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Nonlinear waves in twirling plasmas

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Abstract. The dynamics of nonlinear two-dimensional plasma waves under non-equilibrium conditions is studied in the framework of the two-fluid hydrodynamic approach. The field of initial velocities for a twirling cylindrical plasma is considered as the initial non-equilibrium state. The influence of initial conditions on the formation and existence of nonlinear electron waves is discussed.

1. Introduction

The wave phenomena in cylindrical plasma columns have always been of fundamental and applicational interest (see, for example, Mikhailovskii 1992; Priest and Forbes 2000; Cramer 2001 and references therein). In the present paper we study the dynamics of nonlinear plasma waves that may emerge in neutral, collisionless plasmas having cylindrical form and being a homogeneous medium in the z-direction.

As was shown in Stenflo (1990), Stenflo and Yu (1995) and Stenflo and Yu (2002), in the above-described type of plasmas strong nonlinear cylindrical waves may be excited. The papers cited have studied the plasma media with an immobile background of positive charged ions where there exist conditions that ensure the excitement of electron waves only. In the present paper, we consider a plasma with electrons and singly charged ions twirling around the z-axis. It is clear that in this case, unlike in the above-mentioned research, we impose strong non-equilibrium initial conditions on the system studied.

As is well-known, the characteristic times of electron and ion plasma-related processes can differ by their order of magnitude. In ordinary conditions (not far from equilibrium), this peculiarity allows one to consider electron dynamics and ion dynamics as two independent processes. This is done on the basis of the assumption that there exists a quasi-equilibrium of the fastest processes and a frozen state of the slowest processes. However, for a nonlinear system being initially in a state far from equilibrium it is impossible to treat the slow and the fast subsystems as independent entities; moreover, one is forced to consider the behavior of electron and ion fluid simultaneously taking into account the strong inherent nonlinearity of the system as well as the collective processes occurring in the system.

So, we can expect both the electron motion and the ion motion to emerge simultaneously even for the electron scales of time because of the self-consistent field of the system. This raises the question of whether the electron and ion fluids could effectively interact and influence each other. It is therefore of interest to examine what new physical features emerge in such coupled ion and electron flows due to the initial non-equilibrium velocity field.

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2. Formulation of the initial value problem

The motions of all charged particles in the above-described type of plasma are assumed to occur in the x^0y plane under the Lorentz force alone, with the basic equations in dimensionless form being

$$\partial_t n_s + \operatorname{div}(n_s \mathbf{v}_s) = 0, \quad s = \mathrm{e}, \mathrm{i}, \tag{2.1}$$

$$\partial_t \mathbf{v}_\mathrm{e} + \mathbf{v}_\mathrm{e} \cdot \nabla \mathbf{v}_\mathrm{e} = -\mathbf{E} - \frac{\omega_\mathrm{c}}{\omega_\mathrm{pe}} \mathbf{v}_\mathrm{e} \times \mathbf{B}, \tag{2.2}$$

$$\partial_t \mathbf{v}_\mathrm{i} + \mathbf{v}_\mathrm{i} \cdot \nabla \mathbf{v}_\mathrm{i} = \mu \left(\mathbf{E} + \frac{\omega_\mathrm{c}}{\omega_\mathrm{pe}} \mathbf{v}_\mathrm{i} \times \mathbf{B} \right), \tag{2.3}$$

$$\operatorname{div} \mathbf{B} = 0, \tag{2.4}$$

$$\operatorname{rot} \mathbf{E} = -\frac{\omega_\mathrm{c}}{\omega_\mathrm{pe}} \partial_t \mathbf{B}, \tag{2.5}$$

$$\operatorname{div} \mathbf{E} = n_\mathrm{i} - n_\mathrm{e}, \tag{2.6}$$

$$\partial_t \mathbf{E} + n_\mathrm{i} \mathbf{v}_\mathrm{i} - n_\mathrm{e} \mathbf{v}_\mathrm{e} = \frac{\omega_\mathrm{c}}{\omega_\mathrm{pe}} \frac{\omega_R^2}{\omega_\mathrm{pe}^2} \operatorname{rot} \mathbf{B}, \tag{2.7}$$

where the electron and ion fluids are marked by the indices e and i, respectively; $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mu = m_\mathrm{e}/m_\mathrm{i}$, m_e is the electron mass and m_i is the ion mass. Here we take the initial electron density n_0, the radius of plasma circle R and the inverse electron frequency $\omega_\mathrm{pe} = (4\pi n_0 e^2/m_\mathrm{e})^{1/2}$, where e is the electric charge, as the scale of all densities, lengths and time. All velocities and electric and magnetic fields are normalized by $R\omega_\mathrm{pe}$, by $4\pi e n_0 R$ and by the initial value of the magnetic field B_0, respectively. Also, we use the cyclotron frequency $\omega_\mathrm{c} = eB_0/m_\mathrm{e}c$, where c is the light velocity and the characteristic frequency $\omega_R = c/R$.

It is assumed that at the initial moment of time $t = 0$ the plasma is limited by $r = R$. For simplicity, we shall consider a plasma possessing equal initial constant densities of electrons and ions $n_\mathrm{e} = n_\mathrm{i} = n_0$ with initial velocities of electrons and ions having the same value $\mathbf{v}_\mathrm{e}(\mathbf{r}, t = 0) = \mathbf{v}_\mathrm{i}(\mathbf{r}, t = 0) = \mathbf{U}$. In the general case, the value of $\mathbf{U}$ as the initial non-equilibrium state of the twirling plasma may coincide with any velocity field describing a rotating fluid flow. In particular, we may take some simple, although not trivial, distribution of velocities for some two-dimensional rotating flow as the initial state for the system to be investigated here.Let $\mathbf{U}$ be the Eulerian representation of the incompressible velocity field which is coupled with vortices $\boldsymbol{\omega} = \operatorname{rot} \mathbf{U}$ via the equations

$$\operatorname{div} \mathbf{U} = 0, \quad \operatorname{rot} \mathbf{U} = \hat{\omega}, \quad \partial_t \hat{\omega} + (\mathbf{U} \cdot \nabla)\hat{\omega} = 0. \tag{2.8}$$

As is known, since the velocity field is solenoidal, $\nabla \cdot \mathbf{U} = 0$; one can then introduce the stream function Ψ so that

$$\mathbf{U} = \mathbf{e}_z \times \nabla \Psi, \tag{2.9}$$

which is related to the vorticity through

$$\nabla^2 \Psi = \omega. \tag{2.10}$$

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This equation allows one to determine the stream function Ψ if the vorticity ω is known and *vice versa* one can determine ω if the stream function Ψ is known. In particular, taking $\omega = 0$ for the inside domain, for $r \leqslant R$ we get

$$\Psi = \sum_{k=1} A_k r^k \sin(k\varphi) + B_k r^k \cos(k\varphi), \tag{2.11}$$

where A_k and B_k are arbitrary constants whose form is defined by the boundary or initial conditions.

For simplicity, instead of the full series (2.11), in what follows we shall consider only one element of the series (2.11) under $k = 2$. Besides, such a form of the ansatz allows us to compare our results with the solutions of Stenflo and Yu (Stenflo 1990; Stenflo and Yu 1995; Stenflo and Yu 2002). Insertion of this term into (2.9) yields

$$U_\varphi(r,\varphi) = Ar\sin(2\varphi) + Br\cos(2\varphi), \quad U_r(r,\varphi) = Br\sin(2\varphi) - Ar\cos(2\varphi), \tag{2.12}$$

where we denote $A = A_2$ and $B = B_2$, which can be accepted as an initial velocity distribution for the set (2.1)–(2.7).

3. Evolution equations

Moreover, this consideration suggests that one should look for the real time-dependent velocity field which is described by (2.1)–(2.7) in a similar way since we can expect that for small $t > 0$ the real velocity field might inherit the same structure. So let us suppose that the temporal dependence of the velocity contains the part which is directly described by the same expression (2.12) but where $A = A(t)$ and $B = B(t)$ and where there are some additional terms reflecting the dynamical nature of the system. In other words, we propose to use the following ansatz for the electron and ion velocities:

$$\mathbf{v}_\mathrm{e} = \mathbf{U}(t,r,\varphi) + r\mathbf{u}(t), \quad \mathbf{v}_\mathrm{i} = \mathbf{U}(t,r,\varphi) + r\mathbf{w}(t), \tag{3.1}$$

where $\mathbf{u}(t)$ and $\mathbf{w}(t)$, as well as $A(t)$ and $B(t)$ in (2.12), are some unknown functions yet to be determined. We shall associate the terms $\mathbf{u}$ and $\mathbf{w}$ with the influence of the electromagnetic forces immediately affecting the velocity fields of the electrons and ions.

If we want (3.1) to describe the initial state of the system defining (2.12) with $A(t=0) = A_2$ and $B(t=0) = B_2$ then the functions $\mathbf{u}(t)$ and $\mathbf{w}(t)$ must satisfy $\mathbf{u}(0) = 0$ and $\mathbf{w}(0) = 0$. However, in the general case this requirement is not the necessary condition when the initial states are different from those fixed in (2.12).

It is worth pointing out that in the case $\mathbf{v}_\mathrm{i} \equiv 0$ the ansatz (2.12)–(3.1) is immediately reduced to the relations which have been used by Stenflo and Yu for studying the electron waves in a cylindrical, unmagnetized plasma where the positive ions form an immobile background, and the initial boundary of plasma $r = R$ remains constant for any moment of time.

Similar to (2.12) let us suppose that the electric field has the form

$$E_r = Cr\cos(2\varphi) + Dr\sin(2\varphi) + \frac{r\varepsilon_r}{2}, \quad E_\varphi = Dr\cos(2\varphi) - Cr\sin(2\varphi) + \frac{r\varepsilon_\varphi}{2}, \tag{3.2}$$

where $C = C(t)$, $D = D(t)$, $\varepsilon_r = \varepsilon_r(t)$ and $\varepsilon_\varphi = \varepsilon_\varphi(t)$ are some unknown functions still to be determined.

It should be noted that the relation (3.2) implies that

$$\operatorname{div}\mathbf{E} = \varepsilon_r, \quad \operatorname{rot}\mathbf{E} = \varepsilon_\varphi \mathbf{e}_z. \tag{3.3}$$

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Therefore, from the second expression of (3.2) and (2.5) it follows that

$$\partial_t B_z = -\varepsilon_\varphi. \tag{3.4}$$

Here and in what follows, without loss of generality we set $\omega_{\rm pe} = \omega_{\rm c}$. In that case the magnetic field is some function of time only and, hence, the condition $\mathrm{rot}\,\mathbf{B} = 0$ always holds true. Then instead of (2.2), (2.3) and (2.7) we have

$$\partial_t \mathbf{v}_{\rm e} + \mathbf{v}_{\rm e} \cdot \nabla \mathbf{v}_{\rm e} = -\mathbf{E} - \mathbf{v}_{\rm e} \times \mathbf{B}, \tag{3.5}$$

$$\partial_t \mathbf{v}_{\rm i} + \mathbf{v}_{\rm i} \cdot \nabla \mathbf{v}_{\rm i} = \mu(\mathbf{E} + \mathbf{v}_{\rm i} \times \mathbf{B}), \tag{3.6}$$

$$\partial_t \mathbf{E} = n_{\rm e}\mathbf{v}_{\rm e} - n_{\rm i}\mathbf{v}_{\rm i}. \tag{3.7}$$

This mathematical description relates to the situation when the displacement current is completely compensated by the conduction currents, while the temporal dependence of a magnetic field is completely defined by the azimuthal component of the electric field.

Using the first relation of (3.2) in (2.6) we find the local relation between the electron and ion densities:

$$n_{\rm i} = n_{\rm e} + \varepsilon_r. \tag{3.8}$$

Moreover, following Stenflo (1990), Stenflo and Yu (1995) and Stenflo and Yu (2002), we consider the case of a plasma cylinder with

$$n_{\rm e} = n(t), \tag{3.9}$$

i.e. we restrict our consideration by the spatial uniformity of plasma.

At this point we have given up the quasi-neutrality condition which was used for the description of the initial state. It is easy to show that for $n_{\rm i} = n_{\rm e}$ and for (3.1) it is impossible to sustain the continuity equation (2.1) for $t > 0$. Besides, this feature imposes an additional requirement on the value $\varepsilon_r(t = 0)$ at the initial moment of time. In this case one can expect to get new features of the plasma dynamics that are brought about by the condition (3.8).

In order to determine the unknown functions of time in relations (3.8), (3.1) and (3.2) we substitute these relations into the corresponding hydrodynamic and electrodynamic equations. Then equating terms with the same angular dependence from (3.7) we get

$$\partial_t \varepsilon_r = 2n(u_r - w_r) - 2\varepsilon_r w_r, \tag{3.10}$$

$$\partial_t \varepsilon_\varphi = 2n(u_\varphi - w_\varphi) - 2\varepsilon_r w_\varphi, \tag{3.11}$$

$$\partial_t D = -\varepsilon_r B, \tag{3.12}$$

$$\partial_t C = -\varepsilon_r A. \tag{3.13}$$

From (2.1) and (3.5) it follows for electrons that

$$\partial_t n + 2u_r n = 0, \tag{3.14}$$

$$\partial_t u_r + A^2 + B^2 + u_r^2 - u_\varphi^2 = -\frac{\varepsilon_r}{2} - B_z u_\varphi, \tag{3.15}$$

$$\partial_t u_\varphi + 2u_r u_\varphi = -\frac{\varepsilon_\varphi}{2} + B_z u_r, \tag{3.16}$$

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$$\partial_t A + 2u_r A = -C - B_z B, \tag{3.17}$$

$$\partial_t B + 2u_r B = -D + B_z A. \tag{3.18}$$

Similarly, for ions, we get from (3.6),

$$\partial_t w_r + A^2 + B^2 + w_r^2 - w_\varphi^2 = \mu\left(\frac{\varepsilon_r}{2} + B_z w_\varphi\right), \tag{3.19}$$

$$\partial_t w_\varphi + 2w_r w_\varphi = \mu\left(\frac{\varepsilon_\varphi}{2} - B_z w_r\right), \tag{3.20}$$

$$\partial_t A + 2w_r A = \mu(C + B_z B), \tag{3.21}$$

$$\partial_t B + 2w_r B = \mu(D - B_z A). \tag{3.22}$$

The relations (3.17) and (3.21), and (3.18) and (3.22) describe the dynamics A and B, respectively. Hence these equations in each pair should be equivalent and we can combine the equations of each pair. As a result, from (3.17) and (3.21) we obtain

$$\partial_t A = -\frac{2A}{1+\mu}(w_r + \mu u_r) \tag{3.23}$$

and from (3.18) and (3.22) we get

$$\partial_t B = -\frac{2B}{1+\mu}(w_r + \mu u_r). \tag{3.24}$$

Thus, the spatial and temporal parts of all physical values are kept apart and the set of ordinary differential equations (3.4), (3.10)–(3.16), (3.19), (3.20), (3.23) and (3.24) form the basis for most of the analysis to follow.

We would like to lay special emphasis on the key feature of the above-presented set of equations describing the full dynamics of the system. Owing to the combination of (3.17) and (3.18), (3.21) and (3.22) we eliminate the angular effect of the electric field which is described in (3.2) with the help of $C(t)$ and $D(t)$. For the chosen form of the velocity and electric fields (see (2.12), (3.1) and (3.2)) there is no influence of C and D on the velocities via A, B, $\mathbf{u}$ and $\mathbf{w}$. In this case the effect of the electric field on the electrons and ions comes about via the dynamical electric values ε_r and ε_φ. Thus, the effect of the magnetic field on the dynamics of the system via (3.15), (3.16), (3.19) and (3.20) is very weak.

In order to make our model complete one should define the limits for electron and ion fluids forming the plasma medium. For this we use the total number of particles of kind s,

$$N_s = \int_0^{2\pi}\int_0^{R_s} n_s(t) r\, dr\, d\varphi, \quad s = \mathrm{e}, \mathrm{i}. \tag{3.25}$$

Here $R_e(t)$ is the limit for the electronic fluid and $R_i(t)$ is the limit for the ionic fluid, which limit plasma from the vacuum.

For (3.8) and (3.9), (3.25) yields

$$N_s = \pi R_s^2 n_s, \quad s = \mathrm{e}, \mathrm{i}, \tag{3.26}$$

which represents the law of particle conservation since there is no particle loss or particle creation in the volume $R_s(t)$. Then, from the condition

$$\partial_t N_s = 0$$

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and (2.1), it is easy to determine the velocity of the boundaries $R_{\rm i}$ and $R_{\rm e}$ as

$$\partial_t R_s = v_{rs}(t, R_s) \quad s = {\rm e}, {\rm i}. \tag{3.27}$$

From (3.26) for an initially neutral plasma with $R_e(0) = R_i(0) = R$ we have

$$R_e = \left(\frac{n_0}{n}\right)^{1/2} R, \quad R_i = \left(\frac{n_0}{n+\varepsilon_r}\right)^{1/2} R. \tag{3.28}$$

4. Numerical analysis

Now let us pass over to the consideration of the dynamics of coupled waves in a rotating plasma cylinder. For this purpose we shall rely on the numerical simulations for the model (3.4), (3.10), (3.11), (3.14), (3.15), (3.16), (3.19), (3.20), (3.23), (3.24) and (3.28) with the following initial data. In all computations we shall use $\mu = 10^{-5}$. We shall restrict our numerical simulation to the cases in which always $A(0) = B(0)$ and there is no initial compression, i.e. $u_r(0) \geqslant 0$ and $w_r(0) \geqslant 0$. This means that the initial values $R_{\rm e}(0) = R_{\rm i}(0) = 1$, $n(0) = 1$, $\varepsilon_r(0) = 0$, $\varepsilon_\varphi(0) = 0$, $B_z(0) = 0$ will remain constant in all the present computations and the values $A(0)$, $u_r(0)$, $w(0)$, $u_\varphi(0)$ and $w_\varphi(0)$ will be variable.

First, it should be noted that if we take any initial state in which, in contrast to all other initial parameters, the values of $A(0)$ and $B(0)$ are distinct from zero, then from the set (3.4), (3.10), (3.11), (3.12), (3.14), (3.15), (3.16), (3.19), (3.20), (3.23) and (3.24) it follows that only A, B, u_r and w_r might be changed. In this case the values $u_r(t)$ and $w_r(t)$ will be negative and their absolute value will monotonically increase while the values A and B will increase by exponential rate with respect to $u_r(t)$ and $w_r(t)$ (see (3.23) and (3.24)). Fig. 1, calculated for $A(0) = 5 \times 10^{-3}$, illustrates such behavior. Note that here and in all further calculations we have normalized all dependencies of the velocities by the value $A(0)$. In this case, a monotonic decrease of $R_{\rm e}$ and $R_{\rm i}$ occurs (see curve 1 in Fig. 3).

It is worth noting that all of these tendencies become more pronounced with the growth of $A(0)$. The similar dependencies for $A(t)$, $u_r(t)$ and $w_r(t)$ hold as long as the value $A(0)$ is smaller than 7×10^{-1}. Computations show that for $A(0) > 7\times10^{-3}$ the solutions become the singular solutions that is always the result of compression. In this case self-compression arising from the growth of A_0 is a parasitic, harmful process which leads to a break up in the flow of both ions and electrons. This peculiarity restricts and defines the choice of value $A(0)$ in further calculations.

As is seen from (3.10)–(3.12) the dynamics similar to that presented in Fig. 1 is observed with initial conditions $u_r(0) = w_r(0) \neq 0$ and $u_\varphi(0) = w_\varphi(0) \neq 0$. From (3.10)–(3.12) it follows that in this case the values of ε_r, ε_φ and B_z cannot be changed, and the change of values of A, u_r, w_r, u_φ and w_φ does not depend on the former. To illustrate such a behavior, Fig. 2 presents the typical picture corresponding to $A(0) = 5\times10^{-3}$, $u_r(0) = w_r(0) = 10^{-3}$ and $u_\varphi(0) = w_\varphi(0) = 10^{-3}$. This process is accompanied by a monotonic increase in the skeleton radius over the course of time as is shown in graph 2 in Fig. 3.

Thus, we come to the conclusion that in the framework of the proposed model it is impossible to observe the wavelike behavior without violating the symmetry of the initial velocity field.

Now, let us proceed to considering what happens if we bring some asymmetry into the initial conditions for the velocity field. In the present paper we restrict our

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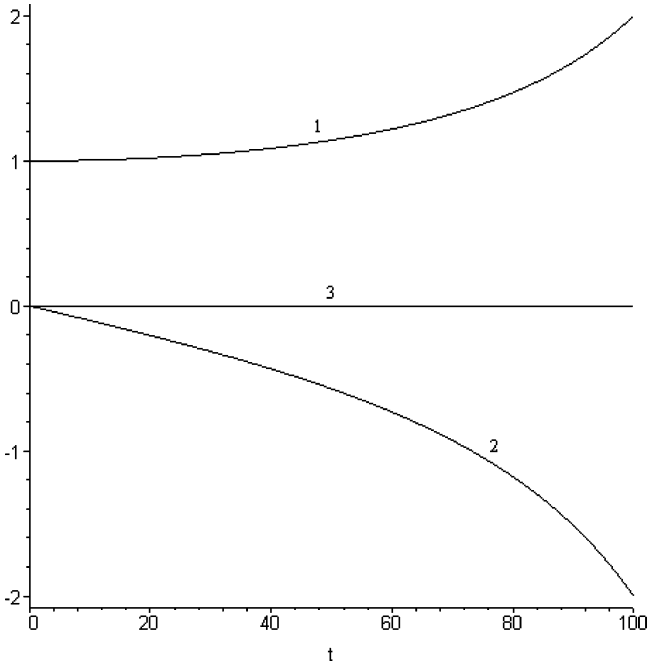


Figure 1. The time dependence of A (curve 1), u_r and w_r (curve 2), and u_φ and w_φ (curve 3) for initial data $A(0) = 5 \times 10^{-3}$ and $u_r(0) = u_\varphi(0) = w_r(0) = w_\varphi(0) = 0$.

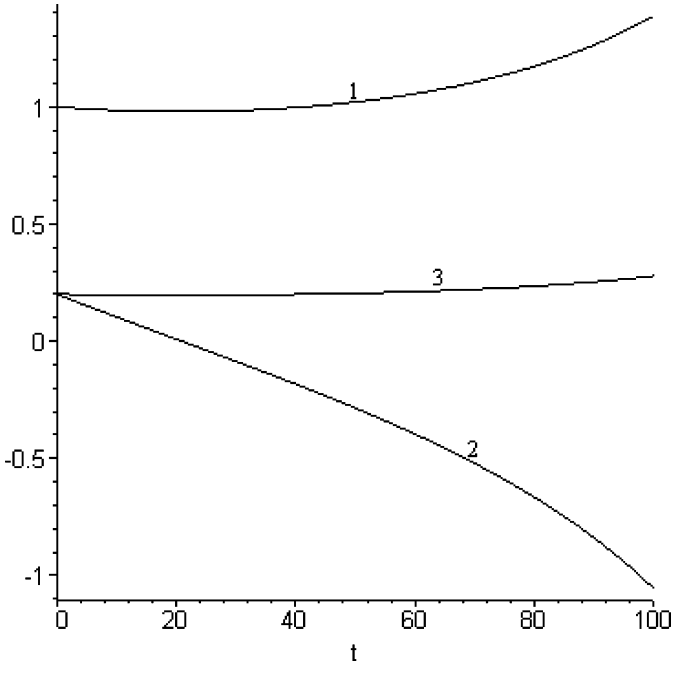


Figure 2. The time dependence of A (curve 1), u_r and w_r (curve 2), and u_φ and w_φ (curve 3) for initial data $A(0) = 5 \times 10^{-3}$ and $u_r(0) = u_\varphi(0) = w_r(0) = w_\varphi(0) = 10^{-3}$.

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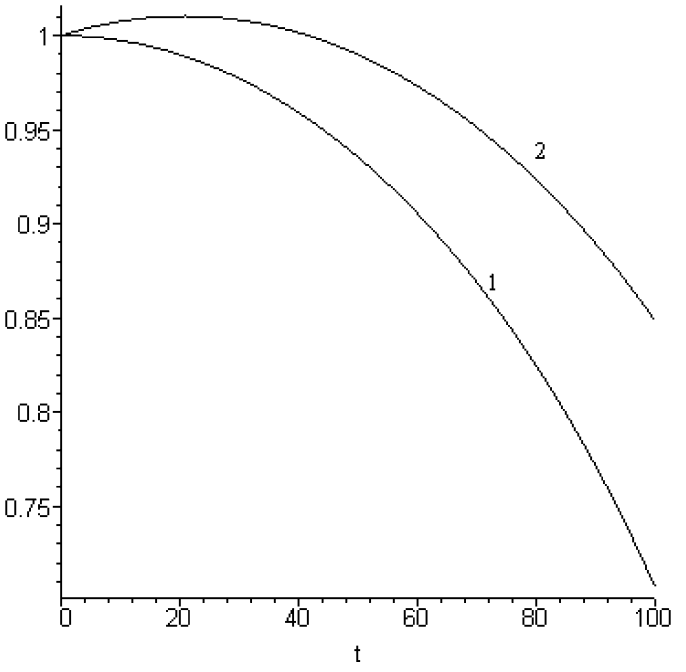


Figure 3. The characteristic radii R_{e} and R_{i} (here they coincide) as a function of time for $A(0) = 5 \times 10^{-3}$. Curve 1 corresponds to $u_r(0) = w_r(0) = u_\varphi(0) = w_\varphi(0) = 0$; curve 2 corresponds to $u_r(0) = w_r(0) = u_\varphi(0) = w_\varphi(0) = 10^{-3}$.

consideration to the case when the radial components of the initial electron and ion velocities have perturbative elements and the initial azimuthal components remain unperturbative.

In Figs 4 and 5 u_r, u_φ, w_r, w_φ and A are plotted when $u_r(0) \geqslant w_r(0)$, and in Fig. 6 the same dependencies are presented for the opposite case when $u_r(0) \leqslant w_r(0)$. In Fig. 7 the corresponding time dependencies of R_{i} and R_{e} are shown. As is seen from these graphs, in all these cases we observe a typical nonlinear oscillatory behavior for the electron component of the velocity while the ion component of the velocity is described by a monotonic temporal dependence for the electron scale of time. In this case, the shape of the electron wave changes over the course of time.

It should be noted that the general character of dynamics for the electron and ion components coincide with the temporal behavior of A. This means that there is a strong dependence of A on the oscillating function $u_r(t)$ (see (3.23) and (3.24)). Although initially we always have an expanding plasma, one can see the monotonic decrease of the radial velocities over the course of time brought about by the strong nonlinearity of the problem (see (3.15) and (3.19)). As is seen from Fig. 7 this effect can lead to the skeleton compression (curves 1 and 2). Also, we see that the curves of R_{i} and R_{e} presented in Fig. 7 are much the same, which reflects the coupling of electron and ion fluids for the initial conditions used.

From the comparison of Fig. 4 and Figs 5 and 6, it follows that there exists a strong influence of the initial radial perturbations $u_r(0)$ and $w_r(0)$ on the dynamics of the azimuthal part. The effect of the initial perturbation values shows itself in the growth of the amplitude of the electron oscillations. However, the swinging

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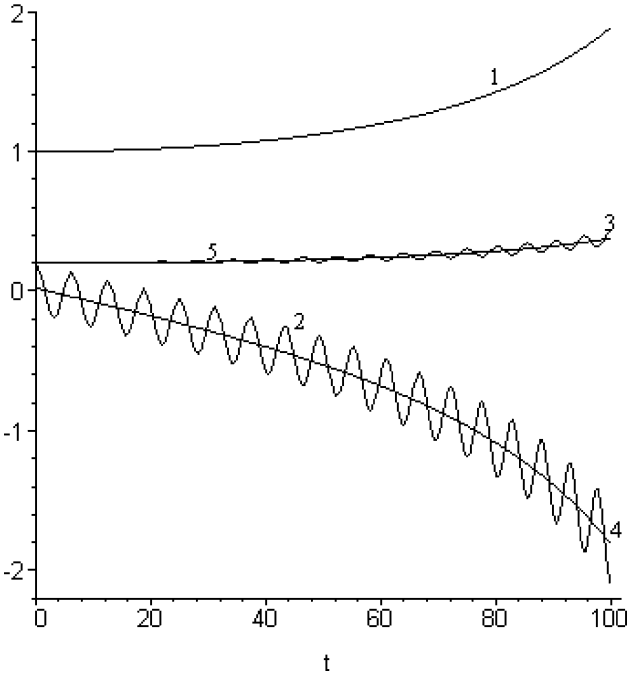


Figure 4. The time dependence of A (curve 1), u_r (curve 2), u_φ (curve 3), w_r (curve 4) and w_φ (curve 5) for $A(0) = 5 \times 10^{-3}$, $u_r(0) = u_\varphi(0) = w_\varphi(0) = 10^{-3}$ and $w_r(0) = 10^{-4}$.

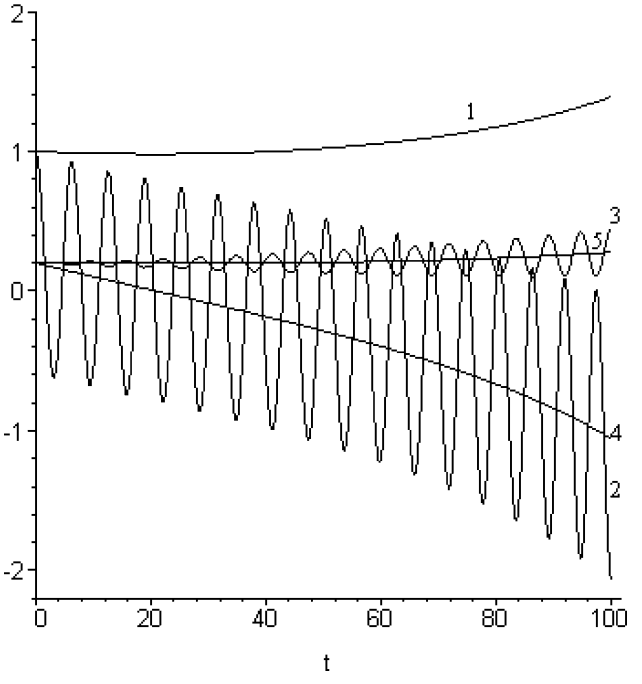


Figure 5. The time dependence of A (curve 1), u_r (curve 2), u_φ (curve 3), w_r (curve 4) and w_φ (curve 5) for $A(0) = 5 \times 10^{-3}$, $w_r(0) = u_\varphi(0) = w_\varphi(0) = 10^{-3}$ and $u_r(0) = 5 \times 10^{-3}$.

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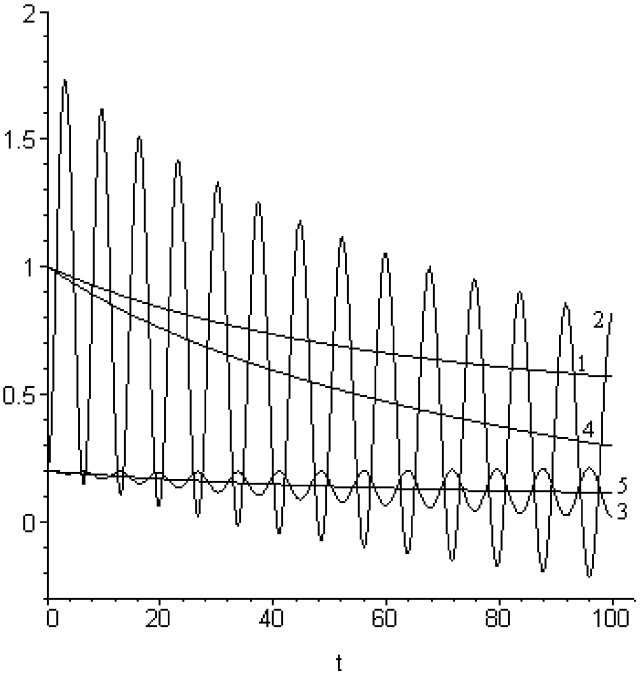


Figure 6. The time dependence of A (curve 1), u_r (curve 2), u_φ (curve 3), w_r (curve 4) and w_φ (curve 5) for $A(0) = 5 \times 10^{-3}$, $u_r(0) = u_\varphi(0) = w_\varphi(0) = 10^{-3}$ and $w_r(0) = 5 \times 10^{-3}$.

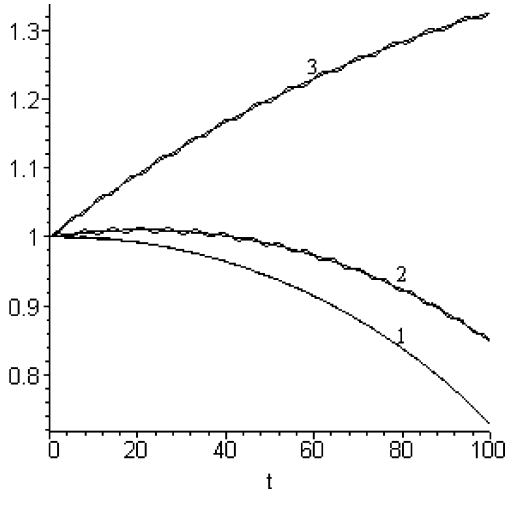


Figure 7. The characteristic radius R_e (wave line) and R_i (straight line) as a function of time for $A(0) = 5 \times 10^{-3}$, $u_\varphi(0) = w_\varphi(0) = 10^{-3}$. Curves 1 correspond to $u_r(0) = 10^{-3}$ and $w_r(0) = 10^{-4}$; curves 2 correspond to $u_r(0) = 5 \times 10^{-3}$ and $w_r(0) = 10^{-3}$; curves 3 correspond to $u_r(0) = 10^{-3}$ and $w_r(0) = 5 \times 10^{-3}$.

of the electron wave shape, which is permanent, over the course of time does not depend on the values $u_r(0)$ and $w_r(0)$.

As is seen from Figs 4 and 5, when $u_r(0) \geqslant w_r(0)$ there exists a weak monotonic growth of the azimuthal part of the velocity for the electron and ion components

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over the course of time that accompanies the monotonic decrease of the electron and ion radii. Under $u_r(0) \ll w_r(0)$ one can see the monotonic decrease of the azimuthal components of the velocities over the course of time brought about by the effect of growth of the skeleton radius, which is shown in graph 3 of Fig. 7. The change in the form and frequency of the oscillations (see curve 5 in Figs 4 and 5) for the azimuthal electronic component may be caused by the intermode exchange and asymmetrical perturbations in the initial conditions. As a result, some acceleration of the originally twirling plasma may be observed.

Why does it occur? We can point out some reasons. At the initial stage the behaviour of the system is determined by its cylindrical geometry. As is seen from (3.15), (3.19), (3.23) and (3.24), for these times we have

$$\partial_t^2 A \approx 2A(A^2 + B^2), \quad \partial_t^2 B \approx 2B(A^2 + B^2), \tag{4.1}$$

i.e. there is an increase in the values of A and B. This feature determines the further dynamics. It should be noted that such behavior arises from the coupling of electron and ion fluids (see (3.17), (3.18), (3.21) and (3.22)) and the omission of the quasi-neutrality condition. Here we use the local but nonlinear relation (3.8). It thus seems that there occurs an amplification of the azimuthal electric field and a decrease of the radial electric field. As a result the forms of the electric and velocity fields are alike (see (2.12) and (3.1)). So it is this coupling of the electric and velocity fields that causes the increase in the azimuthal velocities.

5. Conclusion

In the present paper, we have studied the dynamics of nonlinear two-dimensional plasma waves in an originally twirling plasma cylinder. In other words, the electron and ion fluids have been treated as a set of coupled nonlinear radial and azimuthal oscillators of different kinds in a two-dimensional plane, having the same non-zero initial hydrodynamic velocities. Namely, we have considered here a special type of non-equilibrium – an initial distribution of velocities (2.12) which is the simplest, although non-trivial, velocity field for an incompressible fluid. However, such a case is not unique and it would be interesting to consider more general initial conditions that may lead to new kinds of nonlinear wave structures.

We have studied the dynamics of the nonlinear cylindrical electron and ion flows coupled via a self-consistent field of the system. As was shown, there exists a strong influence of these flows upon each other when the dynamics of each component is determined by the behavior of other components. In some cases (see Figs 4 and 5) one can see the growth of the magnitude for the azimuthal component of the velocities $u_r(t)$, $w_r(t)$ and the parameter $A(t)$ which defines the behavior of the skeleton velocity in general.

Thus, it is possible to assume that such behavior stems is caused by several factors. First, it might be accounted for by the fact that the energy of the oscillation modes is transformed into the rotation motion of the whole plasma cylinder. It might also be assumed that such a transfer is affected by the excitation of the strong nonlinear radial azimuthal oscillations. Second, the self-compression of the plasma skeleton brought about by its rotation might be instrumental here.

Therefore, we may venture a guess that in ordinary twirling plasmas there is a possibility of energy accumulation in the electron rotation mode which is brought about by the energy transfer from other degrees of freedom.

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To our mind such non-equilibrium states with symmetry violation of the initial condition can easily appear in plasma vortex-like systems initially far from equilibrium (see, for example, Aburjania 2006; Eliasson and Shukla 2006). However, as was shown above, the perturbations of initial conditions can lead to a break in the flow, which might complicate the intermode exchange and induce other harmful processes. Taking into account such high sensitivity of our pattern to perturbations in the initial conditions, one can come to the conclusion that there exists a small interval in the system parameters in which it is possible to see the effect described in reality.

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