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Scattering of electromagnetic waves in a magnetized plasma with an HF pump

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Abstract. The scattering of transverse electromagnetic waves by turbulent density fluctuations in a magnetized plasma in the presence of external pump fields is investigated. The case where a lower-hybrid pump wave decays into daughter and ion-cyclotron waves in a plasma with ion-temperature anisotropy is considered. The situation where an upper-hybrid pump wave parametrically excites modified convective cells and electron drift waves is also analyzed. The differential scattering cross-sections in the region above the parametric instability thresholds for these three cases are calculated.

1. Introduction

It is known that in a medium with a certain fluctuation level, the propagation of electromagnetic waves can lead to radiation of waves with new frequencies and wavenumbers, i.e. scattered waves. A great deal of information about the properties of the plasma may be obtained from the spectral properties of the scattered radiation. Investigations of electromagnetic-wave scattering by density fluctuations are important for studying such problems as plasma diagnostics, wave transformation mechanisms in plasmas, measurements of the efficiency of high-frequency (HF)-pump power dissipation, and the definition of dispersion properties of plasma wave processes.

Obviously, the scattering intensity will be sufficiently large in a non-equilibrium plasma when the level of fluctuations is much higher than that of thermal fluctuations. In such a plasma in which instability can develop, the scattering intensity may increase without limit near the instability threshold (Akhiezer et al. 1978; Sitenko 1982).

Combined wave scattering and transformation in a plasma were predicted by Akhiezer et al. (1957).

The theory of scattering of electromagnetic waves in a plasma was developed by Dougherty and Farley (1960), Rosenbluth and Rostoker (1962) and Salpeter (1960), and is described in detail by Bekefi (1976), Sheffield (1975) and Akhiezer et al. (1978).

The problem of fluctuations in a non-equilibrium plasma was considered by Rosenbluth and Rostoker (1962). These authors showed that their results could be obtained by a simple method known as the dressed particle approach. This ap-

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proach has since become a popular way of deriving expressions for the fluctuation spectrum.

Expressions for the fluctuations in density, current, fields and velocity distributions, all for magnetized equilibrium plasmas, were derived in Akhiezer et al. (1962). The fluctuation–dissipation theorem was used in these derivations, including the derivation of the fluctuations in the velocity distribution.

Scattering and transformation of waves in a magnetized plasma were considered by Sitenko and Kirochkin (1966).

Incoherent scattering was reviewed by Evans and Katzenstein (1969). Expressions for the spectral density of the fluctuations using the dressed particle approach were derived. Most attention was paid to the non-magnetized case.

Golant (1968), Heald and Wharton (1969) and Sheffield (1975) have described the possibilities for the use of scattering in diagnostics of laboratory and space plasmas.

Wave-scattering processes in a plasma in the presence of a pump wave in the lower-hybrid range of frequencies were considered by Pavlenko and Panchenko (1990).

The expression for the density fluctuation spectrum from a generalized dressed particle approach allowing for electromagnetic effects was obtained in Chiu (1992). It was shown that a considerable enhancement of the power density in the fluctuation spectrum is found around frequencies that satisfy the dispersion relation.

In Wilhelmsson et al. (1991a), the scattering of electromagnetic waves by density fluctuations in a magnetized plasma was found to depend significantly upon the presence of parametric decay instabilities driven by a lower-hybrid pump wave. This offers interesting new approaches to plasma diagnostics.

Wave-scattering processes in a plasma with ion-temperature anisotropy were considered by Wilhelmsson et al. (1991b). Note that the anisotropy of ion temperature plays an important role in the stability properties of the fusion plasmas.

In recent years, the kinetic theory of fluctuation and scattering processes in plasma has been intensively investigated by Pavlenko (1998), Pavlenko et al. (1999, 2000), Sitenko (1999, 2000) and Sitenko and Kharchenko (1999).

In the present paper, the scattering of transverse electromagnetic waves by turbulent density fluctuations in a magnetized plasma in the presence of external pump fields is investigated.

Note that the investigation of the scattering of a transverse electromagnetic wave by density fluctuations in a magnetized plasma in the presence of parametric decay of the pump wave is a realistic problem in plasma physics. It is known that in a plasma subjected to an HF pump wave, the excitation of different parametric instabilities is possible, and a considerable enhancement of the plasma-fluctuation level occurs near the parametric instability threshold. Thus the differential scattering cross-section depending significantly upon the presence of parametric instabilities driven by an HF pump exceeds the corresponding cross-section due to thermal noise in a magnetized plasma in the absence of a pump-wave field.

Previously, in Pavlenko and Panchenko (1990), the infinite increase in the scattering cross-section (within the framework of linear theory) near the parametric decay threshold was considered. In order to study scattering in the region above the threshold, it is necessary to take into consideration the stabilization mechanism that leads to saturation of the fluctuation level. In the present paper, a mechanism involving charged-particle scattering by turbulent fluctuations is considered (see Pavlenko et al. 1986). We consider the case where a lower-hybrid pump wave is

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supposed to decay into lower-hybrid and ion-cyclotron waves in a plasma with ion-temperature anisotropy. We also analyze the situation where an upper-hybrid pump wave decays into an upper-hybrid wave and modified convective cells. We investigate the combination scattering of electromagnetic waves by electron-density fluctuations in an inhomogeneous magnetized plasma, assuming that the pump wave may decay into upper-hybrid and electron-drift waves.

The scattering cross-sections of electromagnetic waves by density fluctuations in the region above parametric instability thresholds for these three cases are calculated.

2. Basic equations

Let us consider the propagation of transverse eleotromagnetic waves in a plasma. Wave scattering occurs because there is interaction between fluctuations and electromagnetic waves. In order to investigate the scattering of electromagnetic waves in a magnetized plasma, we confine attention to scattering by electron-density fluctuations. The differential scattering cross-section in this case is given by

$$d\Sigma = \frac{1}{2\pi}\left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2\omega''^2}{\omega_{pe}^4} R|\xi|^2 \langle \delta n_e^2 \rangle_{\Delta\omega,\mathbf{q}}\, d\omega''\, d\Theta, \tag{1}$$

where $\Delta\omega = \omega' - \omega''$, $\mathbf{q} = \mathbf{k}' - \mathbf{k}''$, ω', $\mathbf{k}'$ and ω'', $\mathbf{k}''$ are the frequencies and wave vectors of the incident and scattered waves respectively, Θ is the space angle, and $\langle \delta n_e^2 \rangle_{\Delta\omega,\mathbf{q}}$ is the correlator of the electron-density fluctuations at the combination frequency $\Delta\omega$. The factor $R|\xi|^2$ depends on the incident and scattered wave directions with respect to the magnetic field direction, and has a well-known form (Sitenko 1967; Akhiezer et al. 1978).

Note that (1) holds when the frequencies of the incident and scattered waves are much greater than the electron Langmuir frequency, i.e. ω', $\omega'' \gg \omega_{pe}$.

It follows from (1) that to calculate the differential cross-section, the expression for electron-density fluctuations must be derived.

We shall consider fluctuations in an electron–ion magnetized plasma (with constant magnetic field $\mathbf{B}_0$ directed along the z axis) under the influence of a radiofrequency pump-wave field $\mathbf{E}_0(t) = E_0\mathbf{y}\cos\omega_0 t$, which is taken in the dipole approximation.

The procedure for deriving the equations for a fluctuating electric field in such a plasma is presented in detail by Pavlenko et al. (1986). The fluctuation of the distribution function satisfies the equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{e_\alpha}{c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial}{\partial \mathbf{p}} + e_\alpha \mathbf{E}_0(t)\cdot\frac{\partial}{\partial \mathbf{p}}\right](\delta f_\alpha - \delta f_\alpha^s) = -e_\alpha n_\alpha \delta\mathbf{E}\cdot\frac{\partial f_\alpha}{\partial \mathbf{p}}, \tag{2}$$

where δf_α^s is the distribution-function fluctuation with the interaction between particles neglected, $\delta\mathbf{E}$ is the fluctuating electric field ($\alpha = e, i$). For the latter we find

$$\nabla\cdot\delta\mathbf{E} = 4\pi\sum_\alpha e_\alpha \int d\mathbf{p}\,\delta f_\alpha. \tag{3}$$

Solving (3) using the method of characteristics for the Fourier component of the

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distribution-function fluctuation, we have

$$\delta f_\alpha(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p}) = \delta f^s_\alpha(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p}) - e_\alpha n_\alpha \sum_{n,m} J_n(a_\alpha)J_m(a_\alpha)$$
$$\times \int_0^\infty d\tau \exp\left\{ i(\omega + n\omega_0 - k_\parallel v_\parallel)\tau - i\frac{k_\perp v_\perp}{\Omega_\alpha}[\sin(\varphi+\Omega_\alpha\tau) - \sin\varphi]\right\}$$
$$\times \left[\frac{k_\parallel}{k}\frac{\partial}{\partial p_\parallel} + \cos(\varphi+\Omega_\alpha\tau)\frac{k_\perp}{k}\frac{\partial}{\partial p_\perp}\right] f_\alpha(p_\perp,p_\parallel)\,\delta E(\omega+(n-m)\omega_0,\mathbf{k}). \quad (4)$$

Here $\Delta\mathbf{p}$ is the momentum change due to the pump-wave field, φ is the azimuthal angle in velocity space, J_n is the Bessel function of order n, and $a_\alpha = \mu\Omega_\alpha^2/(\omega_0^2-\Omega_\alpha^2)$, where $\mu = k_\perp E_0 c/\omega_0 B_0 \ll 1$ and Ω_α is the cyclotron frequency.

We note that in obtaining (4), we have supposed that trajectories of charged particles are defined by the expressions

$$r_x = R_x + \frac{P_x}{m_\alpha\Omega_\alpha}\sin\Omega_\alpha\tau + \frac{P_y}{m_\alpha\Omega_\alpha}(1-\cos\Omega_\alpha\tau)$$
$$-\frac{e_\alpha E_0}{m_\alpha(\omega_0^2-\Omega_\alpha^2)}\frac{\Omega_\alpha}{\omega_0}[\sin\omega_0 t - \sin\omega_0(t-\tau)],$$

$$r_y = R_y + \frac{P_x}{m_\alpha\Omega_\alpha}\sin\Omega_\alpha\tau - \frac{P_x}{m_\alpha\Omega_\alpha}(1-\cos\Omega_\alpha\tau)$$
$$-\frac{e_\alpha E_0}{m_\alpha(\omega_0^2-\Omega_\alpha^2)}[\cos\omega_0 t - \cos\omega_0(t-\tau)],$$

$$r_z = R_z + \frac{P_z}{m_\alpha}\tau,$$

where $\mathbf{R}$ and $\mathbf{P}$ are the initial values of the coordinate and momentum, and $\tau = t-t_0$.

Substituting (4) into (3) we obtain a system of equations for the fluctuating electric field (Pavlenko et al. 1986):

$$\delta\mathbf{E}(\omega,\mathbf{k}) + \sum_\alpha\sum_{n,m}\chi^n_\alpha J_n(a_\alpha)J_m(a_\alpha)\,\delta\mathbf{E}(\omega+(n-m)\omega_0,\mathbf{k}) = \delta\mathbf{E}^s(\omega,\mathbf{k}), \quad (5)$$

where $\chi^n_\alpha \equiv \chi_\alpha(\omega+n\omega_0,\mathbf{k})$ is the linear plasma susceptibility, defined by the formula

$$\chi^n_\alpha = \frac{4\pi e_\alpha^2 n_\alpha}{k^2}\int d\mathbf{p}\sum_r J_r^2\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right)\frac{1}{\omega - k_\parallel v_\parallel - r\Omega_\alpha + n\omega_0}$$
$$\times\left(\frac{r\Omega_\alpha}{v_\perp}\frac{\partial}{\partial p_\perp} + k_\parallel\frac{\partial}{\partial p_\parallel}\right) f_\alpha(p_\perp,p_\parallel); \quad (6)$$

$\delta\mathbf{E}^s$ is the source fluctuating electric field.

It is known that the spectral density of the electron-density fluctuations can be expressed in terms of the distribution-function fluctuation correlator in the form

$$\langle\delta n_e(\omega,\mathbf{k})\,\delta n_e(\omega_1,\mathbf{k}_1)\rangle = \int\langle\delta f_e(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p})\,\delta f_e(\omega_1,\mathbf{k}_1,\mathbf{p}_1+\Delta\mathbf{p}_1)\rangle\,d\mathbf{p}\,d\mathbf{p}_1. \quad (7)$$

For convenience, we shall calculate the correlator $\mathbf{k}\cdot\mathbf{k}_1\langle\delta f_e(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p})\,\delta f_e(\omega_1,\mathbf{k}_1,\mathbf{p}_1,+\Delta\mathbf{p}_1)\rangle$. We first multiply both sides of (4) by $\mathbf{k}$ and average over the angle φ in the second term on the right-hand side of this equation. Secondly,

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we integrate (4) over τ and $\mathbf{p}$ and take into account that the argument a_α of the Bessel functions is small, i.e. we retain the terms with $n,\ m = 0, \pm 1$ in the sum over n and m. Solving (5), we can express the fluctuating electric field in terms of source-field fluctuations, and it is easy to obtain the equation for the fluctuation of the electron distribution function:

$$\int \mathbf{k}\,\delta f_e(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p})\,d\mathbf{p} = \int \mathbf{k}\,\delta f_e^s(\omega,\mathbf{k},\mathbf{p}+\Delta\mathbf{p})\,d\mathbf{p} + \frac{ik^2}{4\pi e\varepsilon_E(\omega,\mathbf{k})} \times \left[\frac{\mu}{2}\frac{\chi_e^0\chi_i^0\,\delta\mathbf{E}^s_{\omega-\omega_0,\mathbf{k}}}{\varepsilon_{-1}(\omega-\omega_0,\mathbf{k})} + \chi_e^0\,\delta\mathbf{E}^s_{\omega,\mathbf{k}} - \frac{\mu}{2}\frac{\chi_e^0\chi_i^0\,\delta\mathbf{E}^s_{\omega+\omega_0,\mathbf{k}}}{\varepsilon_{+1}(\omega+\omega_0,\mathbf{k})}\right], \tag{8}$$

where

$$\varepsilon_E(\omega,\mathbf{k}) = \varepsilon_0(\omega,\mathbf{k}) + \frac{\mu^2}{4}\chi_e^0\chi_i^0\left[\frac{1}{\varepsilon_{+1}(\omega+\omega_0,\mathbf{k})} + \frac{1}{\varepsilon_{-1}(\omega-\omega_0,\mathbf{k})}\right]$$

and

$$\varepsilon_n(\omega+n\omega_0,\mathbf{k}) = 1 + \sum_\alpha \chi_\alpha^n, \qquad n = 0, \pm 1.$$

Multiplying (8) by the complex-conjugate equation, carrying out the statistical averaging and averaging over the period $2\pi/\omega_0$ of the HF field, we find from (6) the spectral density of the electron-density fluctuations:

$$\langle\delta n_e^2\rangle_{\omega,\mathbf{k}} = \langle\delta n_e^2\rangle^0_{\omega,\mathbf{k}}\left[1 - \frac{\chi_e^{0^*}}{\varepsilon_E^*(\omega,\mathbf{k})} - \frac{\chi_e^0}{\varepsilon_E(\omega,\mathbf{k})}\right] + \frac{k^2}{16\pi^2e^2}\frac{|\chi_e^0|^2}{|\varepsilon_E(\omega,\mathbf{k})|^2} \times \left[\langle\delta\mathbf{E}^2\rangle^0_{\omega,\mathbf{k}} + \frac{\mu^2}{4}|\chi_i^0|^2\left(\frac{\langle\delta\mathbf{E}^2\rangle^0_{\omega+\omega_0,\mathbf{k}}}{|\varepsilon_{+1}|^2} + \frac{\langle\delta\mathbf{E}^2\rangle^0_{\omega-\omega_0,\mathbf{k}}}{|\varepsilon_{-1}|^2}\right)\right], \tag{9}$$

where $\langle\delta\mathbf{E}^2\rangle^0_{\omega,\mathbf{k}}$ and $\langle\delta\mathbf{E}^2\rangle^0_{\omega\pm\omega_0,\mathbf{k}}$ are the common-correlator spectral functions of non-interacting particles in a magnetized plasma in the absence of the pump field (Klimontovich 1982).

We introduce in (9) the correlator of the electron-density fluctuations for non-interacting particles:

$$\langle\delta n_e^2\rangle^0_{\omega,\mathbf{k}} = n_e\sum_r \int 2\pi\rho_\perp\,d\rho_\perp \int d\rho_\parallel\,2\pi\delta(\omega - k_\parallel v_\parallel - r\Omega_e)J_r^2\left(\frac{k_\perp v_\perp}{\Omega_e}\right) f(p_\perp,p_\parallel). \tag{10}$$

Note that from (9) for $E_0 = 0$, we have obtained the correlator of the electron density in a magnetized plasma without a pump (see e.g. Sitenko 1982, equation (6.84)).

3. Scattering processes in a homogenous magnetized plasma

Consider an electron–ion plasma imbedded in an external magnetic field $B_0\mathbf{z}$. Furthermore, the plasma is subjected to an HF pump field, whose electric field is directed perpendicular to the external magnetic field. For a long-wavelength ($\mathbf{k}_0 = 0$),

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lower-hybrid pump, we can write $\mathbf{E}(t) = E_0\mathbf{y}\cos\omega_0 t$, where the pump frequency ω_0 is chosen to be near the lower-hybrid resonance frequency defined by $\omega_{LH} = \omega_{pi}(1+\omega_{pe}^2/\Omega_e^2)^{-1/2}$.

It is known that in such a plasma subjected to a lower-hybrid pump wave, the excitation of different parametric instabilities is possible (Porkolab 1974, 1977). We study the parametric coupling of the lower-hybrid pump with the harmonics of the ion-cyclotron frequency ($\omega \approx n\Omega_i$). The frequency and the damping rate of these oscillations at $n = 1$ are determined by (Wilhelmsson et al. 1991b)

$$\omega^{(1)} = \Omega_i[1 + A_1(\beta_{\perp i})], \tag{11}$$

$$\operatorname{Im}\omega \approx \gamma_t^{(1)} = \left(\frac{\pi}{2}\right)^{1/2} \frac{A_1^3\Omega_i^2}{k_\parallel v_{\parallel i}} \frac{T_{\perp i}}{T_{\parallel i}} \exp\left(-\frac{A_1^2\Omega_i^2}{2k_\parallel^2 v_{\parallel i}^2}\right). \tag{12}$$

In (12), $A_n(\beta_{\perp i}) = I_n(\beta_{\perp i})e^{-\beta_{\perp i}}$, where I_n is the modified Bessel function, and $\beta_{\perp i} \approx (k\rho_{\perp i})^2 > 1$.

We study electromagnetic-wave scattering by density fluctuations when parametric decay of the pump wave into lower-hybrid and ion-cyclotron waves occurs:

$$\omega_0 = \omega_{LH} + \omega^{(1)}. \tag{13}$$

Substituting (9) into (13), we can find the differential scattering cross-section.

It is shown in Pavlenko and Panchenko (1990) that low-frequency oscillations make the main contribution to the correlator of the electron-density fluctuations, because the contribution of lower-hybrid fluctuations is small. Thus, the differential scattering cross-section after integrating over ω'' can be written in the form

$$\frac{d\Sigma}{d\Theta} = \frac{d\Sigma_+}{d\Theta} + \frac{d\Sigma_-}{d\Theta},$$

where

$$\frac{d\Sigma_\pm}{d\Theta} = \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2(\omega' \pm \omega^{(1)})}{\omega_{pe}^4} R|\xi|^2 \frac{\pi}{\operatorname{Im}\varepsilon_E(\Delta\omega,\mathbf{q})\frac{\partial \operatorname{Re}\varepsilon_E}{\partial\Delta\omega}\Big|_{\Delta\omega'=\pm\omega^{(1)}}}$$
$$\times\left(\left|1+\chi_i^0+\frac{\mu^2}{4}\chi_e^0\chi_i^0\frac{1}{\varepsilon_{\mp 1}}\right|^2\langle\delta n_e^2\rangle_{\Delta\omega,\mathbf{q}}^0 + |\chi_e^0|^2\langle\delta n_i^2\rangle_{\Delta\omega,\mathbf{q}}^0\right.$$
$$\left.+\frac{\mu^2}{4}\frac{q^2}{16\pi^2e^2}|\chi_e^0|^2|\chi_i^0|^2\frac{1}{|\varepsilon_{\mp 1}|^2}\langle\delta\mathbf{E}^2\rangle_{\Delta\omega\mp\omega_0,\mathbf{q}}\right). \tag{14}$$

It is shown in Pavlenko and Panchenko (1990) that in the region near the instability threshold $\operatorname{Im}\varepsilon_E \approx (1 - E_0^2/E_{\text{th}}^2)$, where E_{th} is the threshold electric field strength value relative to the excitation of the parametric instability:

$$E_{\text{th}}^2 = 8\frac{\omega_0^2 B_0^2 r_{De}^2}{c^2}\frac{1}{A_0A_1}\frac{\gamma_i^{(1)}\gamma_{LH}}{\omega^{(1)}\omega_{LH}}. \tag{15}$$

It follows from (14) that anomalous growth of the intensity of the scattering electromagnetic wave will occur (i.e. there is critical opalescence) if the pump-field amplitude $E_0 \to E_{\text{th}}$.

In order to study scattering in the region above threshold, i.e. for $E_0 > E_{\text{th}}$, it is necessary to take into consideration the nonlinear stabilization mechanism

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that leads to saturation of the fluctuation level. This mechanism involving charged-particle scattering by turbulent fluctuations has been presented in detail by Pavlenko et al. (1986). According to this stabilization mechanism, we can describe wave scattering from fluctuations in terms of some effective collision frequency ν_{eff}. We also take into consideration the quantities $\tilde{\omega} = \omega + i\nu_{\text{eff}}$ as the eigenfrequencies of the plasma for calculating the density and electric field fluctuation correlators.

For describing the stationary state of the turbulent plasma, we use the redefined threshold value $\tilde{E}_{\text{th}} \approx E_{\text{th}}^2 \nu_{\text{eff}}^2 / \gamma_i^{(1)} \gamma_{LH}$ under the assumption that $\nu_{\text{eff}} > \gamma_i^{(1)}, \gamma_{LH}$. It has been shown by Pavlenko et al. (1992) that ν_{eff} is governed by

$$\nu_{\text{eff}} \approx \frac{E_0}{E_{\text{th}}} (\gamma_i^{(1)} \gamma_{LH})^{1/2}. \tag{16}$$

Taking into account that

$$\left.\begin{aligned} &\chi_e^0 = \frac{1}{q^2 r_{De}^2}, \qquad \chi_i^0 = -\frac{T_{\|i}}{T_{\perp i}} \frac{1}{q^2 r_{D\|i}^2} A_0, \\ &\operatorname{Im} \varepsilon_E \approx \frac{\gamma_i^{(1)} \omega^{(1)}}{\Omega_I^2 q^2 r_{D\|i}^2} \frac{T_{\|i}}{T_{\perp i}} \left(1 - \frac{E_0^2}{\tilde{E}_{\text{th}}^2}\right), \\ &\left.\frac{\partial \operatorname{Re} \varepsilon_E}{\partial \Delta\omega}\right|_{\Delta\omega = \omega^{(1)}} \approx \frac{\omega^{(1)}}{q^2 r_{Di}^2 \Omega_i^2} \frac{1}{A_1} \frac{T_{\|i}}{T_{\perp i}}, \end{aligned}\right\} \tag{17}$$

after a little algebra, we obtain from (14) the formula for the differential scattering cross-section:

$$\begin{aligned} \frac{d\Sigma_\pm}{d\Theta} = {} & \frac{1}{4\pi} \left(\frac{e^2}{mc^2}\right)^2 \frac{(\omega' \pm \omega^{(1)})^2}{\omega_{pe}^4} R |\xi|^2 \\ & \times \frac{q^2}{e^2} \frac{1}{q^2 r_{De}^2} \frac{T_{\|i}}{T_e} \frac{T_{\perp i}}{q^2 \rho_{\perp i}^2} \left[1 + \frac{\mu^2}{16\pi} \frac{T_e}{T_{\perp i}} \frac{\Omega_i^2}{\gamma_{LH} \gamma_i^{(1)}} \frac{1}{q^4 r_{D\|i}^2 \rho_{\perp i}^2}\right]. \end{aligned} \tag{18}$$

For the characteristic parameters of fusion plasmas and for the fields of the pump waves that are used for lower-hybrid heating of plasmas and for diagnostic purposes, we can suppose that $\mu \approx 10^{-1}$, $T_e/T_{\perp i} \approx 10$, $\Omega_i/\gamma_i^{(1)} \geqslant 10^3$ and $\Omega_i/\gamma_{LH} \approx 10$. It follows from comparison of terms in the square brackets in (18) that the term due to the pump-wave field exceeds the usual thermal term for the case where the external electric field is absent, i.e. when $\mu = 0$. It can be seen from (18) that the scattering cross-section depends explicitly on the temperature anisotropy.

As our second example, we study the case where the pump-wave frequency is close to the upper-hybrid frequency:

$$\omega_{UH1} \approx (\omega_{pe}^2 + \Omega_e^2)^{1/2}. \tag{19}$$

Here $\omega_{pe} > \Omega_e$, i.e. we have the case of a weakly magnetized plasma. The important role of parametric instabilities in the region of upper-hybrid resonance was pointed out in Sharma and Shukla (1983) and Pavlenko et al. (1992, 1999).

We consider the decay of the pump wave into an upper-hybrid wave and modified convective cells:

$$\omega_0 = \omega_{UH1} + \omega_c. \tag{20}$$

Here $\omega_c (m_i/m_e)^{1/2} \cos\theta\, \Omega_i$ is the real part of the frequency of the modified convec-

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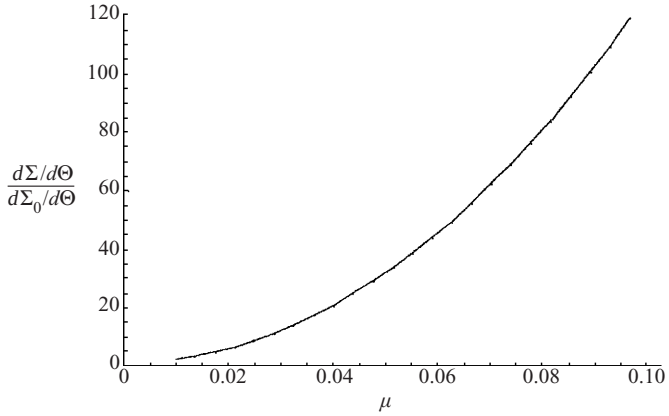


Figure 1. The dependence of the ratio $(d\Sigma/d\Theta)/(d\Sigma_0/d\Theta)$ on the parameter $\mu(E_0)$. Here $d\Sigma/d\Theta$ is the 'pump-field' contribution to the differential scattering cross-section, and $d\Sigma_0/d\Theta$ is the corresponding contribution due to thermal noise when the external electric field is absent.

tive cell, $\mathrm{Im}\,\omega \equiv \gamma_c \approx \frac{1}{2}\nu_{ei}$ (where ν_{ei} is the electron–ion collision frequency), and θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$. It should be noted that convective modes arise in a magnetized plasma with a small ratio of the plasma pressure to the magnetic pressure, and can occur in the ionospheric plasma (Okuda and Dawson 1973; Pavlenko et al. 1989).

The dipole approximation is assumed for the pump wave, because the typical ionospheric plasma parameters satisfy the condition $k_0/k_{0\perp} \ll 1$. Here, $k_0 = \omega_0/c$ (with $\omega_0 \approx \omega_{pe}$ for the upper-hybrid wave) is the wave vector of the pump upper-hybrid wave, and the wavenumber $k_{0\perp}$ satisfies the decay condition (20) (we are assuming that $k_{0\perp} \leqslant 1/\rho_e$). Thus, we have

$$\frac{k_0}{k_{0\perp}} \approx \frac{\omega_{pe} v_{Te}}{\Omega_e c} \ll 1.$$

In the region above the instability threshold, the ratio of the differential scattering cross-section $d\Sigma_\pm/d\Theta$ to the corresponding cross-section due to thermal noise, $d\Sigma^0_\pm/d\Theta$, is determined by

$$\frac{d\Sigma_\pm/d\Theta}{d\Sigma^0_\pm/d\Theta} \approx 1 + \frac{\mu^2}{8}\frac{\omega_{pi}^2}{\Omega_i^2}\frac{\omega_{pe}^2\omega_c^2}{\omega_{UH1}^2\nu_{ei}^2\sin^4\theta} + \frac{\mu^4}{32}\frac{\omega_{pi}^4}{\Omega_i^4}\frac{\omega_{pe}^4\sin^2\theta}{\omega_{UH1}^2\nu_{ei}^2}. \tag{21}$$

For typical ionospheric plasma parameters in the F layer at about 250 km, $n_0 = 10^6\,\mathrm{cm}^{-3}$, $T_e \approx T_i = 0.1\,\mathrm{eV}$, $B_0 = 0.45\,\mathrm{G}$ and $\nu_{ei} = 5 \times 10^2\,\mathrm{s}^{-1}$, we show that the 'pump-field' terms in (21) (proportional to μ) become much greater than unity (Fig. 1).

4. Scattering of electromagnetic waves in an inhomogeneous plasma

In this section, we investigate the scattering of a transverse electromagnetic wave by turbulent density fluctuations in an inhomogeneous magnetoactive plasma with an upper-hybrid pump wave. The function $f_{0\alpha}(\mathbf{p}, y)$ in this case depends on the integrals of motion: the particle energy $W = p^2/2m_\alpha$, the quantity $p_x = m(v_x - \Omega y)$, and the particle longitudinal momentum $p_\parallel = mv_\parallel$. As a result, the distribution function

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(which is Maxwellian in the absence of the density gradient) can be written as

$$f_{0\alpha}(p,y) = \frac{n_0}{(2\pi m_\alpha T_\alpha)^{3/2}} \exp\left[-\alpha\left(\frac{p_x}{m_\alpha \boldsymbol{\Omega}_\alpha} - y\right)^2\right] \exp\left(-\frac{p^2}{2m_\alpha T_\alpha}\right), \tag{22}$$

where $p^2 = p_x^2 + p_y^2 + p_z^2$ and $\alpha = n_0^{-1}\, dn_0/dy$ is the parameter characterizing the plasma density inhomogeneity.

We suppose that such a plasma subjected to a magnetic field directed along the z axis and an external pump-wave electric field excites upper-hybrid and electron-drift waves. Consider an upper-hybrid wave satisfying the dispersion relation

$$\omega_{UH2} = \boldsymbol{\Omega}_e \left(1 + \frac{\omega_{pe}^2 \sin^2\theta}{2\boldsymbol{\Omega}_e^2}\right). \tag{23}$$

It should be noted that the expression (23) is valid in a strongly magnetized plasma for the case $\omega_{pe} \ll \boldsymbol{\Omega}_e$. We assume that the damping rate of the upper-hybrid wave $\gamma_{UH} \approx \nu_{ei}$.

The frequency and the damping rate of the electron-drift wave are (Krall and Trivelpiece 1973)

$$\omega_D \approx -\frac{k_\perp \alpha T_e}{m_e \boldsymbol{\Omega}_e}, \tag{24}$$

$$\gamma_D = -\left(\frac{\pi}{2}\right)^{1/2} \omega_D \left[\frac{-\omega_D k_x^2 \rho_i^2 (1 + T_e/T_i)}{2k_\parallel v_{Te}} + \frac{T_e}{T_i}\frac{\omega_D (1 + T_i/T_e)}{2k_\parallel v_{Ti}} \exp\left(-\frac{\omega_{De}^2}{2k_\parallel^2 v_\parallel^2}\right)\right]. \tag{25}$$

According to (25), the interaction of plasma eigenmodes with electrons is a destabilizing factor; i.e. oscillations build up owing to convection of resonant electrons, because, according to (25), the ion damping is exponentially small. Note that ion Landau damping can become substantial for plasmas in small devices in which the longitudinal component of the wave vector, $k_\parallel$, can be relatively large. Moreover, in magnetic-shear devices, the damping is associated with convection of ions toward regions with large $k_\parallel$, and regions with small $k_\parallel$ are relatively narrow (Pavlenko et al. 1999).

In this work, we consider only the case when drift oscillations are damped, i.e. when the contribution from the second (ion) term in (25) is dominant.

Let us consider the scattering process when there is parametric decay of the pump wave into upper-hybrid and electron-drift waves:

$$\omega_0 = \omega_{UH2} + \omega_D. \tag{26}$$

It should be noted that the pump-wave frequency ω_0 must be slightly above ω_{UH2}, because $\omega_D \ll \omega_0, \omega_{UH2}$. Then the expression (14) can be written as

$$\frac{d\boldsymbol{\Sigma}_\pm}{d\boldsymbol{\Theta}} = \pi \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2(\omega' \pm \omega_D)}{\omega_{pe}^4} R|\xi|^2 \times \frac{|\chi_e^0|^2 \langle \delta n_i^2\rangle_{\Delta\omega,\mathbf{q}}^0 + \dfrac{\mu^2}{4}\dfrac{\boldsymbol{\Omega}_e^2}{\omega_{pe}^2}\dfrac{q^2}{16\pi^2 e^2}|\chi_e^0|^2\,|\chi_i^0|^2 \dfrac{\langle \delta \mathbf{E}^2\rangle_{\Delta\omega\mp\omega_0,\mathbf{q}}^0}{|\varepsilon_{\mp 1}|^2}}{\operatorname{Im}\varepsilon_E(\Delta\omega,\mathbf{q}) \dfrac{\partial \operatorname{Re}\varepsilon_E}{\partial \Delta\omega}\Big|_{\Delta\omega = \pm\omega_D}}. \tag{27}$$

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Here we do not present the expressions for the density and electric field fluctuation correlators that we obtained earlier for a spatially homogeneous plasma. Note that the spectral density of the field and the density fluctuations depends on the correlator of source fluctuations. For an inhomogeneous plasma, the correlator of source fluctuations depends not only on the difference $\mathbf{r}-\mathbf{r}'$ but also on the coordinate $\mathbf{r}$. Since we consider the case of a weakly inhomogeneous plasma, we may assume that the dependence of the correlator of source fluctuations on $\mathbf{r}-\mathbf{r}'$ is stronger than the dependence on the coordinate $\mathbf{r}$, because the former dependence is related to the wavelength of drift oscillations, λ, whereas the latter is related to the characteristic scale length of plasma inhomogeneity, L. Note that we take into consideration the case when $\lambda \ll L$.

We may suppose that

$$\left.\begin{aligned} &|\chi_e^0| \approx \frac{1}{q^2 r_{De}^2}, \qquad |\chi_i^0| \approx \frac{1}{q^2 r_{Di}^2}\frac{T_i}{T_e}\left(1+\frac{k_\parallel^2 v_{Ti}^2}{\omega^2}\right)(1-\beta_{\perp i}),\\ &\operatorname{Im}\varepsilon_E \approx \frac{1}{q^2 r_{De}^2}\frac{\gamma_D}{\omega_D}\left(1-\frac{E_0^2}{\tilde{E}_{\rm th}^2}\right), \qquad \frac{\partial \operatorname{Re}\varepsilon_E}{\partial\omega_D} \approx \frac{1}{q^2 r_{De}^2}\frac{1}{\omega_D},\\ &\langle \delta \mathbf{E}^2\rangle^0_{\Delta\omega\mp\omega_0,\mathbf{q}} \approx 16\pi T_e \frac{\gamma_{UH}}{\omega_{pe}^2}, \qquad \langle \delta n_i^2\rangle^0_{\omega_D,\mathbf{q}} = \frac{q^2}{16\pi e^2}\frac{8\pi}{\omega_D} T_i \frac{1}{q^2 r_{De}^2}\frac{\gamma_D}{\omega_D}, \end{aligned}\right\} \tag{28}$$

and $\tilde{E}_{\rm th}^2 = E_{\rm th}^2 \nu_{\rm eff}^2/\gamma_D\gamma_{UH}$, where the threshold electric field $E_{\rm th}$ for this instability is

$$E_{\rm th}^2 = \frac{8\omega_0^2 B_0^2}{k^2 c^2}(q r_{De})^2 \frac{\omega_{pe}^2}{\Omega_e^2}\frac{\nu_{ei}}{\Omega_e}\frac{\gamma_D}{\omega_D}. \tag{29}$$

Then after lengthy but straightforward calculations, we find from (27) the dominant contribution to the differential scattering cross-section:

$$\begin{aligned} \frac{d\Sigma_\pm}{d\Theta} \approx{}& \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega'^2(\omega'\pm\omega_D)^2}{\omega_{pe}^4} R|\xi|^2 \frac{T_i}{T_e} n_e \\ &\times\left[1+\frac{\mu^2}{8}\frac{1}{q^2 r_{De}^2}\frac{T_e}{T_i}\frac{\Omega_e^2\omega_D^2}{\omega_{UH2}\gamma_{UH}\gamma_D}\right]\left[1+8\frac{q^2 r_{De}^2}{\mu^2}\frac{\omega_{pe}^4\gamma_{UH}^2}{\Omega_e^5\omega_D}\right]. \end{aligned} \tag{30}$$

For typical hot plasma parameters, i.e. $qr_{De}=0.2$, $B_0 = 50\,\mathrm{kG}$, $n_e = 10^{14}\,\mathrm{cm}^{-3}$ and $T_e = 5\,\mathrm{keV}$, the term dependent on pump wave in first set of square brackets in (28) becomes dominant (Fig. 2). We note also that in this case the cross-section for electromagnetic-wave scattering is comparatively sensitive to the density gradient.

Note that the scale size of the density gradient, $L = 1/\alpha$, must satisfy the condition of weak inhomogeneity, $\lambda \ll L$, where $\lambda = 2\pi/k_\perp$ is the wavelength of drift plasma oscillations.

Finally note that the expressions (18), (21) and (30) for the differential scattering cross-section of incident electromagnetic waves of arbitrary polarization with frequency ω' ($\omega' \gg \omega_{pe}$) are valid for scattering by coherent fluctuations on the plasma eigenmode frequencies (upper- and lower-hybrid, drift-wave, convective-cell modes, etc.).

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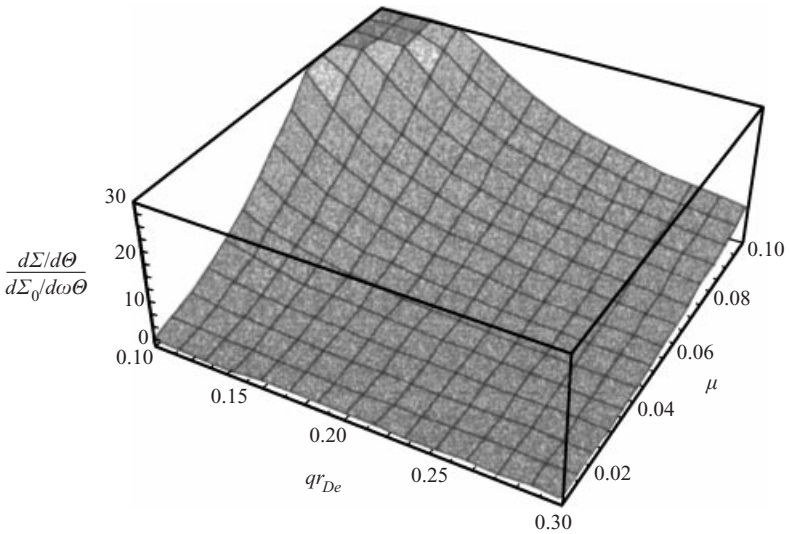


Figure 2. The ratio $(d\Sigma/d\Theta)/(d\Sigma_0/d\Theta)$ as a function of the parameters μ and qr_{De}.

5. Conclusions

To summarize, we have investigated the processes of electromagnetic-wave scattering in both homogeneous and inhomogeneous plasmas with lower-hybrid and upper-hybrid pump waves.

We have shown that the scattering coefficient depends not only on such parameters as the plasma density and temperature and the external magnetic field, but also on the pump-wave amplitude.

Numerical estimations (see Figs 1 and 2) show that the 'pump-field' contributions to the scattering cross-section can exceed the contribution due to thermal noise for typical parameters of laboratory and ionospheric plasmas. We note that in the case of an inhomogeneous plasma, the scattering coefficient depends also on the density gradient.

The results of this paper can be important for the study of the behavior of laboratory and space plasmas, and also for diagnostic purposes.

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