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WHY DO RISK PREMIA VARY OVER TIME? A THEORETICAL INVESTIGATION UNDER HABIT FORMATION

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We study the dynamics of risk premia in a model with external habit formation and highlight the significance of "recession predictability". Although under the specification of Campbell and Cochrane, [*Journal of Political Economy* 107, 205–251 (1999)] the equity risk premium is countercyclical because increases in risk aversion are reinforced by rising recession risks, this need not be the case more generally. We show analytically that in endowment economies procyclical recession expectations can outweigh countercyclical changes in risk aversion, generating counterfactual risk-premium behavior. However, allowing shocks or habits to be sufficiently persistent, or explicitly accounting for the impact of habits on consumption, suffices to generate countercyclical recession risks and risk premia.

Keywords: Risk Premium, Habits, Countercyclicality, Macrofinance

1. INTRODUCTION

Existing empirical evidence suggests that risk premia vary countercyclically over time. [Harvey (1989) and Li (2001) focus on equities; Campbell and Shiller (1991) and Cochrane and Piazzesi (2005) present evidence for the term premium; Lustig

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and Verdelhan (2007) document strong countercyclicality in the exchange rate risk premium]. The two most seminal representative-agent models proposed in this context seem to rely on different mechanisms of accounting for the empirical regularities. In Campbell and Cochrane (1999), risk aversion fluctuates countercyclically as agents' consumption moves relative to a reference habit level. In Bansal and Yaron (2004), shifts in premia are driven by predictable components in consumption as well as time-varying volatility. In this paper we focus on the habit mechanism and show that it also relies heavily on cyclical variation in underlying state variables. In particular, it depends on the fact that recession risks change predictably over the cycle.[1]

Although Campbell and Cochrane (2000) have also stressed the role of predictability in the context of consumption-based asset pricing models, the quantitative importance of the recession risk channel has not been studied extensively.[2] The likely reason is that under the original specification of Campbell and Cochrane (1999), slow-moving habits ensure that recession expectations vary countercyclically. Accordingly, the recession risk channel reinforces the risk-aversion channel and amplifies countercyclical variation in risk premia. Our first contribution is to document that under simpler, but commonly used, habit specifications this need not be the case. In particular, we show that in a model with *nonpersistent* habits and trend-stationary consumption, recession risks vary procyclically, offsetting the impact of countercyclical risk aversion and generating *procyclical* premia. We also demonstrate how increasing the degree of persistence in consumption and habits can overturn this result.

The benchmark endowment-economy framework we adopt is that of Lucas's (1978) and Mehra and Prescott's (1985) asset-pricing model. Although the parsimony of our setup keeps the analysis tractable and enables us to prove most results analytically, the danger is that misspecifying the process for consumption might invalidate the conclusions. To address this point, we verify our findings under a wide range of specifications—including the standard "finance" assumption of i.i.d. consumption growth, persistent consumption growth dynamics [Carroll et al. (2011)], and allowing for alternative trend specifications [den Haan (1995)]. The finding that increasing shock and habit persistence changes the cyclical properties of recession risks and is more likely to make risk premia vary countercyclically survives irrespective of the exact set of underlying assumptions.

Because a slowly time-varying, countercyclical risk premium is key for matching asset pricing data [Campbell and Cochrane (1999, p. 207)], any model with procyclical recession risks might fail along the asset pricing dimension. This finding could potentially have important implications for dynamic macroeconomic models that rely on *nonpersistent* habit specifications [Christiano et al. (2005); Smets and Wouters (2007); Uhlig (2007)] and that are increasingly frequently used to address asset pricing questions [Jermann (1998); De Paoli et al. (2010); Hordahl et al. (2008); Rudebusch and Swanson (2008)].[3] In the remainder of this paper we thus try to clarify risk-premium implications of habit formation in production-economy models.

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An important complication that arises in nonendowment economy models is that habits—in addition to affecting the degree of fluctuations in risk aversion and recession risks—also have an endogenous impact on the dynamics of consumption [in particular, they have a well-documented role in helping to generate hump-shaped consumption impulse responses; see also Fuhrer (2000); Christiano et al. (2005); Smets and Wouters (2007)]. To avoid obfuscating these different channels, we start by proving results analytically in an endowment economy setup under the assumption of hump-shaped consumption dynamics (proxy for the general equilibrium impact of habits). We then verify these results numerically in a fully specified dynamic general equilibrium model. In summary, our second contribution is to show that allowing habits to feed back endogenously on consumption is likely to make recession risks vary countercyclically regardless of the degree of persistence in the habit process. This suggests a potentially important role for habit formation in helping to match risk-premium dynamics in production-economy models.

Several existing contributions explore issues related to those we analyze. Li (2007) documents that premia in the framework of Campbell and Cochrane (1999) are not robustly countercyclical—a point similar to the one we make, though in a different setup. den Haan (1995) considers a related set of driving processes for consumption, but focuses on their implications for the slope of the yield curve. In addition, there are many papers showing how habits in the utility help to match empirical properties of anything from exchange rate risk premia [Verdelhan (2010)] to yield curve dynamics [Wachter (2006)], though we are unaware of any that explicitly investigate the implications of time-varying recession-risks in a general equilibrium context.

In the remainder, we set up the model, present the analytical results, and discuss the underlying intuition, before summarizing and concluding.

2. MODEL AND NOTATION

Our analysis proceeds in the simplest possible setup—that of Lucas's (1978) asset pricing model augmented by allowing for external habits.[4] Agents, indexed by $i \in [0, 1]$ choose consumption C_t^i, investment in riskless bonds B_t^i, and investment in risky assets S_t^i to maximize expected lifetime utility,

$$\max_{C_t^i,\ B_t^i,\ S_t^i} \mathbf{E}\left[\sum_{t=0}^{+\infty} \beta^t \frac{\left(C_t^i - hX_t\right)^{1-\rho} - 1}{1-\rho}\right], \tag{1}$$

$$C_t^i + V_t^f B_t^i + V_t^r S_t^i = B_{t-1}^i + S_{t-1}^i\left(V_t^r + D_t\right), \tag{2}$$

where X_t denotes the external habit, V_t^f is the time-t price of a one-period bond paying a unit of the consumption good next period, and V_t^r is the price of a perfectly divisible risky asset entitling its owner to the stream of dividends $D_{t+1}, D_{t+2} \ldots$.[5]

The way in which habits X_t are defined has important implications for the dynamics of asset prices and risk premia. We choose not to adopt the nonlinear

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specification of Campbell and Cochrane (1999) because the latter has not been used in any larger production-economy DSGE models, is not particularly tractable or tightly parameterized,[6] and finally—in contrast to simpler linear specifications—the rationale for referring to X_t as "habits" is not immediately clear.

Instead, our approach is to posit that habits are an average of past levels of aggregate consumption C_t, with a single parameter $\phi \in [0, 1]$ controlling persistence:

$$X_t := (1 - \phi)C_{t-1} + \phi X_{t-1}. \tag{3}$$

By the setup deliberately being kept simple, our results can shed light on the relative importance of various channels driving the results in Campbell and Cochrane (1999), who use an equation like (3) as a foundation of their model. Furthermore, when $\phi = 0$, the specification simplifies to one in which habits are purely a function of the last period's aggregate consumption, which has frequently been used in the macro literature. Finally, despite the simplicity, the implied coefficient of risk aversion varies countercyclically, and so we retain crucial features of the successful nonlinear specification.[7]

The standard first-order conditions with respect to asset holdings are

$$R^f_{t+1} \cdot \mathbf{E}_t \mathcal{M}^i_{t+1} = 1, \qquad \mathbf{E}_t \mathcal{M}^i_{t+1} R^r_{t+1} = 1, \tag{4}$$

where the stochastic discount factor $\mathcal{M}^i_t$, the marginal utility of consumption Λ^i_t, and gross returns on bonds R^f_t and risky assets R^r_t are given by[8]

$$\mathcal{M}^i_{t+1} := \beta \cdot \frac{\Lambda^i_{t+1}}{\Lambda^i_t}, \qquad \Lambda^i_{t+1} := \left(C^i_{t+1} - hX_{t+1}\right)^{-\rho},$$

$$R^f_{t+1} := \frac{1}{V^f_t}, \qquad R^r_{t+1} := \frac{V^r_{t+1} + D_{t+1}}{V^r_t}.$$

We then let excess consumption C^e_t and the surplus ratio S_t be defined as

$$C^e_t := C_t - hX_t, \qquad S_t := \frac{C_t - hX_t}{C_t} = \frac{C^e_t}{C_t}. \tag{5}$$

As in Campbell and Cochrane (1999), (p. 220), we refer to periods with low values of the surplus consumption ratio as "recessions" and the term $\mathbf{E}_t 1/S_{t+1}$ (or $S_t \mathbf{E}_t 1/S_{t+1}$) will be used to characterize the "fear of recessions" channel.[9]

The textbook definition of the equity risk premium rp_t is

$$\mathrm{rp}_t := \mathbf{E}_t \left[\log\left(R^r_{t+1}\right)\right] - \log\left(R^f_{t+1}\right) = \mathbf{E}_t r^r_{t+1} - r^f_{t+1}, \tag{6}$$

where lowercase letters denote logs.

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Finally, we can define the coefficient of relative risk aversion as[10]

$$\eta(C_t, X_t) := -C_t \cdot \frac{U_{cc}(C_t, X_t)}{U_c(C_t, X_t)},$$

where $U_y(\cdot, \cdot)$ denotes the partial derivative of the utility function $U(\cdot, \cdot)$ with respect to y. Because this coefficient measures agents' willingness to enter pure consumption gambles, given habits fixed at reference level X_t, this can be referred to as *consumption* risk aversion. It is easy to show that if the utility function and external habits are as in equations (1)–(3), then the coefficient of consumption risk aversion is countercyclical.[11]

3. RESULTS

To analyze the determinants of risk premium dynamics in the model, we can derive a second-order approximation to the first-order conditions. This approximation implies that

$$\mathrm{rp}_t + \frac{1}{2}\mathrm{var}_t\, r^r_{t+1} \approx \rho\, \mathrm{cov}_t\left(c^e_{t+1}, r^r_{t+1}\right). \tag{7}$$

Jensen's inequality term aside, the risk premium is proportional to *excess consumption* relative risk aversion ρ and the conditional covariance of returns r^r_{t+1} with excess consumption c^e_{t+1}. Following Li (2001), and under the assumptions discussed therein (see the Appendix for a brief discussion), we can apply Stein's lemma and express $\mathrm{cov}_t(c^e_{t+1}, r^r_{t+1})$ as

$$\mathrm{cov}_t\left(c^e_{t+1}, r^r_{t+1}\right) = \mathrm{cov}_t\left(c_{t+1}, r^r_{t+1}\right) \mathbf{E}_t \frac{1}{S_{t+1}}. \tag{8}$$

Mechanically, equation (8) demonstrates that agents' expectations about recessions matter because they affect the covariance of excess consumption and returns. Combined with equation (7) this shows that if $\mathrm{cov}_t(c_{t+1}, r^r_{t+1})$ is time-invariant, then only changes in these expectations are going to affect risk premium cyclicality.[12]

Crucially, equation (8) can be rewritten as

$$\mathrm{rp}_t + \frac{1}{2}\mathrm{var}_t\, r^r_{t+1} \approx \eta_t\, \mathrm{cov}_t\left(c_{t+1}, r^r_{t+1}\right) \mathbf{E}_t \frac{S_t}{S_{t+1}}, \tag{9}$$

which demonstrates that the risk premium is determined by the coefficient of risk aversion η_t, the covariance of consumption, and returns, as well as the "fear of recessions" variable $\mathbf{E}_t(1/S_{t+1})/(1/S_t)$—which determines whether agents expect their living conditions to improve or deteriorate in the next period. Because the coefficient of relative risk aversion η_t is countercyclical and the covariance term approximately constant, the only factor with the potential to distort the cyclicality of rp_t is the recession expectation term.

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In the remainder of this paper, we shall therefore study how assumptions made about consumption and habits affect the cyclicality of $\mathbf{E}_t S_t / S_{t+1}$. In the first section, we shall implicitly ignore the fact that habits have the potential to endogenously affect the dynamics of consumption. In the second section, we explicitly try to account for such feedbacks and investigate the impact of hump-shaped consumption responses on the cyclicality of recession expectations (in an endowment and production economy).

3.1. No Endogenous Feedback of Habits on Consumption

In this section we investigate factors driving risk premium cyclicality under different specifications for the endowment process. Following den Haan (1995), we examine the case in which consumption is stationary around a linear trend and a case in which it is growth-rate stationary (the case of i.i.d. consumption growth is nested by both of these specifications). We shall show that if consumption is not sufficiently persistent, then habits need to adjust slowly, as otherwise procyclical changes in recession expectations $\mathbf{E}_t S_t / S_{t+1}$ could more than offset the impact of countercyclical risk aversion, leading to procyclical risk premium dynamics.

Trend-stationary consumption process.

PROPOSITION 1. *If the conditional variance of returns* $\mathrm{var}_t(r^r_{t+1})$ *and their conditional covariance with consumption* $\mathrm{cov}_t(r^r_{t+1}, c_{t+1})$ *are constant and log-consumption follows*

$$c_t = \kappa_0 + \kappa_1 t + \gamma c_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \text{i.i.d.}(0, \sigma^2), \qquad \gamma \in [0, 1], \quad \textbf{(10)}$$

then the derivative of the risk premium rp_t *with respect to the current shock realization can be expressed as*[13,14]

$$\frac{\partial \mathrm{rp}_t}{\partial \varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi)\left(C_t - \gamma \sum_{s=0}^{t} \phi^s C_{t-s}\right). \quad \textbf{(11)}$$

The proof of this and all subsequent propositions can be found in the Appendix. Because, in general, the sign of the derivative of the risk premium is ambiguous, in order to build some intuition we now focus on two popular nested specifications—one in which habits fully adjust in a single period and another in which log-consumption is a random walk.

COROLLARY 1. *Under the assumptions of Proposition 1, if habits only depend on past periods' consumption* ($\phi = 0$)*, then the premium is procyclical:*

$$\frac{\partial \mathrm{rp}_t}{\partial \varepsilon_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\gamma) \geq 0. \quad \textbf{(12)}$$

Under the assumptions of Corollary 1, following an adverse shock, recession risks abate and agents expect future conditions to improve [given the AR(1)

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nature of the endowment process, consumption is expected to increase, whereas habits will unambiguously fall]. Because habits adjust fully in a single period, excess consumption, which is all agents care about, is expected to *increase* following the negative shock. Thus, even more risk-averse agents will require lower compensation for bearing risk, and consequently risk premia fall.

On the face of it, the fact that the risk premium is procyclical if $\phi = 0$ could be important, as that assumption is frequently used in macro models. As demonstrated in the following sections, however, this conclusion does not survive extensions to production economies—where the internal propagation mechanism starts playing a role—or alternative assumptions about consumption dynamics.

COROLLARY 2. *Under the assumptions of Proposition 1, if log-consumption follows a random walk ($\kappa_0 = \kappa_1 = 0, \gamma = 1$) and habits are persistent ($\phi > 0$), then the risk premium is countercyclical, as*

$$\frac{\partial \mathrm{rp}_t}{\partial \varepsilon_t} = -E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \sum_{s=1}^{t} \phi^s C_{t-s} < 0. \tag{13}$$

Under the assumptions of Corollary 2, shocks to consumption are permanent and habits adjust gradually. In this setting, after adverse shocks—as expected excess consumption falls—expectations of future conditions deteriorate (recession risks become larger), leading to an increase in the risk premium. This shows that a combination of permanent shocks and persistent habits generates countercyclically varying risk premia.

Equation (11) generalizes this point and shows that a sufficiently persistent shock yields countercyclical premium variation. Although the effect of increasing habit persistence ϕ might seem less clear-cut, evaluating expression (11) for plausible parameter values suggests that raising ϕ has a similar effect.

Growth-stationary consumption process. The final specification we focus on assumes that log consumption *growth* is an AR(1) process:

$$\Delta c_t = \kappa + \delta \Delta c_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}.\mathrm{i.d.}(0, \sigma^2), \qquad \delta \in [0, 1]. \tag{14}$$

This formulation is supported by recent empirical evidence on sticky consumption growth—see, e.g., Carroll et al. (2011).

PROPOSITION 2. *If the conditional variance of returns* $\mathrm{var}_t(r^r_{t+1})$ *and their conditional covariance with consumption* $\mathrm{cov}_t(r^r_{t+1}, c_{t+1})$ *are constant and consumption follows a stochastic process, as in equation (14), then*

$$\frac{\partial \mathrm{rp}_t}{\partial \varepsilon_t} = -\mathbf{E}_t S_{t+1}^2 (1-\phi) \cdot \frac{C_t}{C_{t+1}} \cdot \left[\delta + (1+\delta) \cdot \sum_{s=1}^{+\infty} \phi^s \frac{C_{t-s}}{C_t} \right] < 0. \tag{15}$$

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Under this endowment specification, in contrast to the case of autoregressive detrended consumption (see Corollary 1), risk premia are countercyclical even under a nonpersistent habit specification ($\phi = 0$).[15]

3.2. Allowing for Endogenous Feedback of Habits on Consumption

As argued previously, more sophisticated, production-economy general equilibrium models are likely to introduce a variety of state dependencies into consumption's law of motion.[16] Importantly, Fuhrer (2000) documented that habits help generate hump-shaped consumption impulse responses. We now inspect the "recession risk" channel under hump-shaped consumption dynamics.

In what follows we show that—relative to the specifications of the previous section—the perceived risk of recession under hump-shaped consumption dynamics is more likely to be countercyclical, amplifying the countercyclical risk-aversion channel. So, in a fully general equilibrium setting, habits help generate countercyclically varying risk premia not only because they imply a countercyclical risk aversion, but also because they generate countercyclical recession risks.

To illustrate this point, we first derive analytical results in an endowment economy in which the *exogenous* law of motion for consumption implies hump-shaped impulse responses. Arguably, this analysis has a partial equilibrium flavor, as habit persistence does not map into the shape of the "hump"—which is controlled by different parameters. Accordingly, we conclude by numerically analyzing risk premium dynamics in a full-fledged production economy model. The latter confirms that *endogenous* hump-shaped consumption responses are associated with countercyclical risk premium variation, even for nonpersistent habits.

Endowment Economy. To capture the idea that the peak response of consumption is not attained immediately after the shock hits (i.e., to model "hump-shaped" consumption responses), we now assume that the detrended endowment follows an ARMA(1,1) process.[17]

$$c_t = \kappa_0 + \kappa_1 t + \gamma c_{t-1} + \varepsilon_t + \theta \varepsilon_{t-i}, \qquad \varepsilon_t \sim \mathcal{N}.\text{i.d.}(0, \sigma^2), \qquad \gamma \in [0, 1]. \quad \textbf{(16)}$$

PROPOSITION 3. *If the conditional variance of returns* $\text{var}_t(r^r_{t+1})$ *and their conditional covariance with consumption* $\text{cov}_t(r^r_{t+1}, c_{t+1})$ *are constant and log-consumption follows the specification in equation (16), then*

$$\frac{\partial \text{rp}_t}{\partial \varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi)\left[C_t - (\gamma + \theta) \sum_{s=0}^{t} \phi^s C_{t-s}\right]. \qquad \textbf{(17)}$$

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In particular, if habits only depend on past periods' consumption ($\phi = 0$) and $\gamma + \theta \geq 1$, then

$$\frac{\partial \mathrm{rp}_t}{\partial \varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h C_t [1 - (\gamma + \theta)] \leq 0. \quad \textbf{(18)}$$

Equation (17) demonstrates that θ—the parameter controlling the size of the hump—can play a role similar to that of γ—i.e., consumption persistence. It suggests that models with hump-shaped consumption responses may be able to generate countercyclical premia without persistent habits or shocks. In the case of nonpersistent habits, described in equation (18), if $\gamma + \theta > 1$ then the risk premium is unambiguously countercyclical.

Under this specification, log-consumption decreases further in the period after an adverse shock, following which it converges back to its steady state (i.e., it bottoms out in period two). That is, after a bad shock, agents expect the future to get worse (i.e., recession risks increase) and therefore require higher compensation for bearing risk.

Production economy. We now scrutinize the dynamics of the equity risk premium in a production economy with real rigidities. In particular, we use the model documented in De Paoli et al. (2010).[18] The setup features nonpersistent habit formation (i.e., $h \neq 0$ but $\phi = 0$) and explicitly models capital and capital adjustment costs. Although the structural shocks are AR(1), the model's internal propagation mechanism generates a hump-shaped response of consumption to productivity shocks—see also Figure 1.

To compute the response of the equity risk premium to a productivity shock, we used a third-order perturbation approximation to the model's policy function.[19] Figure 1 confirms that in a general equilibrium framework with habit formation, consumption has a sluggish response to the shock, generating countercyclical recession risks and thus countercyclical risk premia. This suggests that the findings of Proposition 3 also apply to a production-economy model.

4. SUMMARY AND CONCLUSIONS

We have used the simplest possible framework to analyze the determinants of risk premium dynamics. We demonstrated that because of changing recession risks, risk premia can be *procyclical* even though the volatility of consumption is *constant* and despite a *countercyclically* varying risk aversion coefficient. We have also documented that persistent habits, shocks, or features generating hump-shaped consumption responses are all likely to make the premium countercyclical.

Fundamentally, the countercyclicality of the premium rests on agents' belief that changes in economic conditions are persistent. In other words, after an adverse shock, more risk-averse agents will only require a larger premium on risky assets if they do not expect their future conditions to improve massively. Expressed alternatively, our work explicitly explores the role of countercyclical recession risks—a

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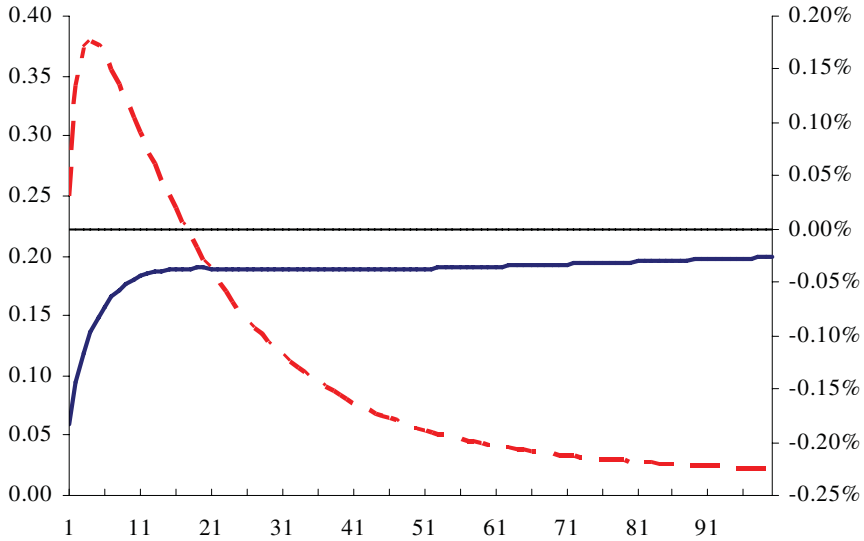


FIGURE 1. Response of consumption (solid line, LHS) and the equity risk premium (dashed line, RHS) to a productivity shock in the model of De Paoli et al. (2010).

feature that is implicit in Campbell and Cochrane (1999), and similar in spirit to the mechanism driving the results in Bansal and Yaron (2004). Our results suggest that factors that help match activity data—i.e., allowing for consumption habits and persistent shocks—are also likely to help along the asset pricing dimension.

Changes in premia substantially contribute to asset price volatility, and so having a good understanding of factors driving them is crucial for modeling asset prices. Given the increasing frequency with which macroeconomic models are being used to address asset pricing puzzles, it is key to clarify how and why changes in standard modeling assumptions translate into different dynamics of premia. Although this study attempts to clarify these issues, further analysis of risk premium dynamics could still be undertaken. For example, in production-economy models, the dynamics of term premia or the inflation risk premium would depend on other aspects of the transmission mechanism. We believe that examining these aspects would be of interest.

NOTES

1. Our use of the terms "recession risks" and "risk aversion" follows Campbell and Cochrane (1999). Both terms are further disambiguated in Section 2.

2. Collard et al. (2006) examine the implications of consumption predictability in an *internal* habit model where risk aversion is, by construction, constant. Thus, implicitly, the variation in equity risk premia in their model is driven entirely by changing recession risks.

3. Both Smets and Wouters (2007) and Uhlig (2007) consider consumption–leisure nonseparabilities, which are absent from our setup.

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4. Several studies argue that disaggregated data offer little empirical support for the external habit specification [Dynan (2000), Chen and Ludvigson (2009)]. Despite this, aggregate data can still behave as if generated in a representative agent habit model. The study of Garleanu and Panageas (2008) demonstrates this point clearly.

5. There are a few reasons why we choose the external rather than internal habit specification. As noted in Campbell and Cochrane (1999), external habits simplify the analysis by eliminating terms in marginal utility by which extra consumption today raises habits tomorrow. Crucially, even though marginal utility under the two specifications differs, what matters for asset prices is the ratio of marginal utilities. As the NBER working paper version of the Campbell and Cochrane article suggests—under linear habits, similar to those we consider—the ratios behave very much alike. Accordingly, we chose to focus on the more tractable external habit setup.

6. Strictly speaking, the Campbell and Cochrane (1999) parameterization is "infinitely dimensional" as, other coefficients aside, it depends on a whole sensitivity function $\lambda(\cdot)$.

7. The parsimony of our setup comes at a cost. For example, to ensure that habits never exceed consumption and so utility remains well defined, we would need restrictions on the support of shocks driving consumption growth. Although Zabczyk (2008) derives such conditions, in the remainder we ignore the problem and proceed under the assumption of normality. Furthermore, unlike Campbell and Cochrane (1999), we have no means of using precautionary saving motives to exactly offset the intratemporal substitution effect, implying that our risk-free rate is going to be volatile. Given that the stated goal of this paper is analyzing premium cyclicality rather than exactly matching asset price dynamics, we consider the implied detriment in fit a small price to pay for tractability.

8. For analytical tractability, we shall restrict attention to a symmetric equilibrium; i.e., we assume that individual consumption C_t^i, marginal utility Λ_t^i, and $\mathcal{M}_t^i$, the stochastic discount factor, equal their respective aggregate equivalents C_t, Λ_t, and $\mathcal{M}_t$.

9. High values correspond to expectations of deteriorating living conditions. In terms of interpretation, the mechanism is similar to the one in Kandel and Stambaugh (1990).

10. As noted in Campbell et al. (1997), "risk aversion may also be measured by the normalized curvature of the value function . . . or by the volatility of the stochastic discount factor. . . . While these measures of risk aversion are different from each other in this model, they all move inversely with S_t."

11. Even though risk aversion η_t and recession risks, as summarized by $\mathbf{E}_t 1/S_{t+1}$, are both functions of the surplus ratio S_t, their dynamics and cyclicality can be very different.

12. Many partial-equilibrium finance papers assume that the covariance of consumption and returns is constant. Numerical simulations conducted on our model suggest that fluctuations in these covariances are small, and for this reason, we impose the assumption of time-invariance in subsequent propositions.

13. In the absence of output in our simple model, we use consumption as a measure of cyclical stance. Hence if variables positively co-move with consumption—as measured by their derivative with respect to shocks to consumption growth ε_t—we say they are procyclical.

14. Equality (11) holds exactly under the additional assumption that excess consumption and risky returns are jointly conditionally lognormal and that consumption is also conditionally lognormal. See Appendix for details.

15. Note that, irrespective of the consumption specification, cyclicality of premia is defined as their "on-impact" response to the white-noise disturbance term ε.

16. Even though the dynamics of aggregate consumption might, particularly given limited data, be indistinguishable from that generated by the processes considered earlier, the asset pricing implications of different specifications could very well differ.

17. The results discussed in this section naturally generalize to arbitrary ARMA(1,K) processes.

18. We use the flexible price version of the model. The calibration is detailed in Table 1, p. 2059.

19. Third order is the lowest that allows for time variation in risk premia. As discussed in Schmitt-Grohe and Uribe (2004), a first-order approximation would imply that premia are zero at all times, whereas a second-order approximation would only allow for constant

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premia. The approximation was computed using Dynare++ (www.dynare.org) and Perturbation AIM (http://www.ericswanson.us/perturbation.html).

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APPENDIX

A.1. PROOF OF PROPOSITION 1

Li (2001) demonstrates that under certain distributional assumptions on c_t^e, r_t^r, and C_t,

$$\mathrm{rp}_t = -\frac{1}{2}\mathrm{var}_t\left(r_{t+1}^r\right) + \delta\,\mathrm{cov}_t\left(c_{t+1}, r_{t+1}^r\right) + \lambda_t\delta\,\mathrm{cov}_t\left(c_{t+1}, r_{t+1}^r\right), \tag{A.1}$$

where $\lambda_t = E_t(\partial s_{t+1}/\partial c_{t+1}) = E_t(1/S_{t+1}) - 1$. Repeated use of the definition of habits—equation (3)—and the endowment specification—equation (10)—makes it possible to express S_t as

$$S_{t+1} = \left[1 - \left(C_t^{\gamma} e^{\kappa_0+\kappa_1(t+1)+\varepsilon_{t+1}}\right)^{-1} h(1-\phi)\sum_{s=0}^{+\infty}\phi^s C_{t-s}\right]. \tag{A.2}$$

Thus, computing the derivative of the above expression, we have

$$\frac{\partial\lambda_t}{\partial C_t} = E_t - S_{t+1}^{-2}\frac{\partial S_{t+1}}{\partial C_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi)\left(1 - \gamma\, C_t^{-1}\sum_{s=0}^{+\infty}\phi^s C_{t-s}\right),$$

which can be plugged into the chain rule $\partial\lambda_t/\partial\varepsilon_t = \partial\lambda_t/\partial C_t \cdot \partial C_t/\partial c_t \cdot \partial c_t/\partial\varepsilon_t$ to yield

$$\frac{\partial\lambda_t}{\partial\varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi)\left(C_t - \gamma\sum_{s=0}^{+\infty}\phi^s C_{t-s}\right). \tag{A.3}$$

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A.2. PROOF OF PROPOSITION 2

Denoting gross consumption growth by $c_t^g := C_t/C_{t-1}$ and defining $x_t := X_t/C_t$, equations (14) and (3) can be written as

$$c_{t+1}^g = \exp(\kappa + \varepsilon_{t+1}) \cdot (c_t^g)^{\delta} \tag{A.4}$$

$$x_t = \left((1-\phi) + \phi x_{t-1}\right) \cdot \left(c_{t-1}^g\right)^{-1}. \tag{A.5}$$

Iterating on (A.5), we can express x_t as

$$x_t = (1-\phi)\sum_{i=1}^{+\infty}\phi^{i-1}\left[\prod_{j=0}^{i-1}\left(c_{t-j}^g\right)^{-1}\right] = (1-\phi)\sum_{s=0}^{+\infty}\phi^{s}\left[\prod_{j=0}^{s}\left(c_{t-j}^g\right)^{-1}\right], \tag{A.6}$$

which implies that

$$S_{t+1} = (1-hx_{t+1}) = \left\{1 - h(1-\phi)\sum_{s=0}^{+\infty}\phi^{s}\left[\prod_{j=0}^{s}\left(c_{t+1-j}^g\right)^{-1}\right]\right\}. \tag{A.7}$$

Similarly to the case of a deterministic trend in consumption, we can now compute the derivative,

$$\frac{\partial S_{t+1}}{\partial c_t^g} = \frac{\partial}{\partial c_t^g}\left\{1 - h(1-\phi)\left(c_{t+1}^g\right)^{-1} - h(1-\phi)\left(c_{t+1}^g c_t^g\right)^{-1}\sum_{s=1}^{+\infty}\phi^{s}\left[\prod_{j=2}^{s}\left(c_{t+1-j}^g\right)^{-1}\right]\right\},$$

where $\prod_{k=2}^{1} a_k = 1$. Exploiting equation (A.4), we know that

$$\frac{\partial (c_{t+1}^g)^{-1}}{\partial c_t^g} = -\left(c_{t+1}^g\right)^{-2}\delta\frac{c_{t+1}^g}{c_t^g} = -\delta\left(c_{t+1}^g c_t^g\right)^{-1} \tag{A.8}$$

$$\frac{\partial (c_{t+1}^g c_t^g)^{-1}}{\partial c_t^g} = -(c_{t+1}^g c_t^g)^{-2}\left(c_{t+1}^g + c_t^g\delta\frac{c_{t+1}^g}{c_t^g}\right) = -(1+\delta)(c_{t+1}^g)^{-1}(c_t^g)^{-2} \tag{A.9}$$

and so

$$\frac{\partial S_{t+1}}{\partial c_t^g} = h(1-\phi)\left(c_{t+1}^g c_t^g\right)^{-1}\left\{\delta + (1+\delta)\cdot\sum_{s=1}^{+\infty}\phi^{s}\left[\prod_{j=1}^{s}\left(c_{t+1-j}^g\right)^{-1}\right]\right\}. \tag{A.10}$$

Accordingly

$$\frac{\lambda_t}{\varepsilon_t} = \mathbf{E}_t - S_{t+1}^2(1-\phi)\left(c_{t+1}^g\right)^{-1}\left\{\delta + (1+\delta)\cdot\sum_{s=1}^{+\infty}\phi^{s}\left[\prod_{j=1}^{s}\left(c_{t+1-j}^g\right)^{-1}\right]\right\}. \qquad \blacksquare$$

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A.3. PROOF OF PROPOSITION 3

Given that

$$\lambda_t = E_t\left[1 - C_{t+1}^{-1}h(1-\phi)\sum_{s=0}^{+\infty}\phi^s C_{t-s}\right]^{-1} - 1 \tag{A.11}$$

and treating λ_t as a function of two variables, $\lambda(C_t, C_{t+1})$,

$$\frac{\partial \lambda_t}{\partial C_t} = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \qquad \frac{\partial \lambda_t}{\partial C_{t+1}} = -E_t S_{t+1}^{-2} C_{t+1}^{-2} h(1-\phi)\sum_{s=0}^{+\infty}\phi^s C_{t-s},$$

we can apply the chain rule,

$$\frac{\partial \lambda_t}{\partial \varepsilon_t} = \frac{\partial \lambda_t}{\partial C_t}\cdot\frac{\partial C_t}{\partial c_t}\cdot\frac{\partial c_t}{\partial \varepsilon_t} + \frac{\partial \lambda_t}{\partial C_{t+1}}\cdot\frac{\partial C_{t+1}}{\partial c_{t+1}}\cdot\left(\frac{\partial c_{t+1}}{\partial \varepsilon_t} + \frac{\partial c_{t+1}}{\partial c_t}\frac{\partial c_t}{\partial \varepsilon_t}\right),$$

to obtain that

$$\frac{\partial \lambda_t}{\partial \varepsilon_t} \approx E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi)\left[C_t - (\theta+\gamma)\sum_{s=0}^{+\infty}\phi^s C_{t-s}\right]. \tag{A.12}$$

■

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