J. Plasma Physics (2010), *vol.* 76, *parts* 3&4, *pp.* 395–401. © Cambridge University Press 2010
doi:10.1017/S0022377809990766

Transition between Raman and Compton regimes in laser pulse amplification

R. A. CAIRNS

School of Mathematics and Statistics, University of St Andrews, St Andrews, Fife, KY16 9SS, UK
(rac@st-andrews.ac.uk)

(Received 24 November 2009 and accepted 13 December 2009, first published online 21 January 2010)

Abstract. Amplification of a short laser pulse by means of its interaction with a counter-propagating long pulse in a plasma has been suggested as a way of reaching high intensities. Two regimes have been discussed in the literature. The first is the Raman regime, where the process is backward Raman scattering, described by the standard equations for resonant three-wave coupling, while the second is the Compton regime, in which electron dynamics is dominated by the ponderomotive force generated by the high-frequency wave, and electrons behave as single particles rather than producing a Langmuir wave. Our aim here is to use a simple model of electron dynamics to investigate the transition between these regimes.

1. Introduction

Amplification of a short laser pulse by a counterpropagating long pump beam has been suggested as a way of producing very short high-intensity pulses while bypassing limitations on the size and power handling of the compression grating used in conventional chirped pulse amplification. One of the first papers to suggest this discussed a regime, which, by analogy with free electron lasers, we call the Compton regime [1]. Here it was supposed that the nonlinear combination of the backward and forward propagating waves produced a periodic ponderomotive force proportional to $\sin(kx-\omega t)$ with k and ω, respectively, the sum of the wave numbers and the difference of the frequencies of the pulse and the pump wave. The self-interaction amongst the plasma electrons was neglected on the assumption that the ponderomotive force was much larger than the electric field produced by the collective effect of the electrons. The electrons then behave like single particles and it was shown that if they are trapped in the moving potential wells, then over the time scale of half a bounce period there is a transfer of energy and momentum from the pump wave to the pulse if the former has the higher frequency. The result for a pulse of duration around the half period is a steady transfer of energy to the pulse leading to superradiance, i.e. the amplitude growing linearly in time, while the pulse length scales growing inversely with time. Slightly later it was shown [2] that the same scaling could be obtained on the assumption that the waves interacted through backward Raman scattering. In this case the frequency difference corresponds to the plasma frequency and the pump wave decays into the pulse and a Langmuir wave. In this case the plasma response is a Langmuir wave, whose behavior is just that of the linear wave slowly modulated by the resonant

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press

nonlinear interaction with the two electromagnetic waves. In this case the electron behavior is dominated by the collective interaction and the periodic potential acts as a weak driving force in resonance, or close to resonance, with the plasma frequency. Subsequent work on the problem taking into account effects like density gradients, frequency chirped pump or Landau damping [3–6] have generally assumed that the system is in the Raman regime and described by the standard equations for weak three-wave interaction. Here we consider a very simple model of electron dynamics produced by the nonlinear interaction between the electromagnetic waves with a view to investigating the relation between these two regimes and obtaining an idea of how the transition from behavior dominated by collective wave effects goes over to behavior determined by single particle dynamics.

2. Model equation

We look at an electron whose initial position is x and displacement ξ. As is well known, if we assume that the electron displacements are such that orbits do not cross, the restoring force with this Lagrangian displacement is exactly $m\omega_p^2\xi$ and the particle oscillates around its initial position with the plasma frequency ω_p. While an individual particle clearly gains no average momentum, a traveling wave does carry momentum. To see this, suppose that a wave going as $e^{ikx-i\omega_p t}$ is set up, then the displacement of an electron centered at x is of the form $\xi_0 e^{ikx-i\omega_p t}$ and the electric field at the position of the particle is $-\frac{m}{e}\omega_p^2\xi$. The density pertubation is

$$n = \frac{\varepsilon_0}{e}\frac{\partial E}{\partial x} = ikE,$$

from which we get, neglecting the difference in position between x and the particle, the expression for the wave momentum density

$$P = \left\langle mn\dot{\xi} \right\rangle = \frac{1}{2}\frac{k}{\omega_p}\varepsilon_0 \left\langle E^2 \right\rangle = \frac{k}{\omega_p} \times \text{ wave energy density.}$$

We, thus, get the standard relation between energy and momentum and the resonance condition for the three-wave interaction just imposes the condition that the Langmuir wave carries away the energy and momentum difference between the electromagnetic waves.

If we have a forcing term of the form $A\sin(kx-\omega t)$, then the electron equation of motion is

$$\ddot{\xi} + \omega_p^2\xi = \frac{1}{m}A\sin(k\,(x+\xi) - \omega t).$$

Normalizing times to $1/\omega_p$ and distances to $1/k$ we get the equation

$$\ddot{\xi} + \xi = a\sin(\xi - \Omega t + \theta), \tag{1}$$

where $\Omega = \omega/\omega_p$, $a = \omega_B^2/\omega_{p^2}$, ω_B is the bounce frequency in the bottom of the potential well produced by the periodic driving force, and θ is a phase factor depending on the initial position of the particle.

3. Solutions of equation

Equation (1) is, of course, an approximation in which the simple harmonic restoring force produced by the collective interaction of the electrons is an approximation

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press

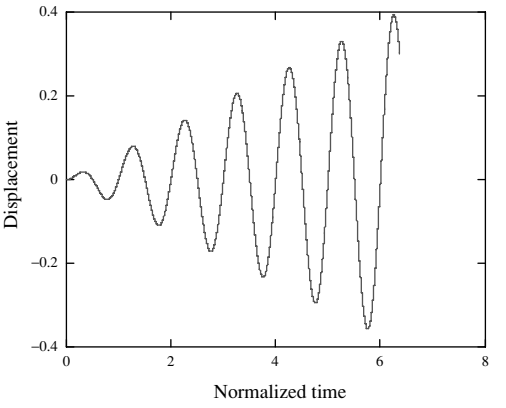


Figure 1. Diplacement as a function of time for $a = 0.02$.

only valid for small amplitude waves. Nevertheless, the equation should give some idea of the way in which the Raman and Compton regimes merge. Throughout we take the initial condition of a particle at rest, neglecting any thermal motion. For small a the behavior is just like a driven oscillator with secular growth in time. This corresponds to the regime of a weak coupling where it is valid, with a time- dependent driving force, to regard the plasma response as a Langmuir wave with slowly time-dependent amplitude. If a is large, then we are in the Compton regime, where the field produced by the electrons is negligible and the dynamics is dominated by the driving force. The question we wish to answer is for what values of a these two limits hold and what happens in the transition region between them. This may give some insight into more detailed investigations of the evolution of compressed laser beams and as to whether in practice a smooth transition from one regime to the other might be expected as the pulse intensity grows. For small a we expect a simple response corresponding to linear growth and this is verified by the solution for $a = 0.02$ and $\Omega = 1$ as shown below. The time is in units of $2\pi/\omega_p$ making it clear that what we are seeing is simply oscillation at the plasma frequency with amplitude growing linearly in time. With a slow time-dependent amplitude of the driving force we would expect the usual slowly varying amplitude approximation for the Langmuir wave to be appropriate and the usual three-wave weak coupling equations to be a good description of the Raman scattering process.

If we go to the other extreme of large a, then the behavior is as in Fig. 2.
It is now clear that the behavior is dominated by the much faster time scale of the oscillations within the potential wells. Interestingly, there is an occasional jump when the particle escapes from one potential well and is trapped by the one behind. Since the amplification process described in [1] depends only on trapping for half a bounce period, this is not likely to be important. Also, in the linear regime the linear growth depended on having $\Omega = 1$ so that the oscillation was driven resonantly. In the regime of high-bounce frequency, the natural frequency is not so important, as is illustrated by the solution below with $\Omega = 0.5$. The precise behavior depends not only on a and Ω but also on the initial starting point of the particle with respect to the phase of the wave. However, for a large value of a the behavior is determined

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press

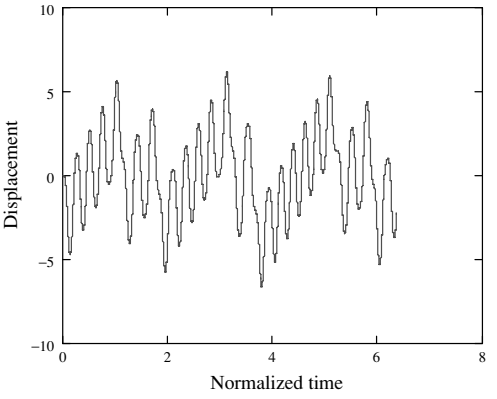


Figure 2. Diplacement as a function of time as in Fig. 1, with $a = 50$.

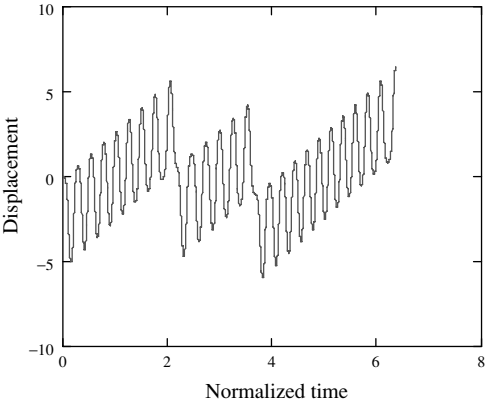


Figure 3. Diplacement as a function of time as in Fig. 2, but with $\Omega = 0.5$.

largely by the trapping, and over the first half of the trapping,period certainly behaves in the way assumed in the theory of the Compton regime.

Having verified that the behavior in the two extreme regimes is as expected, we now look at what happens when a is of the order of unity, going back to $\Omega = 1$ so that we are in resonance. Figure 4 shows what happens with $a = 0.5$. The response can be seen to be at the plasma frequency, with no obvious sign of the bounce frequency playing a role and, at least for a few periods the response is linear. Once the amplitude becomes sufficient to carry the particle across a distance comparable to the spatial period of the driving force, the linear growth saturates and the wave amplitude, if looked at over a longer time, oscillates. When the wave amplitude reaches this level, it is, in any case, doubtful whether the linear approximation for the restoring force remains valid, and we could well be entering a regime of particle orbit crossing and wave breaking. If, however, we have a short pulse

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press

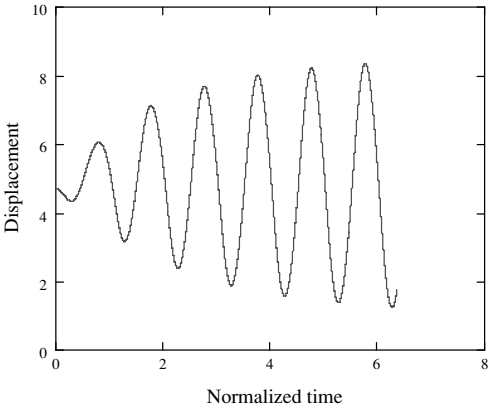


Figure 4. Displacement with $\Omega = 1$, $a = 0.5$.

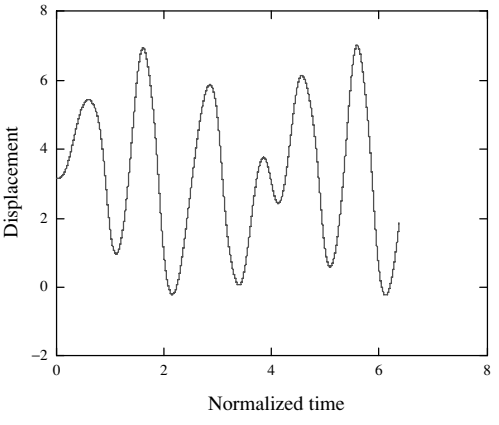


Figure 5. Displacement as in Fig. 4 with $a = 1.5$.

driving the Langmuir wave over a short time, then the three-wave weak interaction approximation probably remains valid.

If we make $a = 1.5$ we see no clear signof trapping, but the regular response to the driving force is clearly disturbed. For this value the exact behavior depends quite strongly on the starting position of the particle, but is overall at the frequency of the driving force, but with rather irregular peaks and troughs.

A further increase to $a = 3$ gives a response which is typically as shown in figure 6. Here there is a clear indication of trapping over the first half cycle, though the particle then escapes from the potential well and follows a rather irregular trajectory. So long as the pulse duration has contracted to around half the trapping time, we would appear to have reached a regime in which the Compton type of amplification can occur.

Once the value goes up to around 5 and above, there is, for most starting points, at least a few trapped oscillations, though at later times the particle may be repeatedly trapped and untrapped, usually slipping backwards as shown already for a much higher value of a. As pointed out previously, this will not affect the Compton

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press

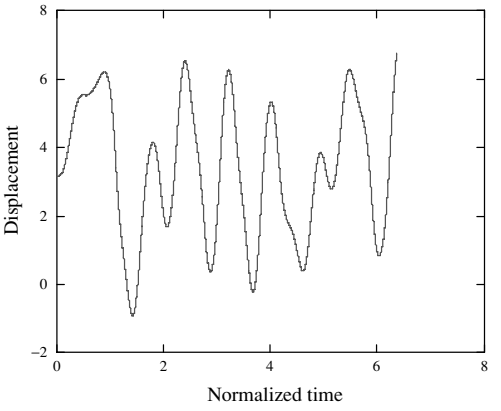


Figure 6. Displacement as in Fig. 5 with $a = 3$.

amplification regime if the Langmuir wave is being driven by a pulse which only lasts about half a cycle.

4. Conclusion

We have set up a simple model equation looking at the response of an electron to the driving ponderomotive force produced by oppositely propagating electromagnetic waves, with a view to investigating the transition between Raman scattering, when the response is dominated by the interaction amongst the electrons setting up a Langmuir wave, and the Compton regime, where the electron dynamics is dominated by the ponderomotive force and electrons effectively respond as single particles. In terms of the parameter $a = \frac{\omega_B^2}{\omega_p^2}$ we have shown that if a is significantly less than unity, then we are in the Raman regime, with the electrons responding as resonantly driven oscillators. For a greater than around 3 , the initial response of the particles is dominated by the ponderomotive potential, though the collective restoring force can lead to detrapping at later times. However, the basic conditions of the theory underlying the growth and shortening of a pulse are satisfied. In the region around and just above 1, there is a response which corresponds neither to trapping of particles nor to a resonant response at the driving frequency. The question thus arises, probably only answerable by much more complex simulations of the plasma behavior, as to what happens in this region and if, as a pulse increases in amplitude, there can be a smooth transition between the regions or if there is some sort of barrier to growth at this value. It does, however, appear that the Compton amplification regime requires only a rather modest ratio of bounce frequency to plasma frequency. The particle is not completely trapped, but does satisfy the condition that its dynamics are dominated by the driving force over at least the first half cycle.

Acknowledgements

This work was carried out as part of the Alpha-X project led by Professor D. A. Jaroszynski of the University of Strathclyde. This project is supported by the UK Engineering and Physical Sciences Research Council.

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press

References

[1] Shvets, G. et al. 1998 *Phys. Rev. Lett.* **81**, 4879.
[2] Malkin, V. M., Schvets, G. and Fisch, N. J. 1999 *Phys. Rev. Lett.* **82**, 44–48.
[3] Malkin, V. M., Schvets, G. and Fisch, N. J. 2000 *Phys. Rev. Lett.* **84**, 1208.
[4] Malkin, V. M. and Fisch, N. J. 2005 *Phys. Plasmas* **12**, 044507.
[5] Ersfeld, B. and Jaroszinsky, D. A. 2005 *Phys.Rev. Lett.* **95**, 165002.
[6] Yampolsky, N. A. and Fisch, N. J. 2009 *Phys. Plasmas* **16**, 072105.

https://doi.org/10.1017/S0022377809990766 Published online by Cambridge University Press