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ECONOMETRIC THEORY MEMORIAL TO ALBERT REX BERGSTROM –INTRODUCTION



We are honored to present this special Memorial Issue of *Econometric Theory* dedicated to Albert Rex Bergstrom, who passed away in May 2005. Rex Bergstrom was a founding member of the Advisory Board of *ET*, he chaired the Tjalling Koopmans Prize Committee on several occasions, and he helped the journal in many other ways as an advisor, a referee, and a regular contributing author. He was fondly known among all his colleagues, friends and students as Rex and was deeply respected by all for his scholarship and humanity. Throughout his life, Rex was an enthusiastic supporter of econometric research, emphasizing its vital links to mathematical statistics and its energizing symbiosis with economics. His professional career spanned some five decades, which were spent in almost equal parts in New Zealand and the United Kingdom.

Rex was deeply committed to research. He made foundational contributions in the areas of empirical simultaneous equations modeling, finite sample distribution theory, and continuous time modeling, the latter being the area for which he is justly credited as a leading pioneer for his pathbreaking research. Most of the articles contained in this issue were presented at a Memorial Conference held at the University of Essex in May 2006, whose complete program of activities is given following this introduction. The articles included in this issue have all been processed through the journal's usual review process, and we have served as joint editors for this special issue. The intention has been to bring together a collection

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of articles that are dedicated to Rex, relating directly to his research interests and falling within the regular scope of the journal.

Comprehensive accounts of Rex's life and work can be found in several recent obituaries and memorial writings (Phillips, 2005a, 2005b, 2005c). Rex's own views about his research and econometrics more generally can be found in the interview published some two decades ago in this journal (Phillips, 1988). The Introduction to this issue will not attempt to cover the same ground, and readers are referred to those earlier memorial writings for further background information. We will focus here on the breadth of Rex's interests as an academic economist and the example that his life and work present to us. Perhaps uniquely in our profession, Rex dedicated the last 40 years of his working life to pursuing a single research goal—the methodological development of econometric models in continuous time and the empirical application of these models to macroeconomic activity. This long-term project evolved into the centerpiece of his life as a researcher.

CAREER OVERVIEW

Rex Bergstrom was born in Christchurch, New Zealand, in 1925. He obtained his first academic appointment at Massey College in 1948, moved to Auckland University College in 1950, and took leave over 1952–54 at Cambridge to complete a Ph.D. under Richard Stone. His thesis on supply and demand for New Zealand's food exports was an extensive and original undertaking that broke new ground in empirical econometrics. The model had 27 equations and 55 parameters. It was the first large-scale macroeconometric model to be estimated by the new method of limited information maximum likelihood (LIML). Impressive in scale, design and econometric methodology, the undertaking was equally impressive in its execution, much of the calculation being done by Bergstrom's hand on an electronic desk calculator.

Published as path-breaking work in *Econometrica*, Bergstrom's (1955) paper set the stage for the ambitious empirical modeling projects that characterized the rest of his career. Many of his contributions to econometric theory, particularly those related to the estimation of continuous systems, were motivated by the need to understand the models and the methods so that they could be more confidently implemented in practical work. Indeed, his curiosity about the working merits of the LIML procedure led to his later investigation (Bergstrom, 1962) into the finite sample properties of the maximum likelihood estimator (MLE) and ordinary least squares (OLS) estimator of the marginal propensity to consume in a simple stochastic closed economy. Under Gaussianity, Bergstrom derived the exact mathematical forms of the density functions of the two estimators so that they could be directly compared in terms of their concentration probabilities. The results pointed clearly to the superiority of simultaneous equations methods like LIML over single equation methods such as OLS. Even in very small samples of 10 observations, the dominance was evident. Bergstrom's (1962) paper in *Econometrica*, along with

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Basmann (1961), forged the gateway to a new research agenda concerned with the analysis of the finite sample distributions of econometric estimators.

In 1962 Bergstrom took up a Readership at the London School of Economics (LSE), and it was at LSE that his interest in continuous time models originated, stimulated by the work of Bill Phillips, a colleague at LSE and a fellow New Zealander. His initial research was on cyclical growth. This interest led to a masterly monograph, *The Construction and Use of Economic Models* (Bergstrom, 1967), that overviewed a compendium of partial and general equilibrium models leading up to models of cyclical growth and methods for their econometric implementation. The latter topic quickly became an absorbing interest and evolved into a lifetime mission in econometric methodology concerned with the econometric estimation of systems of stochastic differential equations using data observed at discrete intervals in time. The subject was in its infancy, an open beach for a fertile imagination.

In 1964 Bergstrom returned to Auckland as Professor of Econometrics, and his first published paper on continuous time econometrics appeared shortly after in *Econometrica* (Bergstrom, 1966). This article demonstrated how a recursive system of stochastic differential equations may lose its recursive structure when the variables are sampled at discrete time intervals, a phenomenon that reflects interactions between variables within the sampling interval and leads to simultaneity. Bergstrom accordingly proposed a nonrecursive approximation in the form of a system of simultaneous equations that could exploit methods of econometric estimation for simultaneous equations. The intent, no doubt already in mind, was to build a methodology suitable for empirical work with such systems in the future.

Bergstrom's work on continuous time econometrics intensified when he returned to England in 1970 to take up the Keynes Visiting Professorship at the University of Essex and subsequently in 1972 a permanent Chair in Economics, a position that he held until his retirement in 1992. During the early years at Essex he collaborated with Clifford Wymer in developing and estimating the first continuous time macroeconometric model of the United Kingdom (Bergstrom and Wymer, 1976). The model, now known as the Bergstrom-Wymer (BW) model, develops the cyclical growth model of Bergstrom (1967) into a system of 13 stochastic differential equations whose parameters are subject to complicated nonlinear restrictions arising from the underlying neoclassical growth theory. The model was estimated using quarterly data for the postwar period, analyzed to determine its long run and stability properties, and used to generate impressive postsample forecasts. The BW model proved to have a lasting legacy, serving as a prototype for many subsequent continuous time macroeconometric models of the U.K. and other developed economies.

During much of the 1970s and the first half of the 1980s Bergstrom held administrative posts at the University of Essex, serving as a dean, department chair, and pro-vice-chancellor. During a period of almost unrelenting university duty, he managed to find time to produce a further major development in continuous time econometric methodology and an overview of the field. The first was an article in

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Econometrica (Bergstrom, 1983) giving a uniqueness and representation theorem for the solution of higher-order systems of stochastic differential equations and using the theory of random measures to specify the system errors. This specification provided a foundation for likelihood-based estimation methods for continuous time systems based on the exact discrete model derived from the system's solution. This approach formed the basis of Bergstrom's subsequent theoretical and empirical work on continuous time systems. The second was an article in the *Handbook of Econometrics* (Bergstrom, 1984) overviewing the field of continuous time econometrics to the early 1980s, surveying its technical methodology, and motivating the use of these systems in practical work. These two articles are still widely cited in research on continuous time models and temporal aggregation issues.

The 1980s was a productive decade for Bergstrom's research, bringing further developments in the methodology of exact discrete modeling that put this approach in the forefront. These developments included: extending the algorithms of likelihood construction to allow for flow variables, mixtures of stocks and flows, and exogenous variables (Bergstrom, 1985a, 1986); the optimal control of stationary systems driven by random measures (Bergstrom, 1987); and optimal forecasting from higher order systems with mixed sample data (Bergstrom, 1989). The decade of the 1980s also witnessed the beginnings of a new era of empirical implementation, this time involving exact discrete methods for estimating continuous time systems. The work was conducted in collaboration with two of Bergstrom's Ph.D. students at Essex, Ben Nowman and Marcus Chambers. The first of these applications (Bergstrom and Chambers, 1990) studied the demand for consumer durables and was one of two new contributions in a volume of otherwise previously-published work on continuous time models (Bergstrom, 1990). The second constituted a significant development of the empirical modeling agenda that had begun in the 1970s with the appearance of the BW model. In collaboration with Ben Nowman and Clifford Wymer, Bergstrom developed a second-order model of the U.K. economy comprising 14 equations and 63 structural parameters. The Bergstrom, Nowman, and Wymer (1992) model, or BNW model as it has become known, set a new standard for continuous time macroeconometric modeling in terms of the complexity of its underlying economic theory and the sophistication of its econometric methodology.

Bergstrom retired from the University of Essex in 1992 at the age of 67 and was honored with the title of Emeritus Professor, the first time an economist had received this recognition at Essex. At this point he moved to London on a permanent basis, having sold his residence in Colchester, although he maintained his association with the university through regular trips to the campus every three weeks or so. On these visits Bergstrom would deal with his mail, talk with colleagues, and check the latest issues of journals in the university library, all the while attired in his familiar and distinguished outfit of black suit, white shirt, and black tie.

The idea of retirement was only a notional one for Bergstrom as he continued to contemplate yet another ambitious project in continuous time modeling.

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Not completely satisfied with the performance of the BNW model, he decided that the model would benefit from the presence of deeply embedded unobservable stochastic trends. To estimate such a model would require the derivation of a new exact discrete time representation to take account of the additional complexity of the new formulation. The new algorithm, which also accounted for different orders of stochastic differential equations, was published in this journal in Bergstrom (1997) and formed the basis for the empirical estimation of the model. Some of Bergstrom's notes on the form of the likelihood function are contained in this memorial issue and serve as testimony to the scholarly thoroughness that characterized his work. An early version of the new macroeconometric model was presented by Bergstrom at the Cowles Conference on *Recent Developments in Time Series Econometrics* in 1999, and his intention was to present the development and specification of the model, along with the estimated parameters and the associated stability analysis and production of forecasts, in the form of a monograph. Fortunately, the vast majority of the book had been written by the time of his death in May 2005, and his co-author, Ben Nowman, was able to guide the project through to its final publication by Cambridge University Press in 2007 (Bergstrom and Nowman, 2007). This final monograph concluded a long and distinguished career by New Zealand's pre-eminent econometrician.

CONTENTS OF THIS VOLUME

In the course of preparing the final version of his 1997 *Econometric Theory* article, Rex Bergstrom produced a set of handwritten notes demonstrating that the value of the Gaussian likelihood remains unchanged irrespective of whether all equations are differenced or the differencing takes place only in the direction of the stochastic trends. These notes were prepared in response to a question raised by the Editor and referees concerning the apparent excess differencing in the exact discrete time model that he had derived; see equation (32) of Bergstrom (1997). In his covering letter Rex wrote:

> As my simple example (on the hand written pages) illustrates, it makes no difference to the computed value of the Gaussian likelihood whether it is derived from the discrete time model obtained by differencing all equations or from the one obtained by differencing only in the direction of the stochastic trends.
>
> . . . I have not the slightest doubt that my algorithm yields the correct value of the Gaussian likelihood, taking account of all information in the sample and all restrictions implied by the continuous time model.

We thank Christine Bergstrom for her kind permission to publish these notes here. The material is of interest in its own right and because it illustrates the care and clarity of vision with which Rex approached all his work.

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The papers that appear in this memorial issue deal with aspects of Rex Bergstrom's research and relate in various ways to his central academic interests in econometric theory and empirical modeling applications. These contributions are described briefly in the following paragraphs.

The first two papers develop exact distribution theory for structural equation systems and relate to Bergstrom's pioneering work in this field. Grant Hillier derives the exact conditional distribution function of the conditional likelihood ratio (CLR) test of Moreira (2003), providing critical value functions that are needed for practical implementation and graphical renderings using computerized algebra. These results are extended to cases involving the coefficients of several endogenous variables. Hillier's paper also provides analytic results on the power functions of the CLR and some related tests. This paper provides useful guidance on the performance characteristics of testing procedures in structural equation models with possibly weak instruments, giving exact distributional results on these testing procedures for the first time.

Peter Phillips develops exact distribution theory for LIML, IV, and OLS estimators in structural equation models with and without identities. The work relates to Bergstrom's (1962) early contribution on the exact distribution of structural estimators in a simple stochastic Keynesian model in which the national income identity occurs. Phillips shows that general IV exact distribution formulas for models without identities also hold for models with identities by specializing along a particular asymptotic parameter sequence implied by the identity. The result involves the use of a uniform asymptotic expansion and is the first instance of a uniform approximation delivering an exact expression for a probability density. Given recent advances in our knowledge of the properties of estimators and test statistics in structural equations, the Phillips paper highlights the continuing legacy of Bergstrom's contributions to the econometrics literature.

The next group of papers in this issue relate to Bergstrom's work on continuous time modeling, the corresponding discrete time analogues, and matters of econometric estimation and inference in such systems.

Since data in economics and finance are always available in discrete time format, the problem of how to discretize continuous time models inevitably represents a key step in addressing parameter estimation. Another key element involves the nature of the parametric specification in the continuous system. Most continuous time models that have been used in practical work assume parameter constancy, which is an important and useful simplification but one which is often shown to be too strong in the context of discrete time modeling. Peter Robinson's paper extends the literature by obtaining the exact discrete time representation of a multivariate linear continuous time model with time varying coefficients. This contribution develops Bergstrom's work on exact discrete equivalent systems and is suggestive of generalizations that can open many avenues of useful work.

Building on the earlier continuous time models of, inter alia, Bergstrom (1966), Bergstrom and Wymer (1976), Bergstrom, Nowman, and Wymer (1992), Berg-

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strom (1997), and Bergstrom and Nowman (2007), Joanne Ercolani extends the trend component to allow for the presence of continuous time cyclical trends. She outlines both differential-difference equation (DDE) and stochastic differential equation (SDE) formulations for the cyclical trend component. An exact discrete time analogue for the SDE formulation is given, although for the DDE approach this issue remains unsolved. Comparison of the two formulations for both stock and flow variables is made through Monte Carlo simulation, using an approximation to the DDE model. Ercolani's paper is important in that it introduces an empirically useful extension to the corpus of continuous time models available for the trend component. Ercolani's work provides a basis for future research to hopefully build upon and further improve the modeling and forecasting abilities of Bergstrom's differential equation systems.

Marcus Chambers' paper also builds on previous pioneering work of Rex Bergstrom; in particular, Bergstrom (1997), and Bergstrom and Nowman (2007). Chambers details algebraic steps for computing a discrete-time representation of a linear continuous-time co-integrated process, whose process vector, y, is observed in a "mixed" form in which some variables are observed directly as "stocks" or values generated by the continuous-time process and indirectly as "flows" or integrals over sampling periods of the continuously generated values. More specifically, Chambers extends some of his own and Bergstrom's results in such a way that over-differencing, as a way of accounting for stochastic trends, a problem inherent in the earlier contributions, does not occur. Issues relating to the estimation, inference, and computation of the model are dealt with in useful detail.

In estimating continuous systems with discrete data, Bergstrom favored the use of exact discrete time models when these were compatible with the available data, or approximations to the exact discrete models when the nature of the data or nonlinearities made use of the exact system infeasible. Roderick McCrorie's paper studies various aspects of this approach to continuous system estimation and identification, outlining difficulties such as aliasing induced lack of identification (Phillips, 1973), multimodality in the likelihood, particular problems in higher order systems (Bergstrom, 1983, 1986), and the absence of a fully developed asymptotic theory covering stochastically nonstationary cases.

Two articles in this issue are closely related to Bergstrom's research in the area of continuous time macroeconometric models, and are written by his two principal co-authors in this area: Clifford Wymer and Ben Nowman. Wymer investigates the presence of aperiodic dynamics in the nonlinear Bergstrom-Wymer model. He finds that the Lyapunov exponents computed from the estimated model suggest that it has strange attractors in the neighborhood of the estimated parameters, implying that the model is stable but subject to aperiodic oscillations. This finding is robust to various changes in some of the model's parameters that are related to economic policy.

Nowman uses his special position as Bergstrom's final research collaborator and co-author to provide an overview of Bergstrom's work in continuous time macroeconometrics and to offer a personal reflection on his work with him that

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spanned almost twenty years. He also discusses the influence of Bergstrom on his research students and comments on their recent contributions that have helped to further the study of continuous time econometrics and issues of temporal aggregation.

In addition to his mainline interests in econometric modeling and continuous systems, Bergstrom made fundamental contributions to the theory of cyclical growth and economic policy. The paper by Viv Hall and John McDermott examines the properties of business cycles in New Zealand over the post World War II period. Since quarterly GDP data for the New Zealand economy is available only from 1977, one of the major contributions of their paper is to disaggregate annual data before this period into quarterly data, thereby producing a new postwar quarterly real GDP series for New Zealand. The paper establishes a business cycle chronology, summarizes the main empirical features of the cycles, and reveals asymmetries in terms of both amplitude and duration. Little evidence is found for duration dependence in either expansions or contractions indicating that New Zealand business cycles tend to be driven by shocks to aggregate economic activity.

An important facet of modern empirical macroeconomic modeling is the increasing use of mechanisms that allow for trend breaks and stochastic trends. Trends and stochastic trends played a role in Bergstrom's empirical work on modelling the U.K. economy, and that work has been extended by Nowman (1998) to allow for segmented trends. The paper by David Harvey, Stephen Leybourne, and Robert Taylor in this issue proposes a new testing procedure for the trend break hypothesis that is robust to whether the shocks are I(0) or I(1), as well as being robust to the form of serial correlation. For the known break date case, the test is based on a simple linear combination of two t-test statistics, one from the level specification and the other from the difference specification. The weight function in the test is itself based on a function of two unit root test statistics and is designed to place a unit weight on the asymptotically normal test statistic in the limit, depending on the integration order of the error term. Simulations and an empirical application to macroeconomic and financial data highlight the usefulness of these procedures.

In an interesting departure from his mainline research, Bergstrom (1985b) developed a nonparametric estimator for nonlinear models with fixed regressors and bounded sample spaces using techniques of Hilbert spaces. The paper by Graciela Chichilnisky in this issue extends Bergstrom's results to the case of unbounded sample spaces using the same orthogonal series estimator as Bergstrom but working in a weighted Hilbert space. Necessary and sufficient conditions are given for the extension of the results from bounded intervals to the positive real line. In practical terms, the regression is based on a transformation of the original regressors, and the conditions essentially require that unbounded sets eventually become less likely than any bounded set, effectively ruling out heavy tailed data. The conditions are related to certain criteria in choice under uncertainty concerning insensitivity to rare events.

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CLOSING REMARKS

Rex Bergstrom's passing brought great sadness to a wide community of professional economists throughout the world. As a colleague he was deeply respected for upholding the highest standards of scholarship; as a teacher and supervisor he was generous, caring, and inspirational; and as a person he held a quiet moral authority that guided his life and career. The economics profession will continue to benefit from his exemplary training of young economists and the legacy of his research accomplishments. Cohorts of students and colleagues from New Zealand and the United Kingdom owe Rex a great debt of gratitude for his guiding influence on their own academic careers. He is sadly missed and prominent in our thoughts as we seek to continue his traditions in our own teaching and research. He will always be remembered with great fondness and respect.

We are delighted to bring together this collection of articles in memory of our friend and colleague, Rex Bergstrom, and we thank all who have contributed in so many ways to this special Memorial Issue of *Econometric Theory*.

Marcus J. Chambers, *University of Essex*
Peter C.B. Phillips, *Yale University, University of Auckland, University of York, and Singapore Management University*
A.M. Robert Taylor, *University of Nottingham*

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