. PRELIMINARY REVIEW

The bipedal system studied in this work has five rigid links connected by pin joints as shown in Figure 1. One link represents the upper body and two links are used for each leg. l_i ($i=1,2,\ldots,5$) are the link lengths that $l_4=l_2$ and $l_5=l_1$. The biped moves steadily in the sagittal plane, that is, the joint profile is continuous and repeatable. Only the single support phase is considered in Hurmuzlu's work.[5,6] Figures l(a) and l(b) show the postures at the-start-of-step and the-end-of-step, respectively. $\theta_i(0_n)$ and $\theta_i(T_n)$ ($i=1,2,\ldots,5$) are absolute angles at the beginning and end of the nth step. The upward vectors represent the supporting leg, and the downward vectors represent the swing leg. The single support phase consists of a swing phase and an impact phase. The swing phase is a continuous forward motion during which the biped is pivoted on one limb (stance limb) and the other limb (swing limb) is moving in the forward direction. The impact phase arises when the swing limb comes into contact with the walking surface. A detailed derivation of the dynamic equations for the swing phase and the impact phase can be found in references by Tzafesta *et al.*[1] and is not repeated here.

2.1. Hurmuzlu's constraint equations

The five constraint functions used by Hurmuzlu[5,6] to determine joint profiles for a 5-link bipedal robot, walking on the horizontal surface, are:

(a) *Erect Body Posture*: The biped maintains an erect posture during locomotion. Referring to Figure 1, this relation can be written as

$$\theta_3(t)=0 \tag{1}$$

(b) *Walking speed*: The walking speed has been defined as the speed of the centre of mass of the upper body in the horizontal direction. According to the kinematical relation, the corresponding constraint function is

$$S_2=l_1\cos\theta_1\dot{\theta}_1+l_2\cos\theta_2\dot{\theta}_2+d_3\cos\theta_3\dot{\theta}_3-V_p=0 \tag{2}$$

where V_p is the desired walking speed.

(c) *Traiectory of the Swing Limb*: A parabola has been chosen as the trajectory of the tip of the swing leg in advance. By choosing the step length S_L and the maximum tip clearance H_m, the trajectory of the swing limb can be determined below:

$$S_3=y_T-\left(\frac{4H_m}{S_L^2}x_T^2+H_m\right)=0 \tag{3}$$

where x_T and y_T are the co-ordinates of the tip of the swinging leg.

(d) *Bias of the Stance Knee*: The stance knee has been fixed at a certain angle during the single support phase. The corresponding relation is

$$S_4=(\theta_2-\theta_1)-\sigma=0 \tag{4}$$

where σ is a given bias angle.

(e) *Coordination of the Motion of the Limbs*: To define the direction of the swing limb's motion, the constraint function has been given as

$$S_5=x_T-2x_{c3}=0 \tag{5}$$

where x_{c3} and x_T are the x-coordinates of the centre of mass of the upper body and the tip of the swing leg, respectively. It also implies that the tip moves twice as fast as the centre of mass of the upper body.

2.2. Requirement of repeatable gait

For a continuous and steady gait, a basic requirement is that the gait is repeatable. Strictly speaking, this condition requires that at any time t, the posture must be exactly same as that at time $t+kT$, where k is an arbitrary integer and T is the step period.

Satisfaction of the requirement of repeatable gait can make the design of joint profiles extremely challenging. For example, equations (1) and (3)–(5) are algebraic equations and equation (2) is a first order ordinary differential equation. Thus, only one initial angle of the nth step, for example, $\theta_1(\theta_n)$, can be chosen and the rest of the initial angles will be fixed by equations (1) and (3)–(5). Once the initial angle is chosen, the solution to the constraint equations (1)–(5) is uniquely determined. Since the impact, at the beginning of the next step, does not alter the posture (joint and segmental angles), but only the angular velocities, the requirement of repeatable gait on flat ground indicates that the posture at the end of each step must be the same as the one at the beginning of each step. For the case of the 5-link bipedal model, shown in Figure 1, we have

$$\theta_1(0_n)=\theta_5(T_n)\ \theta_2(0_n)=\theta_4(T_n)\ \theta_4(0_n)=\theta_2(T_n)\ \theta_5(0_n)=\theta_1(T_n) \tag{6}$$

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In order to have the solution to further satisfy equation (6), the initial angles, constraint functions and their associated gait parameters must be carefully selected. So far, no research has been carried out on how to select these parameters to produce repeatable gait.

3. CORRELATION BETWEEN THE INITIAL POSTURE AND GAIT PARAMETERS FOR REPEATABLE GAIT

In this section, the relationship among the initial angles and the gait parameters associated with the constraint functions used by Hurmuzlu[5] will be investigated. This relationship will be generalised to a certain class of constraint functions. According to equation (1) and (4), equation (2) can be written as

$$\frac{d\theta_1}{dt} = \frac{V_p}{l_1 \cos\theta_1 + l_2 \cos(\theta_1 - \sigma)} \tag{7}$$

Note that the right-hand side of equation (7) is continuous when $|t - t_0| \le T, |, |\theta_1(t)| \le \frac{\pi}{2} - \sigma$, and satisfies Lipschitz condition. There exists a unique solution to equation (7). Solving equation (7), we have

$$\sin\theta_1(t) = \frac{(V_p t + C_0)(l_1 + l_2\cos\sigma)}{A'C'^2} - \frac{l_2 \sin\sigma\sqrt{A'C'^2 - (V_p t + C_0)^2}}{A'C'^2} \tag{8}$$

where C_0 is a constant shown below:

$$C_0 = l_1 \sin\theta_1(0_n) + l_2 \sin[\theta_1(0_n) + \sigma] \tag{9}$$

and $A'C' = \sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos\sigma}$ (see Figure 1). Since $\theta_1(t_n)$ must be real on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, it requires that

$$A'C'^2 - (V_p t + C_0)^2 \ge 0$$

Hence, the step period

$$T \le \frac{A'C' - C_0}{V_p} \tag{10}$$

Based on the geometric relation (see Figure l(a)), it is easy to prove that $A'C' > C_0$ always holds. Equation (10) implies that there is an upper boundary for a step period with V_P not approaching zero. According to equation (5), the upper boundary for the step length also exists:

$$S_L = 2V_p T \le 2(A'C' - C_0) \tag{11}$$

3.1. The relationship among the initial angles, step period and stance knee bias angle

In this section, we develop the explicit relationship between the initial angles, the step period, T, and stance knee bias angle, σ, using the constraint equations (1) to (5). In general, the step period involves two parts: the period of single support phase T_1, the period of double support phase T_2, where $T = T_1 + T_2$. Since only the single support phase is considered here, the step period in the following discussion only includes the period of single support phase. For convenience, we substitute T_1 by T.

At the final moment of the single support phase, the following angular relation confines that two feet touch the ground:

$$y_e(T_n;\theta_i) = l_1\cos\theta_1(T_n) + l_2\cos\theta_2(T_n) + l_4\cos\theta_4(T_n) + l_5\cos\theta_5(T_n) = 0 \tag{12}$$

where $y_e(T_n;\theta_i)$ is the y co-ordinate of the tip of the swing leg. Since the step length is given in Hurmuzlu's constraint equations, we have

$$l_1\sin\theta_1(T_n) + l_2\sin\theta_2(T_n) + l_4\sin\theta_4(T_n) + l_5\sin\theta_5(T_n) - \frac{S_L}{2} = 0 \tag{13}$$

From equations (1), (2) and (4), we may find the final values of every angle, which is detailed below. Equation (8) can also be written as

$$t_n = \frac{1}{V_p}\{l_1\sin\theta_1(t_n) + l_2\sin[\theta_1(t_n) + \sigma] - C_0\} \tag{14}$$

Replacing $\theta_1(t_n)$ by $\theta_1(T_n)$, we find the step period of single support phase. Since the repeatability condition is satisfied, using equations (6), the relationship between the step period and initial posture is obtained:

$$T = \frac{1}{V_p}\{l_1\sin\theta_5(0_n) + l_2\sin\theta_5(0_n) + \sigma] - C_0\} \tag{15}$$

Note that C_0 is a function of $\theta_1(0_n)$, as shown in equation (9), and $\theta_5(0_n)$ is a function of $\theta_1(0_n)$. Thus the relationship between the step period and the initial condition is established in (15).

3.2. The relationship among the initial angles, step length and stance knee bias angle

In this section, we shall discuss the relationship between the initial angles, step length, S_L, and stance knee bias angle, σ, using Hurmuzlu's constraint equations. This relationship can be determined based on the geometry shown in Figure 1.

Since, in Hurmuzlu's work, the upper body is assumed to be in the upright position and the mass center of the upper body to be the midpoint of the tips of the two legs, it follows that the hip joint remains the midpoint of the tips of the two legs. From the geometric relationship shown in Figure 1, we have $|\theta_1(0)| = \angle EAC - \angle ACB$ and

$$\angle EAC = \arcsin\left(\frac{S_L}{4AC}\right) \tag{16}$$

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$$\angle ACB = \arcsin\left(\frac{l_2}{AC}\sin(\sigma)\right) \tag{17}$$

From equations (16) and (17), we have

$$\theta_1(0) = -\arcsin\left(\frac{S_L}{4AC}\right) + \arcsin\left(\frac{l_2}{AC}\sin(\sigma)\right) \tag{18}$$

where $AC = \sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos\sigma}$.

Equation (18) shows the explicit relationship between the initial angle, $\theta_1(0)$, step length, S_L, and supporting knee bias angle, σ. It indicates that if the link lengths and gait parameters, S_L and σ, are given, the initial angle is fixed, as is the solution to the constraint equations (1)–(5) for the following step. On the other hand, for a given physical system and steady gait, once the initial angle and the bias angle of the supporting knee are chosen, the step length cannot be selected arbitrarily.

Since equation (18) has been derived based on the geometrical relationship among the gait parameters and link lengths at the beginning of the step and only equations (3)–(5) were involved, the relationship between the initial angle, $\theta_1(0)$, and gait parameters, shown in equation (18), is valid for more general constraint functions than those used by Hurmuzlu.[5,6] This class of constraint functions should include stance knee bias angle and step length. Furthermore, equation (18) is also valid when the double support phase is considered.

4. THE GENERAL REPEATABILITY CONDITION

In this section, a sufficient and necessary condition for repeatable gait for a 5-link bipedal system (repeatability condition) is developed. This condition will provide a guideline for selecting constraint functions and their associated gait parameters, which lead to a proper initial condition for repeatable gait.

4.1. Repeatability condition

Theorem: For the 5-link bipedal system shown in Figure 1, given a set of constraint equations, which consists of at least one first-order ordinary differential equation, let T and $P(t_n)$ be the step period and instant posture of the nth step, respectively, where $0 < t_n < T$. Triangle $\Delta ACC'$ can be determined by connecting the hip joint A and the tips of two legs, C and C′, as shown in Figure 1. Assuming that neither knee can be bent backward, the sufficient and necessary conditions that lead to a repeatable gait are: (i) $\Delta ACC'(0_n) \cong \Delta ACC'(T_n)$ and (ii) $\theta_3(t)$ is a periodical function with period of T, where 0_n and T_n denote the beginning and end of the nth step.

Note that since the joint profiles are always continuous, it follows that the posture at the end of each step must be identical to the one at the beginning of the next step, which guarantees that triangle $\Delta ACC'(T_n) \cong ACC'(0_{n+1})$.

Proof: Sufficiency

Referred to Figure 1, since $\Delta ACC'(0_n) \cong \Delta ACC'(T_n) \cong \Delta ACC'(0_{n+1})$, we have

$$AC(0_n) = AC(T_n) = AC(0_{n+1}) \tag{19a}$$

$$\angle ACC'(0_n) = \angle ACC'(T_n) = \angle ACC'(0_{n+1}) \tag{19b}$$

Since all links are rigid, $l_2 = l_4$ and $l_1 = l_5$, we have $\Delta ABC(0_n) \cong \Delta ABC(T_n) \cong \Delta ABC(0_{n+1})$. Thus,

$$\angle ACB(0_n) = \angle ACB(T_n) = \angle ACB(0_{n+1}) \tag{20a}$$

$$\angle ABC(0_n) = \angle ABC(T_n) = \angle ABC(0_{n+1}) \tag{20b}$$

Since neither of the knees can be bent backward, from the geometrical relationship shown in Figure 1, we have

$$\theta_1(0_n) = \frac{\pi}{2} - \angle ACC'(0_n) - \angle ACB(0_n) \tag{21a}$$

$$\theta_1(0_{n+1}) = \frac{\pi}{2} - \angle ACC'(0_{n+1}) - \angle ACB(0_{n+1}) \tag{21b}$$

From the relationships shown in (19b) and (20a), we have

$$\theta_1(0_n) = \theta_1(0_{n+1}) \tag{22}$$

Similarly, we can prove that $\theta_5(0_n) = \theta_5(0_{n+1})$. From Figure 1, we see that

$$\theta_2(0_n) = \theta_1(0_n) + \pi - \angle ABC(0_n) \tag{23a}$$

$$\theta_2(0_{n+1}) = \theta_1(0_{n+1}) + \pi - \angle ABC(0_{n+1}) \tag{23b}$$

From equations (20b) and (22), we have

$$\theta_2(0_n) = \theta_2(0_{n+1}) \tag{24}$$

Following the same procedure, we can prove that $\theta_4(0_n) = \theta_4(0_{n+1})$. From condition (ii) that $\theta_3(t)$ is a periodical function, we have $\theta_3(0_n) = \theta_3(T_n)$. Based on the above discussion, all initial angles for the nth and $(n+1)th$ step are the same, which guarantees that the solution $P(t_n)$ and $P(t_{n+1})$ are identical and that the gait is repeatable.

Necessity:

If the gait is repeatable, we have $P(t_n) = P(t_{n+1})$ where $0 < t_n < T$. This indicates that motion of upper body must be repeatable during each step, i.e., $\theta_3(t)$ is a periodical function with period T. For repeatable gait, the step length remains the same, which leads to

$$CC'(0_n) = CC'(0_{n+1}) = CC'(T_n) \tag{25}$$

Since the link lengths are fixed and $\angle ABC(0_n) = \angle ABC(0_{n+1}) = \angle ABC(T_n)$, it follows that $\Delta ABC(0_n) \cong \Delta ABC(0_{n+1}) \cong \Delta ABC(T_n)$. Thus,

$$AC(0_n) = AC(0_{n+1}) = AC(T_n) \tag{26a}$$

$$AC'(0_n) = AC'(0_{n+1}) = AC'(0_n) \tag{26b}$$

According to equations (25) and (26),

$$\Delta ACC'(0_n) \cong \Delta ACC'(T_n) \tag{27}$$

Since $P(t_n) = P(t_{n+1})$, we have that $\theta_3(0_n) = \theta_3(T_n)$. Thus, the repeatability conditions have been proven.

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Remarks:

(a) The above repeatability condition is based on the assumption that at least one constraint equation is a differential equation, the highest order of the differential equations is 1, and the solution is uniquely determined by the initial angles.
(b) The above condition for repeatable joint profiles is not restricted to the bipedal system with only the single support phase. This condition can be used for gait, which involves the double support phase as long as the set of constraint equations for double support phase consist of at least one differential equation where the highest order is 1.
(c) The restriction that neither knee joint can be bent backward is important to keep the above proposed condition sufficient. It is not a strong restriction since in most bipedal walking, neither knee is bent backward. The same restriction has been used in other works.[1,5,6]

For the constraint functions used by Hurmuzlu, due to the constraint of the co-ordination of the motion of the limbs, as shown in equation (5), $\Delta ACC'(0_n)$ and $\Delta ACC'(T_n)$ must be an isosceles triangle. The parabolic trajectory of the tip of the swinging limb during the single support phase, as shown in equation (3), guarantees that the lengths of $CC'(0_n)$ and $CC'(T_n)$ remain the same. The constraint of fixed stance knee bias angle, described by equation (4), guarantees that the length of $AC'(0_n)$ and $AC'(T_n)$ are fixed. Therefore, triangles $\Delta ACC'(0_n)$ and $\Delta ACC'(T_n)$ remain identical for every step and the repeatability condition is automatically satisfied.

4.2. Selection of constraint functions satisfying the repeatability condition

The repeatability condition proposed in Section 4.1 is a sufficient and necessary condition, which can be used to examine whether the constraint functions will produce repeatable gait once they are chosen based on tangible gait characteristics. Furthermore, it can provide a guideline for selecting constraint functions for five-link bipeds, which will be demonstrated through the following example.

The walking speed will first be selected as one of the constraint functions; it provides a first-order ordinary differential equation. The upper body movement can be used as another constraint function, which is not necessarily restrained at the upright position. To satisfy the repeatability condition, $\Delta ACC'(0_n) \cong \Delta ACC'(T_n)$, the vertical and horizontal motion of the hip joint, A, can be prescribed as functions of time with the following conditions satisfied:

$$C'E(0_n) = C'E(T_n) \tag{28a}$$

$$CE(0_n) = CE(T_n) \tag{28b}$$

$$AE(0_n) = AE(T_n) \tag{28c}$$

Satisfaction of equations (28) will guarantee that $\Delta ACC'(0_n) \cong \Delta ACC'(T_n)$.

Based on the above discussion, one set of possible constraint functions that produce repeatable gait can be selected as: walking speed, upper body movement with $\theta_3(0_n) = \theta_3(T_n)$ and vertical and horizontal hip movements that satisfy equations (28). The fifth constraint function can be selected without the restriction of the repeatability condition.

5. CONCLUSIONS

Parametric formulation, developed by Hurmuzlu,[5,6] is a systematic method of designing joint profiles for bipedal models. However, steady and continuous gait requires that the joint profiles are repeatable, which imposes an additional requirement on the solution. Consequently, specific constraint functions, gait parameters and initial angles have to be chosen. Thus, it is highly desirable to develop a technique for specifying the constraint functions, gait parameters and initial angles such that repeatable gait can be produced.

In this paper, two explicit relationships between the initial angles and gait parameters, used in a specific class of constraint functions, have been developed as shown in equations (15) and (18), which include (i) the relationship between the initial angles and step period, stance knee bias angle and walking speed, and (ii) the relationship between the initial angle and step length and stance knee bias angle. From these results, we found that if the step length and stance knee bias angle are selected, the initial angles are fixed, which determine the solution uniquely. Furthermore, with a fixed stance knee bias angle, increasing the step length requires an increase in the value of the initial angle, $\theta_1(0_n)$. In addition to showing the relationship between the initial angles and gait parameters, equations (15) and (18) further show the dependence among the gait parameters. For example, the walking speed, step length and step period are related. Once two of these are selected, the third one will be determined.

We then developed the necessary and sufficient condition for producing repeatable gait. The condition is valid for the types of constraint equations, which consist of algebraic and at least one first-order ordinary differential equations, and whose the solutions are uniquely determined by the initial angles. This condition can be used in two ways. One way is to examine whether the constraint functions will produce repeatable gait once they are chosen based on tangible gait characteristics and other design criteria; another is to provide a guideline for selecting constraint functions and their associated gait parameters in the context of producing repeatable gait. This point has been demonstrated through an example.

Note that in some previous work on designing optimal gait,[8] the repeatability condition was considered by including the error in the joint angles at the beginning and the end of each step in some type of functional and minimizing them. This method works well when optimization is the only constraint function. If additional and improper constraint functions are selected, which inherently violates the repeatability condition, the approach of minimising the error in the joint angles at the beginning and end of each step will not yield repeatable gait.

We believe this research can provide valuable information about bipedal gait. It is a stepping stone for employing

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Hurmuzlu's parametric formulation with various constraint functions. We are currently extending this work to sets of constraint functions that involving higher order ordinary differential equations.

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