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HF surface cyclotron waves in planar waveguide structures with non-uniform plasma filling

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The effects of dispersion of slow electromagnetic surface waves propagating across an external magnetic field in planar metallic waveguides with dielectric coating and non-uniform plasma filling on the harmonics of the electron cyclotron frequency are studied. The theoretical investigation of the surface modes is carried out using a kinetic description for the plasma particles and a mirror-reflection model for the interaction between particles and plasma boundary. The effect of the transverse plasma density inhomogeneity on the electron surface cyclotron-wave spectrum is investigated.

1. Introduction

Volume cyclotron waves have been known for some time (Gross 1951; Sitenko and Stepanov 1956; Bernstein 1958). They are very important in such areas as nuclear fusion, plasma physics and radiophysics. Under laboratory conditions, plasmas have restricted dimensions, which is why propagation of surface waves (SW), which are located near the plasma boundary, is possible. The dispersion properties of SW are now also being investigated intensively, both theoretically and experimentally (see e.g. Vandenplas 1968; Platsman and Wolf 1973; Agranovich & Mill 1982; Kondratenko 1985; Vucovic 1986; Azarenkov *et al.* 1993, and references therein). SW properties are determined by such waveguide parameters as arrangement of construction elements, external magnetic field $\mathbf{B}_0$, plasma density and temperature profiles, etc. Electrostatic SW in a cold plasma with non-uniform boundary have been studied by Stenflo and Gradov (1990). The case of SW propagation when the plasma density is non-uniform in the direction parallel to the boundary has been also investigated by Gradov and Stenflo (1984).

Surface cyclotron waves (SCW) were found (Kondratenko 1985) to propagate in a dielectric waveguide with uniform plasma filling when the external constant magnetic field is perpendicular to the plasma boundary and the penetration depth of the SCW is greater than their wavelength. In this case the SCW frequencies decrease as the wavenumber grows.

In experimental practice one often deals with metal waveguides with non-uniform plasma filling. As will be shown here, SCW properties under these conditions differ from those in the case considered by Kondratenko (1985). We show in this paper the possibility of SCW propagation in non-uniform plasma waveguide structures with planar geometry such as plasma–dielectric–metal

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when $\mathbf{B}_0$ is perpendicular to the plasma boundary. We also investigate the effect of the transverse plasma inhomogeneity and the value of plasma density on the SCW dispersion properties.

Mathematical simplicity as well as the significant role of hydrodynamic instabilities are the main reasons for the wide application of the magnetohydrodynamic method (see e.g. Vandenplas 1968; Krall and Trivelpiece 1973; Kondratenko 1985; Azarenkov *et al.* 1990). However, this approach allows investigation of electromagnetic waves only on the main cyclotron resonance. There is no opportunity in the framework of magnetohydrodynamics to study electromagnetic perturbations on the harmonics of cyclotron frequencies taking account of finite charged-particle Larmor radius. As far as cyclotron waves are concerned, they can be correctly studied only by a kinetic method. To investigate the SCW properties, one can either solve the kinetic equation under the condition of arbitrary plasma density profile or simulate its non-uniformity by a set of uniform plasma layers with different values of the charged-particle concentration. The second way is more cumbersome than the first, but is less difficult. That is why it has been fruitfully applied for solving similar problems concerned with surface-wave propagation in inhomogeneous plasmas (see e.g. Agranovich and Mill 1982; Kondratenko 1985; Azarenkov *et al.* 1990, 1993). To obtain the SCW dispersion equation for the considered plasma non-uniformity model, we used the traditional boundary conditions: first, continuity of the tangential components of the SCW electric and magnetic fields at the plasma–plasma and plasma–dielectric boundaries, and, secondly, taking the tangential component of the electric field to be zero on the metallic surface.

2. Basic equations

Let us consider a semibounded non-uniform plasma occupying the half-space $z > 0$ and restricted at the plane $z = 0$ by a dielectric slab with dielectric permeability ϵ_d and thickness a_d; a perfectly conducting metal is situated in the region $z < -a_d$. The model of a semibounded plasma is appropriate for the case when the transverse sizes of the waveguide structure are sufficiently greater than the skin depth of the considered eigenmode. This requirement is valid for SCW propagation. The external constant magnetic field $\mathbf{B}_0$ is applied perpendicularly to the plasma surface along the z direction. This orientation of the vector $\mathbf{B}_0$ can be realized by using a ferromagnetic metal for the waveguide wall. Plasma inhomogeneity is modelled by a set of several plasma layers with different densities. We suppose that there is a plasma slab with thickness a_1 and density n_1 in the region $0 < z < a_1$ and that a plasma with density n_0 occupies the half-space $z > a_1$. The SCW dispersion equation under these conditions was studied with the help of a computer.

The plasma-particle motion is described by a kinetic equation. The non-perturbed plasma-particle distribution function is Maxwellian. We consider the mirror model of particle reflection from the plasma boundary, which is widely used for solving similar problems (Kondratenko 1985). The space is supposed to be uniform in the y direction. The plasma dispersion along the z direction is supposed to be weak ($k_3 v_{T\alpha} \ll \omega - s|\omega_{c\alpha}|$), where k_3 is the wave vector of the SCW along the z axis, $v_{T\alpha}$ and $\omega_{c\alpha}$ are the thermal velocity and cyclotron frequency of the α-type plasma particles ($\alpha = e$ for electrons and $\alpha = i$ for ions),

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and s can be zero or an integer. This inequality means that the SCW penetration depth into the plasma is much greater than the Larmor radius $\rho_{L\alpha} = v_{T\alpha}/|\omega_{c\alpha}|$ of the plasma particles. The expressions for the components of plasma dielectric permeability tensor ϵ_{ik} are then obtained. They are the same as in the case of an infinite magnetoactive plasma (see e.g. Platsman and Wolf 1973; Krall and Trivelpiece 1973; Kondratenko 1985, and references therein).

In order to describe the SCW electromagnetic field, we use the Maxwell equations. The dependence of the SCW field on the x coordinate and time t is chosen in the form $E, H \propto \exp(ik_1 x - i\omega t)$, where ω and k_1 are the frequency and the wave vector of the SCW respectively. We consider the case of a slow SCW, i.e. $v_{\rm ph}^2|\epsilon_{ik}| \ll c^2$, where c and $v_{\rm ph} = \omega/k_1$ are the velocity of light and the SCW phase velocity respectively. The Maxwell equation system can be divided under these conditions into two independent systems. One describes the SCW with components E_x, H_y and E_z. Using Fourier transformation, we obtain the corresponding Maxwell equations for the Fourier coefficients of the SCW field in the non-uniform plasma region:

$$\left.\begin{aligned} ik\epsilon_{11}E_1 - ik_3 H_2 + ik\epsilon_{13}E_3 &= -\frac{H_y(0)}{2\pi}, \\ k\epsilon_{13}E_1 + k_1 H_2 + k\epsilon_{33}E_3 &= 0, \\ ik_3 E_1 - ikH_2 - ik_1 E_3 &= \frac{E_x(0)}{2\pi}, \end{aligned}\right\} \tag{1}$$

where $k = \omega/c$. SCW damping caused by plasma dispersion is found to be small (Kondratenko 1985), which is why we have taken into account only the real parts of the components ϵ_{ik} in (1). They are given by

$$\left.\begin{aligned} \epsilon_{11} &= 1 - \sum_\alpha \sum_{n=-\infty}^{\infty} \frac{n^2 I_n(y_\alpha)\,\Omega_\alpha^2 b_\alpha}{y_\alpha \omega(\omega - n\cdot\omega_{c\alpha})}, \\ \epsilon_{13} &= -\sum_\alpha \sum_{n=-\infty}^{\infty} \frac{n I_n(y_\alpha)\,\Omega_\alpha^2 \omega_{c\alpha} k_3 b_\alpha}{\omega(\omega - n\omega_{c\alpha})^2 k_1}, \\ \epsilon_{33} &= 1 - \sum_\alpha \sum_{n=-\infty}^{\infty} \frac{I_n(y_\alpha)\,\Omega_\alpha^2 b_\alpha}{\omega(\omega - n\omega_{c\alpha})}. \end{aligned}\right\} \tag{2}$$

Here Ω_α is the Langmuir frequency, $I_m(x)$ is the modified Bessel function (see e.g. Abramowitz and Stegun 1964), $y_\alpha = \frac{1}{2}k_1^2\rho_{L\alpha}^2$ and $b_\alpha = \exp(-y_\alpha)$. One can use (1) for the description of the SCW field in the region of the dielectric coating of the waveguide metallic wall, changing $\epsilon_{11} \to \epsilon_d$, $\epsilon_{33} \to \epsilon_d$ and $\epsilon_{13} \to 0$. The solution of (1) with the mentioned changes for the dielectric region $0 > z > -a_d$ gives us the possibility of finding the impedance $Z_d(0) = E_x(0)/H_y(0)$ of this dielectric slab:

$$Z_d(0) = -\frac{ik_1}{k\epsilon_d}\tanh(k_1 a_d). \tag{3}$$

The functional relation between the tangential components of the electric $E_x(0)$ and magnetic $H_y(0)$ fields in the plasma layer on the plane $z = 0$ is

$$E_x(0) = \frac{ik_1\,\Theta(n_1)}{k_1\,\epsilon_{11}(n_1)}\left\{\frac{H_y(0)}{\tanh[a_1 k_1\,\Theta(n_1)]} - \frac{H_y(a_1)}{\sinh[a_1 k_1\,\Theta(n_1)]}\right\}, \tag{4}$$

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where

$$\Theta(n_1) = \left[\frac{\epsilon_{11}(n_1)}{\epsilon_{33}(n_1)+2B(n_1)}\right]^{1/2},$$

$$B = \frac{k_1}{k_3}\epsilon_{13} = -\sum_{\alpha}\sum_{n=-\infty}^{\infty}\frac{n\Omega_\alpha^2\omega_{c\alpha}I_n(y_\alpha)b_\alpha}{\omega(\omega-n\omega_{c\alpha})^2}.$$

The value of B does not depend upon k_3. The SCW dispersion relation is obtained by equating the dielectric-slab impedance (3) to the plasma-layer impedance (4) and by equating the semibounded-plasma impedance to the plasma-layer impedance at the plane $z = a_1$. The relation between the SCW fields E_x and H_y for $z = a_1$ is

$$E_x(a_1) = \frac{ik_1 H_y(a_1)}{k\{\epsilon_{11}(n_0)[\epsilon_{33}(n_0)+2B(n_0)]\}^{1/2}}. \tag{5}$$

The SCW dispersion relation in the case of the plasma inhomogeneity model considered here can be written in the form

$$\{\epsilon_{11}(n_0)[\epsilon_{33}(n_0)+2B(n_0)]\}^{1/2} = f(n_1)+\frac{\epsilon_d}{\tanh(k_1 a_d)}, \tag{6}$$

where

$$f(n_1) = \epsilon_{11}(n_1)[\epsilon_{33}(n_1)+2B(n_1)]\tanh[k_1 a_1\,\Theta(n_1)].$$

Equation (6) possesses an important new term compared with the case of a dielectric waveguide with uniform plasma filling (Kondratenko 1985). Of course, this term is connected with the plasma-inhomogeneity model. When the plasma region is supposed to consist of two layers with thicknesses a_1 and a_2 and densities n_1 and n_2 respectively and a semibounded plasma then one more term appears on the right-hand side of the SCW dispersion equation compared with (6). In the case where there are two transition plasma layers, the dispersion equation is

$$\{\epsilon_{11}(n_0)[\epsilon_{33}(n_0)+2B(n_0)]\}^{1/2} = f(n_1)+f(n_2)+\frac{\epsilon_d}{\tanh(a_d k_1)}, \tag{7}$$

where

$$f(n_2) = \epsilon_{11}(n_2)[\epsilon_{33}(n_2)+2B(n_2)]\tanh[a_2 k_1\,\Theta(n_2)].$$

The simple analytical expressions

$$\omega = \frac{s|\omega_{ce}|}{1-h_e}, \qquad |h_e| \ll 1,$$

for the solution of (6) can be obtained only in the limit when there is no influence of the dielectric slab: $a_d k_1 \gg 1$ and both of the contributions of the plasma layers are negligibly small compared with the case of a semibounded plasma:

$$\left.\begin{aligned} h_e &= -(s^2-1)(\tfrac{1}{2}y_e)^{s-1}(2s!)^{-1} < 0, \quad s \geqslant 2, \quad y_e \ll 1,\\ h_e &= \Omega_e^2\omega_{ce}^{-2}(2\pi y_e^3)^{-1/2} > 0, \qquad\quad s \geqslant 1, \quad y_e \gg 1. \end{aligned}\right\} \tag{8}$$

These solutions are similar to those obtained in the case of SCW propagation along a semirestricted plasma–vacuum boundary (Kondratenko 1985). They do not take plasma inhomogeneity into account.

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3. Numerical calculation

One can investigate SCW dispersion for arbitrary values of the waveguide structure parameters only by using computer simulation. We have carried out such an investigation only for the electron SCW, because, as the analysis of (2) shows, the ion and electron SCW dispersion relations do not differ in principle. Increasing the parameter

$$A_d = \epsilon_d / k_1 a_d$$

changes the SCW propagation conditions for the worse. The effect is stronger on long-wavelength SCW. The same consequences for the SCW dispersion were obtained in a rarefied plasma when Ω_e^2 approaches ω_{ce}^2.

A change occurs to the long-wavelength SCW dispersion relation when the dielectric-slab thickness is about the depth $\lambda_\perp$ of the SCW field penetration into the plasma. Here

$$\lambda_\perp = \frac{2\pi}{k_1}\left(\frac{\epsilon_{33}+2B}{\epsilon_{11}}\right)^{1/2}. \tag{9}$$

The results of the numerical calculations of the dispersion relation (6) are presented in Fig. 1 for the case when the plasma density decreases with distance away from the plasmas–dielectric boundary, and in Figs 2 and 3 for the case when the plasma density increases with distance. When the waveguide structure contains two plasma surfaces in the plasma inhomogeneity model under consideration, there are two SCW modes. One of these also exists for a metal–dielectric–uniform-plasma waveguide structure with plasma density n_0. That is why it is natural to call this the 'main mode'. The shape of the dispersion curves for the main SCW mode is determined by the properties of dielectric slab: its thickness a_d and dielectric coefficient ϵ_d. The calculated points for the main mode are indicated by triangles in the figures. The frequency of the second SCW mode depends strongly upon the ratio between the densities n_0 and n_1 and the ratio between the thickness of transition layer a_1 and the Larmor electron radius ρ_L. We call this mode the 'minor mode'.

If the concentration of charged particles in the transition slab is much greater than that in the plasma half-space ($n_1 \gg n_0$), there are no essential distinctions between the two SCW modes for the first electron cyclotron harmonics ($s = 1, 2, 3$). Such distinctions appear and increase when the harmonic number s becomes greater. When the plasma density decreases with distance away from the metal wall and the ratio n_1/n_0 is not too large compared with unity, one can see a discrepancy in the SCW frequency spectrum in the limit of a thin transition slab ($a_1 \rho_L \ll 1$). This phenomenon is represented in Fig. 1.

In the opposite case, when the plasma density increases with distance away from the metal wall, the frequencies of the minor modes differ essentially from those of the main SCW modes (see Figs 2 and 3). This phenomenon can be explained by the fact that the SCW frequency depends upon the plasma density; for example, the resonance shifts h_e of the minor SCW modes frequencies are much less than those of the main mode in the case $n_1 = 0.1 n_0$ presented in Fig. 2.

SCW cannot propagate in a rarefied plasma, when $\Omega_e^2 \ll \omega_{ce}^2$. The frequency of the minor mode decreases as the transition-slab thickness decreases (that is when the ratio a_1/ρ_L decreases).

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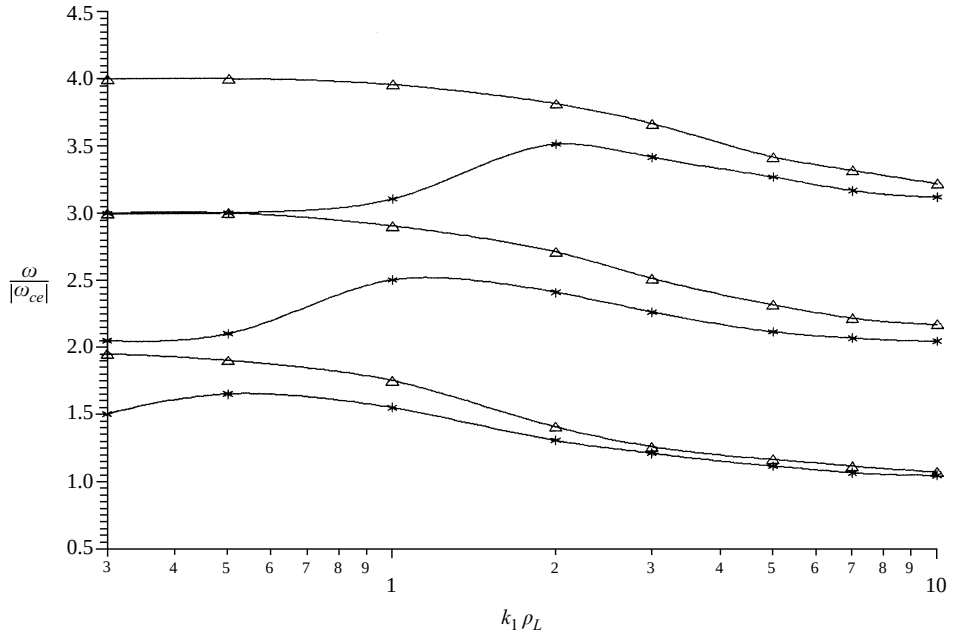


Figure 1. Frequency of electron surface cyclotron waves divided by the electron cyclotron frequency as a function of the wave vector multiplied by the electron Larmor radius. Here $n_1 = 3n_0$, $a_1 = 10^{-6}\rho_L$, $a_d = 10^{-4}\rho_L$, $\epsilon_d = 1$ and $\Omega_e^2 = 10^3\omega_{ce}^2$. Triangles and asterisks correspond to the main and minor modes respectively. The horizontal axis has logarithmic scale.

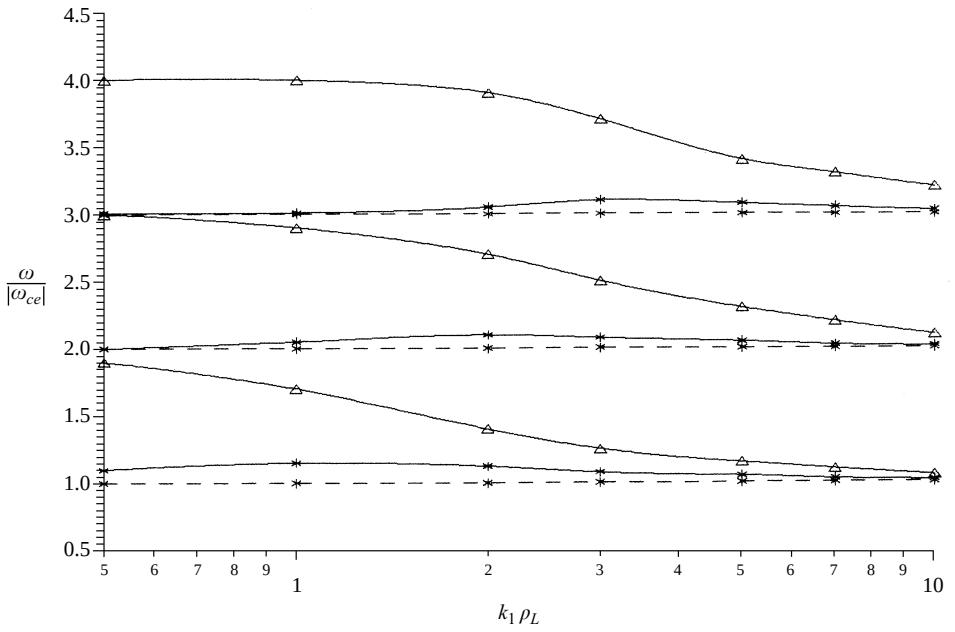


Figure 2. Frequency of electron surface cyclotron waves divided by the electron cyclotron frequency as a function of the wave vector multiplied by the electron Larmor radius. Here $n_1 = 0.1n_0$, $a_d = 10^{-4}\rho_L$, $\epsilon_d = 1$ and $\Omega_e^2 = 10^3\omega_{ce}^2$. Triangles correspond to the frequencies of the main modes. Asterisks correspond to the frequencies of the minor modes when $a_1 = 10^{-6}\rho_L$ (dashed curves) and when $a_1 = 10^{-4}\rho_L$ (solid curves). The horizontal axis has a logarithmic scale.

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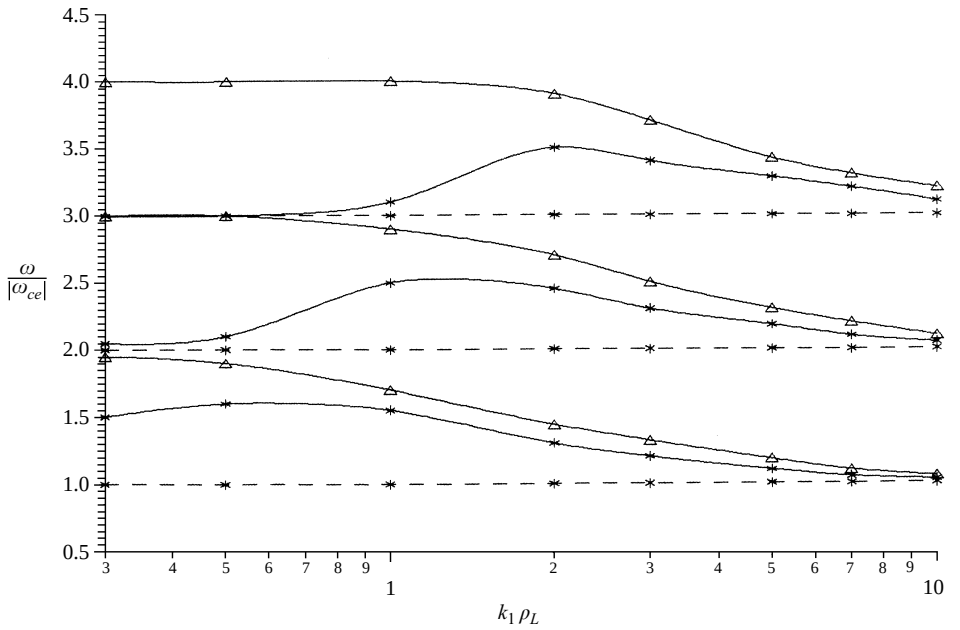


Figure 3. Frequency of electron surface cyclotron waves divided by the electron cyclotron frequency as a function of the wave vector multiplied by the electron Larmor radius. Here $n_1 = 0.3n_0$, $a_d = 10^{-4}\rho_L$, $\epsilon_d = 1$ and $\Omega_e^2 = 10^3\omega_{ce}^2$. Triangles correspond to the frequencies of the main modes. Asterisks correspond to the frequencies of the minor modes when $a_1 = 10^{-6}\rho_L$ (dashed curves) and when $a_1 = 10^{-4}\rho_L$ (solid curves). The horizontal axis has a logarithmic scale.

4. Discussion

SCW dispersion curves and those of volume cyclotron waves have a similar form (see e.g. Gross 1951; Sitenko and Stepanov 1956; Bernstein 1958; Krall and Trivelpiece 1973); nevertheless we must emphasize that the waves considered are just two-dimensional waves, so they are surface waves with a penetration depth $\lambda_\perp$, (9), that is greater than the SCW wavelength $2\pi/k_1$. The validity of the inequality $|\epsilon_{33} + 2B| \gg |\epsilon_{11}|$ is conformed in both analytical and numerical calculations. Plasma inhomogeneity leads to the SCW frequency decreasing compared with the case of a uniform plasma, and also to their spectrum expanding. The latter effect means that SCW damping increases owing to the strong interaction between plasma particles and the SCW.

The considered SCW could be used in some plasma technologies. Surface waves have been applied for the cleaning of semiconductor surfaces (Nonaka 1990, 1992), and it has been experimentally determined that SW have advantages over volume waves in this process. Traditionally, surface modes have been widely used for studying the physical properties of solids (Platsman and Wolf 1973; Agranovich and Mill 1982; Vucovic 1986). Since the SCW frequency depends upon the physical characteristics of the dielectric coating on the metal interface, one can apply these modes for the technical control of such coating. By determining the SCW frequency, we may estimate the thickness of the dielectric layer when the type of insulator material is known, or we may determine the dielectric constant when the thickness of the insulator coating is known. On the other hand, SCW can be used for producing and sustaining gas

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discharges in plasma–metal structures with dielectric sheaths. The results, reported in Vucovic (1986) suggest that an external steady magnetic field facilitates plasma production and prolongs the discharge. And, unlike the SW, which were studied in Laurin *et al.* (1989) and Benova *et al.* (1991), not only are the SCW eigenmodes of the waveguide structure considered here but also their frequencies are multiples of the electron cyclotron resonance. Thus it seems advantagenous to use just SCW for producing gas discharges. The next possible applications of SCW are to radiotechnics and radiophysics. The SCW frequency reaches a value of 100 MHz for magnetic fields $B_0 = 1$–10 kOe. SCW can be excited by parametric pumping. It has been shown theoretically (Girka *et al.* 1986; Girka 1993) that increases in SCW parametric instability can be greater than those of volume cyclotron waves.

5. Conclusion

We have investigated SCW dispersion properties using the Maxwell equations for the SCW field and a kinetic equation for the distribution function of the plasma particles. The results presented here concern mainly SCW propagating at harmonics of the electron cyclotron frequency, but they can easily be generalized to the case of ion SCW. It has been shown that these SCW can propagate in a non-uniform plasma across an external magnetic field if the latter is perpendicular to the metallic wall surface. The SCW penetration depth $\lambda_\perp$, (9), is greater than the wavelength. The plasma non-uniformity has been modelled here by introducing a uniform transition layer between the dielectric and the plasma half-space. In this case two SCW modes occur, with the same harmonic of the cyclotron frequency. Plasma inhomogeneity results in amplification of SCW damping and in a decrease in the SCW frequencies. Simple analytical expressions for the SCW dispersion relation (8) have been found in the limits of small ($k_1 \rho_L \gg 1$) and large ($k_1 \rho_L \ll 1$) wavelengths under the conditions of a narrow ($2\pi a_1 \ll \lambda_\perp$) transition plasma layer and a thick ($a_d k_1 \gg 1$) dielectric slab.

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