Robotica (2005) volume 23, pp. 231–237. © 2005 Cambridge University Press
doi:10.1017/S0263574704001080 Printed in the United Kingdom

The design of redundant isotropic manipulators with special link parameters

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(Received in Final Form: September 28, 2004)

SUMMARY
This paper proposes a general method to develop n-DOF serial isotropic manipulators. The parameters of 6-DOF isotropy generators are used as the initial values in a numerical method to obtain redundant isotropic generators. Additional constraint equations are then added, with the parameters of the obtained generators as initial values, to obtain the isotropy generators with special parameters. Each generator can be developed into many isotropic designs with special or desired link parameters. Two different approaches for developing isotropic manipulators are also investigated.

KEYWORDS: Isotropic manipulators; Link parameters; Redundancy.

I. INTRODUCTION

A manipulator is termed isotropic if it can attain certain isotropic configurations. We can expect better accuracy in instantaneous and kinetostatic analysis, and a manipulator can be well controlled at isotropic configurations. Redundant manipulators are desirable when we deal with obstacle avoidance, energy minimization and increasing dynamic response in trajectory planning. For 6-DOF or redundant serial manipulators, Klein and Miklos introduced a definition for spatial isotropy of a robot, proposing related theorems and developing equations to find a family of 6-DOF isotropic designs and one 7-DOF redundant robot with spatial isotropy.[1] Angeles derived the isotropy conditions for redundant manipulators.[2] Tsai and Huang solved the nonlinear equations (developed from the isotropy conditions) analytically to find many sets of 6 straight lines called isotropy generators.[3] This paper proposes a general method to develop n-DOF redundant isotropic manipulators from 6-DOF isotropy generators. An isotropy generator can be easily obtained by providing three parameters, one of which determines the size while the other two determine the shape of the isotropic designs. Therefore, in theory we can specify only two link parameters in designing 6-DOF isotropic manipulators. More link parameters can be specified for redundant manipulators, but the increase of constraint equations makes the design process more complicated.

The existing methods to find 6-DOF or redundant isotropic designs involve solving about 20 nonlinear equations by numerical techniques. This approach may be very time-consuming if the initial given values are not close to the solutions of the equations. Since 6-DOF isotropy generators can be easily developed, we can use the parameters of the obtained generators as the initial values in a numerical method to develop 7-DOF redundant isotropy generators. Likewise, n-DOF generators can be developed with the parameters of (n-1)-DOF generators as the initial values. We can also use the parameters of n-DOF generators as the initial values to develop (n-1)-DOF generators. A redundant isotropic design with any degree-of-freedom can be easily developed from the proposed method.

There are some preferable link parameters (such as $a_i = 0, \alpha_i = 0$ or 90°, $d_i = 0$) in the kinematic design of manipulators, and these link parameters can be generated if two axes are parallel, perpendicular or intersecting. This paper first finds special 6-DOF isotropy generators whose axes are parallel, perpendicular or intersecting. The obtained generators along with other constraint equations are then used to develop redundant generators with special parameters. A large number of redundant isotropic designs, in theory, can be developed from the proposed method. However, the relative positions of the axes of the obtained manipulators, in general, follow some patterns, so we do not always obtain the designs with desired link parameters.

To find the designs with desired link parameters, we can fix some desired parameters and use the remaining parameters as design variables. However, isotropic designs may not be developed by the method, so the best we can do is to find a manipulator that is as close as possible to an isotropic design. Redundant isotropic manipulators can also be designed in a way that the last six axes of the manipulator constitute a 6-DOF isotropic design, and the first n-6 axes are used as a regional structure to move the isotropic design around. Manipulators developed in this way can attain more isotropic configurations, and the first n-6 links of the manipulators can be any combination of desired link parameters. Some isotropic designs developed from three different approaches are presented for illustration.

II. 6-DOF GENERATORS WITH ORTHOGONAL OR INTERSECTING AXES

Manipulators with parallel or intersecting axes, in general, have better kinematic properties: simplicity, better dexterity, fewer link interactions, etc. This section develops 6-DOF

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https://doi.org/10.1017/S0263574704001080 Published online by Cambridge University Press

Table I. Special generators for $t = 0.2$.

t	ϕ	Intersecting	Perpendicular or orthogonal	Parallel
0.2	0		[1,2,3][4,5,6]	
0.2	75.964		[1,4][2,6][3,5]	
0.2	36.870		[1,5][2,4][3,6]	
0.2	71.565		[1,6][2,5][3,4]	
0.2	45	(1,2,3)(4,5)(4,6)(5,6)		
0.2	30.964	(1,4)(2,6)(3,5)		
0.2	81.870	(1,5)(2,4)(3,6)		
0.2	26.565	(1,6)(2,5)(3,4)		

isotropy generators with orthogonal or intersecting axes. The obtained parameters are then used as the initial values to develop redundant generators with special parameters.

An isotropy generator (called "generator" below) consists of six straight lines. The shape of a generator is determined by two parameters (t, ϕ), while its size is determined by the third parameter ρ. Let S_1 and S_2 denote, respectively, the set of the first three lines and the set of the last three lines. Then the relative position of S_2 to S_1 can be determined by parameter t. The generator with $t = 0$ consists of three pairs of parallel lines. Changing t will make S_2 rotate about the origin O, but the relative positions of the three lines of S_1or S_2 will remain unchanged. Parameter ϕ is the rotation angle of each line about its corresponding offset vector r_i, which originates from O to the ith straight line and is perpendicular to the line. If $\phi = 0$, then the three lines in S_1 or S_2 are always orthogonal but not intersecting. By the solutions developed in Tsai and Huang,[3] the Plücker's coordinates of the ith straight line, denoted by $\begin{bmatrix} \mathbf{e_i'} \\ \mathbf{m_i'} \end{bmatrix}$, can be expressed as functions of t. Furthermore, using the theorem proposed by Klein and Miklos,[1] we can add parameter ϕ to the coordinates by

$$\begin{bmatrix} \mathbf{e_i} \\ \mathbf{m_i} \end{bmatrix} = \begin{bmatrix} \mathbf{rot}(r_i, \boldsymbol{\varphi}) & 0 \\ 0 & \mathbf{rot}(r_i, \boldsymbol{\varphi}) \end{bmatrix} \begin{bmatrix} \mathbf{e_i'} \\ \mathbf{m_i'} \end{bmatrix} \qquad (1)$$

where $r_i = \frac{\mathbf{e_i'} \times \mathbf{m_i'}}{\|\mathbf{e_i'} \times \mathbf{m_i'}\|}$. With this transformation, the coordinates of the three lines in S_1 are functions of ϕ, while the coordinates of the three lines in S_2 are functions of t and ϕ. Therefore, for any t we can solve the equation of mutual moment: $\mathbf{e_i} \cdot \mathbf{m_j} + \mathbf{e_j} \cdot \mathbf{m_i} = 0$ for ϕ to get a generator whose two lines, L_i and L_j, from different sets which are intersecting or parallel. We can also solve equation $\mathbf{e_i} \cdot \mathbf{e_j} = 0$ for ϕ to get a generator with the directions of two lines from different sets which are perpendicular. Some of the results for $t = 0.2$ are shown in Table I, where $(\cdot)$, $[\cdot]$ or $\langle \cdot \rangle$ denote whether the lines in the parenthesis or bracket are intersecting, perpendicular (or orthogonal) or parallel respectively. The three lines in S_1 are functions of ϕ only, so we can have two lines, L_i and L_k, from S_1 intersected using equation $\mathbf{e_i} \cdot \mathbf{m_k} + \mathbf{e_k} \cdot \mathbf{m_i} = 0$ to solve for ϕ. The obtained generator, corresponding to $\phi = 45^\circ$, has the three lines in S_1 intersecting at one point and the three lines in S_2 intersecting at three different points. Furthermore, if $\phi = 45^\circ$ and $t = 0$, then the lines from S_1 and S_2 also intersect, and the six straight lines correspond to the six sides (with equal length) of a tetrahedron. Some special cases of generators with parallel, orthogonal, or intersecting lines are given in Table II. The corresponding configurations are shown in Fig. 1. The results from Tables I and II show that if any two straight lines of a generator are parallel, perpendicular, or intersecting lines, then there exist at least three pairs of straight lines that are parallel, perpendicular, or intersecting.

III. REDUNDANT GENERATORS

Let $\mathbf{H} = [h_{ij}]$ be a 6 by n matrix with Plücker's coordinates of a set of straight lines as its column vectors, where h_{ij} denotes the element in row i and column j of $\mathbf{H}$. Then the set of straight lines is a generator for developing n-DOF serial isotropic designs if $\mathbf{H}$ satisfies the isotropy conditions. In this work, the ith column of $\mathbf{H}$ is expressed as functions of five variables

$$\$_i = [c\alpha_i s\beta_i \quad s\alpha_i s\beta_i \quad c\beta_i \quad x_i \quad y_i \quad z_i]^t \qquad (2)$$

Thus, the number of equations to develop n-DOF generators is 5n. The equations developed from the isotropy conditions are:[1]

(i) The first three rows of $\mathbf{H}$ are equal in length and orthogonal, generating 5 equations
(ii) The last three rows of $\mathbf{H}$ are equal in length and orthogonal, generating 5 equations
(iii) The span of the first three rows and the span of the last three rows are orthogonal, generating 9 equations

Let $\mathbf{H_{i*}}$ denote the ith row of $\mathbf{H}$. Then the nineteen equations can be written as

$$\varepsilon_{ij} = \mathbf{H_{i*}}\,\mathbf{H_{j*}^t} \qquad (3)$$

or

$$\xi_{ij} = \mathbf{H_{i*}}\,\mathbf{H_{i*}^t} - \mathbf{H_{j*}}\,\mathbf{H_{j*}^t} \qquad (4)$$

Furthermore, the equations developed from the definition of a zero-pitch screw can be written as

$$\delta_k = x_k c\alpha_k s\beta_k + y_k s\alpha_k s\beta_k + z_k c\beta_k \qquad k = 1, 2, \ldots n \qquad (5)$$

Table II. Generators with more special parameters.

t	ϕ	Intersecting	Perpendicular or orthogonal	Parallel
0	0	0	[1,2,3][4,5,6]	$\langle 1,4 \rangle \langle 2,5 \rangle \langle 3,6 \rangle$
0	45	(1,2,3)(1,5,6)(2,4,6)(3,4,5)	[1,4][2,5][3,6]	
0.447	0	(1,6)(2,5)(3,4)	[1,2,3][4,5,6]	
0.447	45	(1,2,3)(4,5)(4,6)(5,6)	[1,6][2,5][3,4]	

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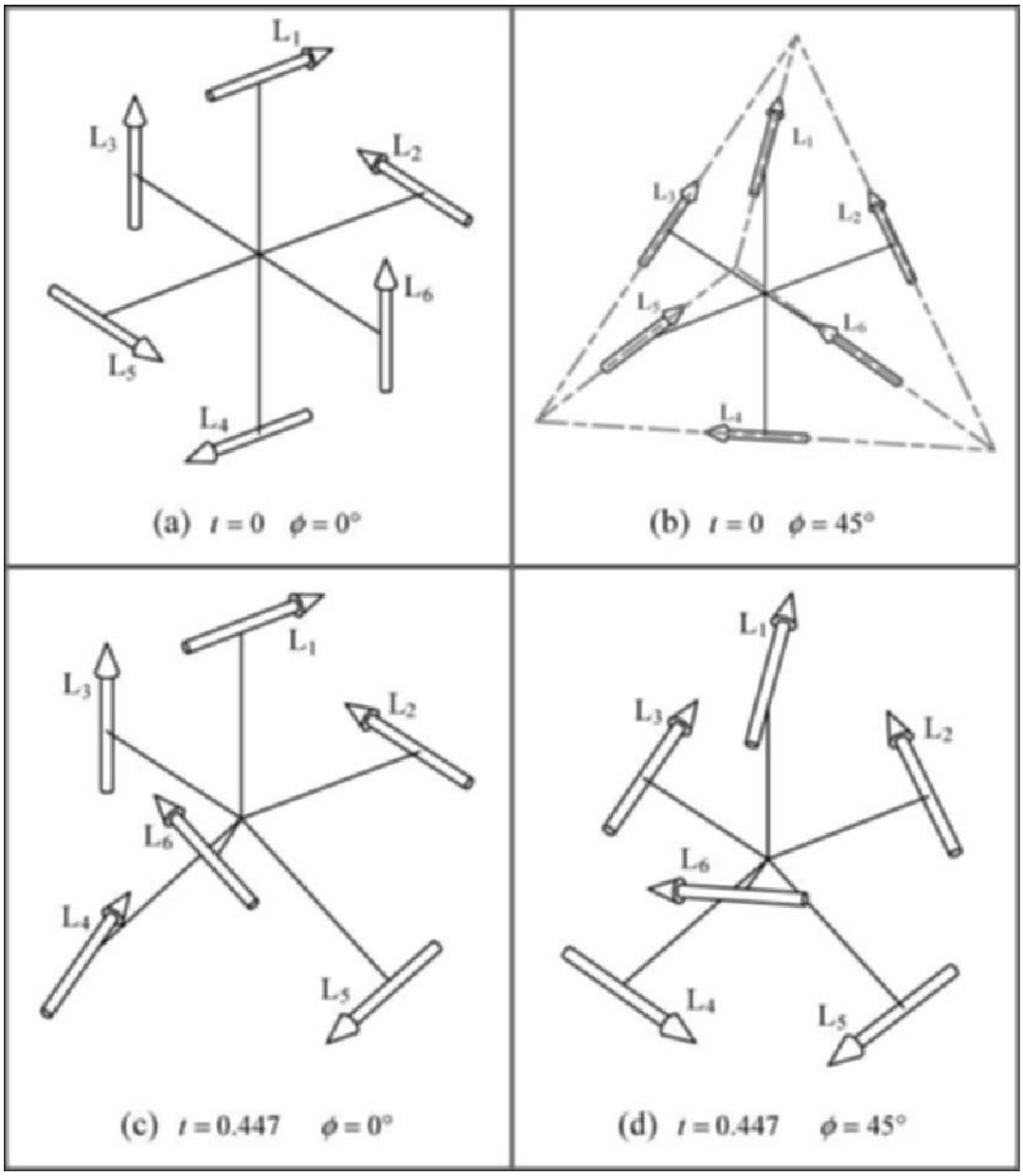


Fig. 1. Configurations of some special generators.

Thus we have (19 + n) equations with 5n variables to develop n-DOF generators. The functions ε_{ij}, ξ_{ij} and δ_k evaluate the residues in a numerical method, and a generator can be developed if all the residues are zeroes.

Some m-DOF generators, especially m = 6k or m = 6k + 3 (k = 1, 2, 3, . . .), are easier to developed so they can be used as the initial generators to find desired generators. Generators with m = 6k consist of k 6-DOF generators, so they can be directly obtained. A special case of generators with m = 6k + 3 can be developed if the first 6k columns correspond to k 6-DOF generators and the last three columns consist of an upper 3 by 3 orthogonal matrix and a lower 3 by 3 zero matrix. If some special designs require that the elements of the lower 3 by 3 matrix can not be a zero matrix, then the last three columns can be replaced by the first or the last three columns of a 6-DOF generator with parameter $\phi = 0$. For the replacement, the residues of the nine equations from condition (iii) are not zeroes. Let **R** and **Q** denote the upper and lower 3 by 3 matrices of the last three columns respectively, where **R** is an orthogonal matrix and **Q** is an matrix whose columns and rows have the same Euclidean norm equal to parameter ρ that determines the size of a generator. Then the residues of the nine equations from condition (iii) are the elements of matrix $\mathbf{RQ^t}$ whose columns or rows also have the same Euclidean norm of ρ. So all the residues are less than ρ, and it is easier for the nonlinear equations to converge if the last three columns are from a smaller 6-DOF generator.

Other degree-of-freedom generators can be developed by adding columns to (or deleting columns from) 6k- or (6k + 3)-DOF generators. If the columns of **H** satisfy the isotropy conditions, then the residues generated by adding one more column to (or deleting one column from) **H** can be estimated as follows

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(i) $|\varepsilon_{12}|$, $|\varepsilon_{23}|$, $|\varepsilon_{13}|$ less than 0.5; $|\varepsilon_{14}|$, $|\varepsilon_{24}|$, $|\varepsilon_{34}|$ less than $|x_n|$; $|\varepsilon_{15}|$, $|\varepsilon_{25}|$, $|\varepsilon_{35}|$ less than $|y_n|$; $|\varepsilon_{16}|$, $|\varepsilon_{26}|$, $|\varepsilon_{36}|$ less than $|z_n|$; $|\varepsilon_{45}| = |x_n\, y_n|$, $|\varepsilon_{56}| = |y_n\, z_n|$, $|\varepsilon_{46}| = |x_n\, z_n|$

(ii) $|\xi_{12}|$ and $|\xi_{23}|$ less than 1; $|\xi_{45}| = |x_n^2 - y_n^2|$, $|\xi_{56}| = |y_n^2 - z_n^2|$

where α_n, β_n x_n, y_n and z_n denote the variables of the added (or deleted) column. Residue $\delta_k = 0$ for all the remaining columns if a column is removed from **H**. Residue $\delta_n = x_n c\alpha_n s\beta_n + y_n s\alpha_n s\beta_n + z_n c\beta_n \neq 0$ for arbitrary values when a column is added. But we can easily find a set of initial values that lead to $\delta_n = 0$. If $x_n = y_n = z_n = 0$, then $|\varepsilon_{12}|$, $|\varepsilon_{23}|$, $|\varepsilon_{13}|$, $|\xi_{12}|$ and $|\xi_{23}|$ are less than one, and the rest of the residues are zeroes. When a generator is developed from a higher degree-of-freedom generator, we can eliminate the column(s) with minimum Euclidean norm and use the remaining columns to find the desired generator. An n-DOF redundant generator can be easily obtained by the proposed method. This generator, in theory, can be developed into $n!$ serial isotropic manipulators. The D-H parameters of the manipulators can be developed by any geometrical approach (for example, that of Gonzalez-Palacios, Angeles, and Ranjbaran).[4]

IV. REDUNDANT GENERATORS WITH SPECIAL PARAMETERS

This section proposes methods to find redundant generators with special parameters. The constraint equations can be developed from the following two equations:

$$\mathbf{e_i \cdot m_j} + \mathbf{e_j \cdot m_i} = a_{ij} \sin \alpha_{ij} \tag{6}$$

and

$$\mathbf{e_i \cdot e_j} = \cos \alpha_{ij} \tag{7}$$

where a_{ij} is the perpendicular distance between lines L_i and L_j, and α_{ij} is the angle between them. Equation (6) gives the constraint equation for two lines to be intersecting if they are not parallel:

$$c\alpha_i s\beta_i x_j + s\alpha_i s\beta_i y_j + c\beta_i z_j + c\alpha_j s\beta_j x_i + s\alpha_j s\beta_j y_i + c\beta_j z_i = 0 \tag{8}$$

Equation (7) provides the constraint equation for two lines to be perpendicular:

$$c\alpha_i s\beta_i c\alpha_j s\beta_j + s\alpha_i s\beta_i s\alpha_j s\beta_j + c\beta_i c\beta_j = 0 \tag{9}$$

or parallel:

$$c\alpha_i s\beta_i c\alpha_j s\beta_j + s\alpha_i s\beta_i s\alpha_j s\beta_j + c\beta_i c\beta_j \pm 1 = 0 \tag{10}$$

Either $+1$ or -1 can be used in Eq. (10), depending on which one yields a smaller residue with the given initial values.

More constraint equations can be added to specify other link parameters. For instance, Eq. (7) can be used as the constraint equation to obtain the desired twist angle α_{ij} between L_i and L_j, and the constraint equation for specific link length a_i between L_i and L_j can be developed from Eqs. (6)–(7):

$$a_i = \frac{\mathbf{e}_i \cdot \mathbf{m}_{i+1} + \mathbf{e}_{i+1} \cdot \mathbf{m}_i}{\sin \alpha_{ij}} = \frac{\mathbf{e}_i \cdot \mathbf{m}_{i+1} + \mathbf{e}_{i+1} \cdot \mathbf{m}_i}{\sqrt{1 - \cos^2 \alpha_{ij}}} = \frac{\mathbf{e}_i \cdot \mathbf{m}_{i+1} + \mathbf{e}_{i+1} \cdot \mathbf{m}_i}{\sqrt{1 - (\mathbf{e_i} \cdot \mathbf{e}_{i+1})^2}}$$

The constraint equation for offset d_i can also be developed. But that equation is much more complicated, and so is not provided here due to limitations of space.

To develop a desired generator, we need a proper initial generator. In this paper, the 6-DOF generators in Table II are used as the initial generators, and we may choose the one with closest parameters as the initial generator. The corresponding values (α_i, β_i, x_i, y_i, z_i) are then used as the initial values to solve the system of nonlinear equations, including the additional constraint equations. There are more design variables if we use a higher degree-of-freedom initial generator, so the nonlinear equations converge more easily. If the equations do not converge, then there is great possibility that the desired generator can not be developed from the initial generator. Using a higher degree-of-freedom initial generator also makes it possible for us to fix some design variables if the number of remained unknowns is still larger than the number of constraint equations. For example, the constraint equations for three axes to be intersecting at one point are very complicated, so we can use the generator with $\phi = 45^\circ$ and fix (α_i, β_i, x_i, y_i, z_i) for the three intersecting lines. Developing a desired generator from a lower degree-of-freedom initial generator, on the other hand, allows us to provide an additional column that simultaneously satisfies many constraint equations. For instance, if the added a column that is the common normal of two straight lines of the initial generator, then the additional axis automatically satisfies four constraint equations to generate perpendicular and intersecting axes. Furthermore, to obtain a generator with three axes intersecting at one point, we can simply add one column representing the straight line through the intersecting point of two straight lines of an initial generator. The residues of this approach are generally larger than those from the method developed with higher degree-of-freedom initial generators, so desired generators may not be obtained by this approach.

V. TWO DIFFERENT APPROACHES FOR DEVELOPING ISOTROPIC DESIGNS

The generators with special parameters can be developed by the methods proposed in the preceding section if these parameters are fairly close to the parameters of an initial generator. From Tables I and II we see that most 6-DOF generators follow some special patterns. Thus, a desired generator may not be developed if we can not find a proper initial generator. An isotropic manipulator is an ideal design, but sometimes it is not practical in applications. For example, in an isotropic configuration, the end-effector is surrounded by several links for lower degree-of-freedom manipulators, which may increase the possibility of link interactions. An isotropic design that is close to an industrial manipulator may

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not be found, so the best we can do is to develop a manipulator that is as close as possible to an isotropic design. An isotropic design can be developed if the residues in Eqs. (3)–(5) are all zeroes. The smaller the residues, the closer we get to an isotropic design. Therefore, the closeness to an isotropic design can be evaluated by, for example, the summation of the absolute values of the residues:

$$\mu = \sum |\varsigma_{ij}| + \sum |\xi_{km}| + \sum |\delta_n| \qquad (11)$$

Since the residues depend on the size of a generator, it is suggested that all the isotropic designs (or nearly isotropic designs) are developed from the same size of generators. After the desired generator is obtained, we can modify the size of the developed generator by multiplying the last three rows of matrix **H** by any non-zero constant. In this paper, the generators with equal row length are used as the initial generators. In this case, measure μ can be used to compare different manipulators to determine which one is closer to an isotropic design. One way of obtaining a nearly isotropic design with desired link parameters is by fixing the desired parameters and solving the nonlinear equations until μ can not longer be reduced. An alternate approach is by putting some weighing functions on the constraint equations corresponding to the desired parameters. Near isotropic designs can also be developed using an optimization technique in which we minimize objective function μ with the desired parameters fixed by constraint equations.

An isotropic manipulator, in general, can only reach finite isotropic configurations C_i and the corresponding configurations obtained by rotating C_i about its first axis. Redundant isotropic manipulators can be designed in a way that they can reach much more isotropic configurations. Suppose that the last 6 axes of an n-DOF manipulator satisfy the isotropy conditions. Then the manipulator is at an isotropic configuration if we fix the first n-6 axes. Therefore, we can design an n-DOF manipulator in which the first n-6 axes can be arbitrary desired link parameters, while the last six axes constitute a 6-DOF isotropic (or nearly isotropic) design. In this case, the manipulator, in theory, can reach up to ∞^{n-5} isotropic (or nearly isotropic) configurations.

VI. NUMERICAL EXAMPLES

VI.1. An 8-DOF isotropic design

To find a redundant isotropic design with three of its axes orthogonal and intersecting at one point, we use the 6-DOF generator with $\rho = 1$, $t = 0$ and $\phi = 45^\circ$ as the initial generator (note that 6-DOF generators with $\rho = 1$ have equal row length). First, letting column 7 = column 6 and using Eq. (8) as the constraint equation, we easily obtain a 7-DOF generator with the first three lines intersecting at one point, denoted by (1, 2), (2, 3) and (1, 3). But the residues of the non-linear equations can not be reduced to zeroes when the constraint equations for orthogonality (Eq. (9)) are added, so the desired generator may not exist. We then add one more column (which is equal to column 7) to the current generator and resolve the equations. In this case, an 8-DOF isotropic design with desired parameters can be developed. Next, we fix the parameters for the three orthogonal and intersecting lines and search for other additional special parameters, trying to get as many pairs of orthogonal and intersecting axes as possible. The final result gives

Table III. D-H parameters of an 8-DOF isotropic manipulator.

Joint i	θ_i	α_i	a_i	d_i
1	0	70.529	0	−21.609
2	180	125.264	0	54.022
3	−114.095	90	16.106	31.190
4	90	48.190	0	−7.203
5	180	37.364	0	13.346
6	90	90	0	23.738
7	−90	90	0	0
8	0	0	21.466	5.062

$$\mathbf{H} = \begin{bmatrix} 0 & -0.624 & 0.782 & 0 & 0.504 & -0.970 & 0.504 & -0.466 \\ 0 & 0.782 & 0.624 & 0 & 0.642 & 0.243 & 0.642 & 0.885 \\ -1 & 0 & 0 & 1 & 0.577 & 0 & -0.577 & 0 \\ -2.725 & 0 & 0 & 1.140 & -3.347 & 0 & 3.347 & 0 \\ -3.468 & 0 & 0 & -4.261 & 0.686 & 0 & -0.686 & 0 \\ 0 & 1.012 & -4.293 & 0 & 2.161 & -1.080 & 2.161 & 1.080 \end{bmatrix}$$

The eight straight lines satisfy (1,2,3), [1,2,3], (2,6), (2,8), (3,6), (3,8), (4,5), (4,7), (5,7), (6,8), [1,6], [1,8], [2,4], [3,4], [4,6], [4,8], $\langle 1, 4 \rangle$. Figure 2 shows the relative positions of the eight straight lines. After increasing the size of the generator by multiplying the last three rows of **H** by 5, the link parameters of a manipulator constructed in the order of 5-7-4-8-6-2-1-3 are given in Table III.

VI.2. 7-DOF isotropic or nearly isotropic designs

This example finds three 7-DOF isotropic or nearly isotropic designs developed by three different approaches and compares their global isotropy. All the design are developed from the same 6-DOF generator with $\rho = 1$, $t = 0$ and $\phi = 45^\circ$. Using an approach similar to the preceding example, we first obtain a 7-DOF generator with (1,2,3), (4,5), (4,6), (4,7), (5,6), (5,7), (6,7), [4,6], [4,7], [5,6], [5,7],$\langle 4, 5 \rangle$, $\langle 6, 7 \rangle$. For the second design, we hope to get an isotropic manipulator whose first two axes are intersecting and perpendicular, and whose last three axes are intersecting and orthogonal. Matrix **H** in the previous example is used as the initial generator. First, we fix the parameters corresponding to the first three columns and delete the eighth column. Next, the two constraint equations for lines 5 and 7 (the two straight lines that are closest to yielding desired parameters) to be perpendicular and intersecting are added with the coefficients of the equations multiplied by 50. We then solve the nonlinear equations by a numerical method until measure μ can not be further reduced. The result gives $\mu = 0.825$ so we only get a nearly isotropic design. For the third design, which can reach more isotropic configurations, we simply add one column to the 6-DOF initial generator, such that the adding line is intersecting and perpendicular to the first line of the initial generator. The link parameters of the three developed manipulators are given in Tables IV–VI.

The global isotropy of a manipulator can be evaluated by the mean value[5] or optimal single worst value[6] of the

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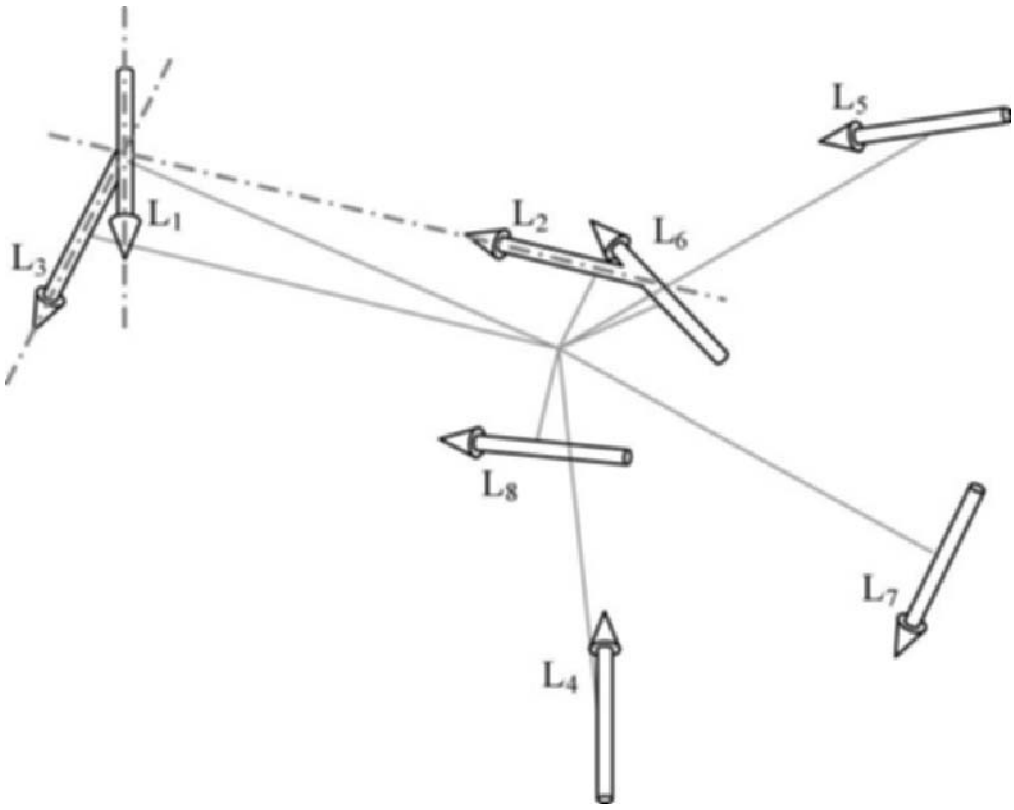


Fig. 2. The 8 axes of an 8-DOF isotropic design.

Table IV. D-H parameters of the first design.

Joint i	θ_i	α_i	a_i	d_i
1	0	−90	0	−0.818
2	0	90	0	1.412
3	180	90	0	1.635
4	68.395	−108.456	0.292	−1.693
5	95.420	131.811	0	−3.118
6	−113.579	131.811	0	0
7	−123.213	0.592	1.155	2.160

Table V. D-H parameters of the second design.

Joint i	θ_i	α_i	a_i	d_i
1	0	90	0	−0.392
2	0	−134.966	0.048	1.974
3	90	90	1.264	1.395
4	−90	22.614	0	−0.105
5	0	90	0	1.429
6	90	90	0	0
7	166.728	0	0.791	0

Table VI. D-H parameters of the third design.

Joint i	θ_i	α_i	a_i	d_i
1	0	90	0	0
2	54.740	120	0	1.414
3	180	60	0	2.828
4	−70.530	60	0	−2.828
5	−70.530	60	0	2.828
6	109.470	60	0	0
7	−125.260	0	1	−1.414

reciprocal of the condition number. But the condition number of the 6×6 Jacobean can not be used as the isotropy measure because orientational and positional isotropy can not be combined naturally into one number. Here we use the mean value and the optimal single worst value of μ to compare global isotropy of the three manipulators. Let θ_{io} denote the displacement of the ith joint at the isotropic configuration. With μ evaluated at $\theta_{io} \pm k * 5^\circ$ for k = 0, 1, 2, 3, 4, the mean values of the three designs are 6.936, 5.158, 8.048, and the optimal single worst values of μ are 17.310, 10.939, 18.372 respectively. This shows that the second design with more desired parameters has better global isotropy.

VII. CONCLUSION

This paper proposes a method in which the parameters of 6-DOF isotropy generators were used as the initial values to develop n-DOF redundant isotropy generators. An n-DOF isotropic design can be easily developed from higher or lower degree-of-freedom generators. The 6-DOF generators whose straight lines are parallel, perpendicular, or intersecting were developed and then used to develop either isotropic designs with special link parameters or nearly isotropic designs with desired link parameters. How to obtain a design that can reach more isotropic configurations was also investigated. For future work, these methods can be extended to find nearly isotropic designs with (i) more desired link parameters; (ii) proper limited joint ranges such that the design can

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generate a specified workspace without link interactions; and (iii) optimal global isotropy.

List of special symbols

ρ	parameter for the size of a generator
t, ϕ	parameters for the shape of a generator
α_i, β_i	angles specify the direction of the ith straight line of a generator
α_{ij}	angle between lines i and j
ζ_{ij}, ξ_{km}, δ_n	residues of nonlinear equations

Acknowledgments

The authors acknowledge the research support from the National Science Council of Republic of China under grant NSC92-2212-E011-044.

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https://doi.org/10.1017/S0263574704001080 Published online by Cambridge University Press