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ESCAPE DYNAMICS AND POLICY SPECIFICATION

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In his monograph *The Conquest of American Inflation* (Princeton, NJ: Princeton University Press, 1999), Sargent suggests that the sharp reduction in U.S. inflation that took place under Volker may vindicate the type of econometric policy evaluation famously criticized by Lucas (*Carnegie–Rochester Conference Series on Public Policy*, 19–46, 1976). At the core of this vindication story is the escape dynamics, recurrent sliding away from the path leading to the time-consistent suboptimal equilibrium level of inflation and toward the low-inflation, optimal, time-inconsistent Ramsey outcome: by recurrently estimating a reduced-form model, in fact, the policy maker could periodically learn an approximate version of the natural rate hypothesis and therefore be induced to disinflate the economy. Two elements seem important in this story: the type of model used by the policy maker to represent the economy, whether structural or reduced-form, and the policy specification, whether derived taking the private sector's expectations as given or as endogenous to the policy design. Although Sargent (1999) stresses the first element, we find that it is instead the second aspect that is crucial to generate recurrent periods of low inflation: the policy maker has to recognize the endogeneity of the private sector's expectations and refrain from exploiting ephemeral short-run trade-offs between inflation and unemployment.

Keywords: Escape Dynamics, Policy Rule, Expectations, Learning

1. INTRODUCTION

The rise in inflation that took place in the United States in the late 1960s and 1970s and its subsequent sharp reduction under Volker's presidency at the Federal Reserve have attracted the attention of many economists over time, and different alternative explanations have been put forward,[1] from price shocks to changes in the natural rate of unemployment.[2] A more convincing account of these historical events is one that establishes a close relation between different historical inflation records and different (indeed, almost opposite) theories that happened to prevail at the time. The rise in inflation that took place starting from the late 1960s was, according to this argument, the result of the view, first developed by economists

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and then spread out among policy makers, that a permanent trade-off between inflation and unemployment was built into the economic system and was, moreover, exploitable. Trying to pursue lower unemployment, policy makers ended up in generating a burst in inflation. The subsequent stabilization was the result of the development by economists of the natural rate hypothesis (NRH), and of its understanding by policy makers. This explanation is comforting, because it implies that those events cannot happen again: policy makers have understood the lesson, and are no longer prone to an inflationary bias.

Sargent (1999) presents instead a different story, one that vindicates the type of econometric policy evaluation famously criticized by Lucas (1976). According to this story, starting from the late 1960s, and following leading economists, policy makers were inclined to believe in the existence of an exploitable trade-off between inflation and unemployment. To gain information about the response of the economy to monetary policy, authorities recurrently estimated a distributed lag Phillips curve, ignoring the warning by Friedman (1979) and Lucas (1976) about the need to identify the private sector's expectations as a relevant variable in positioning (and shifting) the Phillips curve. Fearing parameter drift, policy makers ran their regressions discounting past observation. This practice generated estimates that fluctuated over time, validating the (wrong) initial belief in parameter drift.

At times the estimated Phillips curve would show a trade-off between inflation and unemployment, inducing the policy maker to inflate the economy in order to reduce unemployment. Given that in reality no such trade-off was present in the economy, successive estimations revealed an adverse shift in the Phillips curve: on the basis of this new information, the policy maker discovered the lack of an exploitable trade-off and was induced to disinflate the economy. This dynamic process, Sargent maintains, would account for the burst in inflation during the late 1960s and 1970s, and for its sharp reduction during the 1980s. But it leaves open the possibility of the same events happening again, as soon as data deceive the policy maker by showing a new exploitable trade-off.

The idea that an adaptive policy can temporarily mitigate the inflationary bias first described by Kydland and Prescott (1977) comes from Sims (1988): a policy maker who conducts his policy allowing for uncertainty and tries to exploit a perceived trade-off can learn an approximated version of the NRH and generate nearly optimal outcomes even without explicitly recognizing the role of expectations. Because policy-created variation in inflation has no effects at all on unemployment, a policy maker who periodically reconsiders the statistical relation between these two variables during his policy experiment realizes this lack of correlation and thus reduces inflation and implements a near-optimal policy. This better outcome nevertheless cannot persist: while low inflation is maintained, spurious statistical evidence emerges that leads the policy maker to believe again in an exploitable trade-off and consequently to increase inflation.

Sargent (1999)'s escape dynamics was obtained with the policy maker using a reduced-form Phillips curve, and this was taken as evidence of vindication for the type of reduced-form models that were highly criticized by Lucas (1976). It is our

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opinion, though, that the model specification (structural or reduced-form) is not so important in this story, and it is instead the way in which the policy maker designs his policy that matters. We will in fact show that a policy maker that recursively estimates a reduced-form model, but takes a short-run perspective in policy design and does not consider the effect of his actions on the private sector's expectations, would not generate escape dynamics: for such dynamics to emerge, the policy maker must recognize the endogeneity of the private sector's expectations in his policy design and be concerned with long-run outcomes, instead of trying to exploit ephemeral short-run trade-offs between inflation and unemployment.

The type of model used for the policy design, in contrast, does not play an important role when the policy maker acts as an adaptive learner and recurrently estimates his model. Economists, and consequently policy makers, knew long before the Lucas critique that to do policy analysis structural models were needed.[3] But structural models alone are not enough: we show that, using such models, together with a policy rule derived taking expectations as given, escape dynamics does not appear and the economy displays a persistently high inflation rate. Escape dynamics and the possibility of (temporarily) low inflation reappear, though, when expectations are taken as endogenous in the policy design and the policy maker takes a long-run perspective.

We claim that this aspect has not been adequately recognized in Sargent (1999)'s story, and in the subsequent works on this topic. Treating the private sector's expectations as endogenous in the policy design is essential for a policy maker that is concerned with long-run outcomes, and this allows him to recognize that no exploitable trade-off is built into the economy. If the optimal policy is derived using a recurrently estimated reduced-form model, as in Sargent (1999)'s story, the low-inflation outcome is bound to be only temporary, whereas if instead the optimal policy is derived using a structural model that incorporates the NRH, the low-inflation outcome can be a permanent one. In this sense, the key element for Sargent's vindication story, as for the alternative explanation that relies on advances in economic theory, is to have a policy maker that refrains from exploiting ephemeral short-run trade-offs.

The structure of the paper is as follows: in Section 2 we present Sargent's results in a simplified framework and then show how escape dynamics disappears when the policy maker is concerned only about short-run effects. In Section 3 we endow the central bank with a structural model and show the difference between the policy maker taking a short-run or a long-run perspective in designing his policy. Section 4 discusses the results and Section 5 concludes. Details of policy derivations are confined to the Appendices.

2. REDUCED-FORM MODELS, ADAPTIVE EXPECTATIONS, AND POLICY RULES

The adaptive expectations story put forward in Sargent (1999) departs from the rational expectations paradigm and considers how the process that agents (in this

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case, the policy maker) follow in forming their beliefs about the relevant economic variables can influence the outcome of an economic system.

In the Sargent (1999) setup, the policy maker tries to manage the economy by setting inflation (treated as the policy instrument) at the level that minimizes the expected losses coming from the resulting combinations of unemployment and inflation. To accomplish this task, the policy maker needs to know how the economy will respond to his policy; i.e., he needs to know the Phillips curve. Not being endowed a priori with this knowledge, the central bank (CB) learns it from experience, recurrently estimating from data the parameters in a misspecified model,[4] a distributed-lag Phillips curve.

If the policy maker were to use a decreasing-gain algorithm, such as recursive least squares (RLS), the ultimate outcome would be the Nash self-confirming, time-consistent equilibrium, with unemployment at its natural level and inflation higher than at the optimal level. An analysis of the convergence of this learning rule, using the stochastic approximation approach, is worked out by Evans and Honkapohja (2001, chap. 13.2). Because in equilibrium both the true and the misspecified model give the same prescription in terms of optimal policy to implement, the policy maker is not disappointed by the outcome he observes even though his beliefs about the economy are wrong.

But if the government in each period were to reestimate a distributed-lag Phillips curve using a constant-gain algorithm, an interesting result would emerge: the policy maker would be able to learn from past attempts to exploit a perceived trade-off between inflation and unemployment, and at times would discover a version of the natural-rate hypothesis that would make him (temporarily) reduce inflation and lead the economy toward the Ramsey optimal time-inconsistent outcome (which is not an equilibrium).[5] Sargent calls this sliding away from the path leading to the Nash equilibrium and toward the Ramsey outcome escape dynamics.

The theory of large deviations, as explained in Cho et al. (2002), characterizes the escape dynamics. These are spurred by unusual shocks affecting the learning process of the policy maker, who temporarily gets to learn an approximated version of the NRH and overcome his inflationary bias. A heuristic analysis of the convergence can be found in Evans and Honkapohja (2001, chap. 14.4).[6]

2.1. Escape Dynamics and the Vindication Story

We show here a simple example of an economy that displays escape dynamics. In this static case, the perceived Phillips curve does not contain any lagged term,[7] whereas the true data-generating process is represented by an expectation-augmented Phillips curve that embodies the NRH. Because the public is endowed with rational expectations, no pattern of monetary policy affects unemployment, which fluctuates stochastically around its natural rate as the economy is hit by uncorrelated shocks.

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The actual Phillips curve for this economy is therefore represented by

$$U_t = U^* + \phi(\pi_t - E_{t-1}\pi_t) + \nu_{1,t}, \tag{1}$$

where U^* is the natural rate of unemployment and ν_1 is an i.i.d. zero-mean random shock, whereas the perceived Phillips curve used by the authority to solve his policy problem takes the form

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + \epsilon_{1,t}, \tag{2}$$

with ϵ_1 a white noise error term.

The resulting reduced-form equation for unemployment is

$$U_t = U^* + \phi\epsilon_{2,t} + \nu_{1,t}, \tag{3}$$

where $\epsilon_{2,t}$ is the i.i.d. zero-mean shock in the inflation equation

$$\pi_t = \hat{\pi}_t + \epsilon_{2,t}. \tag{4}$$

Equation (4) says that the central bank can set inflation up to a random component, where $\hat{\pi}_t$ is the policy instrument chosen to minimize the expected loss coming from the resulting combination of unemployment and inflation. Assuming a quadratic loss function, the policy maker solves the infinite horizon problem[8,9]

$$\min_{\hat{\pi}} E_{t-1} \sum_{t=1}^{\infty} \beta^{t-1}\left(U_t^2 + \pi_t^2\right) \tag{5}$$

under the constraint of the perceived Phillips curve. The resulting policy rule is[10]

$$\hat{\pi}_t = \frac{a_{o,t-1}a_{1,t-1}}{1 + a_{1,t-1}^2}. \tag{6}$$

The policy maker adaptively learns from data the reduced-form Phillips curve needed to solve his policy problem by recursively estimating its parameters according to the algorithm

$$a_t = a_{t-1} + \alpha_t R_t^{-1} X_t (U_t - \hat{U}_t), \tag{7}$$

$$R_t = R_{t-1} + \alpha_t (X_t X_t^{'} - R_{t-1}), \tag{8}$$

where a_t is the vector of parameters to be estimated and X_t the vector of regressors. In this case we have

$$a_t = [a_{0,t} \quad a_{1,t}]', \tag{9}$$

$$X_t = [1 \quad \pi_t]', \tag{10}$$

$$\hat{U}_t = a_{t-1}' X_t. \tag{11}$$

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When α_t is a (small) constant, equations (7) and (8) represent a constant-gain algorithm. To obtain a RLS algorithm, simply substitute $1/t$ for α_t in the learning scheme. The matrix R_t in (8) weights the different elements of the vector conveying new information, giving more importance to the components that are less volatile.[11]

The Phillips curve (1) emphasizes that only unexpected inflation can affect the real economy (employment or output): because agents have rational expectations, they can predict, up to a random error, the level of inflation set by the policy maker, and unemployment fluctuates randomly around its natural level: any systematic component of the policy will be correctly anticipated by the public, and will have no effects on real variables.

Of course, if the policy maker knew the true structure of the economy, two situations would possibly arise: the Nash equilibrium, in the case of discretionary policy; or the Ramsey outcome, if credible commitment or reputational considerations were allowed into the problem. But in this setting the policy maker does not know the true structure of the economy and tries to infer it from data. This introduces new and interesting dynamics into the system.

As said before, if the policy maker were to use a RLS algorithm to estimate the perceived Phillips curve, the economy would converge toward the Nash equilibrium, but if instead a constant-gain algorithm were employed, then the economy would be able to periodically escape from such equilibrium. In fact, as simulations show (Figure 1), inflation recurrently slides away from the path leading to the Nash equilibrium (in the simulation, this corresponds to a 5% level of inflation and a 5% level of unemployment) and approaches the Ramsey outcome (inflation at 0%, unemployment at 5%): when the estimated parameters make the policy maker believe in the NRH, it becomes convenient to disinflate the economy. This happens when parameter a_1 in (2) goes to zero (see Figure 2). But as soon as inflation is driven toward zero, the estimated Phillips curve shows a new exploitable trade-off, and the policy maker starts trying to exploit it again.[12]

PROPOSITION 1. *In an economy described by (1) and (4), a policy maker designing his policy by minimizing (5) s.t. (2), which is recurrently estimated using algorithm (7) and (8), would periodically generate episodes of low inflation (escape dynamics). These, though, would not last, as the economy would constantly be attracted toward the high-inflation Nash equilibrium.*

The alternation of these two types of dynamics, mean (which drives the economy toward the Nash equilibrium) and escape (which leads to the Ramsey outcome), Sargent argues, may explain the high inflation during the 1970s and the successive reduction in the 1980s in the United States. Using the type of econometric policy evaluation criticized by Lucas, the Federal Reserve in the late 1960s and 1970s found in the data a positive trade-off to be exploited, and thus inflated the economy; then, during the 1980s, the induction hypothesis in the estimated Phillips curve was satisfied,[13] no trade-off was perceived to exist, and thus the Fed reduced inflation. But if this story is true, and the "conquest" of inflation was not due instead to a sounder understanding of the NRH by the policy maker, we are

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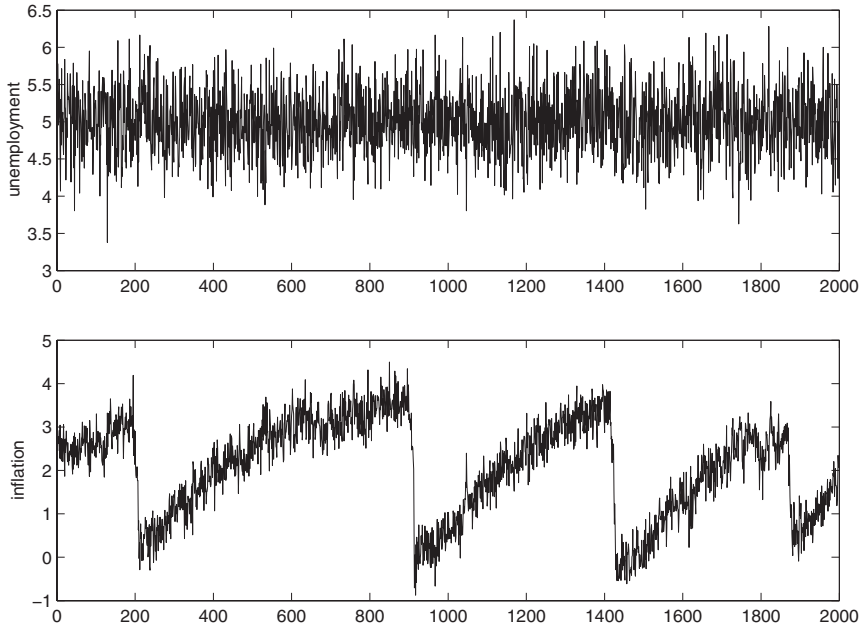


FIGURE 1. Inflation and unemployment under policy rule (6).

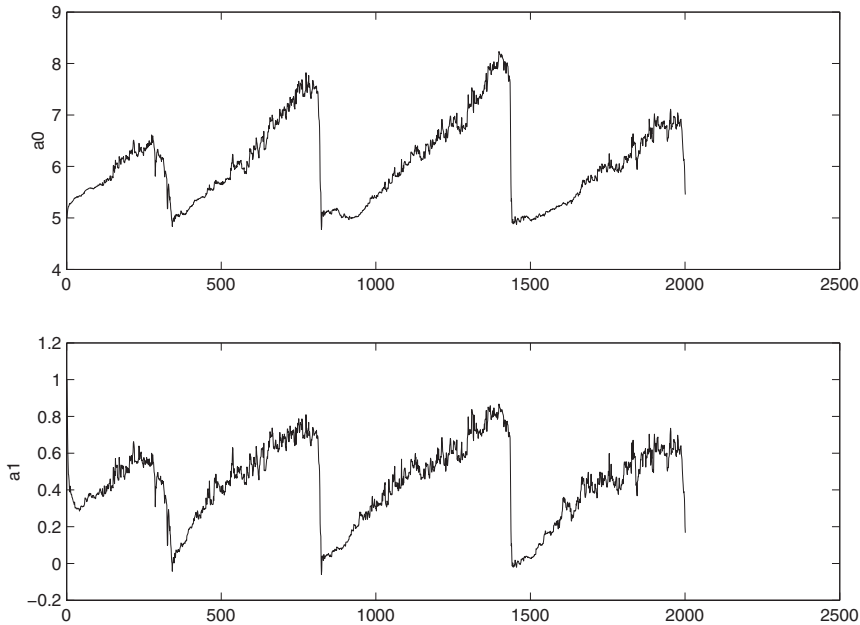


FIGURE 2. Evolution of the estimated parameter values in equation (2).

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doomed to experience high inflation again, as soon as the perceived Phillips curve starts showing a new exploitable trade-off.

2.2. Reduced-Form Models and Short-Run Perspective: A Bad Combination

Sargent (1999)'s results were obtained with the policy maker using a reduced-form Phillips curve, and this was taken as evidence of vindication for the type of reduced-form models that were strongly criticized by Lucas (1976).

It is our opinion, though, that the model specification (structural or reduced-form) is not in fact crucial in this setting, and it is instead the perspective taken by the policy maker when designing his policy that matters. We will argue here that a policy maker who recursively estimates a reduced-form model, but cares only about today's outcome, thus disregarding the effect of his policy on the private sector's expectations, would not generate escape dynamics: in order for them to emerge, the policy maker must be concerned with long-run outcomes and take into account, when designing his policy, the effect of his actions on the private sector's expectations. Moreover, the same results would be obtained by a policy maker who used a structural model: only if he derives his policy by acknowledging the effects that such a policy will have on future expectations can he periodically drive the economy toward a low-inflation outcome.

To make our case, we consider first a policy maker who, using a reduced-form model, solves a simple one-period policy problem

$$\min_{\hat{\pi}_t} E_{t-1}\left(\pi_t^2 + U_t^2\right), \tag{12}$$

s.t.

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + a_{2,t-1}\pi_{t-1} + a_{3,t-1}\pi_{t-2} + a_{4,t-1}\pi_{t-3} + \epsilon_{1,t}. \tag{13}$$

Appendix B shows the derivation of the optimal policy in this case:

$$\hat{\pi}_t = \frac{a_{0,t-1}a_{1,t-1}}{1+a_{1,t-1}^2} + \frac{a_{1,t-1}a_{2,t-1}}{1+a_{1,t-1}^2}\pi_{t-1} + \frac{a_{1,t-1}a_{3,t-1}}{1+a_{1,t-1}^2}\pi_{t-2} + \frac{a_{1,t-1}a_{4,t-1}}{1+a_{1,t-1}^2}\pi_{t-3}. \tag{14}$$

We simulate an economy where the policy maker implements policy rule (14) while adaptively learning the parameters of the perceived Phillips curve (13) with a constant-gain algorithm similar to (7) and (8). Our simulations (Figure 3) reveal that in this case inflation remains persistently around the Nash equilibrium level, and no escapes toward the zero-inflation Ramsey outcome emerge. Moreover, as Figure 3 shows, the inflation path matches closely that of the parameter a_1 on current inflation in the estimated Phillips curve.[14] As this parameter never goes to zero, the policy maker persistently perceives the existence of a short-run trade-off between inflation and unemployment and therefore never drives inflation to zero.

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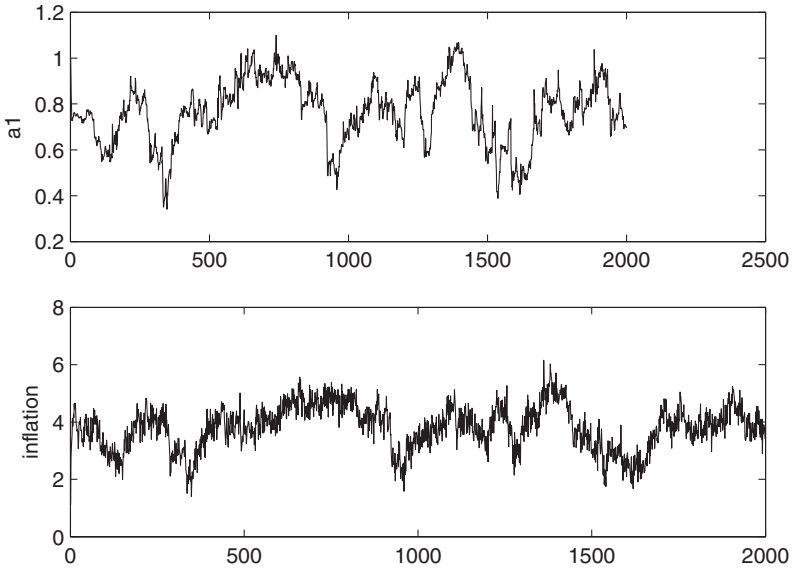


FIGURE 3. Inflation under policy rule (14) and estimated parameter values in equation (13).

PROPOSITION 2. *In an economy described by (1) and (4), a policy maker designing his policy by minimizing (12) s.t. (13), which is recurrently estimated using an algorithm such as (7) and (8), would generate inflation rates persistently close to the high-inflation Nash equilibrium, and never disinflate the economy to the Ramsey outcome.*

3. STRUCTURAL MODELS AND ESCAPE DYNAMICS

To make the point that the policy perspective matters more than the model specification, we consider now a policy maker who uses a perceived Phillips curve that is structural and correctly recognizes the importance of the private sector's expectations. We then solve the policy problem under two alternative scenarios: one in which the CB adopts a short-run perspective and ignores the effects of its policy on expectations, and one where the CB instead takes a long-run perspective and is concerned about the effect of its policy on expectations and therefore on future outcomes.

3.1. Structural Models and Short-Run Perspective: Not Enough

Our economy is still represented by equation (1): unemployment departs from its natural level only when unexpected inflation is generated. But we now endow the policy maker with a correctly specified model, one that takes duly into account

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the role of expectations:[15]

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + a_{2,t-1}\pi_t^e + \epsilon_{1,t}, \tag{15}$$

where π_t^e are the private sector expectations, at time $t-1$, of inflation at time t. The CB thus uses (15) as a constraint in the minimization problem (5),[16] and taking the private sector's expectations as given, obtains the optimal policy

$$\hat{\pi}_t = \frac{a_{o,t-1}a_{1,t-1}}{1+a_{1,t-1}^2} + \frac{a_{1,t-1}a_{2,t-1}}{1+a_{1,t-1}^2}\pi_t^e. \tag{16}$$

To implement this policy, the policy maker needs to have a proxy for the private sector's expectations, and he derives them adaptively according to the equation

$$\pi_t^e = b_{1,t-1}\pi_{t-1} + b_{2,t-1}\pi_{t-2} + b_{3,t-1}\pi_{t-3}, \tag{17}$$

where parameters are recursively estimated from data by regressing inflation on lagged components.

The resulting inflation is shown in Figure 4, together with the evolution of the values for parameter $a_{1,t-1}$ and for the sum $-a_{1,t-1} + a_{2,t-1}$ in (15).[17] Inflation dynamics follow exactly the evolution of the value of parameter a_1. Because the policy maker takes expectations as given, what matters is not the sum $-a_{1,t-1} + a_{2,t-1}$, representing the long-run trade-off in this adaptive model, but just $a_{1,t-1}$, which gives the short-run trade-off. When the CB learns a model in which expectations are explicitly taken into account, and applies a discretionary policy, the sharp escape dynamics that characterize the vindication story do not arise. The policy maker sets inflation always at a positive value, well above the Ramsey zero-inflation level, with the specific magnitude at each time determined by the perceived short-run trade-off as given by the estimated $a_{1,t-1}$. Only if a_1 were to go to zero would the policy maker then perceive that no trade-off was present in the data and set inflation equal to zero, but a_1 never goes to zero (see Figure 4). Thus, a policy maker following the advice coming from an econometric policy evaluation practice with this type of policy rule, even using a structural model, would not have generated the sharp reduction in inflation observed during the 1980s.

PROPOSITION 3. *In an economy described by (1) and (4), a policy maker designing his policy by minimizing (5) s.t. (15), which is recurrently estimated using an algorithm such as (7) and (8), would generate inflation rates persistently close to the high-inflation Nash equilibrium, and never disinflate the economy to the Ramsey outcome.*

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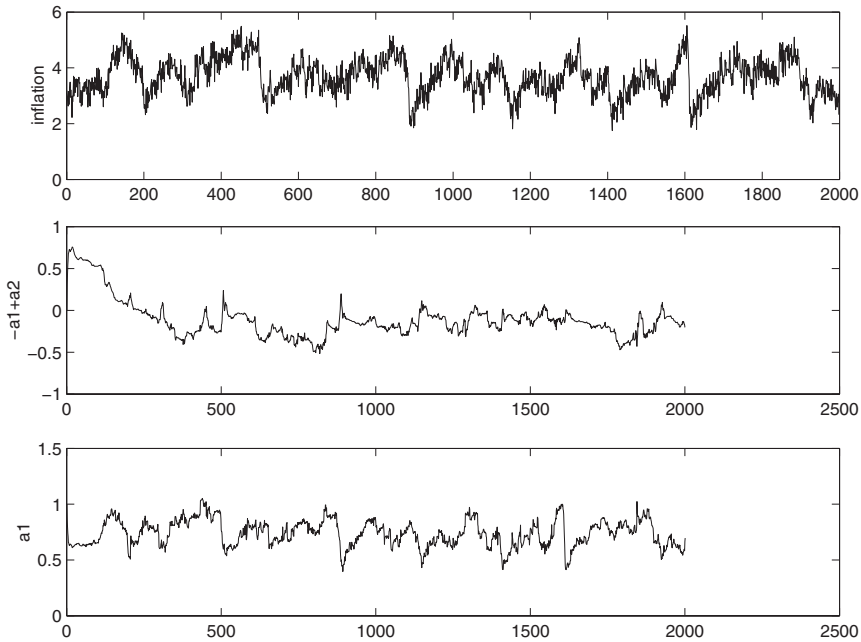


FIGURE 4. Inflation under policy rule (16) and estimated parameter values in equation (15).

3.2. Structural Models and Long-Run Perspective: Escape Dynamics Reappears

Consider now instead the policy problem where the CB minimizes (5), subject to[18]

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + a_{2,t-1}\pi_t^e + \epsilon_{1,t}, \tag{18}$$

$$\pi_t^e = b_{1,t-1}\pi_{t-1}. \tag{19}$$

In this case, expectations are not taken as given and are understood to be affected by past policies. The optimal policy in this case is given by[19]

$$\hat{\pi}_t = \gamma_{0,t-1} + \gamma_{1,t-1}\pi_{t-1}, \tag{20}$$

where $\gamma_{0,t-1}$ and $\gamma_{1,t-1}$ are as in Appendix D.

Note that once intertemporal links are taken into account in the optimization problem, we find from first-order conditions that current policy depends on expected future policy. To actually implement the prescribed policy, this expectational term must be solved out: we do it by deriving the minimum state variable (MSV) solution of the model, using the undetermined coefficient procedure.

We simulate the model, first using a RLS algorithm for the learning of the policy maker, and then a constant-gain algorithm. In the first case, the economy

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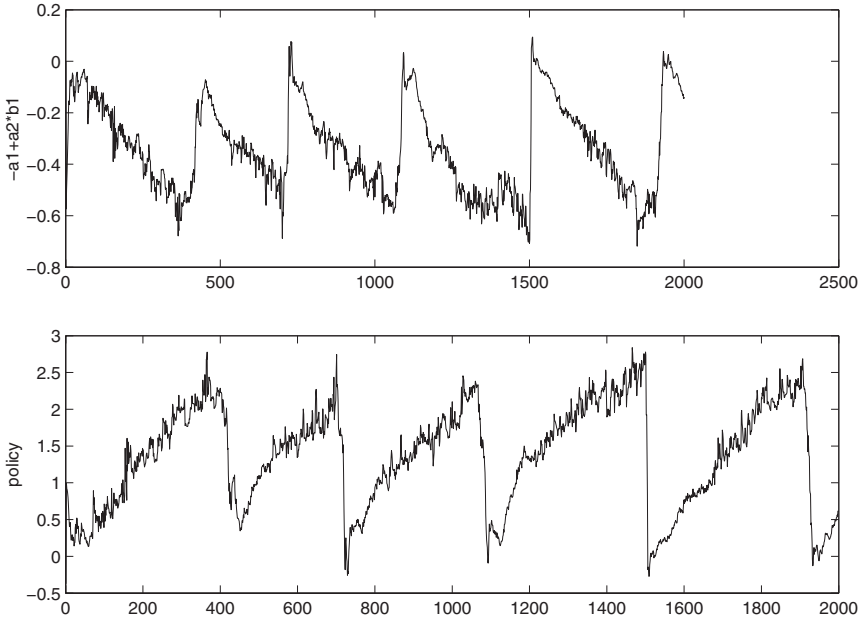


FIGURE 5. Evolution of policy rule (20) and estimated parameter values in equation (18) [after substituting in equation (19)].

converges toward the Nash equilibrium ($U = 5$, $\pi = 5$), but when instead a constant-gain algorithm is implemented, the system does not settle down into an equilibrium, and escape dynamics appear. Figure 5 shows the policy implemented in this last case and the value for the sum $-a_1 + a_2 b_1$:[20] when this value is zero, no long-run trade-off is perceived and inflation is set to zero. Note that a short-run trade-off is always present in the data, because a_1 is systematically different from zero, but the policy maker does not try to exploit it: when the effects of policy on expectations are taken into account, only long-run considerations, as summarized by the induction hypothesis, guide the policy maker's decisions.

PROPOSITION 4. *In an economy described by (1) and (4), a policy maker designing his policy by minimizing (5) s.t. (18) and (19), recurrently estimated using an algorithm such as (7) and (8), would periodically generate episodes of low inflation (escape dynamics). These, though, would not last, as the economy would constantly be attracted toward the high-inflation Nash equilibrium.*

4. DISCUSSION

Given the analysis presented so far, we now try to provide some intuitions for the different results obtained under different policy rules. In the basic case presented first (Section 2.1), the perceived model is static and the short run is

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indistinguishable from the long run. The policy maker sets inflation equal to zero when the coefficient on current inflation in the perceived Phillips curve goes to zero. Because in this model it is not possible to distinguish long-run from short-run considerations, a parameter value of zero on current inflation means at the same time that neither long- nor short-run trade-offs exist. The induction hypothesis, usually referring to a long-run property of the model (requiring a stable policy over many periods), is here given by the single parameter on current inflation, referring to a short-run property.

Using dynamic models, the distinction between short and long run becomes meaningful. When the policy maker implements short-run policies (Sections 2.2 and 3.1), taking the private sector's expectations as given in his policy derivation, what is needed to set inflation to zero is a coefficient on current inflation (the short-run trade-off) equal to zero. But the data never provide such a value. When instead the policy maker considers long-run effects (Section 3.2), what is needed to induce him to drive inflation to zero is the fulfillment of the induction hypothesis. At times, data show this property, leading to disinflationary episodes.[21]

When in Sargent's work the authority estimates a reduced-form Phillips curve with lagged inflation and uses it to find the optimal policy, the policy maker implicitly takes into account the long-run effects of his policy, through the lagged components in future Phillips curves that enter as constraints into the problem. The authority is in effect considering the effects of his policy on expectations, just not explicitly modeling them.[22]

The two things must not be confused: reduced-form vs. structural-form models and short-run vs. long-run considerations. As remarked famously by Lucas (1976), but already noted before by members of the Cowles Commission in the 1950s, in order to carry out policy analysis, it is necessary to have a structural model, one invariant to policy interventions. In Sargent's work, the policy maker uses a reduced-form model, which cannot be regarded as policy-invariant, but reestimates it over time, allowing for parameter drift. In some sense, this procedure fixes the flaw of using a reduced-form model. This explains why, in this setting, using a reduced-form or a structural-form model does not seem to make any difference.

What really matters instead is the type of policy implemented: if the policy maker is short-sighted and tries to exploit short-run trade-offs, high inflation is bound to prevail, no matter what kind of model he uses for policy analysis. Even with the true model known, inflation would be at the nonoptimal Nash equilibrium level. If instead long-run considerations enter into the picture, the policy maker takes into account the effects of his policy on the private sector's expectations and is thus led to reduce inflation: periodically, if an estimated Phillips curve is utilized (either in reduced or structural form), or permanently, if theoretical considerations about the NRH are the guide for his actions.

In this setting, as said, using a reduced-form or a structural-form model does not make much difference in terms of policy recommendations, because the policy maker continuously reestimates his model, in this way accounting for

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parameter drifts. Structural forms are supposed to be policy-invariant, thus suitable for policy analysis (but not necessarily for long-run forecasts, because other sources of parameter drifts may be present); reduced-form models are not meant to be policy-invariant, their parameters varying with variations in policy (at least as long as rational expectations are involved).[23] Continuously reestimating a model compensates, to some degree, for the problem of policy-induced parameter drift in the Phillips curve. This is related to what Lucas (1976) suggested as a possible explanation for the success of short-term econometric forecasting: practising econometricians kept revising their estimated relations used for econometric forecasting, thus practically accounting for non–policy induced parameter drifts.

5. CONCLUSIONS

In this work we have reconsidered the vindication story proposed by Sargent (1999) as a possible explanation for the burst in U.S. inflation during the late 1960s and 1970s and for its subsequent conquest, trying to qualify its relevance. We have shown that the escape dynamics that lies at the core of that story, from the high-inflation Nash equilibrium and toward the low-inflation Ramsey outcome, does not depend on the econometric specification of the model the policy maker uses, but rather depends on the time horizon of the policy he implements, i.e., on whether the authority is concerned only with short-run or also with long-run results. There is no relation between the two aspects: long-run considerations, as well as short-run ones, can be carried out using either a structural or a reduced-form model, as long as those models are recurrently estimated to account for parameter drifts.

The vindication story of Sargent (1999) requires a policy maker that takes a long-run perspective and is able to resist the temptation of a short-sighted management of the economy. In this sense, it is the same requirement necessary for the alternative story, the triumph of the NRH, to hold. Even in that case the policy maker, once it is understood that no permanent trade-off is available, must be able to refrain from generating surprise in inflation in order to temporarily reduce unemployment.[24]

As long as a policy maker is prone to exploit a short-run trade-off, there is no hope of obtaining a low-inflation outcome. When he understands the long-run implications of his policy, better outcomes can be achieved: if this is done through econometric practice, the escape phenomenon is what we expect to see; if instead this understanding comes from more sound economic theory, the Ramsey zero-inflation result is possible.

NOTES

1. For a brief review of possible alternative explanations, see Taylor (1996).

2. This hypothesis has been tested econometrically by Ireland (1999), who found that data on U.S. inflation and unemployment from 1960 to 1997 are consistent with it.

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3. The need for structural models in policy analysis has been recognized at least since the early 1950s. See Marschak (1953).

4. The model is misspecified in the sense that it fails to take explicitly into account the role of expectations in positioning the Phillips curve.

5. The Ramsey outcome is the one that would be chosen by a central planner maximizing social welfare.

6. They note that the phenomenon of escape dynamics seems to be sensitive to the choice of learning scheme: simulations using stochastic-gradient learning with constant gain did not show this phenomenon. The stochastic-gradient learning scheme differs from recursive least squares (RLS), and in general from Newton-type algorithms, because it does not take into account the matrix of second moments in updating beliefs. We tried some simulations using our baseline model, modified to implement a stochastic gradient algorithm, and no escape dynamics showed up. This aspect, we think, deserves further attention.

7. The same basic version of the model has been used in Gerali and Lippi (2002) and Cho et al. (2002).

8. Here the target levels for unemployment and inflation are set to zero. Because the natural rate of unemployment is set to 5%, the policy maker has an inflationary bias.

9. As in Sargent's work, the policy maker disregards uncertainty about parameter values and utilizes the most recent estimates as if they were the true ones. In other words, the authority behaves as if the perceived law of motion were time-invariant, and under this assumption derives the optimal policy. Beck and Wieland (2002) call a rule coming from this procedure a *certainty-equivalent rule*; Kendrick (2002) reports different names for this method: *sequential certainty equivalence*, *update certainty equivalence*, and *heuristic certainty equivalence*; Kreps (1998) refers to this approach as an *anticipated utility* model.

10. For the derivation of this policy, see Appendix A.

11. Initial values for a_0 and R_0 must be set in order to simulate the algorithm. In the example below, we set $a_{0,0} = 5$, $a_{1,0} = 1$, and R_0 equal to the identity matrix (we assign equal weight to the different components, supposing that at the beginning the central bank has no information about their volatility). In addition, ϕ is set equal to 1, $U^* = 5$, $\alpha = 0.0275$. The standard deviation of all the shocks is set at 0.3.

12. Gerali and Lippi (2002) study the effects of different policy objectives on the escape dynamics. They find, somewhat counterintuitively, that a sufficiently inflation-averse policy maker never escapes Nash inflation. Nevertheless, his conservatism does produce a lower inflation rate on average.

13. The induction hypothesis is fulfilled when the sum of coefficients on current and lagged inflation in the Phillips curve is estimated to be zero: this means that inflation has no permanent impact on unemployment.

14. Initial values for the simulations are set as follow: $a_{0,0} = 3$, $a_{1,0} = 1$, $a_{2,0} = 0$, $a_{3,0} = 0$, $a_{4,0} = 0$. All the rest as before.

15. Equation (15) is a generalization of the expectations-augmented Phillips curve, one that allows the coefficients on inflation and expected inflation to differ from each other.

16. For the derivation of the policy rule in this case, see Appendix C.

17. The initial values for simulations are set as follow: $a_{0,0} = 3$, $a_{1,0} = 1$, $a_{2,0} = 1$, $b_{1,0} = 1/3$, $b_{2,0} = 1/3$, $b_{3,0} = 1/3$. The other parameters are set as before.

18. For convenience, we leave out now the extra lags in the estimation of inflation expectations, as they do not make any difference to the argument that we want to make here.

19. The derivation of the optimal policy can be found in Appendix D. An alternative procedure for finding the optimal policy is solving a dynamic programming problem, as is done by Sargent (1999). We tried this way also and obtained similar results.

20. The initial values for the simulations are set as follow: $a_{0,0} = 3$, $a_{1,0} = 1$, $a_{2,0} = 1$, $b_{1,0} = 1$. Other parameters as before, with the addition of β, now relevant, set at 0.98.

21. The same happens in Sargent (1999)'s original formulation, where the Phillips curve includes past inflation and unemployment as regressors: although the coefficient on current inflation never goes

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to zero, the induction hypothesis is at times satisfied, and the policy maker drives the economy toward the Ramsey outcome.

22. Sargent (1999) does not provide an analytical solution for the policy rule in the dynamic model, so we cannot see explicitly how that policy is derived.

23. Evans and Ramey (2006) show that the Lucas critique applies to a broader set of models than the one characterized by rational expectations: in particular, models with adaptive expectations, provided that the updating parameter is optimally tuned, are also subject to the critique.

24. For a discussion of the relevance of the inflationary bias, and why this may not prevent a central bank from implementing time-inconsistent optimal policy, see McCallum (1997).

25. The MSV solution is the bubble-free solution, the one based only on fundamentals. See McCallum (1999).

26. The reason that this is the solution selected by the MSV criterion is that when $b_1 = 0$, and therefore $k_1 = 0$, it must be that $\gamma_1 = 0$. The non-MSV solution is instead given by $\gamma_1 = (1 + \sqrt{1 - 4k_1k_2})/2k_2$ and leads to an explosive path for inflation.

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APPENDIX A

The problem the policy maker has to solve in order to find the optimal policy is the following:

$$\min_{\hat{\pi}} E_{t-1} \sum_{t=1}^{\infty} \beta^{t-1} \left(\pi_t^2 + U_t^2\right), \tag{A.1}$$

s.t.

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + \epsilon_{1,t}, \tag{A.2}$$

$$\pi_t = \hat{\pi}_t + \epsilon_{2,t}. \tag{A.3}$$

Substituting (A.2) and (A.3) into (A.1) and noting that no intertemporal dependence is involved, we simplify the problem as

$$\min_{\hat{\pi}} E_{t-1}[(\hat{\pi}_t + \epsilon_{2,t})^2 + (a_{0,t-1} - a_{1,t-1}(\hat{\pi}_t + \epsilon_{2,t}) + \epsilon_{1,t})^2], \tag{A.4}$$

whose FOC is

$$E_{t-1}[2(\hat{\pi}_t + \epsilon_{2,t}) + 2(a_{0,t-1} - a_{1,t-1}(\hat{\pi}_t + \epsilon_{2,t}) + \epsilon_{1,t})(-a_{1,t-1})] = 0, \tag{A.5}$$

which gives the optimal policy

$$\hat{\pi}_t = \frac{a_{o,t-1}a_{1,t-1}}{1 + a_{1,t-1}^2}. \tag{A.6}$$

APPENDIX B

Because the policy maker in this case is concerned only about today's outcome, his problem is as follows:

$$\min_{\hat{\pi}_t} E_{t-1}\left(\pi_t^2 + U_t^2\right), \tag{B.1}$$

s.t.

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + a_{2,t-1}\pi_{t-1} + a_{3,t-1}\pi_{t-2} + a_{4,t-1}\pi_{t-3} + \epsilon_{1,t}, \tag{B.2}$$

$$\pi_t = \hat{\pi}_t + \epsilon_{2,t}. \tag{B.3}$$

Substituting (B.2) and (B.3) into (B.1), we obtain

$$\min_{\hat{\pi}_t} E_{t-1}\{(\hat{\pi}_t + u_t)^2 + [a_{0,t-1} - a_{1,t-1}(\hat{\pi}_t + u_t) + a_{2,t-1}\pi_{t-1} + a_{3,t-1}\pi_{t-2} + a_{4,t-1}\pi_{t-3} + \epsilon_{1,t}]^2\}. \tag{B.4}$$

The optimality condition is given by

$$E_{t-1}[2(\hat{\pi}_t + u_t) + 2(a_{0,t-1} - a_{1,t-1}(\hat{\pi}_t + u_t) + a_{2,t-1}\pi_{t-1} + a_{3,t-1}\pi_{t-2} + a_{4,t-1}\pi_{t-3} + \epsilon_{1,t})(-a_{1,t-1})] = 0, \tag{B.5}$$

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which implies that

$$\hat{\pi}_t - a_{0,t-1}a_{1,t-1} + a_{1,t-1}^2\hat{\pi}_t - a_{1,t-1}a_{2,t-1}\pi_{t-1} - a_{1,t-1}a_{3,t-1}\pi_{t-2} - a_{1,t-1}a_{4,t-1}\pi_{t-3} = 0, \quad \textbf{(B.6)}$$

and therefore the optimal policy is

$$\hat{\pi}_t = \frac{a_{0,t-1}a_{1,t-1}}{1+a_{1,t-1}^2} + \frac{a_{1,t-1}a_{2,t-1}}{1+a_{1,t-1}^2}\pi_{t-1} + \frac{a_{1,t-1}a_{3,t-1}}{1+a_{1,t-1}^2}\pi_{t-2} + \frac{a_{1,t-1}a_{4,t-1}}{1+a_{1,t-1}^2}\pi_{t-3}. \quad \textbf{(B.7)}$$

APPENDIX C

The policy problem is given by

$$\min_{\hat{\pi}} E_{t-1}\sum_{t=1}^{\infty}\beta^{t-1}\left(\pi_t^2 + U_t^2\right) \quad \textbf{(C.1)}$$

s.t.

$$U_t = a_{0,t-1} - a_{1,t-1}\pi_t + a_{2,t-1}\pi_t^e + \epsilon_{1,t}, \quad \textbf{(C.2)}$$

$$\pi_t = \hat{\pi}_t + \epsilon_{2,t}. \quad \textbf{(C.3)}$$

Here the policy maker takes the private sector's expectations as given, and does not consider the effect of his own policy on them. Substituting (C.2) and (C.3) into (C.1), we get the problem

$$\min_{\hat{\pi}_t} E_{t-1}\sum_{t=1}^{\infty}\beta^{t-1}\{(\hat{\pi}_t + \epsilon_{2,t})^2 + [a_{0,t-1} - a_{1,t-1}(\hat{\pi}_t + \epsilon_{2,t}) + a_{2,t-1}\pi_t^e + \epsilon_{1,t}]^2\}. \quad \textbf{(C.4)}$$

The FOC is

$$E_{t-1}[2(\hat{\pi}_t + \epsilon_{2,t}) + 2(a_{0,t-1} - a_{1,t-1}(\hat{\pi}_t + \epsilon_{2,t}) + a_{2,t-1}\pi_t^e + \epsilon_{1,t})(-a_{1,t-1})] = 0, \quad \textbf{(C.5)}$$

which implies that

$$\hat{\pi}_t - a_{0,t-1}a_{1,t-1} + a_{1,t-1}^2\hat{\pi}_t - a_{1,t-1}a_{2,t-1}\pi_t^e = 0. \quad \textbf{(C.6)}$$

The optimal policy therefore is

$$\hat{\pi}_t = \frac{a_{0,t-1}a_{1,t-1}}{1+a_{1,t-1}^2} + \frac{a_{1,t-1}a_{2,t-1}}{1+a_{1,t-1}^2}\pi_t^e, \quad \textbf{(C.7)}$$

where $\pi_t^e = b_{1,t-1}\pi_{t-1}$.

APPENDIX D

For simplicity of exposition, we drop the time subscript from the parameters. Parameters a and b must be understood to be the most recent estimates available to the policy maker.

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The problem to be solved by the policy maker is as follows:

$$\min_{\hat{\pi}} E_{t-1} \sum_{t=1}^{\infty} \beta^{t-1} \left(\pi_t^2 + U_t^2\right), \tag{D.1}$$

s.t.

$$U_t = a_0 - a_1 \pi_t + a_2 \pi_t^e + \epsilon_{1,t}, \tag{D.2}$$

$$\pi_t = \hat{\pi}_t + \epsilon_{2,t}, \tag{D.3}$$

$$\pi_t^e = b_1 \pi_{t-1}. \tag{D.4}$$

Substituting (D.4) into (D.2) and the resulting expression, together with (D.3), into (D.1), we get

$$\min_{\hat{\pi}_t} E_{t-1} \sum_{t=1}^{\infty} \beta^{t-1} \{(\hat{\pi}_t + \epsilon_{2,t})^2 + [a_0 - a_1(\hat{\pi}_t + \epsilon_{2,t}) + a_2 b_1 \pi_{t-1} + \epsilon_{1,t}]^2\}. \tag{D.5}$$

The FOC is

$$\begin{aligned} &E_{t-1}[2(\hat{\pi}_t + \epsilon_{2,t}) + 2(a_0 - a_1(\hat{\pi}_t + \epsilon_{2,t}) + a_2 b_1 \pi_{t-1} + \epsilon_{1,t})(-a_1) \\ &\quad + 2\beta(a_0 - a_1(\hat{\pi}_{t+1} + \epsilon_{2,t+1}) + a_2 b_1(\hat{\pi}_t + \epsilon_{2,t}) + \epsilon_{1,t+1})(a_2 b_1)] = 0, \end{aligned} \tag{D.6}$$

which can be simplified as

$$E_{t-1}[\hat{\pi}_t + (a_0 - a_1 \hat{\pi}_t + a_2 b_1 \pi_{t-1})(-a_1) + \beta(a_0 - a_1 \hat{\pi}_{t+1} + a_2 b_1 \hat{\pi}_t)(a_2 b_1)] = 0 \tag{D.7}$$

or

$$E_{t-1}\left[\hat{\pi}_t - a_0 a_1 + a_1^2 \hat{\pi}_t - a_1 a_2 b_1 \pi_{t-1} + \beta a_0 a_2 b_1 - \beta a_1 a_2 b_1 \hat{\pi}_{t+1} + \beta a_2^2 b_1^2 \hat{\pi}_t\right] = 0, \tag{D.8}$$

and solving for the policy instrument, we obtain

$$\hat{\pi}_t = \frac{1}{1 + a_1^2 + \beta a_2^2 b_1^2}[(a_0 a_1 - \beta a_0 a_2 b_1) + (a_1 a_2 b_1)\pi_{t-1} + \beta a_1 a_2 b_1 E_{t-1} \hat{\pi}_{t+1}]. \tag{D.9}$$

For simplicity of exposition we write

$$k_0 = \frac{a_0 a_1 - \beta a_0 a_2 b_1}{1 + a_1^2 + \beta a_2^2 b_1^2}, \tag{D.10}$$

$$k_1 = \frac{a_1 a_2 b_1}{1 + a_1^2 + \beta a_2^2 b_1^2}, \tag{D.11}$$

$$k_2 = \frac{\beta a_1 a_2 b_1}{1 + a_1^2 + \beta a_2^2 b_1^2}, \tag{D.12}$$

and the policy rule becomes

$$\hat{\pi}_t = k_0 + k_1 \pi_{t-1} + k_2 E_{t-1} \hat{\pi}_{t+1}. \tag{D.13}$$

Equation (D.13) says that the optimal policy today depends on the optimal policy the authority expects to implement in the next period.

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The true Phillips curve,

$$U_t = U^* + \phi(\pi_t - E_{t-1}\pi_t) + \nu_{1,t}, \tag{D.14}$$

and (D.13), together with condition (D.3), gives a solution for U_t and π_t. But (D.13) is clearly independent from (D.14) and gives a solution for $\hat{\pi}_t$. To find an explicit form for $\hat{\pi}_t$ we look for the MSV solution, using the undetermined-coefficient procedure.

Given that the only state variable in (D.13) is lag inflation, we guess a solution of the form

$$\hat{\pi}_t = \gamma_0 + \gamma_1 \pi_{t-1}. \tag{D.15}$$

Therefore, using condition (D.3), it follows that

$$E_{t-1}\hat{\pi}_{t+1} = \gamma_0 + \gamma_1 E_{t-1}\pi_t = \gamma_0 + \gamma_1 \hat{\pi}_t = \gamma_0 + \gamma_1(\gamma_0 + \gamma_1 \pi_{t-1}), \tag{D.16}$$

and for our guess to be correct it must be that

$$\gamma_0 + \gamma_1 \pi_{t-1} = k_0 + k_1 \pi_{t-1} + k_2[\gamma_0 + \gamma_1(\gamma_0 + \gamma_1 \pi_{t-1})], \tag{D.17}$$

which implies that

$$\gamma_0 = k_0 + k_2\gamma_0 + k_2\gamma_1\gamma_0 \tag{D.18}$$

and

$$\gamma_1 = k_1 + k_2\gamma_1^2. \tag{D.19}$$

From the second equation we have

$$\gamma_1 = \frac{1 \pm \sqrt{1 - 4k_1k_2}}{2k_2}, \tag{D.20}$$

and from the first

$$\gamma_0 = \frac{k_0}{1 - k_2(1 + \phi_1)}. \tag{D.21}$$

The MSV solution[25] is given by[26]

$$\gamma_1 = \frac{1 - \sqrt{1 - 4k_1k_2}}{2k_2}, \tag{D.22}$$

$$\gamma_0 = \frac{k_0}{1 - k_2\left(1 + \frac{1-\sqrt{1-4k_1k_2}}{2k_2}\right)}. \tag{D.23}$$

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