An Analysis of Triode Valve Rectification. By S. E. A. LANDALE, Corpus Christi College. (Communicated by Mr E. B. MOULLIN.)

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1. *Introduction.*

A common method of using a triode valve for rectification is shown in Fig. 1, and is often called Cumulative Grid Rectification.

We may review the function of the grid leak R and the condenser C as follows. The condenser should be sufficiently large so that its impedance to the applied frequency is negligible compared with the resistance R: then the alternating potential between grid and filament is sensibly equal to the applied potential e. Now

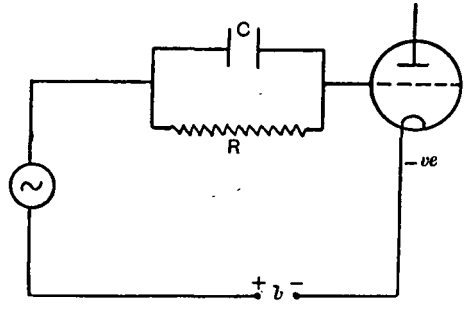


Fig. 1.

since the grid-filament conductivity is not constant this voltage produces a current of complex wave-form of mean value I. This current flows through R and C in parallel: the mean flows through R, and since the impedance of C is very small compared with R, sensibly the whole of the alternating components flow through C. Consequently the P.D. between the condenser terminals is practically constant, and equals RI in value. The effect of the applied alternating voltage is thus to lower the mean potential of the grid by an amount RI.

Let Fig. 2 represent the relation between grid current and grid potential relative to the negative end of the filament. Let the grid voltage be V_1 when no alternating potential is applied. V_1 will depend on the P.D. between the terminals of the grid bias battery b in Fig. 1, the value of R and the characteristics of the valve. When an alternating voltage, $e = A \sin pt + B \sin 2pt + \ldots$

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is applied the mean grid potential will be reduced to V. We desire to calculate the value of $V_1 - V$. To do this we must be able to calculate the mean current I which would result from the applied voltage if the grid had any mean potential V. From this we must derive V from the relation $V_1 - V = RI$. We must, in fact, evolve a method for calculating I for an applied voltage of any magnitude and wave-form superposed on any mean potential V. To do this it is necessary to know the equation which represents Fig. 2. It is very difficult to find an equation to represent a real valve because the curve does not tend to conform to any simple function.

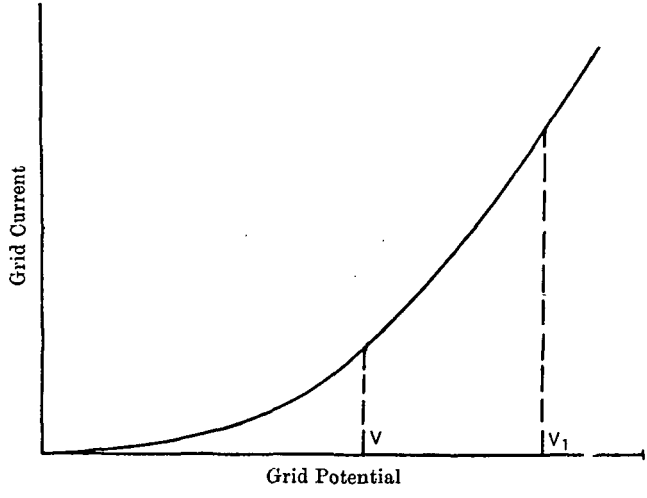


Fig. 2.

If a valve has a very hot tungsten filament the electrons are ejected with a very high initial velocity and there is reason to suppose that the foot of the grid current-grid potential curve will be exponential and may be represented by the equation $I = \alpha e^{bV}$.

It is well known that the foot of the grid current curve for such valves does approximate very closely to an exponential form. The analysis appropriate to an exponential curve has been derived by Colebrook*. Generally speaking, the application of this analysis to any particular valve, which has a high temperature cathode, will give fairly accurate results provided that $E + V$ is negative. V is the mean grid potential when an alternating voltage of

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amplitude E is applied. The similarity between an exponential curve and the grid characteristic of a tungsten filament valve is shown in Fig. 3.

At present it is more usual to employ valves for reception which have thoriated or oxide-coated filaments run at a comparatively low temperature. The velocity of ejection of electrons is small, and experience shows that the grid current curve has an exponential form over so restricted a range that Colebrook's analysis is of little practical interest. The author has examined the foot of the grid characteristic of many dull filament valves and has found that it can be represented with great accuracy over

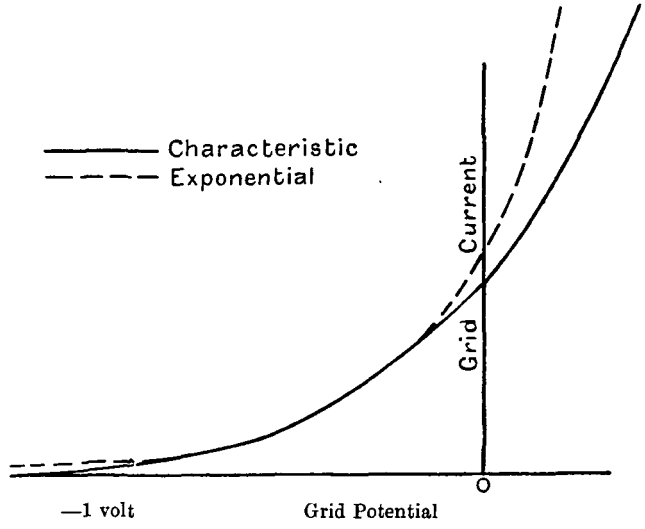


Fig. 3.

a considerable range by a quartic equation. The coefficients of the quartic change widely for different valves, and also with age for the same valve, but nevertheless the range of applicability of the quartic is roughly constant for all triodes.

Fig. 4 shows the foot of the grid characteristic of a certain Mullard P.M. 3 valve, $\mu = 14$, $\rho = 13{,}000\Omega$. In the same figure an exponential curve, a parabola and a quartic curve are plotted. The equation of the quartic is

$$I = 8{\cdot}2 + 27{\cdot}5x + 18{\cdot}3x^2 - 9{\cdot}1x^3 - 4{\cdot}52x^4 \text{ microamperes},$$

where $x = (V - 0{\cdot}8)$ volt. It will be observed that the quartic fits the characteristic with considerable accuracy between $V = 0{\cdot}1$ volt and $V = 1{\cdot}5$ volts, but rapidly becomes grossly inaccurate outside

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those limits. The exponential curve is fairly accurate up to 0·6 volt, and so also is the parabola from 0·4 volt to 1·2 volts.

In the present paper an analysis is developed for calculating the rectified current and change of mean grid potential for a valve in which the grid current curve has the form

$$I = \alpha + \beta V + \gamma V^2 + \delta V^3 + \eta V^4.$$

When this analysis is applied to real valves it is necessary to ascertain that the applied voltage is insufficient to cause the instantaneous grid potential to wander outside the limits between which the quartic is applicable. However, these limits need not enter into our analysis but only in its application to valves.

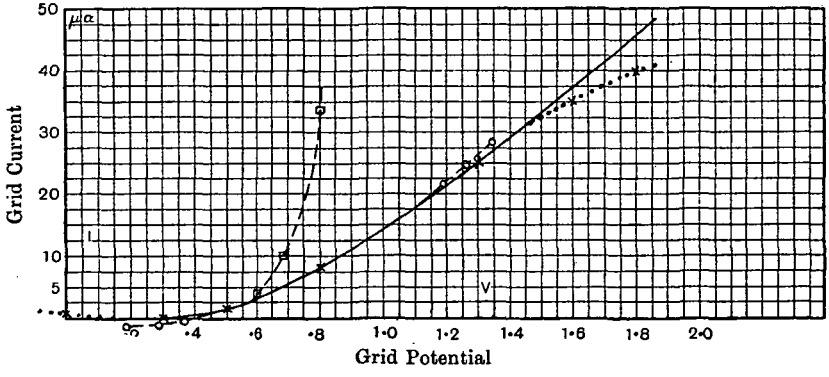


Fig. 4. MULLARD P.M. 3 TRIODE.

———— Grid Current Characteristic.
....×.... Quartic Curve.
— —⊡— — Exponential Curve.
—o—o— Parabola.

2. We first consider the result of applying alternating voltages of various wave-forms to a simple grid rectifier; we shall in fact assume that the resistance R and condenser C in Fig. 1 are short-circuited.

A. The current produced by the application of a sinusoidal voltage, $e = E \sin pt$.

Let the grid current be represented by the equation

$$I = \alpha + \beta V + \gamma V^2 + \delta V^3 + \eta V^4,$$

and let the instantaneous grid potential $V = v + e$, where v is the mean grid voltage. Since we are assuming, at present, that R and C are short-circuited, v will be constant and equal to the P.D. of the bias battery b in Fig. 1.

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Then $I=f(v+e)$, and expanding by Taylor's theorem and putting terms containing the fifth and higher differentials of $f(v)$ equal to zero we have

$$I=f(v)+ef'(v)+\frac{e^2}{2!}f''(v)+\frac{e^3}{3!}f'''(v)+\frac{e^4}{4!}f''''(v)\ldots(1);$$

substituting $E\sin pt$ for e, and simplifying, we have

$$I=f(v)+\frac{E^2}{4}f''(v)+\frac{E^4}{64}f''''(v)+\left[Ef'(v)+\frac{E^3}{8}f'''(v)\right]\sin pt$$
$$-\left[\frac{E^2}{4}f''(v)+\frac{E^4}{48}f''''(v)\right]\cos 2pt-\frac{E^3}{24}f'''(v)\sin 3pt$$
$$+\frac{E^4}{192}f''''(v)\cos 4pt\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

This equation for the current shows that new harmonic terms are produced by the rectifier and also a steady current whose value is $f(v)+\frac{E^2}{4}f''(v)+\frac{E^4}{64}f''''(v)$, but of this current the term $f(v)$ is due to the bias battery alone and not to the applied alternating voltage, hence the rectified current j is given by the expression

$$j=\frac{E^2}{4}f''(v)+\frac{E^4}{64}f''''(v)$$
$$=\tfrac{3}{8}\eta E^4+(3\eta v^2+\tfrac{3}{2}\delta v+\tfrac{1}{2}\gamma)E^2.$$

For any given value of E the rectified current will, therefore, depend on v the bias battery voltage.

B. The current produced by the simultaneous application of three sinusoidal voltages of different frequencies,

$$\frac{1}{2\pi}(p, p-n, p+n).$$

This case is of practical interest since, when $p\gg n$, the sum of these three voltages represents the voltage produced by an acoustically modulated radio signal. The sum may be written in the form $e=(A+B\sin nt)\sin pt$. The expression for I may be calculated by substituting this value for e in equation (1), but the result is very cumbersome. In practice the terms of greatest interest are the steady current terms and the terms of frequency $n/2\pi$, $2n/2\pi$, etc. Terms of frequency $\frac{1}{2\pi}(p, 2p, 2p-n)$, etc., are inaudible but must not be forgotten, as low impedance paths must be provided for them. The following expression shows the value of terms of frequency $n/2\pi$, $2n/2\pi$, etc., the steady current terms

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and a few representative terms of inaudibly high frequency such as $\frac{1}{2\pi}(p,\ 4p,\ 2p+3n)$:

$$I = f(v) + \frac{f''(v)}{2!}\left[\frac{A^2}{2} + \frac{B^2}{4}\right] + \frac{f''''(v)}{4!}\left[\tfrac{3}{8}A^4 + \tfrac{9}{8}A^2B^2 + \tfrac{9}{64}B^4\right]$$
$$+ AB\left[\frac{f''(v)}{2!} + \left\{\tfrac{3}{2}A^2 + \tfrac{9}{8}B^2\right\}\frac{f''''(v)}{4!}\right]\sin nt$$
$$- \frac{B^2}{4}\left[\frac{f''(v)}{2!} + \left\{\tfrac{9}{2}A^2 + \tfrac{3}{4}B^2\right\}\frac{f''''(v)}{4!}\right]\cos 2nt$$
$$- \left[\tfrac{3}{8}AB^3\frac{f''''(v)}{4!}\right]\sin 3nt + \left[\tfrac{3}{64}B^4\frac{f''''(v)}{4!}\right]\cos 4nt$$
$$+ A\left[f(v) + \left\{\tfrac{3}{4}A^2 + \tfrac{9}{8}B^2\right\}\frac{f'''(v)}{3!}\right]\sin pt$$
$$+ \left[\tfrac{1}{8}\left\{A^4 + 3A^2B^2 + \tfrac{3}{8}B^4\right\}\frac{f''''(v)}{4!}\right]\cos 4pt$$
$$+ \tfrac{3}{16}AB^2\frac{f'''(v)}{3!}\sin(2p+3n)t + \ldots \ldots\ldots\ldots\ldots\ldots(3).$$

The complete evaluation shows that the odd terms in the equation $I = \alpha + \beta V + \gamma V^2 + \delta V^3 + \eta V^4$ produce only terms of inaudibly high frequency, and therefore equation (3) is still valid if a quintic equation is used to interpret the characteristic, as far as rectified current and terms of frequency $n/2\pi$, $2n/2\pi$, etc. are concerned.

C. The current produced by the simultaneous application of five sinusoidal voltages of frequencies

$$\frac{1}{2\pi}(p,\ p-m,\ p+m,\ p-n \text{ and } p+n).$$

We will only consider the special case in which

$$e = A\sin pt + \frac{B}{2}\{\sin(p-m)t + \sin(p+m)t\}$$
$$+ \frac{C}{2}\{\sin(p-n)t + \sin(p+n)t\}.$$

This corresponds to the simultaneous modulation of a radio frequency potential $A\sin pt$, by two acoustic frequencies $m/2\pi$ and $n/2\pi$, and may be written in the form

$$e = (A + B\cos mt + C\cos nt)\sin pt.$$

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The evaluation of the current is very laborious, but when it is performed the terms $\cos mt$, $\cos 2mt$, $\cos 3mt$ and $\cos 4mt$, respectively, will be found to have the coefficients shown below:

$$I_m = AB\left[\frac{f''(v)}{2!} + \left\{\tfrac{3}{2}A^2 + \tfrac{9}{8}B^2 + \tfrac{9}{4}C^2\right\}\frac{f''''(v)}{4!}\right],$$

$$I_{2m} = \frac{B^2}{4}\left[\frac{f''(v)}{2!} + \left\{\tfrac{9}{2}A^2 + \tfrac{3}{4}B^2 + \tfrac{9}{4}C^2\right\}\frac{f''''(v)}{4!}\right],$$

$$I_{3m} = \tfrac{3}{8}AB^3\frac{f''''(v)}{4!} \quad \text{and} \quad I_{4m} = \tfrac{3}{64}B^4\frac{f''''(v)}{4!}.$$

The coefficients of the terms $\cos nt$, $\cos 2nt$, etc. may be obtained by interchanging B and C in the equations for I_m, I_{2m}, etc. The following acoustic frequency currents are also produced:

$$\begin{aligned}\tfrac{1}{2}BC\left[\frac{f''(v)}{2!} + \left\{\tfrac{9}{2}A^2 + \tfrac{9}{8}B^2 + \tfrac{9}{8}C^2\right\}\frac{f''''(v)}{4!}\right]\cos(m \pm n)t \\ + BC\left[\tfrac{3}{16}B^2\cos(3m \pm n)t + \tfrac{3}{16}C^2\cos(m \pm 3n)t\right. \\ + \tfrac{9}{32}BC\cos(2m \pm 2n)t + \tfrac{9}{8}AB\cos(2m \pm n)t \\ \left. + \tfrac{9}{8}AC\cos(m \pm 2n)t\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).\end{aligned}$$

Here the sign $\pm$ means both plus *and* minus, not plus *or* minus as the sign usually indicates.

This evaluation is interesting because it shows that not only are currents of frequency $\frac{1}{2\pi}(m, 2m, 3m, 4m, n, 2n, 3n, 4n)$ produced, but also all the combination tones between $\frac{1}{2\pi}(3m \pm n$ and $m \pm 3n)$.

3. So far we have presumed that the grid potential v is not a function of the rectified current. To apply our calculations to the cumulative grid rectifier depicted in Fig. 1 we must make certain modifications, as it has been shown in the introduction that the mean grid potential certainly is then a function of the rectified current; in fact it is the change of mean grid potential that is of primary importance in practice.

A. The Rectified Current produced by a Cumulative Grid Rectifier.

It has been shown in section 1 that the mean grid current will flow through R and that if, as we shall assume, R and C have suitable values, currents having a frequency of the same order as, or higher than, the input frequency will develop no appreciable P.D. across R and C.

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Let i be the initial steady grid current and I the grid current flowing at any instant during the application of an alternating potential e. Let j be the rectified current. Then $i+j$ is the mean grid current, and the instantaneous grid potential will be $b-R(i+j)+e$.

Then $$I=f[b-R(i+j)+e],$$

and the mean grid current

$$i+j=\frac{1}{T}\int_0^T f[b-R(i+j)+e]\,dt.$$

We shall assume that this function is a quartic or quintic expression, then, putting $e=E\sin pt$ and integrating, we have

$$i+j=f[b-R(i+j)]+\frac{E^2}{4}f''[b-R(i+j)]+\frac{E^4}{64}f''''[b-R(i+j)] \quad \ldots\ldots(5).$$

By solving this equation for E and evaluating E for a number of values of j the curve relating j and E can be plotted and used to give values of j for given E as required.

It will be helpful to describe the process more fully. We must first find the initial steady grid current i defined by $i=f(b-Ri)$, where b is the voltage of the grid bias battery: i may be calculated from the quartic equation appropriate to the particular valve, but in practice it is much simpler to read off the value of i from the characteristic curve. We now select suitable values of j and substitute them in equation (5), and derive the corresponding values of E.

This has been done for a certain Mullard P.M. 3 valve whose characteristic was expressed by

$$I=7{\cdot}3+26x+18{\cdot}7x^2-6x^3+1{\cdot}18x^4 \text{ microamperes},$$

where $x=V-0{\cdot}8$ volt. Fig. 5 shows the observed curve relating E and j for this valve, and on the same figure are plotted five values derived from the solution of equation (5). It will be seen that the agreement is within the limits of experimental error up to $E/\sqrt{2}=0{\cdot}9$ volt, that is, $E=1{\cdot}27$ volts. This corresponds to a swing of grid potential of 2·54 volts. It is evident that the deviations of the quartic curve from the characteristic have tended to compensate.

B. Amplitude of Harmonics and the Rectified Current produced by a Cumulative Grid Rectifier when an alternative voltage $e=(A+B\sin nt)\sin pt$ is applied.

A potential of this wave-form is often applied to a cumulative grid rectifier, $n/2\pi$ is generally an acoustic frequency and $p/2\pi$

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a radio frequency. We will consider the effect of the presence of R and C in Fig. 1 on the acoustic frequency output of the rectifier when a wave of this form is applied. Our calculations will be

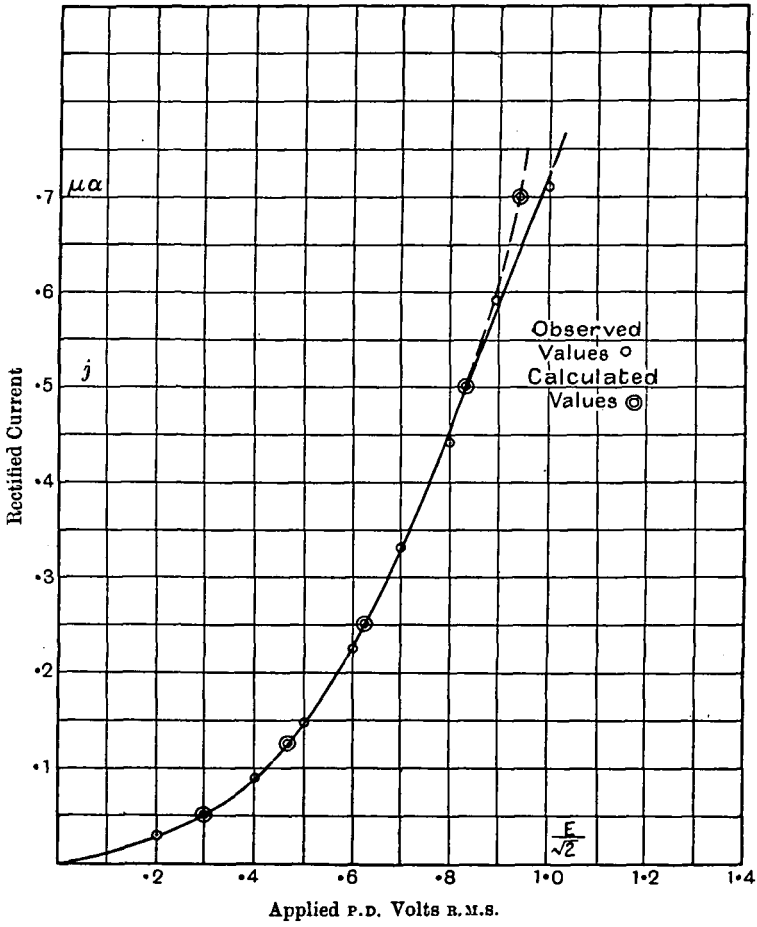


Fig. 5. MULLARD P.M. 3 TRIODE.

90 ~ Signal. $R = \cdot 42$ MΩ. $E = \cdot 25$ μF. Bias Battery Voltage 13. Anode Potential 100 V. Filament P.D. 3·7 V.

based on the assumption that the R and C combination has an impedance of R ohms to currents of frequencies $n/2\pi$, $2n/2\pi$, etc., and a negligible impedance to radio frequencies $p/2\pi$, $2p/2\pi$, etc. This is the ideal condition.

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The presence of R and C complicates our calculations to such an extent that we must neglect the third and higher power terms of the characteristic equation, as otherwise the analysis would become far too cumbersome. We shall, therefore, assume that the grid characteristic may be expressed by the equation

$$I = \alpha + \beta V + \gamma V^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (6),$$

and that the quadratic curve adheres to the grid characteristic for all values of V positive from the value at which grid current first becomes perceptible. That is when

$$V \gtrsim [-\beta + (\beta^2 - 4\gamma\alpha)^{\frac{1}{2}}]/2\gamma.$$

When a voltage $e = (A + B \sin nt) \sin pt$ is applied, the instantaneous grid potential will be

$$V = b + (A + B \sin nt) \sin pt - RI_0,$$

where I_0 is the current in the grid leak.

Substituting this value for V in equation (6), we have

$$\begin{aligned} I = {} & \alpha + \beta b + \beta (A + B \sin nt) \sin pt - \beta R I_0 \\ & + \gamma \{b^2 + (A + B \sin nt)^2 \sin^2 pt + R^2 I_0^2 - 2RI_0 b \\ & - 2RI_0 (A + B \sin nt) \sin pt + 2b (A + B \sin nt) \sin pt\}, \end{aligned}$$

and because I_0 consists only of steady current terms and acoustic frequency terms we may write

$$I_0 = \gamma b^2 + \frac{\gamma}{2} (A + B \sin nt)^2 + \gamma R^2 I_0^2 - 2\gamma R I_0 b + \beta b - \beta R I_0 + a.$$

Then solving this equation as a quadratic in I_0 we have

$$\begin{aligned} I_0 = {} & \frac{1}{2\gamma R^2} [(2\gamma Rb + \beta R + 1) \\ & \pm \sqrt{(\beta R + 1)^2 + 4\gamma Rb - 4\gamma R^2 \alpha - 2\gamma^2 R^2 (A + B \sin nt)^2}] \quad \ldots(7). \end{aligned}$$

The expression under the radical sign will always be positive if the grid potential is not allowed to wander from the parabolic portion of the characteristic, that is, if the mean grid potential

$$V \gtrsim \left[\frac{-\beta + \sqrt{\beta^2 - 4\gamma\alpha}}{2\gamma}\right] + (A + B) + Ri_n \quad \ldots\ldots\ldots (8).$$

where i_n is the amplitude of the alternating currents in R.

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Rearranging equation (7) and writing

$$K = \left\{\frac{(\beta R+1)^2}{4\gamma^2 R^2} + \frac{b}{\gamma R} - \frac{\alpha}{\gamma} - \frac{1}{2}\left(A^2 + \frac{B^2}{2}\right)\right\},$$

we have

$$I_0 = \frac{1}{2\gamma R^2}\left[(2\gamma R b + \beta R + 1) \pm 2\gamma R K^{\frac{1}{2}} \sqrt{1 - \frac{B}{K}\left\{A \sin nt - \frac{B}{4}\cos 2nt\right\}}\right] \quad \ldots\ldots(9).$$

The radical in equation (9) may be replaced by the binomial expansion

$$1 - \frac{1}{2}\frac{x}{K} - \frac{1}{8}\frac{x^2}{K^2} - \frac{1}{16}\frac{x^3}{K^3} - \frac{5}{128}\frac{x^4}{K^4} - \cdots,$$

where $$x = B\left\{A \sin nt - \frac{B}{4}\cos 2nt\right\}.$$

It is evident that the terms x^2, x^3, x^4, etc., in the binomial expansion, will be responsible for harmonics of n up to the 4th, 6th and 8th, etc., respectively. Hence the presence of the grid leak and condenser gives rise to a whole series of harmonics. The importance of these harmonics will depend on the value of K.

We must now consider the values of K which occur in practice. Values of K for several widely differing valves are shown in Table I. To comply with condition (8) the following values of b, R, A and B have been chosen: $b = 15$ volts, $R = 0{\cdot}5$ megohm, $A = B = 0{\cdot}5$ volt.

TABLE I.

Valve		μ	ρ / KΩ	α	β	γ	K
B.T.H.	B. 11	3	3	3·4	13	14	2·2
Triotron	Z.E. 4	8·5	7	4·5	10·2	5·8	5·4
Screened Grid, Marconi	S. 625	110	170	1·3	5·5	10	2·8
Marconi	D.E. 4	7	10	0·25	2·7	7·2	5·4

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The values of α, β and γ are such that

$$i = \alpha + \beta V + \gamma V^2 \text{ microamperes.}$$

If $K = 2$ and $A + B = 1$ volt the second and higher terms in the binomial expansion will not be very important, and our error will not be greater than 1 °/₀ if we write

$$I_0 = \frac{1}{2\gamma R^2}\left[(2\gamma Rb + \beta R + 1) \pm 2\gamma RK^{\frac{1}{2}}\left(1 - \frac{B}{2K}\left\{A \sin nt - \frac{B}{4}\cos 2nt\right\}\right)\right].$$

Then equating steady current terms and alternating current terms we have

$$i + j = \frac{1}{2\gamma R^2}\{(2\gamma Rb + \beta R + 1) \pm 2\gamma RK^{\frac{1}{2}}\} \quad \ldots\ldots(10),$$

and

$$i_n = \frac{B}{2K^{\frac{1}{2}}R^2}\left\{A \sin nt - \frac{B}{4}\cos 2nt\right\} \quad \ldots\ldots\ldots\ldots(11).$$

From these two equations we may calculate the output of any cumulative grid rectifier, when an acoustically modulated radio frequency voltage $e = (A + B \sin nt) \sin pt$ is applied, if we are prepared to assume that the grid current curve of the valve used in the rectifier is parabolic.

4. *Summary.*

It is shown that a quartic equation may be found which expresses the grid characteristic of a valve accurately over a considerable range, and that for modern dull filament valves the grid current curve is exponential over only a very restricted length.

Expressions are given for calculating rectified current and the amplitude of harmonics produced by a simple grid rectifier when voltages of the following wave-forms are applied: $E \sin pt$, $(A + B \sin nt) \sin pt$, and $(A + B \cos mt + C \cos nt) \sin pt$. Further, a formula is given from which we may calculate the rectified current produced by a cumulative grid rectifier when a voltage $E \sin pt$ is applied.

All these formulae are applicable if the grid characteristic is parabolic, cubic, quartic or quintic in form.

It is also shown that when an acoustically modulated radio frequency voltage is applied to a cumulative grid rectifier, even if the valve has a parabolic grid characteristic, an infinite number of harmonics of the modulation frequency are produced. These harmonics are due to the action of the grid leak and condenser.

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It appears, however, that in practice the amplitude of these harmonics will be very small compared with those produced, primarily, by the curvature of the grid characteristic.

In conclusion, I wish to express my indebtedness to Professor C. E. Inglis for placing at my disposal the facilities of the Cambridge University Engineering Laboratory and to Mr L. B. Turner and Mr E. B. Moullin for the interest they have taken in the work.

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