Proc. Camb. Phil. Soc. (1967), **63**, 1049
PCPS 63–121
Printed in Great Britain

Some integrals involving Gauss's hypergeometric function and Meijer's G-function

BY S. D. BAJPAI
Shri G. S. Technological Institute, Indore, India

(*Received* 23 *May* 1966)

1. *Introduction.* The object of this paper is to evaluate some integrals involving the product of Gauss's hypergeometric function and Meijer's G-function by expressing the G-function as a Mellin–Barnes type integral and interchanging the order of integrations. The integrals are important because on specializing the parameters they lead to many results for MacRobert's E-function, Bessel, Legendre, Whittaker functions and other related functions. In what follows δ is a positive integer and $\Delta(\delta, \alpha)$ represents the set of parameters

$$\frac{\alpha}{\delta}, \frac{\alpha+1}{\delta}, \ldots, \frac{\alpha+\delta-1}{\delta}.$$

2. The following formulae have been established

$$\int_0^1 x^{\gamma-1}(1-x)^{\rho-1} e^{-zx}\, {}_2F_1(\alpha, \beta; \gamma; x)\, G_{p,q}^{m,n}\left(\zeta(1-x)^{\delta/t}\middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx$$
$$= (2\pi)^{(1-t)(m+n-\frac{1}{2}p-\frac{1}{2}q)}\, t^{\Sigma b_q - \Sigma a_p + \frac{1}{2}p - \frac{1}{2}q + 1}\, \delta^{-\gamma}\, e^{-z}\, \Gamma(\gamma)$$
$$\times \sum_{r=0}^{\infty} \frac{z^r}{r!}\, G_{lp+2\delta, lq+2\delta}^{tm, tn+2\delta}$$
$$\times \left(\zeta^t t^{t(p-q)} \middle| \begin{matrix} \Delta(\delta, 1-\rho-r), \Delta(\delta, 1+\alpha+\beta-\gamma-\rho-r), \Delta(t, a_1), \ldots, \Delta(t, a_p) \\ \Delta(t, b_1), \ldots, \Delta(t, b_q), \Delta(\delta, 1+\alpha-\gamma-\rho-r), \Delta(\delta, 1+\beta-\gamma-\rho-r) \end{matrix}\right), \tag{2·1}$$

where δ and t are positive integers, and

$$p+q < 2(m+n) \quad |\arg\zeta| < (m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi, \quad \Re(\gamma) > 0,$$
$$\Re\left(\rho + \frac{\delta}{t} b_j\right) > 0, \quad \Re\left(\gamma+\rho-\alpha-\beta+\frac{\delta}{t} b_j\right) > 0 \quad (j = 1, 2, \ldots, m).$$

$$\int_0^\infty x^{\gamma-1}(x+z)^{-\sigma}\, {}_2F_1(\alpha, \beta; \gamma; -x)\, G_{p,q}^{m,n}\left(\zeta(z+x)^{\delta/t}\middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right) dx$$
$$= (2\pi)^{(1-t)(m+n-\frac{1}{2}p-\frac{1}{2}q)}\, t^{\Sigma b_q - \Sigma a_p + \frac{1}{2}p - \frac{1}{2}q + 1}\, \delta^{-\gamma}\, \Gamma(\gamma)$$
$$\times \sum_{r=0}^{\infty} \frac{\delta^r (1-z)^r}{r!}\, G_{lp+2\delta, lq+2\delta}^{tm+2\delta, tn}$$
$$\times \left(\zeta^t t^{t(p-q)} \middle| \begin{matrix} \Delta(t, a_1), \ldots, \Delta(t, a_p), \Delta(\delta, \sigma), \Delta(\delta, \alpha+\beta-\gamma+\sigma+r) \\ \Delta(\delta, \alpha-\gamma+\sigma+r), \Delta(\delta, \beta-\gamma+\sigma+r), \Delta(t, b_1), \ldots, \Delta(t, b_q) \end{matrix}\right), \tag{2·2}$$

where δ and t are positive integers, and

$$p+q < 2(m+n), \quad |\arg\zeta| < (m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi, \quad \Re(\gamma) > 0, \quad |\arg z| < \pi,$$
$$\Re\left(\alpha-\gamma+\sigma-\frac{\delta}{t} a_j\right) > -\frac{\delta}{t}, \quad \Re\left(\beta-\gamma+\sigma-\frac{\delta}{t} a_j\right) > -\frac{\delta}{t} \quad (j = 1, 2, \ldots, n).$$

https://doi.org/10.1017/S0305004100042109 Published online by Cambridge University Press

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha,\beta;\gamma;x)\,G_{p,q}^{m,n}\left(zx^{\delta/t}\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)dx$$
$$= (2\pi)^{(1-t)(m+n-\frac{1}{2}p-\frac{1}{2}q)}\,t^{\Sigma b_q-\Sigma a_p+\frac{1}{2}p-\frac{1}{2}q+1}\,\delta^{-\sigma}\,\Gamma(\sigma)$$
$$\times\sum_{r=0}^{\infty}\frac{(\alpha)_r(\beta)_r}{(\gamma)_r r!}\,G_{tp+\delta,tq+\delta}^{tm,tn+\delta}\left(z^t t^{t(p-q)}\middle|\begin{matrix}\Delta(\delta,1-\rho-r),\Delta(t,a_1),\ldots,\Delta(t,a_p)\\ \Delta(t,b_1),\ldots,\Delta(t,b_q),\Delta(\delta,1-\sigma-\rho-r)\end{matrix}\right),\quad(2\cdot3)$$

where δ and t are positive integers, and

$$p+q<2(m+n),\quad |\arg z|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi\quad \Re(\sigma)>0,$$
$$\Re(\gamma+\sigma-\alpha-\beta)>0,\quad \Re\left(\rho+\frac{\delta}{t}b_j\right)>0\quad (j=1,2,\ldots,m).$$

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha,\beta;\gamma;x)\,G_{p,q}^{m,n}\left(zx^{\delta/t}(1-x)^{-\delta/t}\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)dx$$
$$= (2\pi)^{(1-t)(m+n-\frac{1}{2}p-\frac{1}{2}q)+1-\delta}\,t^{\Sigma b_q-\Sigma a_p+\frac{1}{2}p-\frac{1}{2}q+1}\,\delta^{\sigma+\rho-1}\times\frac{1}{\Gamma(\rho+\sigma)}$$
$$\times\sum_{r=0}^{\infty}\frac{(\alpha)_r(\beta)_r\delta^r}{(\gamma)_r(\rho+\sigma)_r r!}\,G_{tp+\delta,tq+\delta}^{tm+\delta,tn+\delta}\left(z^t t^{t(p-q)}\middle|\begin{matrix}\Delta(\delta,1-\rho-r),\Delta(t,a_1),\ldots,\Delta(t,a_p)\\ \Delta(\delta,\sigma),\Delta(t,b_1),\ldots,\Delta(t,b_q)\end{matrix}\right),\quad(2\cdot4)$$

where δ and t are positive integers, and

$$p+q<2(m+n),\quad |\arg z|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi,\quad \Re(\rho+(\delta/t)b_j)>0\quad (j=1,2,\ldots,m),$$
$$\Re\left(\sigma-\frac{\delta}{t}a_j\right)>-\frac{\delta}{t},\quad \Re\left(\gamma+\sigma-\alpha-\beta-\frac{\delta}{t}a_j\right)>-\frac{\delta}{t},\quad (j=1,2,\ldots,n).$$

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha,\beta;\gamma;x)\,G_{p,q}^{m,n}\left(zx^{\delta}(1-x)^{\lambda}\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)dx$$
$$= (2\pi)^{\frac{1}{2}}\lambda^{\sigma-\frac{1}{2}}\delta^{\rho-\frac{1}{2}}(\lambda+\delta)^{\frac{1}{2}-\rho-\sigma}$$
$$\times\sum_{r=0}^{\infty}\frac{(\alpha)_r(\beta)_r\delta^r}{(\gamma)_r r!\,(\lambda+\delta)^r}\,G_{p+\lambda+\delta,q+\lambda+\delta}^{m,n+\lambda+\delta}\left(\frac{z\lambda^\lambda\delta^\delta}{(\lambda+\delta)^{\lambda+\delta}}\middle|\begin{matrix}\Delta(\lambda,1-\sigma),\Delta(\delta,1-\rho-r),a_1,\ldots,a_p\\ b_1,\ldots,b_q,\Delta(\lambda+\delta,1-\sigma-\rho-r)\end{matrix}\right),\quad(2\cdot5)$$

where δ and λ are positive integers, and

$$p+q<2(m+n),\quad |\arg z|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi,\quad \Re(\rho+\delta b_j)>0,$$
$$\Re(\sigma+\lambda b_j)>0,\quad \Re(\gamma+\sigma-\alpha-\beta+\lambda b_j)>0\quad (j=1,2,\ldots,m).$$

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha,\beta;\gamma;zx)\,G_{p,q}^{m,n}\left(\zeta x^{\lambda}(1-x)^{\delta}\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)dx$$
$$= (2\pi)^{\frac{1}{2}}\delta^{\sigma-\frac{1}{2}}\lambda^{\rho-\frac{1}{2}}(\lambda+\delta)^{\frac{1}{2}-\rho-\sigma}\sum_{r=0}^{\infty}\frac{(\alpha)_r(\beta)_r}{(\gamma)_r r!}\frac{z^r\lambda^r}{\delta^r}$$
$$\times G_{p+2\delta+\lambda,q+2\delta+\lambda}^{m,n+2\delta+\lambda}\left(\zeta\frac{\delta^\delta\lambda^\lambda}{(\lambda+\delta)^{\lambda+\delta}}\middle|\begin{matrix}\Delta(\delta,1-\sigma),\Delta(\delta,1-\sigma),\Delta(\lambda,1-\rho-r),a_1,\ldots,a_p\\ b_1,\ldots,b_q,\Delta(\lambda+\delta,1-\sigma-\rho),\Delta(\delta,1-\sigma-r)\end{matrix}\right),\quad(2\cdot6)$$

where δ and λ are positive integers, and

$$p+q<2(m+n),\quad |\arg\zeta|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi,\quad |\arg(1-z)|<\pi,$$
$$\Re(\rho+\lambda b_j)>0,\quad \Re(\sigma+\delta b_j)>0\quad (j=1,2,\ldots,m).$$

https://doi.org/10.1017/S0305004100042109 Published online by Cambridge University Press

Proof. In the beginning consider the integral (2·1) with $t = 1$. To establish the integral expressing the G-function as a Mellin–Barnes type integral ((1), page 207, (1)) and interchanging the order of integrations, which is justifiable due to the absolute convergence of the integrals involved in the process, we have

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-s)\prod_{j=1}^{n}\Gamma(1-a_j+s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+s)\prod_{j=n+1}^{p}\Gamma(a_j-s)}\,\zeta^s\int_0^1 x^{\gamma-1}(1-x)^{\rho+\delta s-1}e^{-zx}\,{}_2F_1(\alpha,\beta;\gamma;x)\,dx\,ds.$$

Now evaluating the inner integral with the help of ((2), page 400, (8)) and using the multiplication formula for the Gamma-function, we have

$$\delta^{-\gamma}e^{-z}\Gamma(\gamma)\sum_{r=0}^{\infty}\frac{z^r}{r!}\frac{1}{2\pi i}$$

$$\times\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-s)\prod_{j=1}^{n}\Gamma(1-a_j+s)\prod_{k=0}^{\delta-1}\Gamma\left(\frac{\rho+r+k}{\delta}+s\right)\prod_{k=0}^{\delta-1}\Gamma\left(\frac{\gamma+\rho-\alpha-\beta+r+k}{\delta}+s\right)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+s)\prod_{j=n+1}^{p}\Gamma(a_j-s)\prod_{k=0}^{\delta-1}\Gamma\left(\frac{r+\rho-\alpha+r+k}{\delta}+s\right)\prod_{k=0}^{\delta-1}\Gamma\left(\frac{\gamma+\rho-\beta+r+k}{\delta}+s\right)}\zeta^s\,ds$$

On applying ((1), page 207, (1)), the value of the integral (2·1) with $t = 1$ is obtained.

By virtue of the relation ((8), page 39), the integral established above with $t = 1$ in (2·1) can be reduced to the formula (2·1).

Formulae (2·2) and (2·6) can be similarly established on applying the same procedure as above then using ((2), page 400, (10)) and ((2), page 399, (7)) respectively, and formulae (2·3), (2·4) and (2·5) can be similarly obtained with the help of ((2), page 399, (5)).

3. *Particular cases.* 3·1 (i) In (2·1), if we put $m = 1, n = p, b_1 = 0$, replace q by $q+1$, write $1-a_j$ and $1-b_{k-1}$ for a_j and b_k $(j = 1, 2, \ldots, p)$ $(k = 2, 3, \ldots, q+1)$, then by virtue of ((1), page 209, (9)) and ((2), page 444, (2)), we get

$$\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-zx}\,{}_2F_1(\alpha,\beta;\gamma;x)\,E(p;a_u:q;b_v:(1/\zeta)(1-x)^{-\delta/t})\,dx$$

$$= (2\pi)^{\frac{1}{2}(1-t)(p-q+1)}\,t^{\Sigma a_p-\Sigma b_q+\frac{1}{2}(q-p+1)}\,\delta^{-\gamma}\,\Gamma(\gamma)\,e^{-z}$$

$$\times\sum_{r=0}^{\infty}\frac{z^r}{r!}G^{t,tp+2\delta}_{tp+2\delta,\,t(q+1)+2\delta}$$

$$\times\left(\zeta^t t^{t(p-q+1)}\middle|\begin{matrix}\Delta(\delta,1-\rho-r),\Delta(\delta,1+\alpha+\beta-\gamma-\rho-r),\Delta(t,1-a_1),\ldots,\Delta(t,1-a_p)\\ \Delta(t,0),\Delta(t,1-b_1),\ldots,\Delta(t,1-b_q),\\ \qquad\qquad\Delta(\delta,1+\alpha-\gamma-\rho-r),\Delta(\delta,1+\beta-\gamma-\rho-r)\end{matrix}\right), \tag{3·1}$$

where δ and t are positive integers, and

$$p+1>q,\quad |\arg\zeta|<\tfrac{1}{2}(p-q+1)\pi,\quad \Re(\gamma)>0,\quad \Re(\rho)>0,\quad \Re(\gamma+\rho-\alpha-\beta)>0.$$

In (3·1), putting $z = 0$, $\alpha = \beta = 0$, $t = 1$ replacing $1-x$ by x, γ by $\rho-\alpha$ and ρ by α, using ((1), page 209, (9)) and ((2), page 444, (2)), we get a known result ((3), page 190, (77)).

https://doi.org/10.1017/S0305004100042109 Published online by Cambridge University Press

(ii) In (2·1), if we replace $1-x$ by $1/x$, then we get

$$\int_1^\infty (x-1)^{\gamma-1}x^{-\gamma-\rho}\exp\{-z(1-(1/x))\}\,{}_2F_1(\alpha,\beta;\gamma;1-(1/x))\,G^{m,n}_{p,q}\left(\zeta x^{-\delta/t}\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)dx$$
$$= (2\pi)^{(1-t)(m+n-\frac{1}{2}p-\frac{1}{2}q)}\,t^{\Sigma b_q-\Sigma a_p+\frac{1}{2}p-\frac{1}{2}q+1}\delta^{-\gamma}\,e^{-z}\,\Gamma(\gamma)$$
$$\times\sum_{r=0}^{\infty}\frac{z^r}{r!}\,G^{tm,\,tn+2\delta}_{tp+2\delta,\,tq+2\delta}$$
$$\times\left(\zeta^t t^{t(p-q)}\middle|\begin{matrix}\Delta(\delta,1-\rho-r),\Delta(\delta,1+\alpha+\beta-\gamma-\rho-r),\Delta(t,a_1),\ldots,\Delta(t,a_p)\\ \Delta(t,b_1),\ldots,\Delta(t,b_q),\Delta(\delta,1+\alpha-\gamma-\rho-r),\Delta(\delta,1+\beta-\gamma-\rho-r)\end{matrix}\right), \quad (3\cdot2)$$

where δ and t are positive integers, and

$$p+q<2(m+n),\quad |\arg\zeta|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi,\quad \Re(\gamma)>0,$$
$$\Re\left(\rho+\frac{\delta}{t}b_j\right)>0,\quad \Re\left(\gamma+\rho-\alpha-\beta+\frac{\delta}{t}b_j\right)>0\quad (j=1,2,\ldots,m).$$

In (3·2), putting $\delta=t=1$, $z=0$, $\alpha=\beta=0$, $\gamma=\sigma$ replacing $\gamma+\rho$ by ρ using ((1), page 209, (9)), we get a known result ((2), page 417, (2)).

(iii) In (2·1), putting $\delta=t=1$, $z=0$, $\alpha=\beta=0$, $\gamma=\sigma$ replacing $1-x$ by x, we get a known result ((2), page 417, (1)).

3·2. (i) In the left-hand side of (2·2) reducing the G-function to an E-function with the help of ((2), page 444, (2)), we get

$$\int_0^\infty x^{\gamma-1}(x+z)^{-\sigma}\,{}_2F_1(\alpha,\beta;\gamma;-x)\,E(p;a_u:q;b_v:\zeta(z+x)^{\delta/t})\,dx$$
$$= (2\pi)^{\frac{1}{2}(1-t)(p-q+1)}\,t^{\Sigma a_p-\Sigma b_q+\frac{1}{2}(q-p+1)}\,\delta^{-\gamma}\,\Gamma(\gamma)$$
$$\times\sum_{r=0}^{\infty}\frac{\delta^r(1-z)^r}{r!}\,G^{tp+2\delta,\,t}_{t(q+1)+2\delta,\,tp+2\delta}$$
$$\times\left(\zeta^t t^{t(q-p+1)}\middle|\begin{matrix}\Delta(t,1),\Delta(t,b_1),\ldots,\Delta(t,b_q),\Delta(\delta,\sigma),\Delta(\delta,\alpha+\beta-\gamma+\sigma+r)\\ \Delta(\delta,\alpha-\gamma+\sigma+r),\Delta(\delta,\beta-\gamma+\sigma+r),\Delta(t,a_1),\ldots,\Delta(t,a_p)\end{matrix}\right), \quad (3\cdot3)$$

where δ and t are positive integers, and

$$p+1>q,\quad |\arg\zeta|<\tfrac{1}{2}(p-q+1)\pi,\quad \Re(\gamma)>0,\quad |\arg z|<\pi,$$
$$\Re(\alpha-\gamma+\sigma)>0,\quad \Re(\beta-\gamma+\sigma)>0.$$

In (3·3), putting $z=1$, $t=1$, $\zeta=(y/\delta)^\delta$, replacing x by x/y, using ((2), page 439, (5)), we get a known result obtained by Rathi ((7), page 172, (10)).

In (3·3), putting $z=1$, $\delta=t=1$, $\alpha=\beta=0$, $\gamma=\rho$, using ((2), page 439, (5)), we get another known result ((2), page 415, (3)).

(ii) In (2·2), putting $z=1$, $\delta=t=1$, $\gamma=\sigma$, $\sigma=\rho$, $\alpha=k+\sigma-\rho$, $\beta=\lambda+\sigma-\rho$, replacing $1+x$ by x, we get a known result ((2), page 421, (13)).

(iii) In (2·2), putting $z=1$, $\delta=t=1$, $\alpha=\beta=0$, $\gamma=\sigma$, $\sigma=\rho$ replacing $1+x$ by x, we obtain a known result ((2), page 417, (2)).

3·3. In (2·3), putting $t=1$, $\sigma=1+\beta-\gamma-n$, $\alpha=-n$ expressing the G-function on the right-hand side as a Mellin–Barnes type integral, using the multiplication formula

https://doi.org/10.1017/S0305004100042109 Published online by Cambridge University Press

for Gamma-function, interchanging the order of integration and summation, using Saalschutz's theorem ((5), page 360) and ((6), page 32, (9)), then on applying ((1) page 207, (1)), we get a known result obtained by Sharma (9).

3·4. In (2·4), putting $t = 1, \sigma = \beta - \rho$, then applying the same procedure as in 3·3 and using Gauss's theorem, we get a known result obtained by Sharma (9).

3·5. In (2·5), putting $\rho = \sigma = \gamma, \lambda = \delta = l, \gamma = \frac{1}{2}(\alpha+\beta+1)$ then applying the same procedure as in 3·3 and using Whipple's theorem ((5), page 363), we get

$$\int_0^1 x^{\gamma-1}(1-x)^{\gamma-1}\,{}_2F_1(\alpha,\beta;\tfrac{1}{2}(\alpha+\beta+1);x)\,G_{p,q}^{m,n}\left(zx^l(1-x)^l\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)dx$$
$$= \pi l^{-\frac{1}{2}}2^{1-2\gamma}\frac{\Gamma\frac{1}{2}(\alpha+\beta+1)}{\Gamma\frac{1}{2}(\alpha+1)\,\Gamma\frac{1}{2}(\beta+1)}$$
$$\times G_{p+2l,q+2l}^{m,n+2l}\left(z2^{-2l}\middle|\begin{matrix}\Delta(l,1-\gamma),\Delta(l,\frac{1}{2}(\alpha+\beta+1)-\gamma),a_1,\ldots,a_p\\ b_1,\ldots,b_q,\Delta(l,\frac{1}{2}(\alpha+1)-\gamma),\Delta(l,\frac{1}{2}(\beta+1)-\gamma)\end{matrix}\right), \quad (3\cdot4)$$

where l is a positive integer, and

$$p+q < 2(m+n), \quad |\arg z| < (m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi, \quad \Re(\gamma+lb_j) > 0,$$
$$\Re(\gamma-\tfrac{1}{2}(\alpha+\beta)+lb_j) > -\tfrac{1}{2} \quad (j=1,2,\ldots,m).$$

In (3·4), putting $m = 1, n = p, b_1 = 0$, replacing q by $q+1$, writing $1-a_j$ and $1-b_{k-1}$ for a_j and b_k $(j = 1,\ldots,p)$ and $(k = 2,\ldots,q+1)$, then by virtue of ((1), page 209, (9)) and ((2), page 444, (2)) we get a known result obtained by MacRobert ((4), page 197, (7)).

3·6. In (2·6), if we take $z = 0, m = 1, n = p, b_1 = 0$ replace q by $q+1$, write $1-a_j$ and $1-b_{k-1}$ for a_j and $b_k (j = 1,\ldots,p)$ and $(k = 2,\ldots,q+1)$, then using ((1), page 209, (9)) and ((2), p. 444, (2)), it is reduced to a known result ((5), page 415, (55)).

I wish to express my sincere thanks to Dr V. M. Bhise for his help and guidance in the preparation of this paper. I am also grateful to the referee for some useful suggestions for revising the paper. My thanks are also due to Principal Dr S. M. Das Gupta for the facilities he provided for me.

REFERENCES

(1) Erdélyi, A. *Higher transcendental functions*, vol. 1 (1953).
(2) Erdélyi, A. *Tables of integral transforms*, vol. 2 (1954).
(3) MacRobert, T. M. Some integrals involving E-functions. *Proc. Glasgow Math. Assoc.* **1** (1953), 190–191.
(4) MacRobert, T. M. Integrals involving hypergeometric functions and E-functions. *Proc. Glasgow Math. Assoc.* **1** (1958), 196–198.
(5) MacRobert, T. M. *Functions of a complex variable* (1962).
(6) Rainville, Earl D. *Special functions* (1962).
(7) Rathie, C. B. A few infinite integrals involving E-functions. *Proc. Glasgow Math. Assoc.* **2** (1956), 170–172.
(8) Saxena, R. K. Some theorems in operational calculus and infinite integrals involving Bessel function and G-functions. *Proc. Nat. Inst. Sci. India, Part A* **27** (1961), 38–61.
(9) Sharma, K. C. Integrals involving products of G-function and Gauss's hypergeometric function. *Proc. Cambridge Philos. Soc.* **60** (1964), 539–542.

https://doi.org/10.1017/S0305004100042109 Published online by Cambridge University Press