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Large-scale dynamo action driven by forced beating waves in a highly conducting plasma

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The aim of this short paper is simply to demonstrate the effect of interactions between distinct waves induced by an oscillatory forcing on generation of the large-scale electromotive force (EMF) in a highly electrically conducting plasma in the dynamical regime, i.e. when the back reaction of the Lorentz force on the flow is included. The mean EMF is induced by waves with distinct but close phase speeds, which are called 'beating waves' by analogy with the acoustic effect of 'beat', in the presence of a locally uniform seed magnetic field. These waves may have either positive or negative helicity, and it is supposed that waves of a single sign of helicity are preferentially excited by a symmetry-breaking mechanism. The formula for the mean EMF in a highly conducting plasma is derived, conditions relevant to those of the early universe before and during galaxy formation. Within the scope of the weak magnetohydrodynamic turbulence dominated by the linear waves an α-effect, that is generation of the large-scale EMF is studied, which leads to amplification of the seed mean magnetic field. Possible subsequent equilibration due to quenching resulting from the presence of the Lorentz force is briefly discussed.

Key words: astrophysical plasmas, plasma dynamics, plasma instabilities

1. Introduction

The generation of large-scale magnetic fields in natural systems, in particular in astrophysical bodies such as planets, stars and in particular the extremely highly conductive galactic interstellar medium or galaxy clusters etc., has been a long lasting challenge of the dynamo theory. The large-scale field amplification and saturation rely on formation of large-scale electromotive force (EMF) in the system, however, there are various mechanisms that can lead to generative mean EMF, i.e. capable of generation of large-scale fields. The $\boldsymbol{\Omega}(\boldsymbol{x}) \times \boldsymbol{J}$ effect with non-uniform background rotation $\boldsymbol{\Omega}(\boldsymbol{x})$ and $\boldsymbol{J}$ denoting the electrical currents (cf. Rädler 1969; Roberts 1972; Moffatt & Proctor 1982; Rädler 1986; Rädler, Kleeorin & Rogachevskii 2003) or the 'shear-current' effect (cf. Rogachevskii & Kleeorin 2003, 2004) rely on inhomogeneity of the large-scale flows, however in helical (typically also rotating) turbulence the most promising and most frequently addressed is the so-called α-effect (the tensor relating the mean EMF to the mean magnetic field is conventionally denoted by $\boldsymbol{\alpha}$),

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which can also be combined with the effect of non-uniform background rotation producing then the dynamo of $\alpha\boldsymbol{\Omega}$ type; the shear straining and advecting the magnetic field lines in the $\boldsymbol{\Omega}$ effect is, however, often unnecessary and dynamos of α^2 type are known. The α effect is based on influence of nonlinear interactions of the small-scale fluctuating components of the turbulent state on the large-scale magnetic field (cf. e.g. Parker 1955; Braginskii 1964*a*,*b*; Steenbeck & Krause 1966; Steenbeck, Krause & Rädler 1966; Steenbeck *et al.* 1967; Moffatt 1978; Moffatt & Proctor 1982). For dynamo action to occur, the wave field must exhibit chirality (lack of reflexional symmetry), and this requires some mechanism (e.g. gravity $\boldsymbol{g}$ in conjunction with mean rotation $\boldsymbol{\Omega}$, or some type of forcing) that breaks the 'up–down' symmetry of the field. An electromotive force $\mathcal{E}_i = \alpha_{ij}\langle B\rangle_j$ is then generated, where $\langle\boldsymbol{B}\rangle$ is the mean magnetic field, i.e. an α-effect in the terminology of Steenbeck *et al.* (1966), and this leads to exponential growth of magnetic energy until the growing Lorentz force reacts back upon the wave field, leading to a saturated state (Moffatt 1972). It has been demonstrated in Moffatt (1972) and Moffatt (1978, §10.3) and thus known for a long time, that an oscillatory forcing with just one frequency of oscillation induces a non-decaying wave in a resistive fluid, which can create a mean EMF through self-interactions between the kinetic and magnetic components of a single wave; the resistivity is responsible for creation of a necessary phase shift between the small-scale velocity and small-scale magnetic field of the wave. The motivation for this study was based on the simple fact, that every force can be decomposed into spatial and temporal Fourier modes, however, such an expansion involves many waves with different frequencies of oscillation and thus interactions between them are allowed.

In this paper, we describe a new dynamo mechanism, based on interactions between waves induced by a forcing consisting of more than just one frequency of oscillation, which operates in the dynamical regime, thus under inclusion of the effect of the Lorentz force. Similar mechanism, but based on interactions between the system eigen modes in the absence of forcing was considered in Mizerski (2018) and even earlier in Mizerski, Bajer & Moffatt (2012), where the mean EMF generated by the magneto-elliptic instability was calculated (the problem of mean EMF generation by instabilities has been also considered by Lesur & Ogilvie (2008) and Davies & Hughes (2011)). This dynamo mechanism in the terminology of Vainshtein & Zel'dovich (1972) is described as 'fast', that is leads to magnetic field amplification at a rate independent of resistivity, when the resistivity tends to zero.[1] The EMF survives in the limiting situation $\eta = 0$, leading to an α-effect even in ideal fluids. It is supposed, therefore, that the presented theory might be relevant to the dynamo generation of magnetic fields in extremely highly conducting plasma, such as in the galactic interstellar medium or in galaxy clusters, in particular in the early universe, when the galaxies were formed. Indeed, estimates of molecular resistivity for the Milky Way galaxy, $\eta_{\text{galaxy}} \approx 10^3\ \text{m}^2\ \text{s}^{-1}$ and galaxy clusters $\eta_{\text{cluster}} \approx 3\times 10^{-3}\ \text{m}^2\ \text{s}^{-1}$, taken from Spitzer (1962) and Schekochihin & Cowley (2006) respectively, lead to extremely long characteristic diffusive times for the magnetic field on the entire length scale of the system, $t_{\text{galaxy}} \approx 10^{30}\,y$, and $t_{\text{cluster}} \approx 10^{34} - 10^{38}\,y$, many orders of magnitude longer, than the age of the universe $\sim 10^{10}\,y$. Therefore it seems legitimate to neglect the effect of resistivity in such systems and study the generation of large-scale magnetic fields in ideal plasma (a 'fast dynamo'), however, the new mechanism is also related to the standard one in the presence of weak resistivity.

[1]The hitherto studied cases of 'fast' dynamos were in vast majority kinematic and involved amplification of small-scale fields at a rate independent of resistivity.

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In the introduced astrophysical context the external forcing could result from tidal interactions between the galaxies (cf. Wesson (1982) or Rong *et al.* (2015) and references therein) or from large-scale inhomogeneities and rotation, which may force wave propagation in galaxies (cf. Feldman & Lin 1973).

Furthermore, according to the so-called primordial theory of the origin of galactic magnetic fields, the main build-up of the large-scale fields in such systems had taken place billions of years ago, when the galaxies were formed, and it is only maintained by the turbulent flow in the later evolution; the dynamo process, which operated then, no longer needs to operate since the plasma is nearly perfectly conducting and the field cannot decay during the galaxy/cluster lifetime because of the extremely low resistivity (cf. Zweibel & Heiles 1997; Brandenburg & Subramanian 2005). Such a field was termed a 'fossil field' in Mizerski & Moffatt (2018), where the effect of weak plasma viscosity was investigated. There is however an opposite theory in which the galactic dynamos have to continuously operate to sustain the observed steady fields (cf. Ruzmaikin, Shukurov & Sokoloff 1988; Beck *et al.* 1996 and Brandenburg & Subramanian 2005). The reason for invoking the continuously operating dynamo results from the fact that, a primordial field is expected to be strongly wound up after billions of years, which is not observed, however, on the other hand the small-scale effects could decrease the effect of wind up. In any case, a mechanism of large-scale dynamo process operating without the need to appeal to the extremely weak diffusive processes seems strongly desired. We utilize the concept of weak, unbalanced magnetohydrodynamic turbulence (with Coriolis force), which is essentially linear in the turbulent fluctuations and unbalanced so that the up–down symmetry is broken and the net helicity of turbulent flow has a preferred sign (cf. Ng & Bhattacharjee 1996 and Tobias, Cattaneo & Boldyrev 2013). The forcing, therefore, is assumed weak, not to drive the system away from the regime dominated by linear waves. Although the wave packets interact nonlinearly, when their amplitudes are small, they can survive for a long time without getting destroyed, which is the essential feature of the weak magnetohydrodynamic turbulence, as explained in Tobias *et al.* (2013). The effect of the Lorentz force, whose significance in the EMF generation was emphasized in the reviews of dynamo theory by Brandenburg & Subramanian (2005) and Tobias *et al.* (2013) is included and its effect is important in the large-scale dynamo considered here. The weak turbulence is often known to last for long periods of time, which allows for the build up of the large-scale galactic/galaxy cluster field at the early stage of evolution. However, the turbulence eventually becomes 'strong', that is the terms nonlinear in the fluctuations start to play a significant role in the dynamics, and it is not clear what happens then. In particular, numerical simulations of rapidly rotating turbulent convection of Cattaneo & Hughes (2006) and Hughes & Cattaneo (2008) indicate that, when diffusion is very weak compared to advection and straining, the amplitude of fluctuations can be amplified quickly, effectively destroying the linear regime. We emphasize, therefore, that the assumption of weak turbulence may not be valid in some cases, in particular for small-scale systems. Nevertheless, Hughes & Cattaneo (2008) conclude, that at low diffusivity the size of the system plays a crucial role for generation of large-scale fields. Since the results here are thought to be relevant to systems as large scale as galaxies and galaxy clusters, it seems fair to expect that, when the initial amplitudes of waves are weak, there is a long enough period of time when they survive and interact, thus creating the large-scale electromotive force and mean field dynamo. Moreover it is likely, that a force-free mode of the primordial field would survive even in the later stage of evolution, as there are no mechanisms of magnetic energy

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decay, which could suppress such a mode. It is this scenario, which is considered here. It should be emphasized, however, that the effect of gravity, which at the early stage of evolution of the galactic magnetic fields was significant, since it led to strengthening the magnetic field by bringing the field lines closer together, is neglected here for clarity of presentation of the new studied mechanism.

2. Mean and fluctuating equations

We consider the evolution of a magnetic field in a fluid rotating with uniform angular velocity $\boldsymbol{\Omega}$. For simplicity, we suppose that the fluid is inviscid and incompressible with constant uniform density ρ and magnetic resistivity η. Let $\boldsymbol{u}(\boldsymbol{x}, t)$ be the velocity field and $\boldsymbol{B}(\boldsymbol{x}, t) \equiv (\mu_0\rho)^{1/2}\boldsymbol{h}(\boldsymbol{x}, t)$ the magnetic field in the fluid; then the standard governing evolution equations are

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla}P - 2\boldsymbol{\Omega}\times\boldsymbol{u} + (\boldsymbol{h}\cdot\boldsymbol{\nabla})\boldsymbol{h} + \boldsymbol{F}, \tag{2.1a}$$

$$\frac{\partial \boldsymbol{h}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{h} = (\boldsymbol{h}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \eta\nabla^2\boldsymbol{h}, \tag{2.1b}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0, \quad \boldsymbol{\nabla}\cdot\boldsymbol{h} = 0, \tag{2.1c,d}$$

where $P = p/\rho + h^2/2 - (\boldsymbol{\Omega}\times\boldsymbol{x})^2/2$ is the modified (total) pressure field and $\boldsymbol{F}$ is a prescribed body force, independent of the magnetic, velocity and pressure fields. We shall consider the situation in which $\boldsymbol{u}$ and $\boldsymbol{h}$ are turbulent fields, spatially homogeneous on a scale ℓ. Let $\langle\cdot\rangle$ denote a space average over any scale $a \gg \ell$.

Following the 'mean-field electrodynamics' approach of Steenbeck *et al.* (1966) we decompose $\boldsymbol{u}$ and $\boldsymbol{h}$ into mean and fluctuating parts

$$\boldsymbol{u} = \langle\boldsymbol{u}\rangle + \boldsymbol{u}', \quad \boldsymbol{h} = \langle\boldsymbol{u}\rangle + \boldsymbol{h}', \tag{2.2a,b}$$

where $\langle\boldsymbol{u}'\rangle = \langle\boldsymbol{h}'\rangle = 0$. We may further adopt the local frame of reference in which $\langle\boldsymbol{u}\rangle = 0$,[2] and we define $\boldsymbol{H} \equiv \langle\boldsymbol{h}\rangle$. We treat $\boldsymbol{H}$ as locally uniform, but varying weakly on scales much greater than the averaging scale a. We will also neglect the large-scale component of the forcing $\langle\boldsymbol{F}\rangle$ and assume that it is present only on the small scales. Averaging (2.1*a*)–(2.1*d*) then gives

$$\langle(\boldsymbol{u}'\cdot\boldsymbol{\nabla})\boldsymbol{u}'\rangle = -\boldsymbol{\nabla}\langle P\rangle + (\boldsymbol{H}\cdot\boldsymbol{\nabla})\boldsymbol{H} + \langle(\boldsymbol{h}'\cdot\boldsymbol{\nabla})\boldsymbol{h}'\rangle, \tag{2.3a}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = \boldsymbol{\nabla}\times\boldsymbol{\mathcal{E}} + \eta\nabla^2\boldsymbol{H}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{H} = 0. \tag{2.3b,c}$$

Here $\boldsymbol{\mathcal{E}} \equiv \langle\boldsymbol{u}'\times\boldsymbol{h}'\rangle$ is the mean electromotive force induced by interaction between the small-scale fluctuating fields. Subtracting these averaged equations from the full equations (2.1*a*)–(2.1d) gives equations for these fluctuating fields. Linearization which stems from the assumption of weak turbulence discussed in the introduction results in the neglect of squares and products of fluctuating quantities (which might be viewed as a kind of 'first-order smoothing approximation'); the resulting linearized equations are

$$\frac{\partial \boldsymbol{u}'}{\partial t} + \boldsymbol{\nabla}P' - (\boldsymbol{H}\cdot\boldsymbol{\nabla})\boldsymbol{h}' + 2\boldsymbol{\Omega}\times\boldsymbol{u}' = \boldsymbol{F}', \tag{2.4a}$$

[2]This is possible, since the fluctuating fields vary on spatial and temporal scales much smaller than the mean quantities, thus locally, from the point of view of the dynamics of the fluctuations the mean flow can be considered uniform.

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$$\frac{\partial \boldsymbol{h}'}{\partial t} - (\boldsymbol{H}\cdot\nabla)\boldsymbol{u}' - \eta\nabla^2\boldsymbol{h}' = 0, \tag{2.4b}$$

$$\nabla\cdot\boldsymbol{h}' = 0, \quad \nabla\cdot\boldsymbol{u}' = 0. \tag{2.4c,d}$$

The force field $\boldsymbol{F}'$ is assumed sufficiently weak, so that the fluctuations can remain small and the linearization remains valid. Furthermore, as in Moffatt (1978) we suppose, that $\nabla\cdot\boldsymbol{F}' = 0$ and the forcing $\boldsymbol{F}'$ is reflectionally symmetric, thus non-helical, $\langle \boldsymbol{F}'\cdot\nabla\times\boldsymbol{F}'\rangle = \boldsymbol{0}$. In such a way the only source of helicity in the system is the Coriolis effect.

3. Forced waves

We will consider the effect of two Fourier modes of the forcing $\boldsymbol{F}'$, with the same wave vector but distinct frequencies of time oscillation,

$$\boldsymbol{F}' = \mathrm{Re}\{\hat{\boldsymbol{f}}_1 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_1 t)]\} + \mathrm{Re}\{\hat{\boldsymbol{f}}_2 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_2 t)]\}, \tag{3.1}$$

easily generalizable to the case of many temporal Fourier modes. Therefore the linearized equations (2.4*a*)–(2.4*d*) admit solutions in the form of forced waves

$$\begin{aligned}(\boldsymbol{u}',\ \boldsymbol{h}',\ P') \ =\ & \mathrm{Re}\{(\hat{\boldsymbol{u}}_1, \hat{\boldsymbol{h}}_1, \hat{P}_1)\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_1 t)]\} \\ & + \mathrm{Re}\{(\hat{\boldsymbol{u}}_2,\ \hat{\boldsymbol{h}}_2, \hat{P}_2)\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_2 t)]\},\end{aligned} \tag{3.2}$$

which oscillate at the frequencies of the forcing and do not decay even when $\eta \neq 0$. The divergence conditions (2.4*c*,*d*) imply that

$$\boldsymbol{k}\cdot\hat{\boldsymbol{u}} = \boldsymbol{k}\cdot\hat{\boldsymbol{h}} = 0, \tag{3.3}$$

so the waves are transverse and since we assumed $\nabla\cdot\boldsymbol{F}' = 0$, we also get $\boldsymbol{k}\cdot\hat{\boldsymbol{f}}_1 = \boldsymbol{k}\cdot\hat{\boldsymbol{f}}_2 = 0$.

The compact treatment of Moffatt (1978, §10.3) is followed here for ease of comparison. Substitution first in (2.4*b*) gives the familiar relationship between $\hat{\boldsymbol{h}}$ and $\hat{\boldsymbol{u}}$, *viz.*

$$\hat{\boldsymbol{h}}_j = -(\omega_j + \mathrm{i}\,\eta k^2)^{-1}(\boldsymbol{k}\cdot\boldsymbol{H})\hat{\boldsymbol{u}}_j, \tag{3.4}$$

and then substitution in (2.4*a*) and rearrangement of the terms gives

$$-\mathrm{i}\sigma_j\hat{\boldsymbol{u}}_j + 2\boldsymbol{\Omega}\times\hat{\boldsymbol{u}}_j = -\mathrm{i}\,\boldsymbol{k}\hat{P}_j + \hat{\boldsymbol{f}}_j, \tag{3.5a}$$

where

$$\sigma_j = \omega_j - (\omega_j + \mathrm{i}\eta k^2)^{-1}(\boldsymbol{H}\cdot\boldsymbol{k})^2, \tag{3.5b}$$

and $j = 1, 2$. In the considered high conductivity limit, $\eta k^2 \ll |\sigma_j|$, the latter can be simplified to yield

$$\mathrm{Re}\,\sigma_j \approx \omega_j - \frac{(\boldsymbol{k}\cdot\boldsymbol{H})^2}{\omega_j}, \quad \mathrm{Im}\,\sigma_j \approx \eta k^2\frac{(\boldsymbol{k}\cdot\boldsymbol{H})^2}{\omega_j^2}. \tag{3.6a,b}$$

To get the velocity field, we take the double cross-product of (3.5*a*) with $\boldsymbol{k}$, that is we apply to both sides $\boldsymbol{k}\times\boldsymbol{k}\times$; since $\boldsymbol{k}\cdot\hat{\boldsymbol{u}} = 0$, this allows us to obtain

$$\hat{\boldsymbol{u}}_j = D_j^{-1}[2(\boldsymbol{k}\cdot\boldsymbol{\Omega})\boldsymbol{k}\times\hat{\boldsymbol{f}}_j + \mathrm{i}\sigma_j k^2\hat{\boldsymbol{f}}_j], \tag{3.7}$$

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where

$$D_j = \sigma_j^2 k^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2. \tag{3.8}$$

A resonant response is obtained when the frequency of the forcing ω_j coincides with one of the natural (eigen) frequencies of the undamped system (for the given wave vector $\boldsymbol{k}$), that is the frequencies of the Lehnert waves (Magneto–Coriolis waves, cf. Lehnert 1954; Finlay 2008; Mizerski & Moffatt 2018). This happens, when

$$(\mathrm{Re}\,\sigma_j)^2 = 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2/k^2, \quad \text{or} \quad \sigma_j = \mp 2(\boldsymbol{k}\cdot\boldsymbol{\Omega})/k, \tag{3.9a,b}$$

which corresponds to minimum value of $|D_j|$.

We now calculate the helicity of the total forced velocity field $\boldsymbol{u}'$

$$\begin{aligned}\langle \boldsymbol{u}'\cdot\nabla\times\boldsymbol{u}'\rangle &= -\tfrac{1}{2}\,\mathrm{Re}[\mathrm{i}\boldsymbol{k}\cdot(\hat{\boldsymbol{u}}_1^*\mathrm{e}^{\mathrm{i}\omega_1 t}+\hat{\boldsymbol{u}}_2^*\mathrm{e}^{\mathrm{i}\omega_2 t})\times(\hat{\boldsymbol{u}}_1\mathrm{e}^{-\mathrm{i}\omega_1 t}+\hat{\boldsymbol{u}}_2\mathrm{e}^{-\mathrm{i}\omega_2 t})]\\ &= -\mathrm{Re}[\mathrm{i}\boldsymbol{k}\cdot(\hat{\boldsymbol{u}}_1^*\times\hat{\boldsymbol{u}}_2\mathrm{e}^{\mathrm{i}\Delta\omega t})] - \tfrac{1}{2}\,\mathrm{Re}[\mathrm{i}\boldsymbol{k}\cdot(\hat{\boldsymbol{u}}_1^*\times\hat{\boldsymbol{u}}_1)+\mathrm{i}\boldsymbol{k}\cdot(\hat{\boldsymbol{u}}_2^*\times\hat{\boldsymbol{u}}_2)],\end{aligned} \tag{3.10}$$

where $\Delta\omega = \omega_1 - \omega_2$ is the difference between the two frequencies of the forcing. Since $\hat{\boldsymbol{u}}_j$ and $\hat{\boldsymbol{u}}_j^*$ for $j = 1, 2$ are perpendicular to $\boldsymbol{k}$, it follows that $(\hat{\boldsymbol{u}}_i^*\times\hat{\boldsymbol{u}}_j)\parallel\boldsymbol{k}$, so that it is enough to calculate $\hat{\boldsymbol{u}}_i^*\times\hat{\boldsymbol{u}}_j$. The last two terms in the expression (3.10) correspond to the well-known mechanism of mean EMF generation from waves forced by just one frequency, either ω_1 or ω_2, for which the solution was provided in the formula Moffatt (1978, (10.52) of §10.3), and it is recalled here for completeness

$$\mathrm{i}\hat{\boldsymbol{u}}_j^*\times\hat{\boldsymbol{u}}_j = 4|D_j|^{-2}(\boldsymbol{k}\cdot\boldsymbol{\Omega})k^2\,\mathrm{Re}\,\sigma_j|\hat{\boldsymbol{f}}_j|^2\boldsymbol{k}, \tag{3.11}$$

for $j = 1, 2$. The other two terms describe the new mechanism and can be calculated to yield

$$\mathrm{i}\hat{\boldsymbol{u}}_1^*\times\hat{\boldsymbol{u}}_2 = 2(D_1^*D_2)^{-1}(\boldsymbol{k}\cdot\boldsymbol{\Omega})k^2(\sigma_1^*+\sigma_2)(\hat{\boldsymbol{f}}_1^*\cdot\hat{\boldsymbol{f}}_2)\boldsymbol{k}. \tag{3.12}$$

In the two latter formulae we have utilized the assumption of reflectional symmetry of $\boldsymbol{F}$, which implies

$$(\hat{\boldsymbol{f}}_1\mathrm{e}^{\mathrm{i}\omega_1 t}+\hat{\boldsymbol{f}}_2\mathrm{e}^{\mathrm{i}\omega_2 t})\cdot[\boldsymbol{k}\times(\hat{\boldsymbol{f}}_1^*\mathrm{e}^{-\mathrm{i}\omega_1 t}+\hat{\boldsymbol{f}}_2^*\mathrm{e}^{-\mathrm{i}\omega_2 t})] = 0, \tag{3.13}$$

and hence

$$\hat{\boldsymbol{f}}_i^*\times\boldsymbol{f}_j = 0, \tag{3.14}$$

for all combinations of $i = 1, 2$ and $j = 1, 2$. Furthermore, in the weak resistivity limit we get from (3.6)

$$\sigma_1^*+\sigma_2 \approx (\omega_1+\omega_2)\left[1-\frac{(\boldsymbol{k}\cdot\boldsymbol{H})^2}{\omega_1\omega_2}+\mathrm{i}\eta k^2\frac{\Delta\omega(\boldsymbol{k}\cdot\boldsymbol{H})^2}{\omega_1^2\omega_2^2}\right]. \tag{3.15}$$

In order to understand the physical picture it is helpful to consider the limit of weak seed magnetic field $|\boldsymbol{k}\cdot\boldsymbol{H}| \ll \omega_1, \omega_2$, when the helicity is expressed by terms proportional to $(\omega_i+\omega_j)(\boldsymbol{k}\cdot\boldsymbol{\Omega})$. This means, that the helical forced waves propagate either 'upwards' or 'downwards' relative to the direction of $\boldsymbol{\Omega}$. In order to have a non-zero net helicity in a superposition of such waves, the forcing must be such as to preferentially excite waves of either positive or negative helicity, so that the net helicity flux is non-zero. We assume that such symmetry breaking exists in the present cosmological context, and this allows us to focus on waves of, say, positive helicity (corresponding to the choice $\sigma = -2(\boldsymbol{k}\cdot\boldsymbol{\Omega})/k$ in (3.9)).

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We can now write down the general form of the mean EMF generated by waves forced at two distinct frequencies

$$\begin{aligned}\hat{\boldsymbol{\mathcal{E}}}(\boldsymbol{k}) = \langle \boldsymbol{u}' \times \boldsymbol{h}' \rangle &= \tfrac{1}{2}\,\mathrm{Re}[(\hat{\boldsymbol{u}}_1 \mathrm{e}^{-\mathrm{i}\omega_1 t} + \hat{\boldsymbol{u}}_2 \mathrm{e}^{-\mathrm{i}\omega_2 t}) \times (\hat{\boldsymbol{h}}_1^* \mathrm{e}^{\mathrm{i}\omega_1 t} + \hat{\boldsymbol{h}}_2^* \mathrm{e}^{\mathrm{i}\omega_2 t})] \\ &= \hat{\boldsymbol{\mathcal{E}}}_{(1;2)}(\boldsymbol{k}) + \hat{\boldsymbol{\mathcal{E}}}_1(\boldsymbol{k}) + \hat{\boldsymbol{\mathcal{E}}}_2(\boldsymbol{k}), \end{aligned} \tag{3.16}$$

where

$$\hat{\boldsymbol{\mathcal{E}}}_j(\boldsymbol{k}) = \tfrac{1}{2}\,\mathrm{Re}(\hat{\boldsymbol{u}}_j \times \hat{\boldsymbol{h}}_j^*) \tag{3.17}$$

for $j = 1, 2$ is the standard single-frequency contribution and the new effect is contained in

$$\hat{\boldsymbol{\mathcal{E}}}_{(1;2)}(\boldsymbol{k}) = \tfrac{1}{2}\,\mathrm{Re}(\hat{\boldsymbol{u}}_2 \times \hat{\boldsymbol{h}}_1^* \mathrm{e}^{\mathrm{i}\Delta\omega t} + \hat{\boldsymbol{u}}_1 \times \hat{\boldsymbol{h}}_2^* \mathrm{e}^{-\mathrm{i}\Delta\omega t}). \tag{3.18}$$

By the use of (3.4) and in the weak resistivity limit $\eta k^2 \ll |\sigma_j|$ we can simplify the latter expression to

$$\begin{aligned}\hat{\boldsymbol{\mathcal{E}}}_{(1;2)}(\boldsymbol{k}) &= -\frac{\boldsymbol{k}\cdot\boldsymbol{H}\Delta\omega}{2\omega_1^2\omega_2^2}[\omega_1\omega_2\,\mathrm{Im}(\mathrm{i}\hat{\boldsymbol{u}}_1^* \times \hat{\boldsymbol{u}}_2 \mathrm{e}^{\mathrm{i}\Delta\omega t}) - \eta k^2(\omega_1+\omega_2)\,\mathrm{Re}(\mathrm{i}\hat{\boldsymbol{u}}_1^* \times \hat{\boldsymbol{u}}_2 \mathrm{e}^{\mathrm{i}\Delta\omega t})] \\ &\quad + O\left(\frac{\eta^2 k^4}{\omega_1^2}, \frac{\eta^2 k^4}{\omega_2^2}\right). \end{aligned} \tag{3.19}$$

Once again, for completeness we rewrite the single-frequency EMF from Moffatt (1978, (10.46) in §10.3),

$$\hat{\boldsymbol{\mathcal{E}}}_j(\boldsymbol{k}) = -\eta k^2 \frac{\boldsymbol{k}\cdot\boldsymbol{H}}{2\omega_j^2}(\mathrm{i}\hat{\boldsymbol{u}}_j^* \times \hat{\boldsymbol{u}}_j) + O\left(\frac{\eta^3 k^6}{\omega_j^3}\right), \tag{3.20}$$

where only the leading-order term in η was retained. Note that the second term in (3.19) is of the same order in terms of η and of similar form to the single-frequency EMF in (3.20). Equations (3.11), (3.8) and (3.6) allow us to obtain an explicit formula for the single-frequency EMF, however, we can observe already by comparison of the leading-order term in (3.19) and (3.20), that when the plasma resistivity is very weak, $\eta k^2 \ll \Delta\omega$ the single-frequency EMF (3.20) is much weaker than the one generated by interactions between waves with distinct phase velocities (3.19).

The next step is to assume that the frequencies of the forcing are close enough, that $\Delta\omega \ll \omega_1, \omega_2$ and the oscillations can be removed by approximating the exponentials in (3.19) by $\exp(\mathrm{i}\Delta\omega t) \approx 1 + \mathrm{i}\Delta\omega t$ and then demonstrate, that the newly obtained mechanism of EMF generation indeed leads to an α-effect, i.e. a large-scale dynamo. This will be done in the following sections.

4. Mean EMF induced by forced beating waves in an ideal fluid

As argued in the introduction in the case extremely highly conductive plasma, such as the interstellar or intra-cluster plasma, when the characteristic magnetic diffusion times are many orders of magnitude greater than the age of the universe, it seems legitimate to neglect the effect of resistivity and study the case of an ideal fluid. The mean EMF greatly simplifies when $\eta = 0$. Using (3.4) and (3.7), this is given by

$$\hat{\boldsymbol{\mathcal{E}}}(\boldsymbol{k}) = -\frac{\Delta\omega(\omega_1+\omega_2)}{\omega_1^2\omega_2^2 D_1 D_2}(\boldsymbol{k}\cdot\boldsymbol{H})(\boldsymbol{k}\cdot\boldsymbol{\Omega})[\omega_1\omega_2 - (\boldsymbol{k}\cdot\boldsymbol{H})^2]k^2\boldsymbol{k}\,\mathrm{Im}(\hat{\boldsymbol{f}}_1^* \cdot \hat{\boldsymbol{f}}_2 \mathrm{e}^{\mathrm{i}\Delta\omega t}), \tag{4.1}$$

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and now from (3.6) and (3.8)

$$D_j = \frac{k^2}{\omega_j^2}[\omega_j^2 - (\boldsymbol{k}\cdot\boldsymbol{H})^2]^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2. \tag{4.2}$$

We will now consider an asymptotic limit of a very small difference between the frequencies of the forcing ω_1 and ω_2

$$\Delta\omega = \omega_1 - \omega_2 \ll \omega_1, \omega_2, \tag{4.3}$$

which allows us to simplify the expression for the mean EMF at leading order even more

$$\hat{\boldsymbol{\mathcal{E}}}(\boldsymbol{k}) \approx -\Delta\omega \frac{2k^2\omega_1(\boldsymbol{k}\cdot\boldsymbol{\Omega})(\omega_1^2 - (\boldsymbol{k}\cdot\boldsymbol{H})^2)}{[k^2(\omega_1^2 - (\boldsymbol{k}\cdot\boldsymbol{H})^2)^2 - 4\omega_1^2(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2]^2}(\boldsymbol{k}\cdot\boldsymbol{H})\boldsymbol{k}\,\mathrm{Im}(\hat{\boldsymbol{f}}_1^*\cdot\hat{\boldsymbol{f}}_2). \tag{4.4}$$

Such a small difference between the frequencies of waves of the same type corresponds to the effect of 'beat' in acoustics, this is why we adopt the term 'beating waves' in the considered setting.

The EMF (4.4) is present even in the perfectly conducting limit $\eta = 0$. In this limit, it is the interactions between beating waves forced at different but close frequencies, say $\boldsymbol{u}_1'$ and $\boldsymbol{h}_2'$, that produce a non-zero mean $\langle \boldsymbol{u}_1' \times \boldsymbol{h}_2' \rangle$. This is an interesting and novel situation, because the $\eta \to 0$ limit usually possesses severe difficulties in dynamo theory. Here however, we have potentially a well-defined α-effect. We shall explore the consequences in § 5. For now let us write down the final form of the mean EMF in the limit of a weak seed mean magnetic field, which yields

$$\hat{\boldsymbol{\mathcal{E}}}(\boldsymbol{k}) \approx -\Delta\omega \frac{2k^2(\boldsymbol{k}\cdot\boldsymbol{\Omega})}{\omega_1[k^2\omega_1^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2]^2}(\boldsymbol{k}\cdot\boldsymbol{H})\boldsymbol{k}\,\mathrm{Im}(\hat{\boldsymbol{f}}_1^*\cdot\hat{\boldsymbol{f}}_2). \tag{4.5}$$

It is of some interest to comment on the possible case of resonant response of the system defined by (3.9). In an ideal fluid a perfect resonance coincidence of the forcing and eigen frequencies is not possible, as it would lead to an infinite amplitude of the forced wave. This is why only a small departure from resonant condition (3.9) is of interest, since it corresponds to modes with maximal amplitudes in the system. Denoting the eigen frequency of the system i.e. the frequency of Lehnert waves, by $\bar{\omega}$, we can consider modes satisfying

$$\omega_1 = \bar{\omega} + \delta\omega + \Delta\omega, \quad \omega_2 = \bar{\omega} + \delta\omega, \tag{4.6a,b}$$

$$\Delta\omega = \omega_1 - \omega_2 \ll \omega_1, \omega_2, \quad \delta\omega \ll \omega_1, \omega_2 \tag{4.6c,d}$$

and the Lehnert wave frequency is recalled here for clarity

$$\bar{\omega} = -\boldsymbol{k}\cdot\boldsymbol{\Omega}/k \pm \sqrt{(\boldsymbol{k}\cdot\boldsymbol{\Omega}/k)^2 + (\boldsymbol{k}\cdot\boldsymbol{H})^2}, \tag{4.7}$$

and either the plus or minus sign can be used. The mean EMF under near-resonance conditions, at the leading order takes the form

$$\hat{\boldsymbol{\mathcal{E}}}(\boldsymbol{k}) = -\frac{1}{16}\frac{\Delta\omega}{\delta\omega(\Delta\omega + \delta\omega)}\frac{\boldsymbol{k}\cdot\boldsymbol{H}}{(\boldsymbol{k}\cdot\boldsymbol{\Omega}/k)^2 + (\boldsymbol{k}\cdot\boldsymbol{H})^2}\frac{\boldsymbol{k}}{k}\,\mathrm{Im}(\hat{\boldsymbol{f}}_1^*\cdot\hat{\boldsymbol{f}}_2). \tag{4.8}$$

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Note, that the factor $\bar{\omega}\Delta\omega/\delta\omega(\Delta\omega + \delta\omega)$ is large, when $\delta\omega \ll \Delta\omega^{1/2}$. Exact resonance coincidence of either ω_1 or ω_2 with $\bar{\omega}$ is possible in a real fluid, when the mean EMF becomes of the order $\sim 1/\eta k^2$, cf. Moffatt (1978, §10.3). The EMF from beating near-resonant waves in (4.8) is then dominant over the EMF induced by a single wave exactly at resonance with forcing if $\delta\omega \ll \eta k^2 \ll \Delta\omega$ is satisfied. Moreover, in a real fluid the resonant response is mostly probably localized in thin layers of enhanced diffusion (such as current sheets) occurring for short-wavelength waves; then the global dynamical significance of the resonances seems to be reduced.

5. Dynamo action with $\eta = 0$

We will now study in detail the possibility of dynamo action from initially weak seed field, with the EMF given in (4.5), that is without the effect of resonant response of the system to external forcing. First let us note that a spectrum of waves of different $\boldsymbol{k}$ will provide additive contributions to the total EMF $\boldsymbol{\mathcal{E}}$ which then takes the form

$$\boldsymbol{\mathcal{E}} = -\frac{\Delta\omega}{\omega_1} \fint \frac{2k^2(\boldsymbol{k}\cdot\boldsymbol{\Omega})}{[k^2\omega_1^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2]^2}(\boldsymbol{k}\cdot\boldsymbol{H})\boldsymbol{k}\,\mathrm{Im}(\hat{\boldsymbol{f}}_1^*\cdot\hat{\boldsymbol{f}}_2)\,\mathrm{d}^3\boldsymbol{k}, \tag{5.1}$$

where the symbol $\fint$ allows for a discrete and/or continuous spectrum. Let us also assume that ω_1 is such that the resonance coincidence $\omega_1 = -2\boldsymbol{k}\cdot\boldsymbol{\Omega}/k$ is not possible, so that the denominator in the integrand above is strictly positive. To fix ideas we can assume that the two forcing components $\hat{\boldsymbol{f}}_1$ and $\hat{\boldsymbol{f}}_2$ have the same amplitudes and a phase shift of $\pi/2$ i.e. $\hat{\boldsymbol{f}}_1 = \mathrm{i}\hat{\boldsymbol{\mathcal{F}}}(\boldsymbol{k})$, $\hat{\boldsymbol{f}}_2 = \hat{\boldsymbol{\mathcal{F}}}(\boldsymbol{k})$ and $\hat{\boldsymbol{\mathcal{F}}}(\boldsymbol{k}) \in \mathbb{R}$, thus

$$\mathcal{E}_i = \alpha_{ij}H_j \quad \text{where } \alpha_{ij} = \frac{\Delta\omega}{\omega_1} \fint \frac{2k^2(\boldsymbol{k}\cdot\boldsymbol{\Omega})k_ik_j}{[k^2\omega_1^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2]^2}\hat{\mathcal{F}}(\boldsymbol{k})^2\,\mathrm{d}^3\boldsymbol{k}. \tag{5.2}$$

This is an α-effect, in general anisotropic. For simplicity, let us suppose that the force spectrum $\hat{\mathcal{F}}(\boldsymbol{k})^2$ is symmetric about the direction of $\boldsymbol{\Omega}$; then α_{ij} is also axisymmetric. With $\boldsymbol{\Omega} = (0,\ 0,\ \Omega)$ in Cartesian coordinates Oxyz, and with $\hat{\mathcal{F}}(\boldsymbol{k})^2 = \hat{\mathcal{F}}^2(k,\ \theta)$ where θ is co-latitude, we then have

$$\alpha_{ij} = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \tag{5.3}$$

where

$$\gamma = \frac{2\pi\Omega\,\Delta\omega}{\omega_1} \int \mathrm{d}k \int_{-\pi/2}^{\pi/2} \frac{k\cos\theta\,\sin^3\theta}{(\omega_1^2 - 4\Omega^2\cos^2\theta)^2}\hat{\mathcal{F}}^2(k,\theta)\,\mathrm{d}\theta, \tag{5.4a}$$

$$\alpha = \frac{4\pi\Omega\,\Delta\omega}{\omega_1} \int \mathrm{d}k \int_{-\pi/2}^{\pi/2} \frac{k\cos^3\theta\,\sin\theta}{(\omega_1^2 - 4\Omega^2\cos^2\theta)^2}\hat{\mathcal{F}}^2(k,\theta)\,\mathrm{d}\theta. \tag{5.4b}$$

The α-effect becomes isotropic ($\gamma = \alpha$) in the special circumstance that all the constituent wave vectors are concentrated on the cone of angle $\theta = \theta_0$ where $\tan^2\theta_0 = 2$, i.e. $\theta_0 \approx 54.7°$, say positive. In this situation, $\alpha_{ij} = \alpha\delta_{ij}$, and if we

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further assume, that the waves are concentrated around a dominant wavenumber $k = k_0$, then

$$\alpha = \frac{4\pi\Omega\,\Delta\omega}{\omega_1 k_0}\,\mathscr{F}^2, \tag{5.5}$$

where

$$\mathscr{F}^2 = \int \mathrm{d}k \int_{-\pi/2}^{\pi/2} \frac{\cos^3\theta\,\sin\theta}{k(\omega_1^2 - 4\Omega^2\cos^2\theta)^2}\hat{\mathcal{F}}^2(k,\theta)\delta(k-k_0)\delta(\theta-\theta_0)\,\mathrm{d}\theta > 0, \tag{5.6}$$

denotes the effect of the forcing. In order to illustrate the possible behaviour in the simplest way, we now adopt this simple formula.[3]

With $\boldsymbol{\mathcal{E}} = \alpha\boldsymbol{H}$, the mean-field equation (2.3*b*,*c*) (with $\eta = 0$) becomes

$$\frac{\partial \boldsymbol{H}}{\partial t} = \alpha\,\boldsymbol{\nabla}\times\boldsymbol{H}. \tag{5.7}$$

This equation admits growing modes with force-free structure $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{x},\,t) = K\boldsymbol{H}(\boldsymbol{x},\,t)$, where $K \ll k_0$ (to satisfy the scale separation implicit in the mean-field theory). Then (5.7) becomes

$$\frac{\partial \boldsymbol{H}}{\partial t} = K\alpha\,\boldsymbol{H}, \tag{5.8}$$

with solution

$$\boldsymbol{H}(\boldsymbol{x},\,t) = \boldsymbol{H}(\boldsymbol{x},0)\exp\left[\frac{4\pi\Omega\,\Delta\omega}{\omega_1 k_0}\,\mathscr{F}^2 t\right], \tag{5.9}$$

which indicates an exponential growth of the mean field at the initial stage, with a growth rate of $4\pi\Omega\,\Delta\omega\mathscr{F}^2/\omega_1 k_0$. Consideration of a force-free mode is crucial in the considered astrophysical context of early universe. With negligible magnetic diffusion such a force-free mode when amplified by an α-effect can remain undamped, since there is no mechanism of energy transfer from magnetic to kinetic energy and thus no mechanism of magnetic energy loss.

The obtained α-effect is quenched, as indicated by (4.4), and the quenching is of the same type as in the standard diffusive mechanism, since at $|\boldsymbol{k}\cdot\boldsymbol{H}| \to \infty$ the obtained mean EMF behaves like $|\boldsymbol{k}\cdot\boldsymbol{H}|^{-5}$ (cf. Moffatt (1978, §10.3)). This quenching becomes more important as the field grows in magnitude and eventually the nonlinearities lead to equilibration of the field at some level. In the present case there is also another effect, which manifests at large times, namely the time oscillations of the mean EMF on the very long time scale $1/\Delta\omega$, which are negligible at the leading order, but eventually start to matter. When this time scale is much larger than the inverse growth rate of the mean field, the oscillations start to matter only in the already nonlinear regime, when the turbulence had become strong and therefore the first-order smoothing approximation and thus the obtained EMF (4.1) is no longer valid. Surely, eventually some saturated state is obtained, however, such a fully nonlinear regime is outside the scope of this paper.

The question of whether this is a fast dynamo or not is perhaps a matter of semantics. In the kinematic regime in which the Lorentz force is neglected, the conventional distinction between fast and slow dynamos is clear; in the dynamical regime studied here, we have found an α-effect that oscillates on a long time scale.

[3]The assumption of isotropy is not necessary; if $\gamma \neq \alpha$ but $\gamma\alpha > 0$, magnetic energy growth still occurs, as described for the case of constant α_{ij} by Moffatt (1970).

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Starting from an infinitesimal seed field, the long-term behaviour governed by such an α-effect will then also be oscillatory, so this is not a fast dynamo in the strict sense. However, in practice, starting from a field of small but finite amplitude, the growth rate is approximately constant and independent of η at least until nonlinear effects cause saturation, at which stage the fast/slow distinction is no longer relevant. Perhaps we may simply describe this as 'time-limited fast dynamo' action.

6. Conclusions

The simple calculation of the α-effect induced by interactions between distinct waves in the presence of an oscillatory forcing in a highly conducting plasma allowed us to demonstrate a new 'fast' dynamo mechanism (with finite field amplification rate independent of η), that is operating even in the absence of fluid's resistivity. The idea of interacting linear waves stems from the weak magnetohydrodynamic turbulence, which is known to persist in many systems for long times (cf. Ng & Bhattacharjee (1996) and Tobias *et al.* (2013)). The mechanism of mean EMF generation from interactions of waves with distinct but close frequencies (thus the term beating waves) was identified. Such an EMF oscillates in time at a very small frequency equal to the difference between frequencies of the waves, i.e. of the distinct oscillatory components of the forcing. This time dependence is very slow, when the frequencies are close, and it is shown here, that it can be neglected and the EMF leads to amplification of a weak seed field, i.e. dynamo generation of a large-scale magnetic field.

It is hypothesized, that the studied and described mechanism might be relevant to dynamo generation of magnetic fields in extremely highly conducting interstellar or intergalactic plasma in the early universe when the galaxies were formed. It is supposed here, that at that early stage the turbulence is weak, i.e. dominated by terms linear in the turbulent fluctuations. Because of the extremely high conductivity in such systems, giving characteristic diffusion times many orders of magnitude greater than the age of the universe, the so-generated large-scale magnetic field at the early stage could persist even at the later stages of evolution, since diffusive effects are too weak to be observable. Particular attention is focused on the force-free mode of the large-scale field, for which no mechanism of magnetic energy loss exists, when the resistivity is neglected. Such a mode can survive many universe lifetimes without being damped.

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