Macroeconomic Dynamics, **23**, 2019, 384–419. Printed in the United States of America.
doi:10.1017/S1365100516001371

THE GOLDEN RULE OF LONGEVITY

Marias H. Gestsson
Aarhus University
and
University of Iceland

Gylfi Zoega
University of Iceland
and
University of London

How much should society invest in medical care that extends the lives of the older generations? We derive a golden rule for the level of health care expenditures and find that the optimal level of life-extending health care expenditures should increase with rising productivity, increase with the retirement age, and also increase with the population growth rate if a higher growth rate lowers the ratio of retirees to working-age people sufficiently, while the effects of an improvement in medical technology are ambiguous. Moreover, we find that a market economy may be inefficient in terms of the provision of life-extending health care because an individual ignores the effect of his own longevity on the income of others.

Keywords: Health Care Expenditures, Golden Rule, Productivity

1. INTRODUCTION

Advanced economies devote significant resources to health care. A substantial part of the spending is for treatments that will extend the lives of the elderly.[1] In this paper, we address the question how much a nation should sacrifice in terms of the consumption of individuals in order to extend the life of generations and compare the social planner's solution to the market solution. Clearly, a society could sacrifice the consumption of its population in order to devote large resources to treat the old. Alternatively, a nation could choose to live happily, not worrying about the length of the lifespan, as long as they can. We analyze the optimal level of health care spending on old individuals in a balanced-current-account steady state.

Our social planner's problem does not have a time dimension. Instead, we focus on finding the optimal steady state for an economy where the cost of increased life-extending health care—in the form of lower consumption and utility of the

We are much indebted to two referees whose constructive comments helped us improve the paper. Address correspondence to: Gylfi Zoega, Department of Economics, University of Iceland, 101 Reykjavik, Iceland; e-mail: gz@hi.is.

© 2018 Cambridge University Press 1365-1005/18

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

working-age population—is traded off against the utility gained by increasing the lifespan of the old. The possible steady states vary in terms of the consumption per capita and the number of living individuals belonging to the older generations. One possible steady state has higher levels of consumption per capita, while there are relatively few older individuals alive due to a lack of health care. In another steady state, the working-age individuals have to pay higher taxes, hence enjoy lower levels of consumption, but there are more old folks around, kept alive by the health care funded through the taxation of the younger cohorts. We derive an optimality condition for taxation and the provision of health care such that the sum of the utility of all individuals alive at a point in time is maximized. We then solve the private individual's optimization problem and compare it to the social optimum.

2. LITERATURE

A large body of literature exists on the determinants of health care expenditures. What separates this literature from our approach is that we solve the social planner's problem and compare it to the level chosen by individuals acting independently, while this literature mainly approaches the issue as demand for health care by individuals as a function of income and age. Our approach is to derive how much society should spend on extending the lives of its citizens and then show how the private provision of health care may be dynamically inefficient.

The question concerning the optimal level of health care spending is closely related to the golden rule literature that began with Ramsey (1928) on the optimal level of saving and was expanded in papers on the optimal level of research and the optimal level of education (see Phelps, 1966, 1968, among others).[2] Here, investing in future research ideas, education, or the physical capital stock requires a reduction in current levels of consumption, while better education and technology and a larger stock of capital will make possible an increase in the future level of consumption. There is an optimal capital stock, and a corresponding optimal saving rate that maximizes steady-state consumption of the representative individual.

Our optimality condition is similarly a golden rule result. In our analysis, there is a golden rule level of life-extending health care at which the aggregate utility of the currently living individuals, who belong to different generations, is maximized. Increasing the level of taxes and health care beyond this level would reduce aggregate utility in steady state because the reduction in utility of the working-age population would be greater than the utility of individuals who would be kept alive as a result of the higher level of health care spending. We should note that maximizing the aggregate utility of all currently living individuals is equivalent to maximizing the expected lifetime utility of any given individual because he will belong to the different age cohorts as he goes through life.

We are not the first to derive the optimal length of life in an overlapping-generations model. Ehrlich and Chuma (1990) derive a demand function for

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

longevity. They show that the demand for health has to be derived together with the demand for longevity and the related consumption plan and that these choices depend on initial endowments rather than current income. Rosen (1988) derives valuation formulas for age-specific mortality risks. Our analysis is closest to that of Hall and Jones (2007) who derive the demand for life-extending medical care to explain the rising economic resources spent on health care. They find that the rising level of health spending can be explained by the diminishing marginal utility of nonhealth consumption combined with a rising value of life. Becker et al. (2007) model the incentives for medical care at the end of life in order to explain what is frequently perceived as excessive care for end-of-life treatments. Their argument is that the excessive demand for health care can be explained by decreasing marginal benefits of survival. Thus, toward the end of life, the importance of keeping hope alive in a terminal care setting and the low private opportunity cost of medical spending explain the high level of end-of-life spending. Since resources have no value after death, a self-interested individual would be willing to spend his entire wealth to extend his life when dying. Moreover, such life-extending health care may preserve the hope of living, which raises its subjective value to the dying. In a recent paper, Córdoba and Ripoll (2017) provide a model to explain this observation by abandoning the expected utility approach of Hall and Jones and applying the nonseparable utility representation of Epstein and Zin (1989) and Weil (1990) to study preferences toward longevity and mortality risk. In so doing, they relax state separability of preferences and distinguish between the parameters governing mortality aversion and intertemporal substitutions. Using this framework, the authors find that individuals prefer uncertainty about the time of death and that there is a decreasing marginal utility of survival that makes people want to pay more for life extension the closer they get to death.[3] Kuhn et al. (2015) model the simultaneous choice of health care and retirement over the life cycle by making the disutility of work depend negatively on health status. Here, increased private investment in health care has the effect of postponing retirement by lowering the disutility of labor leading to excessive consumption. The authors find that this outcome is inefficient because the individual suffers the disutility of an excessively long working life. In a recent paper, Andersen and Bhattacharya (2014) show how dynamic efficiency considerations can justify public investment in health. First, assuming exogenous mortality risk, they show how publicly-funded health spending on the young can improve welfare if the economy is dynamically efficient—the real rate of interest higher than the rate of population growth. The reason is that the social opportunity costs of public investment in the health care of the young is equivalent to the rate of the population growth, while the private costs is equivalent to the rate of interest.[4] Second, they show that the young may also underinvest in health when mortality risk is endogenous in the presence of mortality-contingent claims. Here, public provision of health care lowers the interest rate on life annuities because of reduced mortality risk, which helps the young if they are net-borrowers. Our analysis differs from both Kuhn et al. (2015) and Andersen and Bhattacharya (2014) by deriving the socially and

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

privately optimal levels of life-extending health care and exploring the effect of changes in different parameters on these optimal levels.

There is a parallel literature on the optimal size of the population in philosophy. Using the utilitarian approach, the optimal population can be determined as the one maximizing total utility, the product of the number of people alive and their personal utility levels. This raises philosophical dilemmas discussed extensively in the literature because an increase in the number of people alive that is accompanied by lower consumption and utility of each of them may increase total utility. Thus, it is possible to increase total utility by making each person worse off. This gives rise to Parfit's (1984) Repugnant Conclusion, according to which a large increase in the number of people alive can increase total utility even though its members have very low levels of consumption, in fact possibly lives barely worth living. Here, population size substitutes for average utility and it is the quality–quantity substitution that is repugnant. A related problem is Methuelah's Paradox, see Cowen (1989). Here, a longer life welfare dominates a shorter life even though the level of consumption in each year is lower. Thus, a very long life during which the individual finds his life almost not worth living is preferred to a shorter life of higher utility per year. As we will show, our economic model will get around these paradoxes by assuming diminishing marginal utility and diminishing marginal productivity of health care spending.

Going back to economics, Becker (1964) introduced the concept of human capital. In his framework, individuals have different levels of human capital, which determines their future earnings potential, and can add to this stock through education and training. Grossman (1972) expanded Becker's notion of human capital to include health capital and made a distinction between health as an output and medical care as one of many inputs in the production function for good health. Here, health is a durable capital stock that gradually depreciates with age but can be augmented through health care and healthy living. Health as capital yields an output of "healthy time" that affects productivity in the workplace, therefore affecting wages, productivity at home, and utility from leisure. We depart from Grossman in not letting health depreciate with age but instead to let retirees face an increasing probability of death. Instead of the capital good "health" depreciating continuously with age we let it work perfectly until it breaks down.

In our model health care spending extends lives. We are concerned about the cost of extending lives rather than improving the quality of life. The cost of extending lives has been shown to be a significant component of overall health care expenditures. Seshamani and Gray (2004) use English longitudinal data and find that approaching death affects costs for up to 15 years prior to the time of death. In particular, the ten-fold increase in costs from 5 years prior to death to the last year of life is much greater than the 30% increase from age 65 to 85 years. Lubitz and Riley (1993) study Medicare payments in the United States and find that around 30% of Medicare payments go to people in the last year of life. These authors find that people in their last year of life make up 35% to 39% of the 5% of beneficiaries with the highest costs. Note that this is clearly an underestimate of

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

the cost of extending lives, as the figure omits the cost of treatment of those whose lives were extended beyond one year. Jones (2003) and Miller (2001) also find that US Medicare expenditures rise rapidly in the years preceding death. The former finds that expenditures rise at the rate of 9.4% per year in the 3–10 years before death and then by 45% in the final two years before death. Jones (2003) argues that the critical determinant of health expenditures as a share of gross domestic product (GDP) is the willingness of society to transfer resources to those at the end of life. Better medical technology that makes it possible to extend lives makes health care expenditures and life expectancy increase over time.

This leads us to the topic of this paper, which is to answer the question how much the working population should pay to extend the lives of the older generation when realizing that they will also receive the same care funded by the next generation. We start by solving the social planner's problem and then move on to model spending on life-extending health care by individuals to compare it to the social optimum.

3. A SIMPLE MODEL

We start with a simple model to describe our basic insights. A more realistic model that also allows for richer comparative statics and a comparison between the socially optimal allocation and an allocation based on individual decision making is presented in the following section. The simple model is similar to that presented in Section 3 of Hall and Jones (2007) with the addition of explicitly taking into account the retirement age. Our more general model in the following section will then show the effect of population growth on the optimal level of health care and make it possible to derive both the socially optimal and the privately optimal level of care and show how the privately optimal level can be excessive.

Consider an economy consisting of individuals who live for A years. The survival probabilities equal unity until age A is reached when death becomes a certainty. Individuals work until they reach the age of retirement R. Assume, for simplicity that one individual is born each unit of time and, hence, the population size is $N = A$, while the number of working and retired individuals are $N_w = R$ and $N_o = A - R$, respectively. It follows that $N = N_w + N_o$.

The production of life-extending health care is given by the health care production function $A = \gamma B(\tau)$, where $\gamma > 0$ is a parameter measuring efficiency in health care production, τ is health care expenditure per retired individual and the function $B(\tau)$ is assumed to be strictly increasing and concave in health care expenditure. We thus envisage all retired individuals receiving health care with the effect of prolonging their lives.

An individual's utility from consumption (c) is given by $U(c)$, which is assumed to be strictly positive, increasing, and concave in consumption. Welfare can therefore be written as

$$W = RU\,(c_w) + (A - R)U\,(c_o)\,, \tag{1}$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

where c_w and c_o are the consumption of working and retired individuals, respectively. A working individual produces y per unit of time.[5] The economy's budget constraint is therefore

$$Ry = Rc_w + (A - R)c_o + (A - R)\tau,$$

where the left-hand side gives total output of the economy, while the right-hand side gives total expenditure on consumption and health care, or

$$R\,(y - c_w) = (A - R)\,(c_o + \tau)\,. \tag{2}$$

To maximize welfare in (1), a social planner chooses consumption of a working individual c_w, consumption of a retired individual c_o, and health care expenditure per retired individual τ, such that welfare W is maximized subject to the economy's budget constraint in (2) and the health care production function. The Lagrangian can we written as

$$\Gamma = RU\,(c_w) + (\gamma B\,(\tau) - R)\,U\,(c_o) + \lambda\,[R\,(y - c_w) - (\gamma B\,(\tau) - R)\,(c_o + \tau)]\,,$$

where λ is the Lagrange multiplier. The first-order conditions give[6]:

$$\frac{\partial U}{\partial c_w} = \frac{\partial U}{\partial c_o} = \lambda, \tag{3}$$

$$\gamma B'\,(\tau)\,U\,(c_o) - \lambda\gamma B'\,(\tau)\,(c_o + \tau) - \lambda(A - R) = 0. \tag{4}$$

The condition in (3) implies consumption smoothing:

$$c_w = c_o = c. \tag{5}$$

Using (5) and (3) in (4) gives

$$U\,(c)\,\gamma B'\,(\tau) = \frac{\partial U}{\partial c}\,(c + \tau)\,\gamma B'\,(\tau) + \frac{\partial U}{\partial c}(A - R). \tag{6}$$

The equation gives optimal spending on health care per retired or old individual, which is the golden rule of health care spending and longevity since health care spending determines longevity A through the health care production function. The left-hand side shows increased social welfare due to increased longevity when each old individual lives and consumes longer. In effect, the number of old individuals receiving utility from consumption is increased and this increases welfare. The first term on the right-hand side shows the lost utility due to increased consumption and health care expenditures of those living longer. The second term on the right-hand side denotes the marginal cost of increased health care for those individuals over retirement age who would have survived in the absence of increased spending on health care.

There is a trade-off in the golden rule of equation (6) between increasing society's welfare by having more old people alive and increasing welfare by

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

having higher consumption for those who would have lived in the absence of increased health care.[7] At the optimum the two are equal at the margin.

Since the second-order conditions for a maximum are fulfilled, the optimality condition in (6), equation (5), and the budget constraint in (2),

$$R\,(y-c) = (A-R)\,(c+\tau)\,, \tag{7}$$

give optimal consumption per capita c and health care expenditure per old individual τ as implicit functions of productivity y, the retirement age R, the efficiency in health care production parameter γ, and the functional forms of the utility function U, and the health care production function A. Comparative static analysis gives the following results[8]:

$$\frac{\partial \tau}{\partial y} = \frac{\frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\,\gamma B'(\tau)\right]R}{\Lambda} > 0 \tag{8}$$

$$\frac{\partial \tau}{\partial \gamma} = \frac{A\frac{\partial U}{\partial c}\frac{R}{\gamma} - \frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\,\gamma B'(\tau)\right](c+\tau)\,B(\tau)}{\Lambda} < 0, \tag{9}$$

$$\frac{\partial \tau}{\partial R} = \frac{\frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\,\gamma B'(\tau)\right](\tau+y) - A\frac{\partial U}{\partial c}}{\Lambda} > 0, \tag{10}$$

where

$$\Lambda \equiv A\frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)} - 2\gamma B'(\tau)\right] + \frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\,\gamma B'(\tau)\right]^2,$$

which is negative, as is shown in Appendix B.

The derivative in (8) is positive implying that it is socially optimal to let health care expenditure and longevity increase following an increase in productivity. This is since the marginal utility of consumption falls with increased income, which raises the optimal level of health care. This result is similar to that derived in Hall and Jones (2007). The derivative in (9) is negative implying that it is socially optimal to let health care expenditure decrease following an increase in efficiency in health care production. Here, an increase in efficiency in old-age health care provision results in a reduction in health care expenditures per old individual needed to maintain unchanged longevity. However, we will show in a richer model introduced below that this is not a general result. Finally, the derivative in (10) indicates that health care expenditure and longevity should increase following an increase in the retirement age. The intuition behind this result is the same as for an increase in productivity above; the marginal utility of consumption decreases with increased income resulting in increased allocation of resources to health care.

We now move on to a richer model with overlapping generations that will allow us to explore the effects of productivity and the efficiency of health care further. The more general model will also enable us to explore the effect of changes in the rate of population growth on the optimal level of health care provision and, most

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

importantly, to derive the privately optimal level of life-extending health care and compare it to the social optimum. We can then draw policy conclusions.

4. THE GOLDEN RULE OF LONGEVITY DERIVED

The model is a continuous-age overlapping-generations model where individuals belonging to different generations (at different ages) are alive at each point in time and the social planner maximizes total utility of all living individuals.[9] An individual works during the first part of his life cycle, and retires at a certain age. Output per worker is the same for all workers and national output is the sum of the output of each working individual. We solve the balanced-current-account steady-state social planner's problem on how to allocate output between consumption and health care expenditures, on one hand, and how to allocate consumable output across individuals, on the other hand. Hence, the social planner's problem does not have a time dimension in that he makes the decision at a point in time facing the currently living generations while being constrained by the total output produced by the economy at that time. Clearly, increasing expenditures would lower consumption but only gradually increase the number of people alive. Our model does not describe this transition.[10] Thus, the paper only describes the optimal steady state, not the transition from one steady state to another.

The social planner can extend the lives of the older generation by reducing consumption and spending more on health care, which generates a higher maximum age and also lowers the mortality of all retirees at a given age. Such a scheme reduces the utility of those who would have lived in the absence of the health care expenditures through lower consumption; that is both the working-age population and the retirees who would have lived in the absence of the increase. The total utility of all living individuals is maximized at the social optimum, which is equivalent to maximizing the total utility of a given individual over his lifespan.

4.1. Demographics

The population at time t is split into two groups: those working (young), whose ages are between 0 and R ($a \in [0, R]$), and those retired (old), whose ages are between R and A ($a \in (R, A]$), where R is the retirement age and A is the maximum age or longevity. Because the main concern of this paper are the effects of health care expenditure for the elderly, it is assumed that survival probabilities are constant and equal to one when an individual is young, and decreasing and concave in age when he is old. Hence, the survival probabilities are as follows:

$$m(a, A) = \begin{cases} 1 & \text{if} \quad a \in [0, R] \\ f(a, A) & \quad\quad\; a \in (R, A] \end{cases}, \tag{11}$$

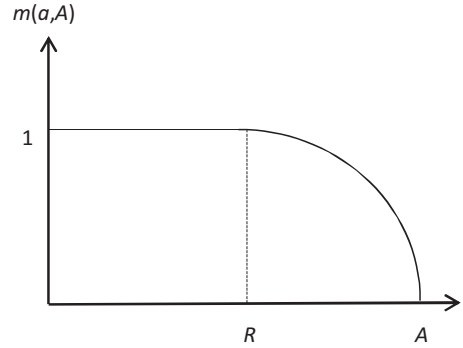
where $\lim_{a \to R^+} f(a, A) = 1$ and $f(A, A) = 0$, which implies that $0 \leq f(a, A) < 1$ must hold for all $a \in (R, A]$. Further, it is assumed that $f(a, A)$ is strictly

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

decreasing and strictly concave in a and strictly increasing in A:

$$\frac{\partial f}{\partial a} < 0, \ \frac{\partial^2 f}{\partial a^2} < 0, \frac{\partial f}{\partial A} > 0.$$

The following figure shows the survival function:



Note that although longevity is uncertain for a given individual, the fraction of individuals reaching a certain age is deterministic for the social planner (see below). Following Boucekkine et al. (2002), the number of individuals born is assumed to grow at a constant rate n, which affects the age distribution in steady state. The number of individuals born at time t is φe^{nt}, where $\varphi > 0$. The number of individuals aged a at time t is, therefore,

$$l\,(t, n, a, A) = \varphi e^{n(t-a)} m\,(a, A) > 0. \tag{12}$$

Note that $l(t, n, 0, A) = \varphi e^{nt} > 0$, $l(t, n, A, A) = 0$, and $\frac{\partial l}{\partial a} < 0$; the number of individuals born at time t is φe^{nt}, the number of individuals exceeding the maximum age A is zero, and the number of individuals aged a is strictly decreasing in a for all $a \in (R, A]$, t and A. In addition, $\frac{\partial l}{\partial A} > 0$ for all $a \in (R, A]$, implying that the number of old individuals at each age level $a \in (R, A]$ is increasing in longevity. Using (12), the population mass in the economy at time t can be written as

$$N\,(t, n, A) = \int_{a=0}^{R} l\,(t, n, a, A)\,da + \int_{a=R}^{A} l\,(t, n, a, A)\,da, \tag{13}$$

and the number of young and old individuals, respectively,

$$N_w\,(t, n, R, A) = \int_{a=0}^{R} l\,(t, n, a, A)\,da, \tag{14}$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

$$N_o(t, n, R, A) = \int_{a=R}^{A} l(t, n, a, A)\, da. \tag{15}$$

It follows from (12)–(15) that the population growth rate and the rate of growth of young and old individuals in the economy follow the growth rate of individuals born: $\dot{N}/N = n$ and $\dot{N}_w/N_w = \dot{N}_o/N_o = n$, where $\dot{N}$, $\dot{N}_w$, and $\dot{N}_o$ are the time derivatives, while the dependency ratio, i.e., the ratio between the number of old and young N_o/N_w, is constant over time.

Furthermore, the number of the young and the old individuals at a given time is strictly increasing in the population growth rate:

$$\frac{\partial N_w}{\partial n} = \int_{a=0}^{R} \varphi(t-a)\, e^{n(t-a)} m(a, A)\, da = \int_{a=0}^{R} (t-a)\, l(t, n, a, A)\, da > 0, \tag{16}$$

$$\frac{\partial N_o}{\partial n} = \int_{a=R}^{A} \varphi(t-a)\, e^{n(t-a)} m(a, A)\, da = \int_{a=R}^{A} (t-a)\, l(t, n, a, A)\, da > 0, \tag{17}$$

where (16) is greater than (17) if $\frac{R}{A} > 1 - \frac{R}{A}$, as is shown in Appendix C.

4.2. Individual Utility

Individuals gain utility from consumption:

$$U(c(a)) \text{ for } a \in [0, A] \tag{18}$$

where $c(a)$ is consumption for an individual aged a at each point in time. Utility from consumption is standard (strictly increasing and concave and satisfies the Inada conditions). As the results below will show the absolute value of utility matters for allocation. Hence, it is necessary to assume that the utility function is such that $U(c(a)) > 0$ for $a \in [0, A]$. As is discussed in Hall and Jones (2007), this requires that for a constant relative risk aversion utility function $\frac{c(a)^{1-\sigma}}{1-\sigma}$, one has to assume that there exists a $\nu > 1$ if $\sigma > 1$ (the case of intertemporal elasticity of substitution being less than one) such that $\nu + \frac{c(a)^{1-\sigma}}{1-\sigma} > 0$ holds. This is assumed in what follow and, as is shown in Appendix F, the results of the paper hold under such a utility function.

4.3. Health Care Expenditure and Longevity

As in the simple model, health care expenditure per old individual τ is assumed to affect longevity and thus the health of old individuals (since $f(a, A)$ is assumed to be strictly increasing in A) in the following way:

$$A = \gamma B(\tau), \tag{19}$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

where $\gamma > 0$ is a parameter measuring efficiency in health care production, and $B' > 0$ and $B'' < 0$ implying positive but diminishing returns to health care expenditure. Furthermore, it is assumed that $\gamma B(0) > R$ ensuring $A>R$ for all $\tau \geq 0$ implying that individuals reach retirement age even though there is no spending on old-age health care.

4.4. Output

Each young individual produces $y > 0$ at each point in time. National output in the economy at time t can therefore be written as follows:

$$N_w(t, n, R, A)\, y, \tag{20}$$

It follows that national output can only increase when the number of working-age individuals increases or if output per working individual goes up.

This and constant dependency ratio ($\frac{N_o}{N_w}$) (see above) ensures stationarity in the model and the existence of steady-state solutions for the optimal consumption per capita and health expenditure per old individual since it implies that output per capita $\frac{N_w y}{N_w+N_o} = \frac{y}{1+\frac{N_o}{N_w}}$ is constant over time.

4.5. The Social Planner's Problem

We assume a balanced current account growth path in steady state and hence balanced budget constraints for the economy at each point in time. The economy-wide budget constraint is therefore in balance at all points in time:

$$\begin{aligned}
&\int_{a=0}^{R} l(t, n, a, A)\, y da \\
&\quad = \int_{a=0}^{R} l(t, n, a, A)\, c(a) da + \int_{a=R}^{A} l(t, n, a, A)\, c(a) da \\
&\qquad + \int_{a=R}^{A} l(t, n, a, A)\, \tau da,
\end{aligned}$$

which can, after multiplying through by e^{-nt}, be written as follows:

$$\begin{aligned}
&\int_{a=0}^{R} \tilde{l}(n, a, A)\, y da \\
&\quad = \int_{a=0}^{R} \tilde{l}(n, a, A)\, c(a) da + \int_{a=R}^{A} \tilde{l}(n, a, A)\, c(a) da + \int_{a=R}^{A} \tilde{l}(n, a, A)\, \tau da,
\end{aligned} \tag{21}$$

where $\tilde{l}(n, a, A) = e^{-nt} l(t, n, a, A)$.

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

Because a balanced budget for the economy is assumed, there is no transfer of resources across time in steady state. The social planner's welfare objective can therefore be written as

$$W = \int_{a=0}^{R} l\,(t, n, a, A)\, U(c(a))da + \int_{a=R}^{A} l\,(t, n, a, A)\, U(c(a))da,$$

which can, after multiplying through by e^{-nt}, be written as

$$\tilde{W} = \int_{a=0}^{R} \tilde{l}\,(n, a, A)\, U(c(a))da + \int_{a=R}^{A} \tilde{l}\,(n, a, A)\, U(c(a))da, \qquad (\mathbf{22})$$

where $\tilde{W} = e^{-nt}W$. In essence, the social planner is maximizing the sum of utilities of all living generations at a point in time, and thus gives a golden rule, taking into account that his decision affects both consumption of each working individual through taxes as well as the number of old individuals who get to live due to the provision of health care. He can choose to increase the number of the living by reducing the consumption of each member of the working-age population and finds the optimal amount of expenditures on health care, hence also the optimal tax rate, by trading off one effect against the other.

The maximization problem solved by the social planner is equivalent to maximizing the expected lifetime utility of a given individual over time in that by always maximizing the sum of utilities of all living individuals of different ages he manages to maximize the expected lifetime utility of each individual.

The maximization of this objective function subject to the budget constraint in (21) gives the social optimum. Note that the welfare function in (22) is strictly increasing in $c(a)$ for all $a \in [0, A]$ and τ (through A). The budget constraint in (21) ensures that a maximum exists to the constrained maximization problem (increased spending on health care per old individual decreases consumption given output and hence raises the marginal utility of consumption, ensuring that a maximum exists).[11] Also note that time t does not appear in (21) or (22) and, hence, the solutions for consumption per capita and spending on health care per old individual are independent of time.

The Lagrangian for the maximization problem is (after using (11), (12), (19), (21), and (22)) as follows:

$$\Gamma = \int_{a=0}^{R} \varphi e^{-na} U(c(a))da + \int_{a=R}^{\gamma B(\tau)} \varphi e^{-na} f\,(a, \gamma B\,(\tau))\, U(c(a))da$$

$$+ \lambda \left[\int_{a=0}^{R} \varphi e^{-na} y da - \int_{a=0}^{R} \varphi e^{-na} c(a)da \right.$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

$$- \int_{a=R}^{\gamma B(\tau)} \varphi e^{-na} f\left(a, \gamma B\left(\tau\right)\right) c(a) da - \int_{a=R}^{\gamma B(\tau)} \varphi e^{-na} f\left(a, \gamma B\left(\tau\right)\right) \tau da \Bigg].$$

Assuming an interior solution, the first-order conditions are derived using that $f(A, A) = 0$. First, the marginal utility is set equal to the shadow price of output λ:

$$\frac{\partial U}{\partial c(a)} = \lambda \text{ for } a \in [0, A]. \tag{23}$$

The second condition makes that marginal benefit of increasing spending on the health care of each older individual τ:

$$\begin{aligned}
&\left[\int_{a=R}^{A} \varphi e^{-na} U(c(a)) \frac{\partial f}{\partial A} da\right] \gamma B'(\tau) \\
&\quad - \lambda \left[\int_{a=R}^{A} \varphi e^{-na} (c(a)) \frac{\partial f}{\partial A} da + \int_{a=R}^{A} \varphi e^{-na} \tau \frac{\partial f}{\partial A} da\right] \gamma B'(\tau) \\
&\quad - \lambda \int_{a=R}^{A} \varphi e^{-na} f(a, A) da, \\
&= 0
\end{aligned} \tag{24}$$

where (19) has been used. The last condition is the budget constraint for the economy that sets total output equal to the sum of the consumption of the young, consumption of the old, and the provision of health care for the old:

$$\int_{a=0}^{R} \varphi e^{-na} y da = \int_{a=0}^{R} \varphi e^{-na} c(a) da + \int_{a=R}^{A} \varphi e^{-na} c(a) da + \int_{a=R}^{A} \varphi e^{-na} \tau da. \tag{25}$$

The first condition implies that the marginal utility of consumption is independent of age (because λ is independent of age). Hence, we have consumption smoothing across generations:

$$c(a) = c \text{ for } a \in [0, A]. \tag{26}$$

Using (23) in (24) after multiplying through by e^{nt}, using (26) and doing some manipulation using (11), (12), (14), and (15) gives our main result:

$$U\left(c\right) \frac{\partial N_0}{\partial A} \gamma B'\left(\tau\right) = \frac{\partial U}{\partial c} \left(c + \tau\right) \frac{\partial N_o}{\partial A} \gamma B'\left(\tau\right) + \frac{\partial U}{\partial c} N_o\left(t, n, R, A\right). \tag{27}$$

The equation gives optimal spending on health care per old individual, which is our golden rule of health care spending. It is analogous to equation (6). The left-hand side shows increased social welfare in terms of a greater number of

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

old individuals reaching each age level and therefore more individuals receiving utility from consumption. The first term on the right-hand side shows the lost utility for all others, whose consumption is reduced due to the consumption and medical needs of those who now reach higher age levels because of the increased provision of health care. The second term denotes the marginal cost of increased health care for all individuals over retirement age. There arises a clear trade-off between increasing spending on life-extending medical care and allowing people to enjoy extended lives, on one hand, and increasing the consumption of those who would have lived in the absence of increased spending on medical care, on the other hand.

We now come back to the repugnant conclusion emphasized in the philosophy literature. This conclusion does not arise in equation (27) above since the maximization of total utility does generate a solution for the optimal size of the population. While increasing expenditures on health care provision does reduce mortality among the old and generate a larger number of living individuals, there is a cost in terms of diminished utility of the working-age population as well as those who are old but would have survived in the absence of the increased expenditures. Moreover, there are diminishing returns to health care spending ($B''(\tau) < 0$). There comes a point at which increasing expected longevity further —and increasing the number of people alive—reduces the utility of these two groups and does not lengthen lives sufficiently to make their marginal utility approach infinity because the utility function satisfies the Inada conditions. Thus, we end up avoiding both Parfit's Repugnant Conclusion as well as Cowen's Methuselah's Paradox.[12]

Finally, the budget constraint in (21) has to hold, which can be written in the following way after multiplying by e^{nt}, using the result in (26) and doing some manipulation using (11), (12), (14), and (15):

$$(y - c)\, N_w\,(t, n, R, A) = (c + \tau)\, N_o\,(t, n, R, A)\,. \qquad \textbf{(28)}$$

The net output consumption of the working-age population must equal the sum of consumption and the health care expenditures of the old generations.

Note that we do not allow for spending on health care to raise the level of health of working-age individuals, only to extend the lives of the old beyond the age at which they would otherwise have died. Moreover, we do not allow for the output from increased health care expenditures to benefit the working-age individuals contemporaneously, only to extend their lives and hence their expected lifetime utility. Another limitation of the analysis is that there is an empirical connection between health levels when young and in later years, which is not present in the model.[13] However, making the utility of the working-age individuals, as well as their life expectancy, depends on their health outcomes while working, and hence the level of health care, would not add insights into our analysis while complicating the formulation of the model unnecessarily.

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

4.6. Comparative Statics

Since the second-order conditions for a maximum are fulfilled, the conditions in (27) and (28) give the optimal steady-state levels of consumption per capita c and health care expenditure per old individual τ as implicit functions of population growth n, productivity y, the retirement age R, the parameters φ and γ, and the functional forms of the utility function U and the health care production function A.[14] Below we analyze how productivity, both in the private sector y and in the public sector γ, as well as the population growth rate n affect the optimal level of health care expenditure per old individual τ.

The effect of increased productivity y is given by[15]:

$$\frac{\partial \tau}{\partial y} = \frac{\frac{\partial^2 U}{\partial c^2}\left(N_o + (c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right) N_w}{\Psi} > 0, \tag{29}$$

where

$$\Psi \equiv (N_w + N_o)\,\frac{\partial U}{\partial c} N_o \left[\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - 2\frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right]$$
$$+ \frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]^2.$$

Note that Ψ is negative, as is shown in Appendix E, the numerator is negative due to diminishing marginal utility, and $\frac{\partial N_o}{\partial A} > 0$, as can be seen from (11), (12), and (15), which gives the sign in (29).

Intuitively, the marginal utility of consumption of each member of the working-age population declines as the level of productivity increases. This is reflected in the negative sign of the numerator, which is negative due to its inclusion of the second derivative of the utility function. As in Hall and Jones (2007),[16] lower marginal utility reduces the burden of paying taxes to support the currently old and to pay for their health care, which increases the golden rule level of health care provision. It follows that people should live longer in more developed countries and poor countries should spend less on extending the lives of their citizens.[17]

The effect of increased efficiency in health care provision γ is given by[18]:

$$\frac{\partial \tau}{\partial \gamma} = -\frac{\begin{array}{r}\left[(N_w + N_o)\,\frac{\partial U}{\partial c} N_o \left(\frac{1}{A} + \frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - \frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\right.\\ \left. + \frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right](c+\tau)\,\frac{\partial N_o}{\partial A}\right] B(\tau)\end{array}}{\Psi} \underset{<}{>} 0. \tag{30}$$

The second term in the numerator is negative, which gives a negative derivative as in the simple model in Section 3. The additional insight given by this model

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

comes from the bracket in the first term, where

$$\frac{\partial\left(\frac{\frac{\partial N_o}{\partial A}}{N_o}\right)}{\partial A}=\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2}-\frac{1}{N_o}\frac{\partial N_o}{\partial A}$$

are the effects of an increase in longevity on the relative increase in the number of old from an increase in longevity, which equals $-\frac{1}{A}$ in the simple model in Section 3 making the first term in the numerator vanish. Here, this is negative (since $\frac{\partial^2 N_o}{\partial A^2}<0$) giving decreasing effects of an increase in longevity on the number of old. However, whether this is greater than or less than $\frac{1}{A}$ is ambiguous leaving the sign of the first term ambiguous resulting in the sign of the derivative being ambiguous. Hence, further specification for the survival function in (11) is necessary for determining the sign of the derivative.

Intuitively, increased efficiency in health care provision results in a reduction in the health care expenditures per capita needed to maintain unchanged longevity and health, which implies a negative sign for the derivative in (30), while the marginal benefit from spending on health care increases, which implies a positive sign for the derivative. It is not clear which effect is stronger.

The effect of increased population growth rate n is given by[19]:

$$\frac{\partial\tau}{\partial n}=-\frac{\left\{\begin{array}{c}(N_w+N_o)\,\frac{\partial U}{\partial c}N_o\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_0}{\partial A\partial n}-\frac{1}{N_o}\frac{\partial N_o}{\partial n}\right)\\+\frac{\partial^2 U}{\partial c^2}\left[N_o+(c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right](c+\tau)\,N_o\left(\frac{1}{N_o}\frac{\partial N_o}{\partial n}-\frac{1}{N_w}\frac{\partial N_w}{\partial n}\right)\end{array}\right\}}{\Psi}\gtreqless 0. \tag{31}$$

Both terms in the numerator are indeterminate. The last bracket in the first term is,

$$\frac{\partial\left(\frac{\frac{\partial N_o}{\partial A}}{N_o}\right)}{\partial n}=\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A\partial n}-\frac{1}{N_o}\frac{\partial N_o}{\partial n},$$

giving the effects of an increase in the population growth rate on the relative increase in the number of old from an increase in longevity. This and hence the first term in numerator are positive (negative) if an increase in the population growth rate accelerates (slows down) the relative growth in the number of old from an increase in longevity implying that old-age health care expenditure should increase (decrease). The last bracket in the second term is,

$$\frac{\partial\left(\frac{N_o}{N_w}\right)}{\partial n}=\frac{1}{N_o}\frac{\partial N_o}{\partial n}-\frac{1}{N_w}\frac{\partial N_w}{\partial n},$$

giving the effects of an increase in the population growth rate on the dependency ratio. Hence, the second term in the numerator is negative (positive) if an increase in the population growth rate results in an increase (decrease) in the dependency ratio (remember that $\frac{\partial^2 U}{\partial c^2}<0$). We can therefore conclude that expenditure on

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

old-age health care should increase following an increase in the population growth rate if it accelerates the relative growth in the number of old from an increase in longevity and if it results in decrease in the dependency ratio. Hence, and increase in the population growth rate should increase spending on old-age health care if there is a sufficient decrease in the dependency ratio.

The intuition behind this result is that an acceleration in the effects of an increase in longevity on the number of old individuals results in spending on old-age health care being more effective in terms of increasing longevity and the number of the old while a decrease in the dependency ratio results in more individuals working relative to being retired and, hence, output per capita increases making it being optimal to spend more resources on old-age health care due to diminishing marginal utility of consumption.

Finally, it is interesting to analyze how an increase in the utility from living, measured by the ν in Hall and Jones (2007) nonhomothetic CRRA (constant relative risk aversion) utility function:

$$U(c(a)) = \nu + \frac{c(a)^{1-\sigma}}{1-\sigma},$$

where $\nu > 1$ if $\sigma > 1$ (the case of intertemporal elasticity of substitution being less than one) such that $\nu > 0$ holds, affects the optimal spending on old-age health care. This gives the following results:

$$\frac{\partial \tau}{\partial \nu} = \frac{-\left(N_w + N_o\right) \frac{\partial N_o}{\partial A} \gamma B'(\tau)}{\Psi} > 0$$

Not surprisingly, when life becomes more valuable (an increase in ν), it is socially optimal to respond by spending more on life-extending health care.

5. INDIVIDUAL DECISION MAKING

We now leave the social planner's problem and describe the decision by a representative agent on how much to spend on life-extending health care.

Due to the uncertain lifetime of an individual, it is assumed that there exists an annuities market [as in Yaari (1965)] providing an individual with an instrument to insure himself against an uncertain lifetime [see, for example, Blanchard (1985) and Sheshinski (2008)]. This implies that an individual buys annuities at any age a from an insurance company earning rate of return $r_b(a)$ and the insurance company invests at an exogenous rate of return r for the amount of annuities bought. A zero-profit condition for the insurance company—due to the assumption of free entry into the competitive annuity market—gives the following in equilibrium:

$$r_b(a) = r + \rho(a, A) \tag{32}$$

where $\rho(a, A) = -\frac{\frac{\partial m(a,A)}{\partial a}}{m(a,A)}$ is the hazard rate for an individual aged a, i.e., the probability that an individual dies at age a conditional on being alive at age a. It

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

follows that the higher the mortality rate the higher is r_b. Hence, using equation (11) gives

$$\rho(a, A) = \begin{cases} 0 \\ -\frac{\frac{\partial f(a,A)}{\partial a}}{f(a,A)} \end{cases} \quad \text{for } a \in \begin{matrix} [0, R] \\ (R, A] \end{matrix} \tag{33}$$

and, hence, in equilibrium:

$$r_b(a) = \begin{cases} r \\ r + \rho(a, A) \end{cases} \text{ for } a \in \begin{matrix} [0, R] \\ (R, A] \end{matrix}. \tag{34}$$

The preceding equations show that the higher the mortality rate among retirees, the higher are the earnings on the annuity in equilibrium.

An individual's expected lifetime utility is

$$U = \int_{a=0}^{A} e^{-\delta a} m(a, A) U(c(a)) da,$$

where $\delta > 0$ is the subjective rate of time preference. Using (11), the equation can be written as

$$U = \int_{a=0}^{R} e^{-\delta a} U(c(a)) da + \int_{a=R}^{A} e^{-\delta a} f(a, A) U(c(a)) da. \tag{35}$$

An individual consumes when young and old, works and earns labor income when young and spends on old-age health care when old. His budget constraint therefore reads

$$\int_{a=0}^{R} e^{-\Gamma(a)} y da = \int_{a=0}^{R} e^{-\Gamma(a)} c(a) da + \int_{a=R}^{\gamma B(\tau)} e^{-\Gamma(a)} c(a) da + \int_{a=R}^{\gamma B(\tau)} e^{-\Gamma(a)} \tau da, \tag{36}$$

where $\Gamma(a) = \int_{z=0}^{a} r_b(z) dz$.

An individual's problem is to choose $\{c(a)\}_{a=0}^{A}$ and τ such that (35) is maximized subject to (36) and (19) taking the annuity contract r_b as given (hence, it is independent of longevity A in an individual's optimization problem). The

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

Lagrangian for the problem is

$$\Lambda = \int_{a=0}^{R} e^{-\delta a} U(c(a)) da + \int_{a=R}^{\gamma B(\tau)} e^{-\delta a} f\left(a, \gamma B\left(\tau\right)\right) U(c(a)) da$$
$$+ \lambda \left[\int_{a=0}^{R} e^{-\Gamma(a)} y da - \int_{a=0}^{R} e^{-\Gamma(a)} c(a) da - \int_{a=R}^{\gamma B(\tau)} e^{-\Gamma(a)} c(a) da \right.$$
$$\left. - \int_{a=R}^{\gamma B(\tau)} e^{-\Gamma(a)} \tau da \right].$$

Assuming an interior solution, this gives the following first-order conditions [in addition to the budget constraint in (36)]. The first is the first-order condition for consumption during working years:

$$\frac{\partial \Lambda}{\partial c(a)} = e^{-\delta a} \frac{\partial U}{\partial c(a)} - \lambda e^{-ra} = 0 \quad \text{for } a \in [0, R]. \tag{37}$$

The second condition gives optimal consumption in retirement:

$$\frac{\partial \Lambda}{\partial c(a)} = e^{-\delta a} f\left(a, A\right) \frac{\partial U}{\partial c(a)} - \lambda e^{-\Gamma(a)} = 0 \quad \text{for } a \in [R, A], \tag{38}$$

where (19) has been used. In both cases the marginal utility of consumption is set equal to the shadow price of wealth. The main difference is that the old face a higher interest rate because the annuity companies pay them a higher return on their savings due to their falling numbers.

Using that in equilibrium [using (32)–(34)]:

$$e^{-\Gamma(a)} = e^{-\int_{z=0}^{a} [r+\rho(z,A)] dz} = e^{-\int_{z=0}^{a} \left[r + \frac{-\frac{\delta f(z,A)}{\delta z}}{f(z,A)}\right] dz} = e^{-ra + \int_{z=0}^{a} \left[\frac{\frac{\delta f(z,A)}{\delta z}}{f(z,A)}\right] dz}$$
$$= e^{-ra + \int_{z=0}^{a} \left[\frac{d \ln f(a,A)}{dz}\right] dz} = e^{-ra + \int_{z=0}^{a} d \ln f(z,A)}$$
$$= e^{-ra + [\ln f(a,A) - \ln f(0,A)]} = e^{-ra} e^{\ln f(a,A)} = e^{-ra} f\left(a, A\right),$$

where it is used that $f(0, A) = 1$, gives the condition:

$$\frac{\partial \Lambda}{\partial c(a)} = e^{-\delta a} \frac{\partial U}{\partial c(a)} - \lambda e^{-ra} = 0 \text{ for all } a \in [0, A]. \tag{39}$$

Since we are only interested in comparing allocation under individual decision making with the steady-state social planner allocation, we ignore the possibility of a time-varying consumption profile for an individual being optimal and assume that that the real interest rate r equals the subjective rate of time preference δ and,

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

hence,

$$\frac{\partial U}{\partial c(a)} = \lambda \text{ for all } a \in [0, A],$$

and

$$c(a) = c \text{ for } a \in [0, A]. \tag{40}$$

The first-order condition with respect to τ is as follows:

$$\frac{\partial \Lambda}{\partial \tau} = e^{-\delta a} f(A, A) U(c(a)) \gamma B(\tau) + \gamma B(\tau) \int_{a=R}^{A} e^{-\delta a} \frac{\partial f}{\partial A} U(c(a)) da$$

$$- \lambda e^{-\Gamma(a)} c(a) \gamma B(\tau) - \lambda e^{-\Gamma(a)} \gamma B(\tau) - \lambda \int_{a=R}^{A} e^{-\Gamma(a)} da = 0$$

where (19) has been used. Using (32)–(34) gives in equilibrium:

$$\frac{\partial \Lambda}{\partial \tau} = e^{-\delta a} f(A, A) U(c(a)) \gamma B(\tau) + \gamma B(\tau) \int_{a=R}^{A} e^{-\delta a} \frac{\partial f}{\partial A} U(c(a)) da$$

$$- \lambda e^{-r\mathrm{A}} f(A, A) c(a) \gamma B(\tau) - \lambda e^{-r\mathrm{A}} f(A, A) \gamma B(\tau)$$

$$-\lambda \int_{a=R}^{A} e^{-ra} f(a, A) da = 0$$

or, since $f(A, A) = 0$:

$$\gamma B(\tau) \int_{a=R}^{A} e^{-\delta a} \frac{\partial f}{\partial A} U(c(a)) da = \lambda \int_{a=R}^{A} e^{-ra} f(a, A) da = 0.$$

Finally, using (40) and making the real interest rate r equal the subjective rate of time preference δ, gives

$$U(c) \left[\int_{a=R}^{A} e^{-ra} \frac{\partial f}{\partial a} da \right] \gamma B(\tau) = \frac{\partial U}{\partial c} \left[\int_{a=R}^{A} e^{-ra} f(a, A) da \right] \tag{41}$$

The left-hand side gives the increased expected lifetime utility from increased spending on old-age health care and the right-hand side gives the marginal costs in terms of increased spending on old-age health care. Note that the Methuelah's Paradox does not arise, the individual will not desire to have a very long life at a low level of consumption. As consumption approaches zero, the marginal utility of consumption will approach infinity and so exceed the value of living an extra year.

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

6. SOCIAL OPTIMUM AND INDIVIDUAL DECISION MAKING COMPARED

Equation (41) can be compared to equation (42) that gives the optimal (that is the social planner) level of spending on the health care. Using equations (11), (12), and (15) in (27), multiplying through with e^{-nt} and dividing by the parameter φ gives that spending on old-age health care in social optimum has to fulfill:

$$U(c)\left[\int_{a=R}^{A} e^{-na}\frac{\partial f}{\partial A}da\right]\gamma B'(\tau) = \frac{\partial U}{\partial c}(c+\tau)\left[\int_{a=R}^{A} e^{-na}\frac{\partial f}{\partial A}da\right]\gamma B'(\tau) + \frac{\partial U}{\partial c}\left[\int_{a=R}^{A} e^{-na} f(a, A)\, da\right] \quad (42)$$

The left-hand side has the utility of the individuals who survive due the increased health care spending. The first term on the right has the utility effect of reduced consumption due to the consumption of these individuals as well as their health care spending. The second term denotes the marginal cost of increased health care for all individuals over retirement age. This can be compared to equation (41).

The main differences between equations (41), showing the private decision on health care, and equation (42), showing the social optimum lies (i) in the first term on the right-hand side of equation (42) that is missing from equation (41) and (ii) in differences in discounting. First, assuming that r=n, the missing term in equation (41) measures the effect of lower mortality on the rate of interest paid by insurance companies r_b. In essence, individuals ignore the effect their increased longevity has on the income of other members of their cohort. It follows that people overinvest in life-extending health care by ignoring the negative externality their own longevity has on the consumption of other members of their cohort.

When we relax the assumption that $r = \delta = n$, we get some additional reasons why the individual decisions may differ from the social optimum. An impatient individual whose $\delta > n$ will discount the benefits of a longer life with a discount rate that exceeds the rate used by the social planner. The social planner uses the population growth rate n because the higher is n the larger are the cohorts or generations and the greater is the benefit from extending the lives of every cohort. The individual, in contrast, discounts the future utility from a longer life by his subjective rate of time preferenceδ, which could be different from n. Conversely, if individuals are very patient, have a low δ, then they may discount the benefits of a longer life with a lower discount rate than the social planner. It follows that individuals may spend less or more on life-extending health care than the socially optimal amount. In particular, patient individuals—those having a rate of time preference that is lower than the rate of population growth $\delta < n$—will spend "too much" on life-extending health care compared to the social planner's solution.

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

Second, the costs of increased health care in terms of the lower consumption of the old are discounted by the rate of population growth n in the social planner's problem but by the real interest rate r in the individual optimization. The reason is that the individual finds the discounted future sum of increased health care spending and uses the interest rate to discount this future stream. It follows that if $r < n$ then the future costs for the individual will be greater than those calculated by the social planner and conversely if $r > n$. It follows that in a dynamically efficient economy, where $r > n$, individuals will spend more on life-extending health care than the socially optimal amount.

7. DISCUSSION

We have derived a golden rule for the amount that a nation should sacrifice in terms of the consumption of individuals in order to extend the life of generations. The results show that more productive societies should spend more per capita on health care because the utility loss of individuals is smaller due to a lower marginal utility of consumption. Also, an increase in the retirement age and the rate of population growth raises the optimal level of health care spending, the latter if it accelerates the effects of increased longevity on the number of old and decreases the dependency ratio. However, increased efficiency in health care provision has an ambiguous effect.

Comparing the social optimum with the outcome of utility maximization by individuals shows that there is no guarantee that economies will invest the socially optimal amount in life-extending health care and indeed this would only happen by chance. The main reason for our results is that individuals do not take into account the negative effect their own longevity has on the return to annuities of other members of their cohorts, hence the consumption and lifetime utility of these individuals. This effect is strengthened if individuals have a low rate of time preference ($\delta < n$), and thus do not discount additional time of life heavily, and the rate of interest is high ($r > n$), which makes them discount the costs of extra health care at a higher rate.

There remains the issue what can be done so that a market economy spends the socially optimal amount on life-extending health care. In the case when the rate of time preference equals the interest rate, which equals the rate of population growth, there is excessive spending on health care because individuals do not take the crowding of their longevity into account. Each long life beyond retirement age contributes to a changed demography that entails lower lifetime income and consumption for everyone. Clearly, it is privately optimal in our model for an individual to outlive his cohort and enjoy simultaneously a long life and higher consumption caused by the passing away of many members of his cohort.[20] To paraphrase, devoting resources to extend one's life imposes a financial burden on the rest of society but this is not taken into account by each individual because the alternative would be death or at least a higher probability of death. There are a few individuals that are so selfless as to want to die to raise the standard of living of

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

those surviving. This would also involve problems of collective action: One death would not improve the standard of living of anyone much, while if many people are dying one can put it off knowing that one's own decision will not matter! The tendency to spend excessively on health care is then exacerbated by a dynamic efficient economy with a rate of interest higher than the rate of population growth in a growing economy. In such an economy, individuals discount future spending on health care, hence the marginal costs of extending lives, at a rate which is higher than the population growth rate. Finally, a low rate of time preference would make each individual appreciate more the prospects of longer lives and as a result spend more on health care. So what can a government do in this case?

Leighton and Hughes (1955) describe how Eskimo societies from Greenland to Alaska solved the collective action problem by practicing senicides. In a hunter society people become a burden on society when they stopped being able to hunt. These authors quote Weyer (1932)[21] that "A native of Angmagasalik will throw himself into the sea, often prompted by the admonition from his relatives that he has nothing to live for." Leighton and Hughes describe how in Eskimo literature the killing of aged parents by dutiful children was a common occurrence. An early account of the death of elders by suicide among the Igluik Inuit is that of Rasmussen (1929, 1931).[22] He describes how religious beliefs encouraged old men and women to hang themselves in the belief that the transition from life to death and on to the world of sprit was both brief and painless. The elderly were very rarely coerced to commit suicide, rather they took the initiative propelled by cultural factors and religious beliefs. This is explained by the very low level of production per member of the household, an outcome that comes out of our equation (27) for a low value of consumption per capita when τ is allowed to take a negative value. In this case the marginal benefit of increasing health care expenditures is very low and the marginal cost very high due to low consumption so that the optimal level of τ is zero (or negative when allowed).

In modern societies, the obvious solution to the externality described in our model would be to levy taxes on private old-age health care increasing the private cost from spending on old-age health care and the private cost from increasing longevity. Another way would be to levy taxes on annuities return so as to decrease the return from increasing longevity. A third solution to the problem would be to subsidize health care if individuals do not save adequately for health care due to a high rate of time preference. It would be too low if people are short sighted and do not think much about the last few years of life and also if interest rates are very low so that the expected discounted cost of health care is very high. A public health care system could also be used to provide the socially optimal level of health care.

Currently, most developed countries are either already experiencing or expecting budgetary problems because of the aging of the population. The crowding effects described in this paper would apply not only to the rate of return to annuities, retirement schemes, and pension funds, which affect the level of private consumption, but also, in an extended model, to the provision of public consumption

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

such as old-age care. Public debate in many European countries is centered on these issues. There is the question about taxation, public debt and public services, the return to pension funds' assets, public services, the retirement age, and, last but not least, immigration. One case for accepting immigrants is based on the observation that they tend to be of working age and so will increase the lifetime consumption of the indigenous population. As our model has shown, increased population growth due to immigration could actually increase the level of optimal health care spending due to lower dependency ratio, making it possible for the native population to live longer. Omitting the consideration of population growth, the age structure, and the size of the working-age population from a discussion of health care, immigration, public services, and taxation may lead to incorrect or misleading policy conclusions.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

NOTES

1. According to the World Bank the United States spent on an average from 2001 to 2010 around 16% of gross domestic product (GDP) on health care (sum of public and private costs).

2. Mertens and Rubinchik (2015) derive a golden-rule equilibrium in an overlapping-generations model and show it to be Pareto optimal with constant returns to scale production, transfers, arbitrary life-time productivity, and homogeneous instantaneous felicity.

3. Their approach allows for the case of low intertemporal substitution while maintaining homotheticity and can also explain why poor people value life extension no less than wealthier people.

4. This result mirrors the standard result that a pay-as-you-go pension system is welfare reducing in a dynamically efficient economy.

5. We ignore the effect of health care spending on the health of working-age individuals and hence also their productivity. See Madsen (2016) for evidence on the link between health status and productivity.

6. In Appendix A, it is shown that the first-order conditions derived below in fact give a maximum for the welfare function in (1), subject to the budget constraint in (2).

7. This equation is similar to equation (5) on page 46 in Hall and Jones (2007).

8. See Appendix B for detailed derivations.

9. Our setting is based on Andersen and Gestsson (2016).

10. The setting of our model is directly comparable to the chapter on the golden rule of education in Edmund Phelps's book *Golden rules of economic growth: studies of efficient and optimal investment*. In the education setting, the social planner has to decide how many years people spend in school, the more years the more productive they become but the fewer are the years left to have productive work. There is no time dimension here. If now a social planner were to add one year to everyone's education, the initial effect would be to reduce output because there would be fewer people in the labor market and no one had become more productive. Only gradually would the more productive cohorts enter the labor market and it would take decades for the full effect on output to appear.

11. In Appendix D, it is shown that the first-order conditions derived below in fact give a maximum for the welfare function in (22), subject to the budget constraint in (21).

12. We do not need to resort to the reasons Ng (1989) proposed for people not willing to live very long lives of misery. This author argued that people might be genetically or culturally programmed to

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

desire the longest possible life but not a life of thousands of years and that people may not be able to understand the significance of large numbers.

13. See Almond and Currie (2011).

14. See Appendixes D and E.

15. See Appendix E.

16. Below, we show the social optimum and the comparative statics exercises using a CRRA utility function in order to further compare our results to those of Hall and Jones (2007).

17. Empirical studies have shown that the actual level of health care expenditures is increasing in the level of productivity, hence output. Newhouse (1977) studied the relationship between GDP per capita and per capita medical care expenditures for 13 developed countries and found that more than 90% of the variation in the level of health care expenditures across countries could be explained by differences in the level of GDP per capita. Several other studies found similar results using data for different samples of countries and different time periods. Gerdtham et al. (1992) extended this analysis to pooled data of 20 Organisation for Economic Co-operation and Development (OECD) countries over the period 1960–1987 and also found a positive income elasticity of health care spending with respect to GDP. The same applies to Parkin et al. (1987), who use PPP-adjusted numbers for health care spending and GDP per capita in a cross-section of countries. Hitiris and Posnett (1992) also found a positive effect of income on health care.

18. See Appendix E.

19. See Appendix E.

20. Note that our model omits any positive effect on utility from mingling with people in one's own age group!

21. Page 248.

22. Taken from Kirmayer et al. (1998).

23. In fact, this utility function can be seen as a special case of the utility function in (18) assuming that the utility level is positive $U(c(a)) > 0$ and, hence, using the utility function proposed in Hall and Jones (2007) should not change the results and conclusions drawn.

REFERENCES

Andersen, Torben M. and Joydeep Bhattacharya (2014) A dynamic-efficiency rationale for public investment in the health of the young. *Canadian Journal of Economics* 47(3), 697–719.

Andersen, Torben M. and Marias H. Gestsson (2016) Longevity, growth and intergeneration equity – the deterministic case. *Macroeconomic Dynamics* 20(4), 985–1021.

Baltagi, Badi H. and Francesco Moscone (2010) Health care expenditures and income in the OECD reconsidered: Evidence from panel data. *Economic Modelling* 27(4), 804–811.

Becker, Gary S. (1964) *Human Capital*. New York: Columbia University Press for the National Bureau of Economic Research.

Becker, Gary, Kevin Murphy, and Tomas Philipson (2007) The Value of Life Near Its End and Terminal Care. NBER working paper no. 13333.

Blanchard, Olivier E. (1985) Debt, deficits and finite horizons. *Journal of Political Economy* 93(2), 223–247.

Boucekkine, Rarouf, David de la Croix, and Omar Licandro (2002) Vintage human capital, demographic trends, and endogenous growth. *Journal of Economic Theory* 104, 340–375

Córdoba, Juan C. and Marla Ripoll (2017) Risk aversion and the value of life. *The Review of Economic Studies* 84(4), 1472–1509.

Cowen, T. (1989) Normative population theory. *Social Choice and Welfare* 6, 33–43.

Cullis, John G. and P. West (1979) *The Economics of Health: An Introduction*. Oxford: Martin Robertson.

Currie, Janel and Douglas Almond (2011) Human capital development before age five. In Orly Ashenfelter and David Card (eds.), *Handbook of Labor Economics*, vol. 4, Part B, pp. 1315–1486. Amsterdam: Elsevier Science B.V..

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

Ehrlich, Isaac and Hiroyuki Chuma (1990) A model of the demand for longevity and the value of life extension. *The Journal of Political Economy* 98(4), 761–782.

Epstein, Larry and Stanley E. Zin (1989) Substitution, risk aversion and the temporal behavior of consumption growth and asset returns: A theoretical framework. *Econometrica* 57, 937–969.

Gerdtham, Ulf G. and Mickael Löthgren (2000) On stationarity and cointegration of international health expenditure and GDP. *Journal of Health Economics* 19(4), 461–475.

Gerdtham, Ulf G., Jes Søgaard, Bengt Jönsson, and Frederik Andersson (1992) A pooled cross-section analysis of the health care expenditures of the OECD countries. *Health Economics Worldwide* 287–310.

Grossman, Michael (1972) On the concept of health capital and the demand for health. *Journal of Political Economy* 80(2), 223–255.

Hall, Robert E. and Charles I. Jones (2007) The value of life and the rise in health spending. *The Quarterly Journal of Economics* 122(1), 39–72.

Hitiris, Theo and John Posnett (1992) The determinants and effects of health expenditure in developed countries. *Journal of Health Economics* 11, 173–181.

Jones, Charles I. (2003) Why have health expenditures as a share of GDP risen so much? *New England Journal of Medicine* 328, 215–26.

Kirmayer, Laurence J., Fletcher, Cristopher, and Lucy J. Boothroyd (1998) Suicide among the Inuit of Canada. In A. A. Lenaars, S. Wenckstern, I. Sakinofsky, R. J. Dyck, M. J. Kral, and R. C. Bland (eds.), *Suicide in Canada*, pp. 189–211. Toronto: University of Toronto Press.

Kuhn, Michael, Wrzaczek, Stefan, Prskawetz, Alexia, and Gustav Feichtinger (2015) Optimal choice of health and retirement in a life-cycle model. *Journal of Economic Theory* 158(A), 186–212.

Leighton, Alexander H. and Charles C. Hughes (1955) Notes on Eskimo patterns of suicide. *Southwestern Journal of Anthropology* 11(4), 327–338.

Leu, R. (1986) Public/private provision and health expenditures: A cross-country comparison. In A. J. Culyer and B. Jonsson (eds.), *Public and Private Provision of Health Care*. Oxford: Basil Blackwell.

Lubitz, James D. and Gerald F. Riley (1993) Trends in medicare payments in the last year of life. *New England Journal of Medicine* 328, 1092–1096.

Madsen, Jacob B. (2016) Health, human capital formation and knowledge production: Two centuries of international evidence. *Macroeconomic Dynamics* 20(4), 909–953.

Matteo, Livio Di (2005) The macro determinants of health expenditures in the United States and Canada: Assessing the impact of income, age distribution and time. *Health Policy* 71, 23–42.

Maxwell, Robert J. (1981). *Health and Wealth*. Lexington, MA: Lexington Books.

Mertens, Jean-Francois and Anna Rubinchik (2015) Pareto optimality of the golden rule equilibrium in an overlapping generations model with production and transfers. *Macroeconomic Dynamics* 19(8), 1780–1799.

Miller, Tim (2001) Increasing longevity and Medicare expenditures. *Demography* 38(2), 215–226.

Newhouse, Joseph P. (1977) Medical care expenditures: A cross-national survey. *Journal of Human Resources* 12, 115–125.

Parfit, Derek (1984) *Reasons and Persons*. Oxford: Oxford University Press.

Parkin, David, Alistair McGuire, and Brian Yule (1987) Aggregate health care expenditures and national income. *Journal of Health Economics* 6, 109–127.

Phelps, Edmund S. (1966) *Golden Rules of Economic Growth*. New York: W. W. Norton.

Phelps, Edmund S. (1968) Models of technical progress and the golden rule of research. *Review of Economic Studies* 33, 133–146.

Ponthiere, Gregory (2003) Utilitarian population ethics: A survey. University of Liege. http://www2.ulg.ac.be/crepp/papers/crepp-wp200303.pdf.

Ramsey, Frank (1928) A mathematical theory of saving. *Economic Journal* 38, 543–559.

Rosen, Sherwin (1988) The value of changes in life expectancy. *Journal of Risk and Uncertainty* 1(3), 285–304.

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

Seshamani, Meena and Alastair M. Gray (2004) A longitudinal study of the effects of age and time to death on hospital costs. *Journal of Health Economics* 23, 217–235.
Sheshinski, Eytan (2008) *The Economic Theory of Life Annuities*. Princeton: Princeton University.
Weil, Philippe (1990). Nonexpected utility in macroeconomics. *Quarterly Journal of Economics* 105, 29–42.
Weyer, Edward Moffat (1932) *The Eskimos: Their Environment and Folkways*. New Haven, CT: Yale University Press.
Yaari, Menahem E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *The Review of Economic Studies* 32(2), 137–150.

APPENDIX A: SECOND-ORDER CONDITIONS FOR THE SIMPLE MODEL

Given consumption smoothing in (5), the first-order derivatives of the Lagrangian can be written as follows:

$$\begin{aligned}\frac{\partial \Gamma}{\partial c} &= \left(\frac{\partial U}{\partial c} - \lambda\right)\gamma B(\tau),\\ \frac{\partial \Gamma}{\partial \tau} &= [U(c) - \lambda(c+\tau)]\gamma B'(\tau) - \lambda(\gamma B(\tau) - R),\\ \frac{\partial \Gamma}{\partial \lambda} &= Ry - \gamma B(\tau)c - (\gamma B(\tau) - R)\tau,\end{aligned}$$

and the second-order derivatives are (after evaluating those at maximum, using the health care production function and rewriting) as follows:

$$\frac{\partial^2 \Gamma}{\partial c^2} = \frac{\partial^2 U}{\partial c^2}A,$$

$$\frac{\partial^2 \Gamma}{\partial \tau^2} = \frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)} - 2\gamma B'(\tau)\right],$$

$$\frac{\partial^2 \Gamma}{\partial \lambda^2} = 0,$$

$$\frac{\partial^2 \Gamma}{\partial c \partial \tau} = 0,$$

$$\frac{\partial^2 \Gamma}{\partial c \partial \lambda} = -A,$$

$$\frac{\partial^2 \Gamma}{\partial \tau \partial \lambda} = -\left[(A-R) + (c+\tau)\gamma B'(\tau)\right].$$

Hence, for the bordered Hessian to be positive definite and the first-order conditions being necessary and sufficient for a maximum, the following must hold:

$$-A^2 < 0,$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

which always holds, and

$$-A\left\{A\frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)}-2\gamma B'(\tau)\right]+\frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\gamma B'(\tau)\right]^2\right\}>0.$$

For the second condition to hold, it must hold that

$$A\frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)}-2\gamma B'(\tau)\right]+\frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\gamma B'(\tau)\right]^2<0,$$

which always holds.

APPENDIX B: COMPARATIVE STATIC ANALYSIS FOR THE SIMPLE MODEL

Taking total difference of (6) and (7) with respect to the endogenous variables c and τ and the exogenous variables/parameters R, y, and γ after using the health care production function gives the following:

$$\begin{aligned}&\frac{\partial U}{\partial c}\gamma B'(\tau)\,dc+U(c)\gamma B''(\tau)\,d\tau+U(c)B'(\tau)\,d\gamma=\frac{\partial^2 U}{\partial c^2}(c+\tau)\gamma B'(\tau)\,dc\\&\quad+\frac{\partial U}{\partial c}\gamma B'(\tau)\,dc+\frac{\partial U}{\partial c}\gamma B'(\tau)\,d\tau+\frac{\partial U}{\partial c}(c+\tau)\gamma B''(\tau)\,d\tau\\&\quad+\frac{\partial U}{\partial c}(c+\tau)B'(\tau)\,d\gamma+\frac{\partial^2 U}{\partial c^2}(A-R)dc+\frac{\partial U}{\partial c}\gamma B'(\tau)\,d\tau+\frac{\partial U}{\partial c}B(\tau)\,d\gamma\\&\quad-\frac{\partial U}{\partial c}dR,\end{aligned}$$

and

$$\begin{aligned}&(y-c)\,dR+Rdy-Rdc\\&\quad=(A-R)dc+(A-R)d\tau+(c+\tau)\gamma B'(\tau)\,d\tau+(c+\tau)B(\tau)\,d\gamma\\&\qquad-(c+\tau)\,dR.\end{aligned}$$

Rearranging and collecting terms gives the following:

$$\begin{aligned}&\frac{\partial^2 U}{\partial c^2}\left[(A-R)+(c+\tau)\gamma B'(\tau)\right]dc\\&\quad-\left[\left(U(c)-\frac{\partial U}{\partial c}(c+\tau)\right)\gamma B''(\tau)-2\frac{\partial U}{\partial c}\gamma B'(\tau)\right]d\tau\\&\quad=\left[\left(U(c)-\frac{\partial U}{\partial c}(c+\tau)\right)B'(\tau)-\frac{\partial U}{\partial c}B(\tau)\right]d\gamma+\frac{\partial U}{\partial c}dR,\end{aligned}$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

and

$$Adc + \left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right] d\tau = Rdy - (c+\tau)\,B(\tau)\,d\gamma + (\tau+y)\,dR.$$

Using from (6) that

$$U(c) - \frac{\partial U}{\partial c}(c+\tau) = \frac{\frac{\partial U}{\partial c}(A-R)}{\gamma B'(\tau)}$$

gives

$$\begin{aligned}&\frac{\partial^2 U}{\partial c^2}\left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right] dc \\ &\quad - \frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)} - 2\gamma B'(\tau)\right] d\tau \\ &= -\frac{\partial U}{\partial c}\frac{R}{\gamma} d\gamma + \frac{\partial U}{\partial c} dR,\end{aligned}$$

and

$$Adc + \left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right] d\tau = Rdy - (c+\tau)\,B(\tau)\,d\gamma + (\tau+y)\,dR,$$

where the health care production function is used. The equation system in matrix terms is therefore,

$$\begin{pmatrix} \frac{\partial^2 U}{\partial c^2}\left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right] & -\frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)} - 2\gamma B'(\tau)\right] \\ A & (A-R) + (c+\tau)\,\gamma B'(\tau) \end{pmatrix} \begin{pmatrix} dc \\ d\tau \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -\frac{\partial U}{\partial c}\frac{R}{\gamma} & \frac{\partial U}{\partial c} \\ R & -(c+\tau)\,B(\tau) & \tau + y \end{pmatrix} \begin{pmatrix} dy \\ d\gamma \\ dR \end{pmatrix},$$

Using Cramer's rule gives

$$\frac{\partial \tau}{\partial y} = \frac{\frac{\partial^2 U}{\partial c^2}\left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right] R}{\Lambda},$$

which gives equation (8);

$$\frac{\partial \tau}{\partial \gamma} = \frac{A\frac{\partial U}{\partial c}\frac{R}{\gamma} - \frac{\partial^2 U}{\partial c^2}\left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right](c+\tau)\,B(\tau)}{\Lambda},$$

which gives equation (9), and

$$\frac{\partial \tau}{\partial R} = \frac{\frac{\partial^2 U}{\partial c^2}\left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right](\tau+y) - A\frac{\partial U}{\partial c}}{\Lambda}$$

which gives equation (10), where

$$\Lambda \equiv A\frac{\partial U}{\partial c}\left[(A-R)\frac{B''(\tau)}{B'(\tau)} - 2\gamma B'(\tau)\right] + \frac{\partial^2 U}{\partial c^2}\left[(A-R) + (c+\tau)\,\gamma B'(\tau)\right]^2,$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

which is negative assuming that the second-order conditions for maximum are satisfied (see Appendix A) and, hence, there exist implicit functions $\tau = f(y, \gamma, R)$ and $c = g(y, \gamma, R)$.

APPENDIX C: EFFECTS OF AN INCREASE IN N ON THE NUMBER OF WORKING AND RETIRED INDIVIDUALS

Using equations (14) and (15) gives

$$\frac{\partial N_w}{\partial n} - \frac{\partial N_o}{\partial n} = \int_{a=0}^{R} (t-a)\, l\,(t, n, a, A)\, da - \int_{a=R}^{A} (t-a)\, l\,(t, n, a, A)\, da.$$

Since $\frac{\partial l}{\partial a} < 0$ and $t > a$, it holds that

$$\frac{\partial\,[(t-a)\, l\,(t, n, a, A)]}{\partial a} = -l\,(t, n, a, A) + (t-a)\,\frac{\partial l}{\partial a} < 0.$$

This implies that

$$\int_{a=0}^{R} (t-a)\, l\,(t, n, a, A)\, da > (t-R) l\,(t, n, R, A)\, R,$$

$$\int_{a=R}^{A} (t-a)\, l\,(t, n, a, A)\, da < (t-R) l\,(t, n, R, A)\,(A-R).$$

Hence, a sufficient condition for $\frac{\partial N_w}{\partial n} - \frac{\partial N_o}{\partial n} > 0$ to hold is that

$$(t-R) l\,(t, n, R, A)\, R > (t-R) l\,(t, n, R, A)\,(A-R),$$

or

$$\frac{R}{A} > 1 - \frac{R}{A}.$$

APPENDIX D: SECOND-ORDER CONDITIONS FOR MODEL IN SECTION 4

Given consumption smoothing in (26), the first-order derivatives of the Lagrangian (multiplied by $e^{nt} > 0$ a constant) can be written as follows [using (11), (12), (14),

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

and (15)]:

$$\frac{\partial e^{nt}\Gamma}{\partial c} = \left(\frac{\partial U}{\partial c} - \lambda\right)(N_w + N_o),$$

$$\frac{\partial e^{nt}\Gamma}{\partial \tau} = [U(c) - \lambda(c+\tau)]\frac{\partial N_o}{\partial A}\gamma B'(\tau) - \lambda N_o,$$

$$\frac{\partial e^{nt}\Gamma}{\partial \lambda} = yN_w - c(N_w + N_o) - \tau N_o,$$

and the second-order derivatives are as follows (after evaluating those at maximum and rewriting):

$$\frac{\partial^2 e^{nt}\Gamma}{\partial c^2} = \frac{\partial^2 U}{\partial c^2}(N_w + N_o),$$

$$\frac{\partial^2 e^{nt}\Gamma}{\partial \tau^2} = \frac{\partial U}{\partial c}N_o\left[\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - 2\frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right],$$

$$\frac{\partial^2 e^{nt}\Gamma}{\partial \lambda^2} = 0,$$

$$\frac{\partial^2 e^{nt}\Gamma}{\partial c \partial \tau} = 0,$$

$$\frac{\partial^2 e^{nt}\Gamma}{\partial c \partial \lambda} = -(N_w + N_o),$$

$$\frac{\partial^2 e^{nt}\Gamma}{\partial \tau \partial \lambda} = -\left[N_o + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right].$$

Hence, for the bordered Hessian to be positive definite and the first-order conditions being necessary and sufficient for a maximum, the following must hold:

$$-(N_w + N_o)^2 < 0,$$

which always holds, and

$$-(N_w + N_o)\left\{(N_w + N_o)\frac{\partial U}{\partial c}N_o\left[\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - 2\frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right]\right.$$
$$\left. + \frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]^2\right\} > 0.$$

For the second condition to hold it must hold that

$$(N_w + N_o)\frac{\partial U}{\partial c}N_o\left[\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - 2\frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right]$$
$$+ \frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]^2 < 0,$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

or

$$\frac{\partial^2 N_o}{\partial A^2} < \frac{2}{N_o}\left(\frac{\partial N_o}{\partial A}\right)^2 + \frac{\left(-\frac{\partial^2 U}{\partial c^2}\right)\frac{\left[N_o+(c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]^2}{(N_w+N_o)\frac{\partial U}{\partial c}N_o} + \left(-\frac{B''(\tau)}{B'(\tau)}\right)}{\gamma B'(\tau)}\frac{\partial N_o}{\partial A}.$$

The right-hand side is positive because $\frac{\partial N_o}{\partial A} > 0$, $\frac{\partial U}{\partial c} > 0$, $-\frac{B''(\tau)}{B'(\tau)} > 0$, and $\frac{\partial^2 U}{\partial c^2} < 0$. This implies that, for the first-order conditions being necessary and sufficient for a maximum, there must be an upper bound on $\frac{\partial^2 N_o}{\partial A^2}$, or an upper bound on how longevity affects the effects of longevity on the number of old individuals. A sufficient condition for that is achieved by assuming that there is an upper bound on how longevity affects the number of old individuals or, by using (11), (12), and (15):

$$\frac{\partial^2 N_o}{\partial A^2} = \varphi e^{n(t-A)}\frac{\partial f}{\partial A}_{a=A} + \int_{a=R}^{A} \varphi e^{n(t-a)}\frac{\partial^2 f}{\partial A^2}da \leq 0,$$

which is assumed.

APPENDIX E: COMPARATIVE STATIC ANALYSIS FOR MODEL IN SECTION 4

Taking total difference of (27) and (28) with respect to the endogenous variables c and τ and the exogenous variables/parameters y and γ gives

$$\begin{aligned}
&\frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}\gamma B'(\tau)\,dc + U(c)\frac{\partial^2 N_0}{\partial A \partial n}\gamma B'(\tau)\,dn + U(c)\frac{\partial^2 N_o}{\partial A^2}\gamma^2 B'(\tau)^2 d\tau \\
&\quad + U(c)\frac{\partial^2 N_o}{\partial A^2}\gamma B'(\tau)\,B(\tau)\,d\gamma + U(c)\frac{\partial N_o}{\partial A}\gamma B''(\tau)\,d\tau + U(c)\frac{\partial N_o}{\partial A}B'(\tau)\,d\gamma \\
&= \frac{\partial^2 U}{\partial c^2}(c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\,dc + \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}\gamma B'(\tau)\,dc + \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}\gamma B'(\tau)\,d\tau \\
&\quad + \frac{\partial U}{\partial c}(c+\tau)\frac{\partial^2 N_0}{\partial A \partial n}\gamma B'(\tau)\,dn + \frac{\partial U}{\partial c}(c+\tau)\frac{\partial^2 N_o}{\partial A^2}\gamma^2 B'(\tau)^2 d\tau \\
&\quad + \frac{\partial U}{\partial c}(c+\tau)\frac{\partial^2 N_o}{\partial A^2}\gamma B'(\tau)\,B(\tau)\,d\gamma + \frac{\partial U}{\partial c}(c+\tau)\frac{\partial N_o}{\partial A}\gamma B''(\tau)\,d\tau \\
&\quad + \frac{\partial U}{\partial c}(c+\tau)\frac{\partial N_o}{\partial A}B'(\tau)\,d\gamma + \frac{\partial^2 U}{\partial c^2}N_o dc + \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial n}dn + \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}\gamma B'(\tau)\,d\tau \\
&\quad + \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}B(\tau)\,d\gamma,
\end{aligned}$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

and

$$N_w dy - N_w dc + (y-c)\frac{\partial N_w}{\partial n}dn$$
$$= N_o dc + N_o d\tau + (c+\tau)\frac{\partial N_o}{\partial n}dn + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\,d\tau$$
$$+ (c+\tau)\frac{\partial N_o}{\partial A}B(\tau)\,d\gamma,$$

where it is used that $\frac{\partial N_w}{\partial A} = 0$ [since survival probabilities are constant and equal to 1 for $a \in [0, R]$, from (14), (12), and (11)]. Rearranging and collecting terms gives

$$\frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]dc$$
$$-\left[\left(U(c) - \frac{\partial U}{\partial c}(c+\tau)\right)\left(\frac{\partial^2 N_o}{\partial A^2}\gamma^2 B'(\tau)^2 + \frac{\partial N_o}{\partial A}\gamma B''(\tau)\right)\right.$$
$$\left. -2\frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]d\tau$$
$$= \left[\left(U(c) - \frac{\partial U}{\partial c}(c+\tau)\right)\left(\frac{\partial^2 N_o}{\partial A^2}\gamma B(\tau) + \frac{\partial N_o}{\partial A}\right)B'(\tau) - \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial A}B(\tau)\right]d\gamma$$
$$+ \left[\left(U(c) - \frac{\partial U}{\partial c}(c+\tau)\right)\frac{\partial^2 N_0}{\partial A \partial n}\gamma B'(\tau) - \frac{\partial U}{\partial c}\frac{\partial N_o}{\partial n}\right]dn,$$

and

$$(N_w + N_o)\,dc + \left[N_o + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]d\tau$$
$$= N_w dy - (c+\tau)\frac{\partial N_o}{\partial A}B(\tau)\,d\gamma - \left[(c+\tau)\frac{\partial N_o}{\partial n} - (y-c)\frac{\partial N_w}{\partial n}\right]dn.$$

Using from (27) and (28) that

$$U(c) - \frac{\partial U}{\partial c}(c+\tau) = \frac{\frac{\partial U}{\partial c}N_o}{\frac{\partial N_o}{\partial A}\gamma B'(\tau)}$$
$$y - c = (c+\tau)\frac{N_o}{N_w}.$$

gives

$$\frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]dc$$
$$-\frac{\partial U}{\partial c}N_o\left[\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - 2\frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right]d\tau$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

$$= \frac{\partial U}{\partial c} N_o \left(\frac{1}{A} + \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) B(\tau)\, d\gamma$$

$$+ \frac{\partial U}{\partial c} N_o \left(\frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_0}{\partial A \partial n} - \frac{1}{N_o} \frac{\partial N_o}{\partial n} \right) dn,$$

and

$$(N_w + N_o)\, dc + \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau) \right] d\tau$$

$$= N_w dy - (c+\tau) \frac{\partial N_o}{\partial A} B(\tau)\, d\gamma$$

$$- (c+\tau) N_o \left(\frac{1}{N_o} \frac{\partial N_o}{\partial n} - \frac{1}{N_w} \frac{\partial N_w}{\partial n} \right) dn$$

where the health care production function in (19) is used. The equation system in matrix terms is, therefore,

$$\begin{pmatrix} \frac{\partial^2 U}{\partial c^2} \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau) \right] & -\frac{\partial U}{\partial c} N_o \left[\left(\frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - 2 \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) \gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)} \right] \\ (N_w + N_o) & \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau) \right] \end{pmatrix} \begin{pmatrix} dc \\ d\tau \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{\partial U}{\partial c} N_o \left(\frac{1}{A} + \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) B(\tau) & \frac{\partial U}{\partial c} N_o \left(\frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_0}{\partial A \partial n} - \frac{1}{N_o} \frac{\partial N_o}{\partial n} \right) \\ N_w & -(c+\tau) \frac{\partial N_o}{\partial A} B(\tau) & -(c+\tau) N_o \left(\frac{1}{N_o} \frac{\partial N_o}{\partial n} - \frac{1}{N_w} \frac{\partial N_w}{\partial n} \right) \end{pmatrix} \begin{pmatrix} dy \\ d\gamma \\ dn \end{pmatrix}.$$

Using Cramer's rule gives

$$\frac{\partial \tau}{\partial y} = \frac{\frac{\partial^2 U}{\partial c^2} \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau) \right] N_w}{\Psi},$$

which gives equation (29), and

$$\frac{\partial \tau}{\partial \gamma} = -\frac{\left\{ \begin{array}{l} (N_w + N_o) \frac{\partial U}{\partial c} N_o \left(\frac{1}{A} + \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - \frac{1}{N_o} \frac{\partial N_o}{\partial A} \right) B(\tau) \\ + \frac{\partial^2 U}{\partial c^2} \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau) \right] (c+\tau) \frac{\partial N_o}{\partial A} B(\tau) \end{array} \right\}}{\Psi},$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

which gives equation (30), and

$$\frac{\partial \tau}{\partial n} = -\frac{\left\{\begin{array}{l}(N_w + N_o)\,\frac{\partial U}{\partial c} N_o \left(\frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_0}{\partial A \partial n} - \frac{1}{N_o}\frac{\partial N_o}{\partial n}\right) \\ \quad + \frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right](c+\tau)\, N_o \left(\frac{1}{N_o}\frac{\partial N_o}{\partial n} - \frac{1}{N_w}\frac{\partial N_w}{\partial n}\right)\end{array}\right\}}{\Psi},$$

which gives equation (31), where

$$\Psi \equiv (N_w + N_o)\,\frac{\partial U}{\partial c} N_o \left[\left(\frac{1}{\frac{\partial N_o}{\partial A}}\frac{\partial^2 N_o}{\partial A^2} - 2\frac{1}{N_o}\frac{\partial N_o}{\partial A}\right)\gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right]$$
$$+ \frac{\partial^2 U}{\partial c^2}\left[N_o + (c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right]^2,$$

which is negative assuming that the second-order conditions for maximum are satisfied (see Appendix D) and, hence, there exist implicit functions $\tau = f(y, \gamma, n)$ and $c = g(y, \gamma, n)$.

APPENDIX F: THE RESULTS ASSUMING A CRRA UTILITY FUNCTION

As is discussed in Hall and Jones (2007), a constant relative risk aversion utility function with the elasticity of intertemporal substitution less than one is most common in quantitative macromodels. Assuming such a function results in the level of utility being negative and, hence, Hall and Jones (2007) propose a nonhomothetic constant relative risk aversion utility function:

$$U(c(a)) = \nu + \frac{c(a)^{1-\sigma}}{1-\sigma},$$

where ν>1 and σ >1 (and elasticity of intertemporal substitution less than one $\frac{1}{\sigma} < 1$). Since allocation in our model depends on the level of utility, it is important to make sure that assuming this utility function does not change the conclusions drawn from the analysis in chapters 4–6 above.[23]

Using the utility function, the social planner's optimal condition for spending on old-age health care in equation (27) becomes

$$\left(\nu + \frac{c^{1-\sigma}}{1-\sigma}\right)\frac{\partial N_0}{\partial A}\gamma B'(\tau) \;= c^{-\sigma}(c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau) + c^{-\sigma} N_o(t, n, R, A),$$

and the comparative static analyses for the golden rule allocation in equations (29)–(31) are the following:

$$\frac{\partial \tau}{\partial y} = \frac{-\sigma c^{-\sigma-1}\left(N_o + (c+\tau)\,\frac{\partial N_o}{\partial A}\gamma B'(\tau)\right) N_w}{\Psi} > 0,$$

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press

$$\frac{\partial \tau}{\partial \gamma} = -\frac{\left\{\begin{array}{l}(N_w + N_o)\, c^{-\sigma} N_o \left(\frac{1}{A} + \frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - \frac{1}{N_o} \frac{\partial N_o}{\partial A}\right) B(\tau) \\ \quad -\sigma c^{-\sigma-1} \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau)\right] (c+\tau) \frac{\partial N_o}{\partial A} B(\tau)\end{array}\right\}}{\Psi} \gtreqless 0,$$

$$\frac{\partial \tau}{\partial n} = -\frac{\left\{\begin{array}{l}(N_w + N_o) \frac{\partial U}{\partial c} N_o \left(\frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_0}{\partial A \partial n} - \frac{1}{N_o} \frac{\partial N_o}{\partial n}\right) \\ \quad -\sigma c^{-\sigma-1} \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau)\right] (c+\tau) N_o \left(\frac{1}{N_o} \frac{\partial N_o}{\partial n} - \frac{1}{N_w} \frac{\partial N_w}{\partial n}\right)\end{array}\right\}}{\Psi} \gtreqless 0,$$

where

$$\Psi \equiv (N_w + N_o)\, c^{-\sigma} N_o \left[\left(\frac{1}{\frac{\partial N_o}{\partial A}} \frac{\partial^2 N_o}{\partial A^2} - 2 \frac{1}{N_o} \frac{\partial N_o}{\partial A}\right) \gamma B'(\tau) + \frac{B''(\tau)}{B'(\tau)}\right]$$

$$-\sigma c^{-\sigma-1} \left[N_o + (c+\tau) \frac{\partial N_o}{\partial A} \gamma B'(\tau)\right]^2 < 0,$$

while the individual's optimal condition for spending on old-age health care in equation (41) becomes

$$\left(\nu + \frac{c^{1-\sigma}}{1-\sigma}\right) \left[\int_{a=R}^{A} e^{-ra} \frac{\partial f}{\partial a} da\right] \gamma B'(\tau) = c^{-\sigma} \left[\int_{a=R}^{A} e^{-ra} f(a, A)\, da\right].$$

Hence, these do not change the results and conclusions drawn.

https://doi.org/10.1017/S1365100516001371 Published online by Cambridge University Press