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Money and the C-CAPM

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Abstract

We consider asset pricing in a monetary economy where liquid assets are held to lower transaction costs. The ensuing model extends the capital asset pricing model (CAPM) and the consumption CAPM by deriving real money growth as an additional factor determining returns. Empirically, the two model versions compare favorably to other theoretical asset pricing models along several dimensions, supporting the traditional intertemporal asset pricing perspective. A value premium arises because value firms are sensitive to liquidity shocks but growth firms are not. Although no alternative factor drives out the money growth factor, the conditioning CAY factors of Lettau and Ludvigson (2001b) add explanatory power.

I. Introduction

The empirical performance of the basic asset pricing theories—the CAPM and the consumption CAPM (C-CAPM)—has been disappointing. Alternative theories such as Cochrane's (1996) investment-based model and Lettau and Ludvigson's (2001a), (2001b) C-CAPM conditional on the consumption-to-wealth ratio perform considerably better, but the purely empirical three-factor model of Fama and French (FF3) (1996) outperforms all theoretical asset pricing formulations. A major challenge to the profession is to provide a set of pricing factors that is both reasonable from a macroeconomic perspective and competitive with the Fama-French model in pricing the cross section of asset returns.

From a macroeconomic perspective, the monetary environment as related to money growth and inflation appears to be a prime candidate for providing a systematic pricing factor. Early empirical results starting with Chen, Roll, and Ross (1986) suggest a role for money in asset pricing by finding that both anticipated

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inflation and unanticipated inflation are significant factors in pricing the cross section of assets. Following these results, a large body of work has investigated the link between equity prices and inflation or money growth.

Boyle and Young (1988) consider a static asset pricing model in which individuals have direct utility over consumption as well as real money balances. In their framework, both monetary shocks and real shocks affect the equity premium. Marshall (1992) obtains similar results but employs a dynamic formulation in which the level of liquidity measured by real money holdings affects the cost of transactions. Both papers focus primarily on the link between stock prices and inflation and do not consider cross-sectional pricing implications. Similarly, Boyle and Peterson (1995), Thorbecke (1997), Jensen and Mercer (2002), and others examine the monetary policy impact on asset pricing without considering cross-sectional asset pricing implications.

Chan, Foresi, and Lang (1996) examine the implications for cross-sectional asset pricing of a liquidity constraint in a cash-in-advance economy in which certain goods can be purchased only with money. In this environment real inside money holdings are equal to the consumption good that requires cash in advance. Due to a separability-in-utility assumption, the stochastic discount factor depends solely on the growth rate of this cash-in-advance consumption good. Hence, the growth rate of real inside money balances becomes the asset pricing factor. Chan et al. (1996) find that their model with real inside money growth as the sole factor outperforms the C-CAPM but performs marginally worse than the market CAPM.[1,2]

We explore a natural improvement to the C-CAPM by arguing that the availability of money as a source of liquidity facilitates transactions and affects the marginal value of wealth in generating consumption. There is broad agreement that liquidity is essential in lubricating the economy, but money contributes to the economy via many mechanisms. However, various theories of money demand commonly imply that money affects the marginal resource cost of raising utility: The Sidrauski formulation views money as directly affecting utility; the Lucas (2000) shopping time model implies that money affects utility by reducing the time lost in the process of completing transactions; Marshall (1992) relies on the benefit of money holdings for lowering transaction costs; Chan et al. (1996)

[1]The money factor, even if it is theoretically equivalent to consumption, can outperform the consumption factor, given that consumption services are difficult to measure. A similar motivation has led to the production-based asset pricing models of Balvers, Cosimano, and McDonald (1990), Cochrane (1991), and Balvers and Huang (2007)—in which output growth, investment growth, and productivity shocks, respectively, are alternative proxy variables measuring the marginal rate of intertemporal substitution in consumption—and the ultimate-consumption CAPM of Parker and Julliard (2005), in which long-run consumption proxies for (short-run) consumption. Our approach entails a much more direct attempt at resurrecting the C-CAPM that does not rely on measurement error in consumption.

[2]From a different angle, Pástor and Stambaugh (2003) find that aggregate liquidity is a systematic asset pricing factor that can also explain a large part of momentum profits. They measure aggregate liquidity as the average price reversals induced by order flow. This concept of liquidity, related to whether *financial assets* can be traded quickly and at low cost, is a micro liquidity concept and quite different from our macro liquidity concept related to the monetary environment, which deals with whether *consumption goods* can be traded easily (at low time and resource cost). Intriguingly, Chordia, Sarkar, and Subrahmanyam (2005) link the two liquidity concepts empirically by showing that improvements in stock market liquidity are tied to (nominal) money growth.

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assume that cash in advance is required before transactions can be completed. Walsh ((2003), chaps. 2 and 3), contains a nice survey of theories of money demand. In each of these perspectives, both consumption and the liquidity available determine the marginal value of financial returns for a representative investor.

We follow the formulation in Marshall (1992) to introduce money to the C-CAPM. In this environment the marginal value of financial returns is determined by the marginal utility of consumption together with the marginal cost of doing transactions, which depends on the liquidity currently available. Accordingly, the stochastic discount factor varies with real consumption growth as well as with real money growth, in which the real money growth factor combines the effects of nominal money growth and inflation. From the envelope condition the marginal utility of wealth alternatively varies with real market returns as well as real money growth rates. A two-factor asset pricing model emerges with either market return and real money growth or consumption growth and real money growth as the factors.

Following Lettau and Ludvigson (2001b), we employ the Fama-MacBeth (1973) method (with Shanken (1992) and generalized method of moments (GMM)-based corrections for measurement error in the betas) to price the cross section of the 25 Fama and French portfolios sorted by size and book-to-market ratio. Our real-money-growth-augmented CAPM explains 64%, and the real-money-growth-augmented C-CAPM explains 58%, of the cross-sectional returns variation, in either case with insignificant alphas. Both compare favorably with the CAPM, the C-CAPM, and the theories of Cochrane (1996), Campbell and Cochrane (1999), and Lettau and Ludvigson (2001a), (2001b). Our models' performances here fall short of that of the Fama and French (1993), (1996) three-factor model, which explains 77% of the returns variation across portfolios. As argued by Lewellen, Nagel, and Shanken (LNS) (2009), the cross-sectional fit is a weak evaluation criterion. Accordingly, we consider the performance of the models along various other dimensions, which include alphas, significance of fit, alternative test assets, and coefficient stability. While none of the models are perfect, we find that our models generally outperform the others, including the Fama-French model.

In Section II, we derive a discrete-time factor model based on the assumption of conditional normality of returns and factors. We derive both a money CAPM (the MCAPM) and a money C-CAPM (the MC-CAPM) and consider unconditional implications of each with and without a constant. Section III empirically evaluates both versions and compares them to alternative asset pricing models. We take into account various evaluation dimensions and also obtain comparable estimates based on a GMM-stochastic discount factor approach. Utilizing J_t (χ^2) difference tests in the stochastic discount factor framework, we further conduct direct comparisons between rival models. Section IV concludes.

II. Asset Pricing in a Monetary Economy

We add money to the standard C-CAPM of Breeden (1979). Money is held because it lowers the transaction cost of purchasing consumption goods, as modeled in Marshall (1992) who, however, does not consider cross-sectional asset

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pricing issues. The specific model introduces one of several possible ways in which money affects the resource cost of marginal utility.

The addition of money is best modeled in discrete time, since money must be acquired before it can be used to facilitate transactions. Analogously to the Brownian motion assumption in continuous time, we assume conditional normality and obtain a specific solution for the asset pricing equation by providing and applying an extension of Stein's (1973), (1981) lemma. The resulting model has substantially different implications compared to Chan et al. (1996)—the only previous paper considering the theoretical cross-sectional asset pricing implications of money—by generating money balances as an additional factor to consumption instead of as an alternative proxy.

A. The Model

A representative consumer-investor in an endowment economy maximizes expected lifetime utility subject to a budget constraint and given that transaction costs to purchase consumption goods are mitigated by money holdings. This is captured by the following formulation:

$$V(w_t, m_t, x_t) = \max_{c_t, \mu_t, \{s_t^i\}_{i=1}^n} \left(u(c_t) + \delta E_t[V(w_{t+1}, m_{t+1}, x_{t+1}]\right), \tag{1}$$

$$\text{subject to} \quad w_{t+1} = R_{t+1}[w_t - c_t - T(c_t, m_t) + m_t - \mu_t], \tag{2}$$

$$R_{t+1} = \sum_{i=0}^{n} s_t^i R_{t+1}^i, \qquad \sum_{i=0}^{n} s_t^i = 1, \quad \text{and} \tag{3}$$

$$m_{t+1} = (\mu_t + z_{t+1})/\pi_{t+1}. \tag{4}$$

Equation (1) represents the representative agent's maximized value of lifetime utility (the derived or indirect utility function) in real terms as given by the Bellman equation of dynamic programming.[3] Equation (2) gives the budget constraint. Here, w_t and m_t represent, respectively, real financial wealth (excluding money holdings) and real money holdings; and x_t indicates a set of state variables, exogenous to the consumer-investor, that is sufficient to represent (unmodeled) changes in the investment opportunity set. The agent in period t chooses current consumption c_t, real money holdings μ_t (for use in the upcoming period), and the portfolio shares s_t^i of a risk-free asset (asset 0) and n risky assets. Lifetime utility is time separable, future utility is discounted with factor δ, and felicity $u(\cdot)$ in each period is a concave function of consumption.

$T(c, m)$ represents the real transaction cost of purchasing the current level of consumption. Indicating partial derivatives by subscripts, the conditions on the transaction cost are that $T(c, m) > 0$, $T_c(c, m) > 0$, $T_m(c, m) < 0$, $T_{cc}(c, m) \geq 0$,

[3] By the implicit function theorem, the state variables in the value function are the predetermined variables in decision problems (1)–(4). We assume a general Markov formulation for the variables exogenous to the representative agent: $R_{t+1}^i = R^i(x_t, \varepsilon_{t+1}^{R^i})$, $\pi_{t+1} = \pi(x_t, \varepsilon_{t+1}^{\pi})$, $z_{t+1} = z(x_t, \varepsilon_{t+1}^{z})$, $x_{t+1} = x(x_t, \varepsilon_{t+1}^{x})$, where all εs are i.i.d. Then x_t, w_t, and m_t are jointly necessary and sufficient for describing the state determining the agent's indirect utility.

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$T_{mm}(c,m) \geq 0$, and $T_{cm}(c,m) \leq 0$. These conditions are intuitive. Transaction costs are positive, increasing in consumption, and decreasing in real balances. The second derivative conditions ensure that increases in consumption and decreases in real money holdings have either a constant effect or become increasingly costly. The cross-derivative condition implies that the marginal cost of consumption stays constant or falls as more real money balances are held.[4]

Investable wealth in equation (2) equals $w_t + m_t$ minus current expenditure on consumption and future money holdings $c_t + T(c_t, m_t) + \mu_t$. The gross real portfolio return is a weighted average of returns on the individual assets as given by equation (3). Possible changes in the distribution over time of the asset returns are captured by the state variables x_t (the Merton (1973) factors). The choice of real money balances at the current price level to be used in the next period, μ_t, differs from the real balances used in the next period at the then-relevant price level, due to the gross inflation rate $\pi_{t+1} \equiv p_{t+1}/p_t$ and due to an assumed transfer of government revenues from money creation, $z_{t+1} = (M_{t+1} - M_t)/p_t$, to the representative consumer-investor, yielding equation of motion (4).

Appendix A shows from the first-order conditions that equations (1)–(4) imply that

$$E_t[r^i_{t+1} V_w(w_{t+1}, m_{t+1}, x_{t+1})] = 0, \quad \forall\, i, \quad \text{and} \tag{5}$$

$$V_w(w_t, m_t, x_t) = \frac{u_c(c_t)}{1 + T_c(c_t, m_t)}, \tag{6}$$

with r^i_{t+1} representing excess returns, $R^i_{t+1} - R^0_{t+1}$. The marginal value of wealth $V_w(w_{t+1}, m_{t+1}, x_{t+1})$ serves as the stochastic discount factor pricing all assets. Equation (6) follows from the envelope condition but is nonstandard in that the marginal utility of consumption is insufficient to capture marginal value of wealth: Marginal value of wealth depends on consumption but also on the real value of money holdings, determining the cost of purchasing consumption goods at the margin. Given consumption, if money holdings turn out to be low after the price level and market returns are realized (a liquidity squeeze), then the marginal cost of consumption is higher at this given consumption level so that the marginal value of wealth is lower from equation (6).

We continue in the spirit of Merton (1973) and Breeden (1979) by developing an equilibrium asset pricing model without committing to a specific (computable) general equilibrium environment. To isolate the contribution of the money factor, we exclude Merton (1973) factors by assuming that there are no changes over time in the exogenous dividend processes, ruling out shifts in the investment opportunities set (except, possibly, through changes in the real money supply). Thus, we

[4]See Feenstra ((1986), p. 278), Marshall ((1992), pp. 1319–1321), and Walsh ((2003), pp. 119–120), on the conditions for the transaction cost function. These conditions together with concave utility guarantee that the second-order conditions hold. Feenstra (1986), building on Brock (1974), shows that the formulation used here can be derived from standard theories of transactions and precautionary demand for money. He also shows that the transaction cost approach is formally equivalent to the Sidrauski assumption of putting money directly in the utility function. Further, the "cash-in-advance" approach is just a limiting case of this approach when $T(c,m)$ becomes infinite for $c > m$. Thus, our model employs a quite general approach to incorporating money.

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have $V(w_t, m_t, x_t) = V(w_t, m_t)$. We further assume that all asset returns and the factors are conditionally multivariate normal. Appendix B then derives and applies an extension of Stein's lemma that, from equation (5), yields the beta formulation for expected asset returns:[5]

$$E_t r^i_{t+1} = \beta^i_t E_t r_{t+1} + \gamma^i_t (E_t g^m_{t+1} - a^m_t), \quad \forall\, i. \tag{7}$$

The β^i_t, γ^i_t coefficients are analogous to those obtained in a multivariate regression. Equation (7) provides a two-factor model of asset returns, the MCAPM. There is a standard market beta and a real money growth "gamma." The two factors are the observable market excess return and the money growth rate minus an unobservable variable a^m_t that is constant across assets and corrects for the fact that money growth is not an asset return.

The risk premium on the real money growth factor (the expected excess return on asset m, $E_t(g^m_{t+1}) - a^m_t$) may be positive or negative. As presented in Appendix A this depends on the sign of $V_{mw}(w, m)$. If the sign is positive (higher money growth raises the marginal value of wealth) then the money growth factor is a "hedging" factor in the sense that positive exposure of a portfolio to money growth shocks mitigates risk because this portfolio tends to pay out well when the marginal value of wealth is high. Then both risk premia are ambiguous. The reason is that a positive hedging benefit may be offset by positive exposure to market risk if the covariance between market return and money growth is positive (as Table 1 shows, this is the case empirically).

A sufficient (but not necessary) condition for both the money growth risk premium and the market risk premium to be positive (when the empirically relevant case also implies that the covariance between market return and money growth is positive) is that $V_{mw}(w, m) \leq 0$. The latter implies that higher money growth decreases (or leaves unchanged) the marginal value of wealth. This occurs from equation (6) when the lower marginal transaction cost raises consumption sufficiently that the decrease in marginal utility offsets the reduction in the transaction cost. For a positive money factor risk premium, $E_t(g^m_{t+1}) - a^m_t > 0$, if asset i has positive exposure to money growth risk, $\gamma^i_t > 0$, then asset i's expected return is higher, since the money factor exacerbates risk. Intuitively, when liquidity is high so that transactions are cheap and, accordingly, optimal consumption is high (lower marginal utility of consumption), the extra wealth from a higher return on asset i positively correlated with liquidity tends to be less useful. Asset i is thus less attractive, so that a higher expected return is required.

[5] An alternative expression can be derived without Stein's lemma if, instead of joint normality of returns, consumption, and money growth, the marginal utility of consumption, and the marginal transaction cost are jointly lognormal. Then $E_t(r^{ic}_{t+1}) + \tfrac{1}{2}\sigma^2_{r^{ic}} = \mathrm{Cov}_t[\ln u_c(c_{t+1}), r^{ic}_{t+1}] + \mathrm{Cov}_t[\ln(1 + T_c(c_{t+1}, m_{t+1}), r^{ic}_{t+1}]$, for the MC-CAPM. Here, returns are continuously compounded. Drawbacks are the additional "nuisance" term and that specific assumptions must be made for the functional forms of the utility and transaction costs to relate returns to the observable consumption and money growth. In addition, for quarterly data, returns are closer to normal than to lognormal. It is moreover impossible to apply this technique to the unspecified indirect utility function as is necessary to obtain equation (7) in the MCAPM case.

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B. From Theory to Measurement

Under the same general equilibrium assumptions that rule out Merton (1973) factors—that nominal money growth and dividend processes are i.i.d.—the betas are constant over time. Taking unconditional expectations, equation (7) then takes the form that we consider empirically:

$$E(r^i) = \beta^i E(r) + \gamma^i [E(g^m) - a^m], \quad \forall\, i. \tag{8}$$

The standard Fama-MacBeth (1973) two-pass "beta" approach under the assumption that the betas are stationary provides consistent estimates of the β^i, γ^i coefficients of each asset in the first pass. In the second pass, the specification makes clear that the constant should be zero (asymptotically only, due to measurement error in the generated regressors) if the estimates of the β^i, γ^i coefficients are used as regressors. Standard errors in tests of significance use a Shanken (1992) or GMM correction to adjust for measurement error.

Instead of using the marginal value of wealth as the stochastic discount factor, based on the envelope condition provided in equation (6) we can directly employ the transaction-cost-adjusted marginal utility of consumption condition instead to derive a C-CAPM with money growth as an additional factor, the MC-CAPM. An analogous derivation to that of the MCAPM yields

$$E(r^i) = \beta^{ic} [E(g^c) - a^c] + \gamma^{ic} [E(g^m) - a^m], \quad \forall\, i. \tag{9}$$

Here, both risk premia are unobservable and again ambiguous. The money growth factor represents the risk of changes in transaction cost. These transaction costs have both an income effect and a substitution effect. The substitution effect occurs because a lower cost of doing transactions implies higher optimal consumption (lower marginal utility). So an asset that pays out mostly during high liquidity is less valuable (and requires a higher return). The confounding income effect occurs for given planned consumption: An asset that pays out most during high liquidity is worth more (and requires a lower return) because it allows more transactions exactly when transaction costs are low. The risk premium on the money growth factor is positive if the substitution effect dominates the income effect.[6]

III. Results and Comparison to Alternative Asset Pricing Models

A. Methodology

We employ the Fama-MacBeth (1973) two-pass "beta" approach to estimate and evaluate the various asset pricing models. Under the assumption of

[6]Equation (8) and similarly equation (9) can be used straightforwardly to derive a stochastic discount factor (see Cochrane (2005), sec. 6.3): $f^* = 1 + ar + bg^m$. This factor is the unique stochastic discount factor (up to linear transformations) that can be mimicked by a portfolio of existing assets. Although it should price all assets just as well as the stochastic discount factor in equation (5), $f = V_w(w, m)$, the latter is generally much different, since it need not be normally distributed and then, given our normality assumption, cannot be generated by a portfolio of existing assets.

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beta stationarity, it is optimal to estimate betas using the full sample. In this case, Cochrane ((2005), sec. 12.3) shows that coefficient estimates from running a cross-sectional second pass are identical to those from averaging the coefficient estimates of the cross-sectional regressions for each time period. We use the latter approach but make Shanken (1992) and GMM corrections to the *t*-statistics to account for measurement error in the generated regressors (the betas) and to provide results robust to heteroskedasticity and serial correlation in the errors.[7] The beta approach under beta stationarity is also equivalent to a stochastic discount factor approach with identity weighting matrix in the GMM estimation. We generally follow Lettau and Ludvigson (2001b) in not conducting second-stage GMM estimation with an optimal weighting matrix. In addition to identity weighting, we consider the Hansen-Jagannathan weights for model evaluation. For a discussion on the choice of weighting matrix, see Lettau and Ludvigson ((2001b), p. 1276) and Cochrane ((2005), chap. 11).

The recent work of LNS (2009) emphasizes that multifactor models typically yield a good fit for any given set of test assets with an approximate factor structure, even if the empirical factors are mostly unrelated to the true factors. Accordingly, in evaluating our models and comparing against other models, we focus on a variety of alternative dimensions that help in identifying a proper model. The beta approach allows evaluation of the results relying on three general criteria: First, the magnitudes, signs, and significance of the coefficient estimates (the risk premia) relative to their theoretical values. The theory implies i) an intercept of zero. As our derivations show, this holds even if the factors are not returns. Additionally, the theory may suggest ii) the sign of each risk premium, or that the difference between the actual value of the risk premium and the average return on the factor should be zero (if the factor is a return). In a one-factor model such as the CAPM, implications i) and ii) are equivalent, in that a deviation from i) implies a deviation from ii) and vice versa. Because most of the signs of the risk premia are ambiguous in our model, we place special emphasis on i), checking the intercept. We also evaluate the risk premia when the regression forces the intercept to equal its theoretical value of zero.

The second evaluation criterion deals with the fit of the model in explaining the returns of the set of test assets. The theory implies that the stochastic discount factor should price all assets. In principle, considering equations (8) and (9), for instance, the average returns should be explained perfectly by the factor sensitivities. Thus, i) a higher R^2 suggests a better model. Further, as a minimum requirement, ii) the R^2 should be significantly positive; that is, the model should explain significantly better than an equal number of random factors. The empirical R^2s are expected to exceed the upper bound of a 90% confidence interval for

[7] As Jagannathan and Wang (1998) point out concerning the measurement error issue, the Fama-MacBeth method need not overestimate the precision of the standard errors under heteroskedasticity. We thus provide the Fama-MacBeth *t*-values as well as the Shanken *t*-values. Cochrane (2005) provides a convenient GMM approach for calculating the *t*-statistics, which we apply here with five lags (see John Cochrane's Web site (http://faculty.chicagobooth.edu/john.cochrane/research/Papers/) for details). These GMM *t*-statistics improve on the Shanken adjusted *t*-values in the presence of heteroskedastic or autocorrelated errors.

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R^2s generated by random factors. Lastly, iii) additional factors should have no significant impact in explaining the cross-sectional returns.

As a third evaluation criterion, we exploit the implication that a proper model should in principle explain and yield the same risk premia for any set of test assets. Thus: i) we consider all evaluation criteria for several sets of test assets, and ii) we check whether the risk premia estimates are of similar magnitude for each of the groups of test assets.

In summary, to deal with the LNS (2009) critique, we evaluate the results for each model along a variety of dimensions. In addition to the adjusted R^2 measure of the goodness of fit and the average pricing error, we also consider: the significance of the intercept; the influence of additional factors; confidence intervals for the R^2s; fits for various test asset groups, including some with little factor structure; and the stability of risk premia estimates across test asset groups.

B. Data

As test assets, we initially employ the time series of the returns on the 25 portfolios sorted by size and book-to-market ratio that are provided by Fama and French (available from Kenneth French's Web site at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). These are the assets that are most commonly used to test asset pricing models and that present the most dramatic challenge to asset pricing theory. The three-month T-bill rate is used as the risk-free asset to generate excess returns. Because the 25 Fama-French portfolios clearly have an approximate factor structure, we also consider them jointly with the 30 industry portfolios (also available from Kenneth French's Web site), as recommended by LNS (2009). In addition, we consider the 25 "cluster" portfolios of Ahn, Conrad, and Dittmar (2009) (available from Robert Dittmar's Web site at http://webuser.bus.umich.edu/rdittmar/) that have little factor structure and in addition do not have the propensity for systematic deviations from expected returns that may affect industry portfolios.

Nominal money growth is based on the time series of M2 and is deflated by the consumer price index, consumption is proxied by quarterly nondurables consumption and services. These series are available from the Federal Reserve Bank of St. Louis. Market returns are the value-weighted returns index from CRSP. Due to the quarterly availability of the consumption data, all series are compounded from monthly to quarterly.[8] The availability of M2 data further limits our sample to start in 1959Q1. The sample ends in 2004Q4.[9]

[8]Monthly consumption growth data are available and are used by Hodrick and Zhang (2001). These data may be unreliable as a measure of the stream of consumption services (see also Parker and Julliard (2005) on this point); in particular, they exhibit measurement error, as evidenced by a significant first-order correlation coefficient of -0.35. We follow the previous literature (e.g., Lettau and Ludvigson, (2001a), (2001b)) in focusing on quarterly consumption data, but the results for monthly data are quite similar and are available from the authors.

[9]Other data used for evaluating competing models are the CAY ratio from Sydney Ludvigson's Web site (http://www.econ.nyu.edu/user/ludvigsons/), the book-to-market (BM) and size SIZE variables (average of logs over time) from Kenneth French's Web site, and the *investment growth* data (nonresidential and residential) and total population from the Federal Reserve Bank of St. Louis Web site (http://www.stlouisfed.org/).

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Table 1 provides descriptive statistics of the factors (money growth, consumption growth, and market return) emphasized in our models: means, standard deviations, cross-correlations, and autocorrelations. The factors mutually have moderate positive correlation. The Jarque-Bera (1980) test indicates that strict normality can be rejected for consumption growth and market returns, but not for money growth. Table 1 further shows realized market returns sorted by consumption and money growth realizations. Market returns are higher in high consumption growth states compared to low consumption growth states. However, sorting by money growth as well implies a large return difference between high money growth and low money growth states. This intimates a role for both consumption and money growth in explaining asset returns and suggests that using money growth as an additional factor may provide better definition to the consumption growth factor.

TABLE 1
Summary Statistics

Summary statistics for the MCAPM and MC-CAPM factors: the consumption growth rate g^c, the real growth rate of the money supply (M2) g^m, and the value-weighted market excess return r, for quarterly data from 1959Q1 to 2004Q4. The Jarque-Bera (1980) statistic provides a test for the null hypothesis that a variable is normally distributed. The bottom entries represent the realized value-weighted market excess returns r for different realizations of consumption and money growth: "+" indicates above median, and "−" below median growth.

Panel A. MCAPM and MC-CAPM Factors

	g^c	g^m	r
Mean (%)	0.55	0.38	1.58
Std. (%)	0.44	1.17	8.55
Jarque-Bera	19.63	0.78	12.5
p-value (%)	0.01	67.61	0.19
Cross-Correlation			
g^c	1	0.455	0.261
g^m	0.455	1	0.275
r	0.261	0.275	1
Autocorrelation			
1 lag	0.404	0.542	0.035
2 lags	0.243	0.38	−0.094
3 lags	0.303	0.377	−0.046
4 lags	0.086	0.232	−0.018

Panel B. Market Excess Return r (%)

	$g^c(+)$	$g^c(-)$
$g^m(+)$	6.19	1.36
$g^m(-)$	1.17	0.32

C. Results

Table 2 provides the results for our models: the money CAPM (MCAPM) and the money C-CAPM (MC-CAPM), as well as a pure money growth model, the money asset pricing model (MAPM) of Chan et al. (1996). Table 2 further includes standard alternative models: the CAPM, the C-CAPM, the Fama-French (1996) three-factor model (FF3), Cochrane's (1996) investment-based model (INV), Lettau and Ludvigson's (2001b) CAY model (the conditional market version (CAYCAPM) and the conditional consumption version (CAYC-CAPM)), and Campbell and Cochrane's (1999) habit C-CAPM (HABIT).

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TABLE 2

Fama-MacBeth Estimates for 10 Models with 25 Size- and Value-Sorted Test Portfolios

The test assets are the 25 portfolios sorted by size and book-to-market ratio available from Kenneth French's Web site. The factors in each model (MCAPM, MC-CAPM, MAPM, CAPM, C-CAPM, FF3, INV, CAYCAPM, CAYC-CAPM, and HABIT) are, respectively, market return and money supply growth (r, g^m); consumption growth and money supply growth (g^c, g^m); money supply growth (g^m); market return (r); consumption growth (g^c); the Fama-French three factors (r, SMB, HML); Cochrane's (1996) investment growth for nonresidential and residential sectors (ΔNRIQ, ΔRIQ); Lettau-Ludvigson's market CAY (CAY, r, CAY·r) and consumption CAY (CAY, g^c, CAY·g^c) models; and the habit model ($g^c(t-1)$, g^c, $g^c(t-1)\cdot g^c$). For each, we report the risk premiums, t-statistics, Shanken's (1992) adjusted t-statistics, GMM t-statistics (based on five lags), and the adjusted R^2. Data are from 1959Q1 to 2004Q4. The confidence intervals are the error bands suggested by LNS (2009) under the assumption that the model is useless: the 90% interval around the median R^2 based on 5,000 draws from random multivariate normally distributed factors with the appropriate number of factors considered for each model. The results on the right side of Table 2 are obtained when the constant is restricted to its theoretical zero value.

		Constant Unrestricted				Constant Restricted to Zero		
MCAPM	Const.	r	g^m		Adjusted R^2	r	g^m	
Coeff.	0.622	1.208	1.589		64.24%	1.884	1.799	
t-value	0.478	0.632	3.473		[1.74%, 57.49%]	2.166	5.054	
Shanken-t	0.281	0.386	2.065			1.480	2.799	
GMM-t	0.381	0.468	2.141			1.550	2.114	
MC-CAPM	Const.	g^c	g^m			g^c	g^m	
Coeff.	−0.371	−0.081	1.544		58.00%	−0.119	1.366	
t-value	−0.420	−0.293	3.305		[1.74%, 57.49%]	−0.498	4.536	
Shanken-t	−0.224	−0.157	1.786			−0.282	2.623	
GMM-t	−0.230	−0.180	1.802			−0.032	0.848	
MAPM	Const.	g^m				g^m		
Coeff.	−0.560	1.425			41.98%	2.260		
t-value	−0.665	2.823			[0.05%, 40.62%]	3.279		
Shanken-t	−0.422	1.809				1.553		
GMM-t	−0.428	2.021				3.697		
CAPM	Const.	r				r		
Coeff.	3.329	−1.372			9.68%	2.727		
t-value	3.210	−0.860			[0.05%, 40.62%]	3.078		
Shanken-t	3.169	−0.851				3.004		
GMM-t	3.193	−0.886				4.270		
C-CAPM	Const.	g^c				g^c		
Coeff.	2.082	0.053			0.37%	0.637		
t-value	2.702	0.194			[0.05%, 40.62%]	3.149		
Shanken-t	2.683	0.192				1.808		
GMM-t	2.981	0.209				0.992		
FF3	Const.	r	SMB	HML		r	SMB	HML
Coeff.	3.825	−3.585	0.754	1.374	77.86%	1.406	1.530	2.000
t-value	4.063	−2.440	1.236	2.221	[6.19%, 70.06%]	1.785	2.412	3.351
Shanken-t	3.546	−2.177	1.152	2.069		1.715	2.278	3.189
GMM-t	3.361	−2.326	1.120	2.225		2.324	2.180	2.797
INV	Const.	ΔNRIQ	ΔRIQ			ΔNRIQ	ΔRIQ	
Coeff.	0.739	−1.178	2.133		20.32%	−1.278	3.096	
t-value	0.824	−1.280	1.439		[1.74%, 57.49%]	−1.280	3.456	
Shanken-t	0.647	−1.011	1.140			−0.914	2.542	
GMM-t	0.585	−0.998	1.017			−0.163	0.287	
CAYCAPM	Const.	CAY	r	CAY·r		CAY	r	CAY·r
Coeff.	3.674	−1.389	−2.792	−0.002	44.52%	−1.613	1.986	0.089
t-value	4.337	−2.988	−2.180	−0.061	[6.19%, 70.06%]	−3.347	2.506	3.204
Shanken-t	2.858	−1.990	−1.547	−0.041		−1.674	1.713	1.624
GMM-t	3.260	−1.915	−1.808	−0.060		−0.143	0.134	0.313
CAYC-CAPM	Const.	CAY	g^c	CAY·g^c		CAY	g^c	CAY·g^c
Coeff.	3.286	−1.320	0.029	−0.004	45.11%	−0.581	0.764	0.001
t-value	4.647	−2.782	0.245	−1.291	[6.19%, 70.06%]	−0.806	4.301	0.674
Shanken-t	3.114	−1.883	0.168	−0.873		−0.393	2.057	0.327
GMM-t	3.363	−2.142	0.155	−1.179		−0.362	2.303	0.382
HABIT	Const.	$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$		$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$
Coeff.	2.256	0.063	0.002	0.003	27.67%	−0.319	0.749	0.004
t-value	2.815	0.343	0.015	2.694	[6.19%, 70.06%]	−0.829	3.687	1.620
Shanken-t	1.682	0.207	0.009	1.629		−0.395	1.735	0.768
GMM-t	2.521	0.269	0.008	1.516		−0.151	1.011	0.871

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1. Fit and Coefficients

For the 25 Fama-French portfolios in Table 2, the MCAPM yields a good fit, with adjusted R^2 of 64% and positive risk premia for market return and the real money growth rate—with the latter strongly significant, even using the Shanken (1992) t-statistic. The intercept implies a risk-adjusted return, alpha, that is insignificant and equals 0.62% per quarter. The MC-CAPM yields an R^2 of 58% and an insignificant intercept of −0.37% per quarter. Thus, both versions of the model perform similarly, as would be expected (given the model, which implies that both equations (8) and (9) are correct) if consumption growth is measured about as accurately as wealth growth/market return. In the MCAPM, the market risk premium is plausible, implying an annualized equity premium of around 5%. It becomes marginally significant only when the constant is set equal to its theoretical value of zero, in which case the annualized equity premium is close to 8%, also plausible. The money growth risk and consumption growth factors are not assets, so their risk premia do not have a predicted magnitude. The insignificant negative sign for the consumption growth risk premium in the MC-CAPM may occur because: i) the sign is theoretically ambiguous, i.e., the fact that real money growth and consumption growth are positively correlated (see Table 1) means that, in equation (8), the effect of higher money growth raises the adjusted marginal utility of consumption, offsetting (in part) the lower marginal utility caused by higher consumption growth, causing the net effect to be smaller (or even, theoretically, negative) and more difficult to identify empirically; ii) it reflects the deficiency of measured consumption growth as a proxy for theoretical consumption growth; or iii) important (conditioning) variables are omitted, in which case our model may be relevant but may not be the full story.

As shown by the 90% error bands suggested by LNS (2009) for the adjusted R^2s in Table 2, the R^2s for the MCAPM and the MC-CAPM are both large enough to reject the hypothesis that the model is useless. When we drop the market excess return and consumption growth from our MCAPM and MC-CAPM, respectively, we obtain the MAPM, which produces an R^2 of 42%. This is substantially lower than that of our models, yet outside the 90% error band for a one-factor model. The MAPM further has a significant risk premium on money growth and insignificant alpha.

The regular C-CAPM and CAPM both perform poorly, as is typically observed, with adjusted R^2 below 1% for the C-CAPM and below 10% for the CAPM, significant intercepts implying a risk-adjusted return per quarter of above 2% for the C-CAPM and above 3% for the CAPM, and insignificant or incorrect signs for the factors. The FF3 model has adjusted R^2 of 78% (outside the 90% error band) but has a risk-adjusted return of 3.83% per quarter, which is economically and statistically significant. The R^2s for each of the other models (INV, CAYCAPM, CAYC-CAPM, and HABIT) are well inside their error bands.

Figure 1 illustrates the fit for each of the models by showing the deviation of each test asset's return from that predicted by the model. For our models, the pricing errors are largest for the "35" and "15" portfolios (smaller size quintiles and largest book-to-market ratio value), which differs from the FF3 model and

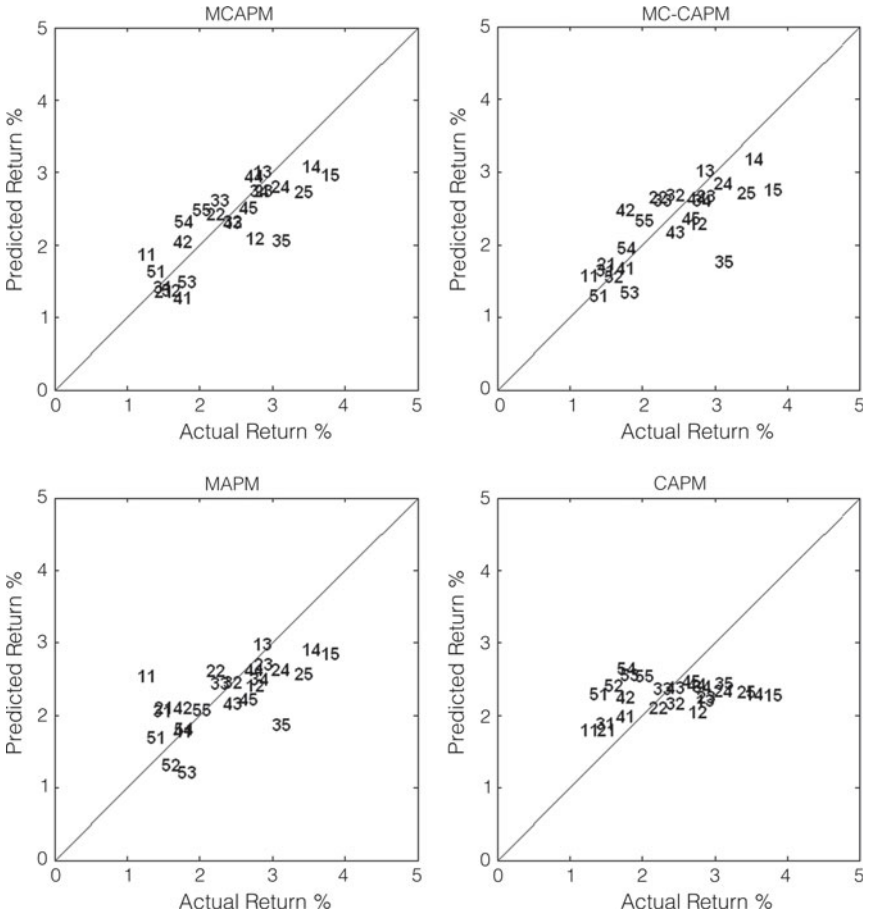
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most of the other models for which the pricing error of the "11" portfolio (smallest size and smallest book-to-market ratio growth) is largest.

FIGURE 1

Actual and Predicted Returns for 10 Linear Factor Models in the Case of the 25 Size- and Value-Sorted Test Assets

The actual predicted returns for the 10 linear factor models with the 25 Fama-French portfolios sorted by size and book-to-market ratio as the test assets. The models are, in order: MCAPM, MC-CAPM, MAPM, CAPM, C-CAPM, FF3, INV, CAY-CAPM, CAYC-CAPM, and HABIT. The two-digit numbers denote the Fama-French 25 portfolios. The first digit refers to the size quintile and the second digit refers to the book-to-market quintile.



(continued on next page)

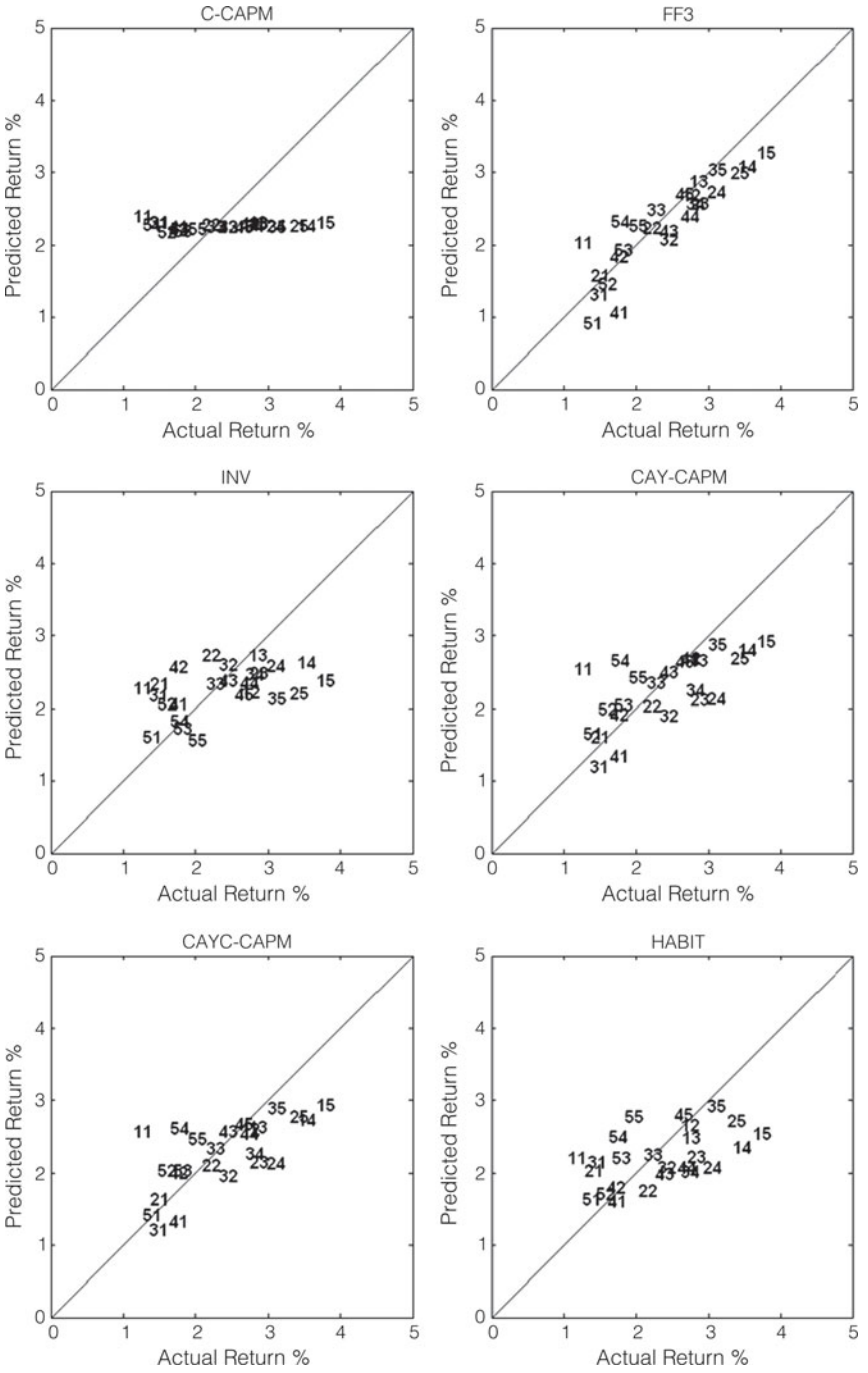
2. Alternative Test Assets

LNS (2009) suggest considering the 55 portfolios obtained by combining the 25 Fama-French portfolios with the 30 industry portfolios. Results are in Table 3. The adjusted R^2 for each of the models is now substantially lower. Only the R^2 for the MCAPM exceeds the LNS 90% error bands, although the R^2s of the MC-CAPM, FF3, and CAYCAPM models are very close to the upper bounds of their respective error bands. The alphas are positive and quite large for each of the

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FIGURE 1 (continued)

Actual and Predicted Returns for 10 Linear Factor Models in the Case of the 25 Size- and Value-Sorted Test Assets



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TABLE 3

Fama-MacBeth Estimates for 10 Models with 55 Industry Plus Size/Value Test Portfolios

The test assets are the 30 industry portfolios and 25 portfolios sorted by size and book-to-market ratio available from Kenneth French's Web site. The factors in each model (MCAPM, MC-CAPM, MAPM, CAPM, C-CAPM, FF3, INV, CAYCAPM, CAYC-CAPM, and HABIT) are, respectively, market return and money supply growth (r, g^m); consumption growth and money supply growth (g^c, g^m); money supply growth (g^m); market return (r); consumption growth (g^c); the Fama-French three factors (r, SMB, HML); Cochrane's (1996) investment growth for nonresidential and residential sectors (ΔNRIQ, ΔRIQ); Lettau-Ludvigson's market CAY (CAY, r, CAY·r) and consumption CAY (CAY, g^c, CAY·g^c) models; and the habit model ($g^c(t-1)$, g^c, $g^c(t-1)\cdot g^c$). For each, we report the risk premiums, t-statistics, Shanken's (1992) adjusted t-statistics, GMM t-statistics (based on five lags), and the adjusted R^2. Data are from 1959Q1 to 2004Q4. The confidence intervals are the error bands suggested by LNS (2009) under the assumption that the model is useless: the 90% interval around the median R^2 based on 5,000 draws from random multivariate normally distributed factors with the appropriate number of factors considered for each model. The results on the right side of Table 3 are obtained when the constant is restricted to its theoretical zero value.

		Constant Unrestricted				Constant Restricted to Zero		
MCAPM	Const.	r	g^m		Adjusted R^2	r	g^m	
Coeff.	1.614	0.270	0.842		30.22%	1.706	1.092	
t-value	2.046	0.270	3.052		[0.56%, 26.84%]	2.599	4.114	
Shanken-t	1.645	0.234	2.498			2.515	3.084	
GMM-t	1.900	0.294	1.973			2.866	2.312	
MC-CAPM	Const.	g^c	g^m			g^c	g^m	
Coeff.	0.933	−0.026	0.692		25.01%	0.032	1.090	
t-value	1.492	−0.224	2.366		[0.56%, 26.84%]	0.284	3.897	
Shanken-t	1.225	−0.186	1.970			0.204	2.810	
GMM-t	1.170	−0.228	2.087			0.014	1.215	
MAPM	Const.	g^m				g^m		
Coeff.	0.990	0.471			15.78%	2.075		
t-value	1.571	1.433			[0.02%, 17.88%]	2.998		
Shanken-t	1.458	1.336				1.534		
GMM-t	1.389	1.414				3.594		
CAPM	Const.	r				r		
Coeff.	2.080	−0.005			0.00%	1.891		
t-value	2.697	−0.005			[0.02%, 17.88%]	2.866		
Shanken-t	2.697	−0.005				2.860		
GMM-t	2.481	−0.005				3.132		
C-CAPM	Const.	g^c				g^c		
Coeff.	1.845	0.044			1.33%	0.361		
t-value	3.165	0.375			[0.02%, 17.88%]	2.863		
Shanken-t	3.149	0.374				2.244		
GMM-t	3.059	0.352				0.587		
FF3	Const.	r	SMB	HML		r	SMB	HML
Coeff.	2.234	−0.524	0.500	0.847	29.18%	1.681	0.466	1.114
t-value	2.476	−0.481	1.093	1.703	[2.00%, 31.85%]	2.626	1.017	2.177
Shanken-t	2.435	−0.475	1.091	1.697		2.622	1.012	2.146
GMM-t	2.092	−0.416	1.017	1.515		2.928	0.942	1.768
INV	Const.	ΔNRIQ	ΔRIQ			ΔNRIQ	ΔRIQ	
Coeff.	1.426	−0.161	0.698		5.23%	−0.415	2.176	
t-value	2.337	−0.305	0.704		[0.56%, 26.84%]	−0.733	2.666	
Shanken-t	2.298	−0.301	0.693			−0.647	2.376	
GMM-t	2.116	−0.254	0.699			−0.109	0.501	
CAYCAPM	Const.	CAY	r	CAY·r		CAY	r	CAY·r
Coeff.	1.276	−0.424	0.446	0.050	31.07%	−0.344	1.584	0.068
t-value	1.772	−1.341	0.461	2.564	[2.00%, 31.85%]	−1.116	2.460	3.231
Shanken-t	1.482	−1.136	0.413	2.192		−0.879	2.427	2.561
GMM-t	1.467	−1.284	0.494	2.088		−0.056	0.290	0.680
CAYC-CAPM	Const.	CAY	g^c	CAY·g^c		CAY	g^c	CAY·g^c
Coeff.	1.598	−0.255	0.137	0.001	15.21%	−0.265	0.508	0.002
t-value	2.656	−0.891	1.039	0.677	[2.00%, 31.85%]	−0.445	3.473	1.282
Shanken-t	2.358	−0.800	0.929	0.608		−0.283	2.220	0.823
GMM-t	2.600	−0.676	0.919	0.473		−0.297	2.017	0.641
HABIT	Const.	$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$		$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$
Coeff.	1.913	−0.114	0.038	−0.001	15.80%	0.017	0.202	−0.002
t-value	2.935	−0.964	0.366	−0.544	[2.00%, 31.85%]	0.133	2.131	−2.159
Shanken-t	2.794	−0.921	0.350	−0.519		0.087	1.413	−1.399
GMM-t	2.880	−0.841	0.282	−0.471		0.007	0.796	−0.650

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models but not significant for the MAPM, the MC-CAPM, the CAYCAPM, and (marginally) the MCAPM. The risk premia for the MCAPM are similar to those in the 25 Fama-French assets case: both are greatly significant in the version without constant and with similar magnitude for the market risk premium, but the significance is of somewhat smaller magnitude for the money growth risk premium. For the MC-CAPM, the coefficients have similar magnitudes, but the consumption growth risk premium is still insignificant. The FF3 model presents a bit of a coefficient instability problem in that the risk premia for the size and value factors are, respectively, a factor three and a factor two smaller than in the 25 Fama-French test assets case.

Ahn et al. (2009) construct portfolios by sorting on historical correlations of return. We use their 25 "cluster" portfolios as alternative test assets. Table 4 shows that only the MC-CAPM, the C-CAPM, and the FF3 models have an adjusted R^2 outside of the LNS (2009) error bands. Except for the MAPM, the alphas for all models are insignificant but economically quite high, although the MCAPM has an alpha of only 0.046%. Of all models' risk premia, only consumption growth and size are significant unless the constant is restricted to zero, in which case the money growth risk premium for the MCAPM and the market risk premium (for all models with a market return) are significant as well. The risk premia for the MCAPM are quite similar in magnitude compared to the 55 and 25 Fama-French asset cases, especially for the case without a constant. For the MC-CAPM, however, the coefficients vary substantially, to the extent that the money growth risk premium becomes negative for the 25 cluster assets. Generally, the risk premia for the other models are also less stable than those of the MCAPM.

The negative value for the money risk premium in the cluster portfolios case is troublesome for our model: not so much for the MCAPM version, in which case it is insignificant by each of the three t-statistics and turns significant with the expected positive sign (and similar size as in the other test cases) when the constant is set to its theoretical zero value, but in the MC-CAPM version, in which the money risk premium is significantly negative when the constant is set to zero for both the uncorrected and the GMM t-statistics. This result adds to the suspicion that measured consumption is a poor proxy for theoretical consumption. Given the strong correlation between money growth and the consumption growth proxy (see Table 1), the money growth risk premium estimate may be biased, especially when particular portfolio sorts magnify otherwise small pricing errors, as may be the case for the cluster portfolios.

3. GMM Distance Tests

An alternative evaluation method concerns the pricing errors based on the distance between each model's stochastic discount factor and a true stochastic discount factor, as given by the Hansen-Jagannathan (HJ) (1997) distance. This measure is obtained as the square root of the minimized GMM objective when we use the second moment matrix of asset returns to weight the moment conditions. The HJ distance is reported in Table 5 for each of the 10 models, together with p-values for the null-hypothesis that the distance is zero. None of the models survive the HJ test. Ahn and Gadarowski (2004) point out, however, that the

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TABLE 4

Fama-MacBeth Estimates for 10 Models with 25 Cluster Test Portfolios

The test assets are the 25 clustering portfolios from Ahn et al. (2009) available from Robert Dittmar's Web site. The factors in each model (MCAPM, MC-CAPM, MAPM, CAPM, C-CAPM, FF3, INV, CAYCAPM, CAYC-CAPM, and HABIT) are, respectively, market return and money supply growth (r, g^m); consumption growth and money supply growth (g^c, g^m); money supply growth (g^m); market return (r); consumption growth (g^c); the Fama-French three factors (r, SMB, HML); Cochrane's (1996) investment growth for nonresidential and residential sectors (ΔNRIQ, ΔRIQ); Lettau-Ludvigson's market CAY (CAY, r, CAY·r) and consumption CAY (CAY, g^c, CAY·g^c) models; and the habit model ($g^c(t-1)$, g^c, $g^c(t-1)\cdot g^c$). For each, we report the risk premiums, t-statistics, Shanken's (1992) adjusted t-statistics, GMM t-statistics (based on five lags), and the adjusted R^2. Data are from 1959Q1 to 2004Q4. The confidence intervals are the error bands suggested by LNS (2009) under the assumption that the model is useless: the 90% interval around the median R^2 based on 5,000 draws from random multivariate normally distributed factors with the appropriate number of factors considered for each model. The results on the right side of Table 4 are obtained when the constant is restricted to its theoretical zero value.

		Constant Unrestricted				Constant Restricted to Zero		
MCAPM	Const.	r	g^m		Adjusted R^2	r	g^m	
Coeff.	0.046	2.327	−0.433		11.07%	2.118	0.999	
t-value	0.034	1.518	−1.226		[0.49%, 22.35%]	3.133	2.798	
Shanken-t	0.030	1.369	−1.092			3.077	2.146	
GMM-t	0.026	1.259	−1.025			3.478	2.382	
MC-CAPM	Const.	g^c	g^m			g^c	g^m	
Coeff.	2.215	0.342	−0.707		36.10%	4.252	−1.303	
t-value	2.597	2.903	−1.947		[0.49%, 22.35%]	4.952	−3.749	
Shanken-t	1.575	1.807	−1.204			0.441	−0.333	
GMM-t	1.588	1.600	−1.253			3.488	−2.791	
MAPM	Const.	g^m				g^m		
Coeff.	2.708	−0.178			0.42%	2.345		
t-value	3.140	−0.526			[0.03%, 16.05%]	3.482		
Shanken-t	3.105	−0.520				1.631		
GMM-t	3.388	−0.504				3.966		
CAPM	Const.	r				r		
Coeff.	−0.313	2.660			7.51%	2.345		
t-value	−0.227	1.717			[0.03%, 16.05%]	3.482		
Shanken-t	−0.217	1.654				3.470		
GMM-t	−0.210	1.660				3.966		
C-CAPM	Const.	g^c				g^c		
Coeff.	0.586	0.340			19.01%	2.345		
t-value	0.767	2.890			[0.03%, 16.05%]	3.482		
Shanken-t	0.610	2.332				0.672		
GMM-t	0.646	1.842				3.966		
FF3	Const.	r	SMB	HML		r	SMB	HML
Coeff.	−0.477	1.622	4.350	0.271	49.84%	1.001	3.928	1.401
t-value	−0.313	1.018	4.023	0.261	[1.74%, 29.91%]	1.475	4.241	1.331
Shanken-t	−0.251	0.841	3.328	0.217		1.424	3.460	1.113
GMM-t	−0.250	0.815	3.245	0.233		1.513	2.820	0.994
INV	Const.	ΔNRIQ	ΔRIQ			ΔNRIQ	ΔRIQ	
Coeff.	1.229	1.042	1.492		8.68%	1.806	0.672	
t-value	1.298	1.790	1.285		[0.49%, 22.35%]	1.892	0.624	
Shanken-t	1.157	1.611	1.157			1.475	0.499	
GMM-t	1.184	1.591	1.169			1.953	0.639	
CAYCAPM	Const.	CAY	r	CAY·r		CAY	r	CAY·r
Coeff.	−0.704	−0.126	2.978	0.011	9.02%	−2.269	4.371	0.045
t-value	−0.524	−0.430	1.959	0.633	[1.74%, 29.91%]	−1.557	2.740	2.684
Shanken-t	−0.485	−0.401	1.838	0.594		−0.647	1.214	1.210
GMM-t	−0.488	−0.419	1.942	0.787		−1.233	2.390	3.153
CAYC-CAPM	Const.	CAY	g^c	CAY·g^c		CAY	g^c	CAY·g^c
Coeff.	0.784	−0.423	0.395	−0.002	22.70%	−0.747	0.673	0.000
t-value	0.986	−1.365	3.271	−1.279	[1.74%, 29.91%]	−0.954	5.109	−0.279
Shanken-t	0.721	−1.018	2.435	−0.959		−0.506	2.704	−0.155
GMM-t	0.801	−1.169	1.872	−1.058		−0.601	3.031	−0.164
HABIT	Const.	$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$		$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$
Coeff.	0.536	0.259	0.364	0.002	24.67%	−0.390	0.640	0.001
t-value	0.718	1.842	3.043	2.060	[1.74%, 29.91%]	−0.516	4.897	1.696
Shanken-t	0.518	1.347	2.237	1.506		−0.191	1.826	0.659
GMM-t	0.547	1.558	1.924	1.706		−0.356	2.716	0.996

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HJ distance causes the correct model to be rejected too often for commonly used sample sizes.

TABLE 5

Distance Tests for 10 Linear Factor Pricing Models with Three Sets of Test Portfolios and HJ and Identity Weighting Matrices

The test assets are the 25 Fama-French portfolios sorted by size and book-to-market ratio, the 55 Fama-French portfolios (30 industry portfolios plus 25 portfolios sorted by size and book-to-market ratio), and the 25 Ahn-Conrad-Dittmar clustering portfolios. The distances and their statistical significance are provided for each case for both the HJ weighting matrix (Hansen and Jagannathan (1997)) and the Identity weighting matrix (Lettau and Ludvigson (2001b)). The *p*-values for the weighted χ^2-statistic of Jagannathan and Wang (1996) to test for the significance of the distance measure (for both weighting matrices) are based on 10,000 simulations. The factors in each model (MCAPM, MC-CAPM, MAPM, CAPM, C-CAPM, FF3, INV, CAYCAPM, CAYC-CAPM, and HABIT) are, respectively, market return and money supply growth (r, g^m); consumption growth and money supply growth (g^c, g^m); money supply growth (g^m); market return (r); consumption growth (g^c); the Fama-French three factors (r, SMB, HML); Cochrane's (1996) investment growth for nonresidential and residential sectors (ΔNRIQ, ΔRIQ); Lettau-Ludvigson's market CAY (CAY, r, CAY·r) and consumption CAY (CAY, g^c, CAY·g^c) models; and the habit model ($g^c(t-1)$, g^c, $g^c(t-1)\cdot g^c$).

	MCAPM	MC-CAPM	MAPM	CAPM	C-CAPM	FF3	INV	CAYCAPM	CAYC-CAPM	HABIT
25FF										
HJ-Matrix	0.5836	0.5976	0.6156	0.5903	0.6086	0.5413	0.6132	0.5547	0.6050	0.6193
p-value	0.08	0.08	0.00	0.09	0.02	0.67	0.00	0.96	0.00	0.01
I-Matrix	0.0210	0.0230	0.0270	0.0325	0.0345	0.0160	0.0313	0.0268	0.0253	0.0293
p-value	23.53	41.37	28.35	0.12	0.08	1.18	2.16	0.28	1.41	1.19
55FF										
HJ-Matrix	1.0892	1.0961	1.0962	1.1042	1.1034	1.0760	1.0994	1.0985	1.0825	1.1065
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I-Matrix	0.0395	0.0412	0.0437	0.0471	0.0469	0.0395	0.0461	0.0394	0.0436	0.0463
p-value	24.87	20.18	9.53	2.76	3.09	3.68	2.66	8.62	1.29	2.03
25Clustering										
HJ-Matrix	0.5125	0.4882	0.5133	0.5181	0.5161	0.4282	0.5182	0.5025	0.5018	0.4914
p-value	0.83	24.91	1.77	0.36	0.65	30.62	0.29	0.43	0.64	6.20
I-Matrix	0.0542	0.0451	0.0560	0.0555	0.0516	0.0410	0.0544	0.0511	0.0503	0.0498
p-value	0.22	64.10	0.06	0.02	2.78	19.17	0.09	0.49	2.17	3.61

As recommended by Altonji and Segal (1996), Cochrane (1996), (2005), and Lettau and Ludvigson (2001b), we compute an alternative distance measure using the identity matrix instead of the HJ weighting matrix. The distance is then given by the square root of the minimized GMM objective using an identity weighting matrix. Table 5 now shows that for the 25 test portfolios, the models with the smallest distance are our two models (MCAPM and MC-CAPM) plus the Fama-French model. While the distance measures are not significantly different from zero for both of our models or for the MAPM, the Fama-French model is rejected based on the distance with identity weighting matrix. Lettau and Ludvigson (2001b) also report rejecting the Fama-French model based on this measure and point out that this may be in part because models with returns as factors have smaller sampling errors in the estimated betas. For the 55 test portfolios, similar results hold except that now the CAYCAPM model survives. For the 25 cluster portfolios, only the MC-CAPM and the FF3 model are not rejected for the identity weighting matrix. (For the HJ weighting matrix, the Habit model also is not rejected in the 25 cluster assets case.)

4. Model Comparisons

We conduct some direct model comparisons in Table 6 in order to see the marginal impact of the relevant risk factors more clearly and ascertain whether the money growth factor substitutes for well-known alternative factors. To this end we conduct J_t-difference tests (discussed in Cochrane (2005), p. 259). These

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TABLE 6

Head-to-Head Model Comparisons via J_t-Difference Tests

The J_t-difference statistic provides the increase in the J_t value (an aggregate measure of weighted pricing errors) when we exclude a factor (more generally, a set of factors) from an unrestricted model that consists of all factors of two models combined. The p-values are shown in percentages and give the probability that the omitted factor has zero impact on the pricing errors. Panel A of Table 6 compares market return, r, and consumption growth, g^c, with real money growth, g^m; Panel B compares real money growth with the two CAY models; Panel C compares real money growth, g^m, with lagged inflation, $\pi(t-1)$. Inflation data are constructed from the consumer price index for quarterly data from 1959Q1 to 2004Q4. We find lagged inflation by calculating a weighted average of past inflation rates using a half-life of three years to determine the weights following Cohen et al. ((2005), p. 652). We include all factors of the MCAPM or MC-CAPM and the Cohen et al. (2005) model combined: the excess market return, real money growth, and lagged inflation. We exclude, in turn, real money growth and lagged inflation. The "high inflation" regime includes all time periods during which lagged inflation exceeds the overall average inflation rate; the "low inflation" regime includes all time periods during which lagged inflation is below the overall average inflation rate.

	One-Stage				Two-Stage			
Panel A. The MCAPM and MC-CAPM versus Simpler Alternatives								
	Model							
	MCAPM		MC-CAPM		MCAPM		MC-CAPM	
	Var. Dropped							
	r	g^m	g^c	g^m	r	g^m	g^c	g^m
25FF	2.19	4.16	6.36	11.29	5.04	5.09	9.06	17.73
p-value	13.91	4.14	1.17	0.08	2.48	2.40	0.26	0.00
55FF	26.31	46.62	26.55	64.73	23.09	44.99	27.65	64.38
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25Clustering	3.18	8.66	15.63	20.71	1.79	10.08	15.17	21.41
p-value	7.44	0.32	0.01	0.00	18.04	0.15	0.01	0.00
Panel B. The MCAPM and MC-CAPM versus Two CAY Model Versions								
	Model							
	CAYCAPM + g^m		CAYC-CAPM + g^m		CAYCAPM + g^m		CAYC-CAPM + g^m	
	Var. Dropped							
	CAY + CAY·r	g^m	CAY + CAY·g^c	g^m	CAY + CAY·g^c	g^m	CAY + CAY·g^c	g^m
25FF	3.06	6.29	8.86	13.84	3.33	6.32	7.27	15.86
p-value	21.62	1.22	1.19	0.02	18.91	1.20	2.64	0.01
55FF	8.16	40.50	8.51	54.31	7.81	39.10	10.32	53.47
p-value	1.69	0.00	1.42	0.00	2.02	0.00	0.57	0.00
25Clustering	2.82	10.64	2.22	12.35	5.48	9.89	3.39	14.91
p-value	24.43	0.11	32.92	0.04	6.45	0.17	18.36	0.01
Panel C. The MCAPM and MC-CAPM versus the Money Illusion Model								
	Model							
	MCAPM + $\pi(t-1)$		MC-CAPM + $\pi(t-1)$		MCAPM + $\pi(t-1)$		MC-CAPM + $\pi(t-1)$	
	Var. Dropped							
	g^m	$\pi(t-1)$	g^m	$\pi(t-1)$	g^m	$\pi(t-1)$	g^m	$\pi(t-1)$
High Inflation								
25FF	33.88	0.86	22.28	0.05	36.29	1.56	24.61	0.00
p-value	0.00	35.33	0.00	81.60	0.00	21.16	0.00	98.18
55FF	144.58	3.64	183.34	2.56	138.64	2.67	180.21	2.69
p-value	0.00	5.63	0.00	10.95	0.00	10.21	0.00	10.11
25Clustering	1.39	0.08	0.05	0.12	1.62	0.01	0.24	0.09
p-value	23.85	77.13	81.95	72.56	20.27	92.43	62.36	76.53
Low Inflation								
25FF	51.03	1.42	40.04	1.41	49.35	0.61	35.66	2.23
p-value	0.00	23.30	0.00	23.57	0.00	43.34	0.00	13.51
55FF	14.10	10.79	9.25	9.76	12.50	11.09	8.20	10.19
p-value	0.02	0.10	0.24	0.18	0.04	0.09	0.42	0.14
25Clustering	12.14	2.95	13.09	3.15	15.52	1.61	14.73	2.14
p-value	0.05	8.56	0.03	7.59	0.01	20.44	0.01	14.37

tests measure the joint pricing errors of a combination of two models and use a χ^2 test to see if the pricing errors are significantly higher when one of the models (or a single factor) is dropped from the combination.

Panel A of Table 6 considers the MCAPM and MC-CAPM as the combination models from which, respectively, the market excess return or consumption growth factor is dropped to yield the MAPM and from which the money growth factor is dropped to yield, respectively, the CAPM or the C-CAPM. Dropping money growth significantly (p-values less than 5%) raises pricing errors in all cases: for the 25FF, 55FF, and 25Cluster test asset cases; for both the MCAPM and the MC-CAPM; and whether we employ an identity weighting matrix (one-stage) or GMM weighting matrix (two-stage). Dropping consumption growth from the MC-CAPM also raises pricing errors significantly in all three test asset cases for both one- and two-stage weighting matrices. Dropping the market factor raises pricing errors significantly for the 25FF asset case, the two-stage weighting matrix, and the 55FF asset case with both weighting matrices. For the 25Cluster assets, the increase in pricing errors is not quite significant. Thus both consumption growth and money growth are essential factors in the MC-CAPM, and the same is true, although not as obviously for all test assets, for the market factor and money growth in the MCAPM.

To determine if money growth substitutes for other known factors, we check the correlation between money growth and each of the other model factors. Except for the significant positive correlation shown in Table 1 between money growth and consumption growth or the market factor, money growth does not correlate significantly with other model factors. We also check specifically to see to what extent money growth contributes to both CAY models, which are the most prominent conditional versions of the CAPM and C-CAPM. Panel B of Table 6 shows the J_t-difference test results for combinations of the CAYCAPM (CAYC-CAPM) model with the MCAPM (MC-CAPM), or, put alternatively, combinations of the CAY models with the money growth factor. Dropping the money growth factor again raises pricing errors significantly in all cases. On the other hand, dropping all CAY model factors except the market factor from the CAYCAPM plus money growth (so the MCAPM remains) raises pricing errors as well, but not by as much as when money growth is dropped. The increase is strongly significant for the 55FF case. Similarly, dropping all CAY model factors except consumption growth from the CAYC-CAPM plus money growth (so the MC-CAPM remains) raises pricing errors significantly for both the 25FF and 55FF cases.

We conclude that, although money growth adds important explanatory power that is distinct from the CAY model factors, it does not drive out the CAY factors. Thus, our (unconditional) MCAPM and MC-CAPM models are not the full story: Accounting for the variation of factor sensitivities and risk premia over time with a common factor, such as the CAY variable, further increases the explanatory powers of both versions.

5. Explaining the Value and Size Premia

Since our models explain well the 25 Fama-French assets sorted by size and value, they provide, in principle, an explanation for the size and value effects. To examine the specific contributions of each factor, we consider the factor

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sensitivities. Table 7 presents the betas of both of our models together with the standard errors of the estimations. As is typical, the betas of the macro factors are estimated less precisely.

TABLE 7

Factor Sensitivities with Standard Errors and Contributions of Each Factor to Average Returns for the MCAPM and MC-CAPM Models

The betas, factor contributions to average returns, and standard errors of the betas for each of the 25 Fama-French portfolios sorted by size and value in Panel A of Table 7 for the MCAPM with market return and money supply growth (r, g^m) as the factors, and in Panel B for the MC-CAPM with consumption growth and money supply growth (g^c, g^m) as the factors.

Panel A. MCAPM

	r					g^m				
	Small		Medium		Big	Small		Medium		Big
Beta										
Growth	1.12	1.13	1.07	0.99	0.76	−0.07	−0.40	−0.32	−0.34	0.05
	0.93	0.88	0.84	0.78	0.67	0.21	0.46	0.43	0.30	−0.04
Blend	0.79	0.76	0.68	0.68	0.56	0.90	0.76	0.74	0.54	0.11
	0.72	0.70	0.65	0.62	0.48	1.00	0.84	0.84	0.99	0.70
Value	0.74	0.70	0.64	0.61	0.55	0.91	0.79	0.42	0.73	0.75
Return Contribution										
Growth	1.35	1.37	1.30	1.20	0.92	−0.11	−0.64	−0.51	−0.55	0.09
	1.12	1.06	1.01	0.95	0.81	0.34	0.73	0.68	0.47	−0.07
Blend	0.95	0.92	0.82	0.82	0.68	1.43	1.21	1.17	0.86	0.18
	0.87	0.84	0.79	0.75	0.58	1.58	1.34	1.33	1.57	1.11
Value	0.89	0.85	0.77	0.74	0.66	1.45	1.25	0.66	1.15	1.20
Beta Standard Error										
Growth	0.12	0.09	0.08	0.07	0.05	0.90	0.68	0.57	0.49	0.34
	0.11	0.07	0.06	0.06	0.05	0.78	0.53	0.46	0.43	0.37
Blend	0.09	0.07	0.06	0.06	0.05	0.64	0.50	0.44	0.42	0.39
	0.08	0.07	0.06	0.05	0.05	0.60	0.48	0.44	0.39	0.38
Value	0.09	0.08	0.07	0.07	0.07	0.65	0.58	0.54	0.49	0.48

Panel B. MC-CAPM

	g^c					g^m				
	Small		Medium		Big	Small		Medium		Big
Beta										
Growth	4.10	2.25	2.34	1.43	2.26	1.47	1.48	1.43	1.40	1.19
	1.58	1.19	0.59	0.10	0.27	1.80	2.02	2.01	1.85	1.26
Blend	1.31	1.41	0.76	1.13	0.59	2.26	2.04	1.97	1.71	1.14
	0.62	0.69	0.94	1.27	0.74	2.33	2.12	1.98	2.02	1.54
Value	1.63	0.92	1.44	0.78	0.44	2.11	2.04	1.45	1.81	1.77
Return Contribution										
Growth	−0.33	−0.18	−0.19	−0.12	−0.18	2.28	2.29	2.21	2.16	1.84
	−0.13	−0.10	−0.05	−0.01	−0.02	2.79	3.11	3.10	2.86	1.95
Blend	−0.11	−0.11	−0.06	−0.09	−0.05	3.49	3.15	3.04	2.64	1.76
	−0.05	−0.06	−0.08	−0.10	−0.06	3.59	3.27	3.06	3.12	2.38
Value	−0.13	−0.07	−0.12	−0.06	−0.04	3.26	3.15	2.25	2.80	2.74
Beta Standard Error										
Growth	3.10	2.63	2.33	2.09	1.53	1.16	0.99	0.87	0.78	0.58
	2.67	2.04	1.87	1.73	1.49	1.01	0.77	0.70	0.65	0.56
Blend	2.20	1.85	1.64	1.59	1.41	0.83	0.70	0.62	0.60	0.53
	2.05	1.76	1.63	1.47	1.32	0.77	0.66	0.61	0.55	0.50
Value	2.20	1.98	1.85	1.70	1.61	0.83	0.75	0.69	0.64	0.61

For the MCAPM, the market betas for small firms are a little higher than for large firms, explaining a small quarterly size premium of around 0.3%. The market betas are larger for growth firms than for value firms, explaining a negative value effect of around −0.4%. However, the differences in exposure to the money

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factor are around one for each size class. This accounts for a clear quarterly value risk premium of around 1.5%, robust across each of the five size classes. Small firms generally have somewhat higher exposure to money growth risk. The size premium is quite variable across the value classes but around 0.4% on average.

For the MC-CAPM, the differences between the consumption betas explain a small size premium of about 0.1% per quarter and a similar value risk premium of around 0.15%. The money growth factor accounts for a quarterly value premium of approximately 0.7% and for a quarterly size premium of close to 1.0%.

In our MCAPM, the explanation for the value premium is exclusively due to differences in sensitivity to the money growth factor: Growth firms have zero or very low sensitivity, but value firms have high sensitivity to money growth. Given the positive risk premium on money growth, value firms are riskier due to higher exposure to "liquidity risk." A plausible reason for the differences in sensitivity is that value firms are expected to generate much of their revenues currently. Higher real money balances lower the current cost of transactions, which benefits those firms that rely more on current trade and revenue—value firms. Growth firms are largely unaffected by current transaction costs, since most of their anticipated value hinges on future transactions.[10] The size premium in the MCAPM is explained in about equal parts by differences in market risk and money growth exposure. For the MC-CAPM, the value premium is explained similarly as for the MCAPM, but the size premium is almost fully due to differences in money growth exposure.

6. Characteristics

To test for the influence of "characteristics" (that is, nonrisk factors), we include the size and book-to-market variables as additional "factors" (characteristics) in the second pass, as suggested by Jagannathan and Wang (1998): We separately add a measure for size and book-to-market for the firms in each portfolio to the second-pass regressions. As seen in Table 8, each of the 10 models is rejected based on both of these added variables (except for the MC-CAPM and the MAPM, which are not rejected based on the size variable), with negative coefficients ("characteristics premia") on the size variable and positive coefficients on the book-to-market variable for all models. This includes also the FF3 model. The blanket rejection of all models along this dimension suggests the need for a further search for a specification that more fully absorbs the size and value effects, and implies again that our money-modified CAPM is not the full story (as, in the end, no model ever can be). In particular, the fact that our real money factor and the Fama-French factors independently add explanatory power suggests that

[10]The referee provides the interesting observation that our explanation may tie into the Bansal, Dittmar, and Lundblad (2005) perspective on the value premium. Bansal et al. find that the cash flows of value firms are more sensitive to consumption. From our perspective, the observed increase in cash flow sensitivity to consumption may reflect the impact of the omitted variable, money growth, which has significant positive correlation with consumption growth (see Table 1) and is more important for value firms, particularly because of its impact on current cash flows.

viewing the value and size factors as proxies for liquidity risk may only be part of the explanation for the importance of these factors.[11]

7. Summary and Discussion of the Results

On the whole, our MCAPM outperforms the competing models: It displays close to the highest adjusted R^2s and lowest pricing errors; the alphas are smallest, or close to it, and insignificant; and the risk premia are consistent with predictions and are relatively stable across the test asset groups. The MC-CAPM also performs well, but not as uniformly. Either model does well in direct comparisons with both simpler and more complex models, suggesting that our model factors are all necessary in the sense of adding significant explanatory power and are not driven out by known alternatives. Nevertheless, there are some troublesome failings, typically they are less severe for our models, but common to all models.

First, the adjusted R^2s for the 55 industry plus size/book-to-market assets and the 25 cluster asset cases are relatively low, around 30%–50% for the best models. The diminished fit in the 55 asset case relates to the difficulty in explaining a larger group of test assets: The need for two groups of test assets to have the same risk premia amplifies small errors in factor measurement. As a result, even the relatively low 30% R^2 for our MCAPM exceeds the error bound of 27% for the 55 test assets. The relatively poor fit for the 25 cluster assets is more puzzling. One reason is surely that the signal-to-noise ratio decreases when the factor structure diminishes.

Second, many of the models' risk premia are insignificant. Among our models, this is only an issue for the MC-CAPM, for which the consumption growth risk premium is insignificant and negative for two of the three test asset groups (and the money growth risk premium is negative and significant for one of the three test asset groups). However, the model predicts that the consumption growth risk premium can be either positive or negative. Even if the negative component does not dominate, it will lead to a smaller risk premium, which may not be picked up well empirically. In addition, of course, consumption growth measurement may be a problem, as has been pointed out repeatedly in the literature (most recently by Parker and Julliard (2005)), which could also affect the money risk premium, since the consumption proxy and money are strongly correlated. Lastly, the consumption growth risk premium may be biased if one omits important conditioning variables (such as CAY).

Third, all models display large and statistically significant alphas. Again, this is not a problem for our models, except that the MCAPM has an economically large (if barely insignificant) alpha in the 55 test asset case. Perhaps when

[11] Since the previous literature (e.g., Chen et al. (1986) and Flannery and Protopapadakis (2002)) has illustrated the importance of a variety of macro variables, we also considered various macro variables as factors, such as real GNP, the risk-free rate, inflation, and nominal money growth. Adding any of these factors to consumption growth or the market risk premium yields results that are significantly inferior to our benchmark results, with R^2 values varying from 15% to 48%. When we add both nominal money growth and inflation separately to the consumption growth or the market risk premium model, the risk premia on these factors are of opposite sign and similar in absolute value, as predicted by our model, but a formal test narrowly rejects the restriction that the absolute values are equal (p-value of 0.04 in both cases). These results are available from the authors.

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TABLE 8

The Marginal Importance of Size and Book-to-Market Ratio Characteristics

Panel A of Table 8 adds the size variable (SIZE), and Panel B adds the book-to-market ratio (BM) in the second pass for the 10 linear factor models. Test assets are the 25 portfolios sorted by size and book-to-market ratio available from Kenneth French's Web site. The factors in each model (MCAPM, MC-CAPM, MAPM, CAPM, C-CAPM, FF3, INV, CAYCAPM, CAYC-CAPM, and HABIT) are, respectively, market return and money supply growth (r, g^m); consumption growth and money supply growth (g^c, g^m); money supply growth (g^m); market return (r); consumption growth (g^c); the Fama-French three factors (r, SMB, HML); Cochrane's (1996) investment growth for nonresidential and residential sectors (ΔNRIQ, ΔRIQ); Lettau-Ludvigson's market CAY (CAY, r, CAY·r) and consumption CAY (CAY, g^c, CAY·g^c) models; and the habit model ($g^c(t-1)$, g^c, $g^c(t-1)\cdot g^c$). For each, we report the risk premiums, t-statistics, and the adjusted R^2. Data are from 1959Q1 to 2004Q4.

	Panel A. The Size Characteristic						*Panel B. The Value Characteristic*					
MCAPM	Const.	r	g^m	SIZE		R^2	Const.	r	g^m	BM		R^2
Coeff.	6.909	−3.242	0.060	−0.389		80.83%	0.359	2.423	0.859	0.895		79.47%
t-value	4.042	−2.419	0.187	−2.658			0.269	1.147	2.681	2.988		
MC-CAPM	Const.	g^c	g^m	SIZE			Const.	g^c	g^m	BM		
Coeff.	2.431	−0.322	0.755	−0.196		63.43%	0.489	0.131	1.054	0.683		78.92%
t-value	1.200	−2.510	1.638	−1.168			0.515	0.564	2.095	2.365		
MAPM	Const.	g^m	SIZE				Const.	g^m	BM			
Coeff.	−0.366	1.367	−0.014			42.01%	0.523	1.014	0.724			78.71%
t-value	−0.132	2.142	−0.065				0.512	1.685	2.060			
CAPM	Const.	r	SIZE				Const.	r	BM			
Coeff.	7.602	−3.754	−0.427			49.23%	0.942	2.321	1.316			72.39%
t-value	5.224	−2.552	−3.558				0.768	1.108	3.664			
C-CAPM	Const.	g^c	SIZE				Const.	g^c	BM			
Coeff.	6.479	−0.549	−0.418			19.01%	1.717	0.269	0.980			67.97%
t-value	5.338	−2.688	−3.575				2.311	1.045	3.573			
FF3	Const.	r	SMB	HML	SIZE		Const.	r	SMB	HML	BM	
Coeff.	7.009	−1.112	−1.592	1.241	−0.597	82.63%	3.778	−2.383	0.542	−0.042	0.776	80.13%
t-value	4.691	−0.716	−1.559	2.014	−3.111		4.009	−1.500	0.905	−0.044	2.122	
INV	Const.	ΔNRIQ	ΔRIQ	SIZE			Const.	ΔNRIQ	ΔRIQ	BM		
Coeff.	10.706	−2.002	−5.655	−0.723		58.12%	0.794	−0.186	2.442	0.935		81.71%
t-value	4.556	−2.080	−2.784	−4.122			0.883	−0.233	1.667	3.477		
CAYCAPM	Const.	CAY	r	CAY·r	SIZE		Const.	CAY	r	CAY·r	BM	
Coeff.	8.057	0.407	−3.105	0.014	−0.536	83.25%	1.003	−0.037	2.145	0.002	1.264	75.50%
t-value	6.338	1.217	−2.423	0.542	−4.821		1.126	−0.101	1.369	0.080	4.314	
CAYC-CAPM	Const.	CAY	g^c	CAY·g^c	SIZE		Const.	CAY	g^c	CAY·g^c	BM	
Coeff.	5.465	−0.156	−0.217	0.002	−0.376	70.87%	2.334	0.115	0.055	−0.003	1.477	22.70%
t-value	4.561	−0.431	−2.227	0.809	−3.280		3.393	0.269	0.468	−0.881	5.409	
HABIT	Const.	$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$	SIZE		Const.	$g^c(t-1)$	g^c	$g^c(t-1)\cdot g^c$	BM	
Coeff.	7.448	−0.406	−0.280	0.000	−0.501	72.60%	1.283	0.242	0.104	−0.002	1.536	81.73%
t-value	5.239	−2.176	−2.322	−0.486	−4.375		1.677	1.322	0.853	−1.419	4.865	

equally weighted industries are classified with hindsight, successful firms are overweighted. Further, industry portfolios display persistent unanticipated returns due to the fact that over time (and with hindsight) some industries decay and others emerge. The tobacco industry, which is a huge outlier for all models, exemplifies this issue.

Fourth, the risk premia estimates vary, sometimes substantially, across test assets. For instance, the consumption risk premium for each of the consumption-based models is generally small and often insignificant for the 25FF and 55FF cases but large and significant for the 25Cluster assets.

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Fifth, the size and book-to-market variables have significant additional explanatory value when added to all models as characteristics, including not only our models but even the Fama-French model, which includes size and book-to-market factors. This issue may also be part of the explanation for low R^2s.

D. An Alternative to Money Illusion

A paper by Cohen, Polk, and Vuolteenaho (2005) provides a competing theory of how money growth affects asset pricing. It considers the Modigliani-Cohn hypothesis that agents have a type of money illusion that causes stock market investors to incorrectly discount real cash flows with nominal discount rates. In the context of the CAPM, this money illusion implies that higher anticipated inflation is associated across the board with lower stock prices and accordingly higher mean returns. Because mean market returns increase as well, the implication is that stocks with betas below one (above one) have higher (lower) average returns than predicted by the CAPM. Thus,

$$r_t^i = \beta^i r_t + (1-\beta^i)\delta\,(\pi_{t-1} - \bar{\pi}) + \varepsilon_t, \tag{10}$$

where ε_t is a zero-mean error term and $0 < \delta \leq 1$ to reflect the possibilities of complete or partial money illusion. Observed lagged inflation π_{t-1} affects interest rates (discount rates) and matters relative to the long-run average $\bar{\pi}$ that shapes the public's perception of real cash flow growth. On the other hand, our model, restated here for the sake of comparison, rests on a rational explanation of why nominal money growth and inflation matter in addition to the market excess return: Real money growth is a second systematic factor in addition to excess market returns, as it affects the pricing kernel by changing the transaction-cost-adjusted marginal value of wealth:

$$r_t^i = \beta^i r_t + \gamma^i \left(g_t^m - a_{t-1}^m\right) + \varepsilon_t. \tag{11}$$

The variables in addition to excess market returns in both models, though different (lagged inflation vs. current real money growth), both represent an aggregate monetary factor and are statistically correlated: $\mathrm{corr}(g_t^m, \pi_{t-1}) = -0.47$.[12] The negative correlation stems from the facts that real money growth is the difference between nominal money growth and inflation, and that inflation is persistent over time. Hence it is possible that the Cohen et al. (2005) model improves on the CAPM empirically merely because it adds a variable that is correlated with our real money growth factor. Or, vice versa, it is also possible that our model improves on the CAPM because it adds a variable that is correlated with the Cohen et al. (2005) inflation factor.

The model comparison we used in the previous section to compare our model with others does not work for the Cohen et al. (2005) model. To see this, take unconditional expectations in equation (10). The inflation factor risk premium is

[12]For our measure of π_{t-1}, we employ a weighted moving average of past inflation rates using a half-life of three years to infer the weights (as in Cohen et al. (2005), p. 652). Actual lagged inflation produces a stronger negative correlation: $\mathrm{corr}(g_t^m, \pi_{t-1}) = -0.62$.

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zero on average, and the model predicts that lagged inflation has no explanatory power for *unconditional* returns. Instead, we take a conditional approach to compare the two models. We split the sample into three equal regimes: high, medium, and low inflation, and we exclude the medium inflation sample. The high inflation sample includes the one-third of periods t for which π_{t-1} is the highest, and the low inflation sample includes the one-third of periods t for which π_{t-1} is the lowest. In each regime the inflation risk premium $\pi_{t-1} - \bar{\pi}$ now is nonzero (positive for high inflation, negative for low inflation), so the Cohen et al. (2005) model has the definite implication that each asset's inflation beta, $1 - \beta^i$, affects its mean return for the regime (i.e., within each regime, inflation should help explain the cross section of mean returns).

As for the unconditional model comparisons in Section III.C.4, we nest both models here by creating a three-factor model with excess market return, lagged inflation, and real money growth as the factors (i.e., adding lagged inflation to the MCAPM, or, equivalently, adding real money growth to the Cohen et al. (2005) model). We then check in turn whether dropping the real money growth factor or the inflation factor significantly affects the model's pricing errors. To this end, we again use the J_t-difference test. Table 6, Panel C displays the results. In the high inflation regime we find that removing our money growth factor significantly increases pricing errors for the 25 Fama-French assets, with a p-value of 0.0%; while removing the Cohen et al. (2005) inflation factor has a much smaller and insignificant effect on pricing errors, with a p-value of 35.3%. Similarly, in the low inflation regime, removing the money growth factor increases pricing errors significantly, with a p-value of 0.0%, whereas removing inflation has an insignificant effect (p-value of 23.3%). Generally similar results hold for the other test assets (55FF and 25Cluster), whether we use one-stage or two-stage weights for the pricing errors and whether we consider the MCAPM or the MC-CAPM. (The exceptions are that for the 25Cluster assets in the high inflation regime, dropping real money growth does not increase pricing errors significantly, although it has more effect than dropping inflation; and for the 55FF assets in the low inflation regime, dropping inflation has a significant effect on pricing errors.)

It appears that once we account for real money growth as prescribed by our MCAPM or MC-CAPM, lagged inflation has no marginal explanatory power for the cross section of returns. But when we include lagged inflation as a possible explanatory variable, adding real money growth significantly reduces cross-sectional pricing errors. What looks to be the effect of money illusion may in fact be the equilibrium asset pricing impact of money growth on the transaction-cost-adjusted marginal value of wealth.

IV. Conclusion

The promise of the consumption-based asset pricing model of Breeden (1979), that a consumption growth factor alone is adequate for pricing any asset, is based on the fact that the consumption level is a sufficient statistic for capturing marginal utility of the representative consumer, or any consumer in complete financial

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markets. However, monetary theory suggests that a further factor is relevant: Liquidity, as determined by the real quantity of money balances, affects the cost of acquiring consumption goods and thus affects the transaction-cost-adjusted marginal utility of consumption. A generalized version of Stein's (1973), (1981) lemma permits the derivation of a two-factor model, the MCAPM, or, using the envelope condition, an alternative two-factor model, the MC-CAPM. This two-factor model, in both versions, is subjected to the data.

We evaluate our two models and eight competing models by considering their implications for cross-sectional differences in unconditional mean returns. As Lewellen, Nagel, and Shanken (2009) argue, judging merely the model's point estimate for the fit is too weak a criterion when the test assets have an approximate factor structure. Accordingly, we include in our evaluation: other test assets that are unlikely to have much of a factor structure, error bands for the fit measure, risk premium implications (including zero alphas), the importance of alternative variables, and the stability of risk premia estimates across test asset groups.

Although each of the models has some deficiencies, our two versions of the model on balance clearly outperform the alternatives. For each of the three groups of test assets, our models are close to the best in terms of fit and pricing errors. Risk premia generally have the predicted signs and magnitudes; in particular, the alphas are insignificant for our models in all three of the test asset groups. The risk premia in both our models are of similar magnitude across the test asset groups. Each of the risk factors, furthermore, adds significant explanatory power and cannot be driven out by standard alternative factors. On the other hand, a specification test that includes size or book-to-market ratios as characteristics in addition to our theoretical factors suggests shortcomings in our models, as well as in all other models considered here (including the FF3 model). Possibly related to this issue, R^2 for the test asset groups with little factor structure is quite low for all models. Additional research is needed to propose an alternative specification that better clarifies the remaining role of the size and book-to-market variables.

Our model shows further strength in that it is a purely unconditional model. Accordingly, the empirical results are not subject to the Lewellen and Nagel (2006) critique that argues for a further evaluation criterion related to the magnitudes of the risk premia for interaction variables arising in conditional models, a criterion that most of the recent generation of asset pricing models clearly fails to meet. Our model also contributes in that we are able to show that our real money growth variable, based on traditional risk theory, dominates the inflation variable of Cohen, Polk, and Vuolteenaho (2005), whose theory relies on money illusion.

A different contribution arises from the fact that the money growth factor in our MCAPM and MC-CAPM versions can be controlled by policy, which is not so for standard alternative factors. The model thus introduces an additional dimension that may be relevant for monetary policy discussions. While stabilization of the *level* of asset values by central banks is a major issue (see, for instance, Bernanke and Gertler (2001)), we find that monetary shocks may also substantially impact *relative* asset values: Monetary growth shocks affect relative asset prices by raising the realized gap between returns on value and growth stocks.

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In summary, both of our model versions perform well and generally outperform existing models in explaining three groups of test assets: the cross section of the 25 Fama and French portfolio returns of firms sorted by size and book-to-market ratio, the 55 portfolios (30 industry portfolios added to the 25 portfolios as suggested by LNS (2009)), and the 25 cluster portfolios of Ahn, Conrad, and Dittmar (2009). Thus, a logical extension to Breeden's model appears to provide results that avoid most of the puzzles of the original C-CAPM. That such an extension was not attempted previously may be somewhat surprising but is at least in part due to the difficulty of dealing with the cash-in-advance lag in a continuous time framework. As such, the extension of Stein's lemma employed here was an important ingredient in obtaining a theoretical solution.

Appendix A. Deriving Multifactor Beta Expressions in Discrete Time

The first-order conditions for the consumption and portfolio choices derived from (1)–(4) are

$$u_c(c_t) = \delta[1 + T_c(c_t, m_t)]E_t[R_{t+1}V_w(w_{t+1}, m_{t+1}, x_{t+1})] \quad \text{and} \tag{A-1}$$

$$E_t[R^i_{t+1}V_w(w_{t+1}, m_{t+1}, x_{t+1})] = R^0_{t+1}E_t[V_w(w_{t+1}, m_{t+1}, x_{t+1})], \quad \forall\, i. \tag{A-2}$$

The envelope condition for the wealth state variable is

$$V_w(w_t, m_t, x_t) = \delta E_t[R_{t+1}V_w(w_{t+1}, m_{t+1}, x_{t+1})]. \tag{A-3}$$

Combining (A-1) and (A-3) yields equation (6) in the text. Aggregate (A-2) multiplied by the portfolio shares over all i and use (A-3) to obtain

$$V_w(w_t, m_t, x_t) = \delta E_t[R^i_{t+1}V_w(w_{t+1}, m_{t+1}, x_{t+1})], \quad \forall\, i. \tag{A-4}$$

Apply (A-4) for generic asset i and for asset 0 and subtract to produce equation (5).

The discrete time formulation with conditionally normal distributions requires Stein's extended lemma stated in (B-2), which has not appeared elsewhere in the finance or statistics literature. Applying to the trivariate normal case at hand:

Multivariate Extension of Stein's Lemma. If X, Y, and Z are multivariate normal and $h()$ is differentiable, with $E|h_i(Y, Z)| < \infty$ for $i = \{Y, Z\}$, then

$$\text{Cov}[X, h(Y, Z)] = E[h_Y(Y, Z)]\text{Cov}(X, Y) + E[h_Z(Y, Z)]\text{Cov}(X, Z). \tag{B-2′}$$

Proof. See Appendix B.

The derivation of the asset pricing equations now extends a standard approach for deriving single-factor models (see, for instance, Huang and Litzenberger (1988), pp. 100–101) to apply to multifactor models. Apply the definition of covariance to equation (5) and set $V(w_t, m_t, x_t) = V(w_t, m_t)$, as argued in the text, to dispense with the Merton (1973) variables:

$$E_t(r^i_{t+1})E_t[V_w(w_{t+1}, m_{t+1})] = -\text{Cov}_t[V_w(w_{t+1}, m_{t+1}), r^i_{t+1}], \quad \forall\, i. \tag{A-5}$$

Apply the multivariate version of Stein's lemma in (B-6′) to (A-5), noting from equation (2) and our normality assumptions that w_{t+1} is conditionally normal, to produce

$$E_t(r^i_{t+1}) = A_t\text{Cov}_t(w_{t+1}, r^i_{t+1}) + B_t\text{Cov}_t(m_{t+1}, r^i_{t+1}), \quad \forall\, i \tag{A-6}$$

$$\text{with} \quad A_t \equiv \frac{-E_t[V_{ww}(w_{t+1}, m_{t+1})]}{E_t[V_w(w_{t+1}, m_{t+1})]}, \quad B_t \equiv \frac{-E_t[V_{mw}(w_{t+1}, m_{t+1})]}{E_t[V_w(w_{t+1}, m_{t+1})]}. \tag{A-7}$$

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Double subscripts indicate second derivatives. It can be shown that $V_{ww}(w,m) < 0$ (confirming risk aversion) and that the sign of $V_{mw}(w,m)$ is ambiguous.[13] Thus $A_t > 0$ and $B_t \gtreqless 0$.

To derive equation (7), assume (with little loss of generality) that an asset position m can be created that is correlated with money growth but uncorrelated with the market return. Scaling this position can generate an excess return of[14]

$$r^m_{t+1} = g^m_{t+1} - a^m_t + \eta^m_{t+1}, \tag{A-8}$$

where $g^m_{t+1} \equiv m_{t+1}/m_t$; η^m_{t+1} has zero mean and is uncorrelated with both the market return and money growth; and a^m_t is known at time t. Relating the covariances in equation (A-6) to the market excess return r_{t+1} using equation (2) and to the real money supply growth rate g^m_{t+1} gives

$$E_t(r^i_{t+1}) = A_t\bar{w}_t\mathrm{Cov}_t(r_{t+1}, r^i_{t+1}) + B_t m_t \mathrm{Cov}_t(g^m_{t+1}, r^i_{t+1}), \quad \forall\, i, \tag{A-9}$$

where $\bar{w}_t \equiv w_t + m_t - c_t - T(c_t, m_t) - \mu_t$.

Expressions for the risk premia are found by applying (A-9) to the market asset and asset m:

$$E_t(r_{t+1}) = A_t\bar{w}_t\mathrm{Var}_t(r_{t+1}) + B_t m_t \mathrm{Cov}_t(g^m_{t+1}, r_{t+1}), \tag{A-10}$$

$$E_t(g^m_{t+1}) - a^m_t = A_t\bar{w}_t\mathrm{Cov}_t(r_{t+1}, g^m_{t+1}) + B_t m_t \mathrm{Var}_t(g^m_{t+1}), \tag{A-11}$$

where, from equation (2), $\bar{w}_t \equiv w_t + m_t - c_t - T(c_t, m_t) - \mu_t$ is investable wealth. Solve for A_t and B_t from (A-10) and (A-11):

$$\begin{pmatrix} A_t\bar{w}_t \\ B_t m_t \end{pmatrix} = \begin{pmatrix} \mathrm{Var}_t(r_{t+1}) & \mathrm{Cov}_t(r_{t+1}, g^m_{t+1}) \\ \mathrm{Cov}_t(g^m_{t+1}, r_{t+1}) & \mathrm{Var}_t(g^m_{t+1}) \end{pmatrix}^{-1} \begin{pmatrix} E_t r_{t+1} \\ E_t g^m_{t+1} - a^m_t \end{pmatrix}, \tag{A-12}$$

and plug into (A-9) to obtain equation (7) with

$$\beta^i_t = \frac{\mathrm{Var}_t(g^m_{t+1})\mathrm{Cov}_t(r^i_{t+1}, r_{t+1}) - \mathrm{Cov}_t(g^m_{t+1}, r_{t+1})\mathrm{Cov}_t(g^m_{t+1}, r^i_{t+1})}{\mathrm{Var}_t(g^m_{t+1})\mathrm{Var}_t(r_{t+1}) - [\mathrm{Cov}_t(g^m_{t+1}, r_{t+1})]^2} \quad \text{and} \tag{A-13}$$

$$\gamma^i_t = \frac{\mathrm{Var}_t(r_{t+1})\mathrm{Cov}_t(r^i_{t+1}, g^m_{t+1}) - \mathrm{Cov}_t(r_{t+1}, g^m_{t+1})\mathrm{Cov}_t(r_{t+1}, r^i_{t+1})}{\mathrm{Var}_t(g^m_{t+1})\mathrm{Var}_t(r_{t+1}) - [\mathrm{Cov}_t(g^m_{t+1}, r_{t+1})]^2}. \tag{A-14}$$

The derivation of the MC-CAPM expression leading to equation (8) is completely analogous and is omitted.

[13] The sign of $V_{mw}(w,m)$ can be obtained by differentiating $V_w(w,m)$ in equation (6) with respect to m, using the fact that optimal consumption is determined by the state variables, $c = c(w,m)$. The direct effect of higher money holdings is to *raise* the marginal value of wealth as the marginal transaction cost falls (given $T_{cm}(c,m) < 0$). However, higher money holdings also have a price effect in that consumption becomes cheaper due to lower transaction cost. This *lowers* the marginal value of wealth as the higher optimal consumption level reduces the marginal utility of consumption and increases the marginal transaction cost.

[14] The return of some asset j can always be decomposed as $r^j_{t+1} = a^0_t + a^1_t g^m_{t+1} + a^2_t r_{t+1} + \eta^j_{t+1}$. Hold asset j, short a^2_t units of the market asset, and short the risk-free asset to create a zero-investment position. Then scale this position to generate unit exposure to money growth. This creates asset m with $E_t[r^m_{t+1} V_w(w_{t+1}, m_{t+1})] = 0$ and return as in (A-8).

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Appendix B. A Multivariate Extension of Stein's Lemma

In the finance literature, Stein's (1973), (1981) lemma is taken to be the statement that $\text{Cov}[Y, h(X)] = E[h_X(X)]\sigma^{XY}$ when X and Y are bivariate normal (see, for instance, Huang and Litzenberger (1988), pp. 101 and 116, Cochrane (2005), p. 163). The lemma was first used and proven indirectly in the finance literature by Rubinstein (1973) using exact Taylor expansions. Ingersoll ((1987), pp. 13–14) proves this result directly without naming it Stein's lemma. The result is generally attributed to Stein (1973), (1981).

Below we prove a multivariate generalization that to our knowledge has not been proven or used elsewhere: $\text{Cov}[Y, h(\mathbf{X})] = \sum_{i=1}^{n} E[h_{X_i}(\mathbf{X})]\sigma^{X_i Y}$, where $\mathbf{X}$ represents a vector of multivariate normal variables. This form of Stein's lemma is of great use in deriving multifactor asset pricing models as suggested by this paper's model. We first state the existing result:

Stein's (1981) Lemma 2. Assume: i) a random variable Y and random n-vector $\boldsymbol{\varepsilon}$ such that Y and $\boldsymbol{\varepsilon}$ are independent multivariate normal, and ii) the partial derivative $g_Y(Y, \boldsymbol{\varepsilon})$ exists and has $E|g_Y(Y, \boldsymbol{\varepsilon})| < \infty$. Then

$$\text{(B-1)} \qquad \text{Cov}[Y, g(Y, \boldsymbol{\varepsilon})] \quad = \quad E[g_Y(Y, \boldsymbol{\varepsilon})](\sigma^Y)^2.$$

Next we extend the lemma to allow for dependence among the random variables:

Stein's Extended Lemma. Assume: i) a random variable Y and random n-vector $\mathbf{X}$ such that Y and $\mathbf{X}$ are multivariate normal, and ii) the n-vector $h_{\mathbf{X}}(\mathbf{X})$ of partial derivatives exists with $E||h_{\mathbf{X}}(\mathbf{X})|| < \infty$. Then

$$\text{(B-2)} \qquad \text{Cov}[Y, h(\mathbf{X})] \quad = \quad E[h_{\mathbf{X}}(\mathbf{X})]' \sigma^{\mathbf{X}Y}.$$

Proof. It is always possible to find coefficients $\mathbf{b}$ such that

$$\text{(B-3)} \quad \mathbf{X} \quad = \quad \mathbf{b}Y + \boldsymbol{\varepsilon}, \quad \text{with} \quad \text{Cov}(Y, \boldsymbol{\varepsilon}) = 0 \quad \text{and} \quad \mathbf{b} \quad = \quad \text{Cov}(Y, \mathbf{X})/(\sigma^Y)^2.$$

Then

$$\text{(B-4)} \qquad \begin{aligned} \text{Cov}[Y, h(\mathbf{X})] \quad &= \quad \int_{\mathbf{X}} \int_{Y} h(\mathbf{x})(y - \mu^Y) f(y, \mathbf{x}) dy\, d\mathbf{x} \\ &= \quad \int_{\boldsymbol{\varepsilon}} \int_{Y} h(\mathbf{b}y + \mathbf{e})(y - \mu^Y) f(y, \mathbf{b}y + \mathbf{e}) dy\, d\boldsymbol{\varepsilon}. \end{aligned}$$

Set $g(Y, \boldsymbol{\varepsilon}) \equiv h(\mathbf{b}Y + \boldsymbol{\varepsilon})$, so that $g_Y(Y, \boldsymbol{\varepsilon}) = h_{\mathbf{X}}(\mathbf{b}Y + \boldsymbol{\varepsilon})'\mathbf{b}$. Using this, we can apply Stein's lemma to (B-4) to yield

$$\text{(B-5)} \qquad \text{Cov}[Y, h(\mathbf{X})] \quad = \quad E[g_Y(Y, \boldsymbol{\varepsilon})](\sigma^Y)^2 \quad = \quad E[h_{\mathbf{X}}(\mathbf{X})]' \sigma^{\mathbf{X}Y}.$$

To complete the proof, note that $E||h_{\mathbf{X}}(\mathbf{X})|| < \infty$ implies $E|h_{\mathbf{X}}(\mathbf{X})| < \infty$ and $E|h_{\mathbf{X}}(\mathbf{X})'\mathbf{b}| < \infty$ so that $E|g_Y(Y, \boldsymbol{\varepsilon})| < \infty$, which was needed to apply Stein's lemma to (B-4). □

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