Relativistic Invariance and Modal Interpretations*

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A number of arguments have been given to show that the modal interpretation of ordinary nonrelativistic quantum mechanics cannot be consistently extended to the relativistic setting. We find these arguments inconclusive. However, there is a prima facie reason to think that a tension exists between the modal interpretation and relativistic invariance; namely, the best candidate for a modal interpretation adapted to relativistic quantum field theory, a prescription due to Rob Clifton (2000), comes out trivial when applied to a number of systems of physical interest. However, it is far from clear whether this difficulty for the modal interpretation is traceable to relativistic invariance per se or to the infinite number of degrees of freedom involved. In any case, the proponents of the modal interpretation have work to do.

1. Introduction. The modal interpretation of quantum mechanics (QM) was introduced by its progenitors—Kochen (1985), Healey (1989), and van Fraassen (1991)—to solve the quantum measurement problem without the expedient of measurement collapse. Accepting that isolated quantum systems everywhere and always undergo unitary evolution, the modal interpretation replaces the eigenvector-eigenvalue link with the rule that, when a quantum (sub)system is described by a reduced density operator W, the observables determinate on that (sub)system are those that commute with all of the projectors onto an eigenbasis of W; we will say that those observables share a spectral resolution with W.[1] Assuming W to be

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1. The modal interpretation of ordinary quantum mechanics comes in many varieties (documented in Vermaas 1999), of which this is just one. We treat it here as representative.

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nondegenerate (and so suppressing complications which will emerge presently and which are chronicled in the collections of Dieks and Vermaas (1998) and Healey and Hellman (1998)), its spectral resolution is furnished by a unique set of one-dimensional projection operators; the set of observables deemed determinate by the modal interpretation is therefore Abelian (i.e., pairwise-commuting).[2] Given the correspondence quantum logic erects between projection operators and propositions attributing observables determinate values, and adopting the spectrum rule that the value of a determinate observable is one of its eigenvalues, the modal interpretation thereby identifies sets of value-attributing propositions that admit two-valued homomorphisms (i.e., Boolean truth valuations). Because one-dimensional projection operators are atoms in the Boolean algebra of modally determinate projection operators, each element of W's spectral resolution corresponds to a truth valuation on the set of determinate propositions. According to the modal interpretation, the Born Rule probabilities W defines for the projection operators in its spectral resolution give a probability distribution over these possible truth valuations.

To see how the modal interpretation proposes to resolve the measurement problem, consider the unitary evolution of a composite object plus measurement apparatus with the interaction between the two systems designed to establish a perfect correlation between the eigenvectors $\{|o_i\rangle\}$ of an object observable O and the eigenvectors $\{|p_i\rangle\}$ of a pointer observable P of the measuring system. If the object system is originally in a nontrivial superposition of O eigenstates, the composite evolves to the entangled state $|\Psi\rangle = \sum_i c_i|o_i\rangle \otimes |p_i\rangle$. $|\Psi\rangle$ implies the reduced density matrix $W = \sum_i |c_i|^2 E_{|p_i\rangle}$ for the measuring system, where $E_{|p_i\rangle}$ is the projection operator for the subspace spanned by $|p_i\rangle$. The projection operators $E_{|p_i\rangle}$ give a spectral resolution of W, and $\{E_{|p_i\rangle}\}$ also furnishes a spectral resolution for the pointer observable P. According to the modal interpretation, therefore, the pointer observable is determinate on the apparatus system after a unitary measurement, and the weights $|c_i|^2$ give a probability distribution over its possible values. Thus, measurements are shown to have outcomes manifesting Born Rule statistics while leaving the Schrödinger equation inviolate. The measurement problem is solved.

But a complication looms. A variety of results suggest that the modal interpretation violates relativistic invariance. Here we discuss a recent and

2. It is worth noting that when W is degenerate—when, for instance, it is pure—the set of projections determinate according to the modal interpretation is not Abelian. It contains, for instance, all projections, mutually commuting or not, assigned probability 0 by W. For reassurance that this does not embroil the modal interpretation in Kochen-Specker type contradictions, see Dickson 1995 or Bub and Clifton 1996.

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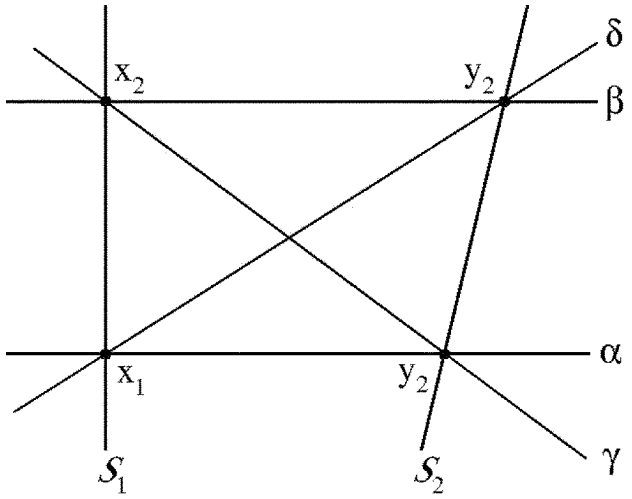


Figure 1. The setup for Myrvold's No-Go result.

elegant one due to Wayne Myrvold (2002). We use it synecdochically for a family of results (including those of Dickson and Clifton (1998) and Arntzenius (1998)) among which there are significant differences, in order to proceed most directly to our main business, which is to introduce members of a new family of results suggesting a tension between modal interpretations and relativistic invariance. Myrvold considers four intersecting hyperplanes in Minkowski spacetime, labeled α, β, γ, and δ in Figure 1. The hyperplanes α and δ intersect at a point x_1, and β and γ intersect at a point x_2. It is arranged that there is a system S_1 whose world line passes through these two points. Similarly, α and γ intersect at a point y_1 and β and δ intersect at a point y_2, through which points the world line of a second system S_2 passes. Because "in a relativistic context, the notion of an instantaneous state of a spatially extended system must be replaced by the notion of a state on a spacelike hyperplane" (Myrvold 2002, 1773), let the state $\psi(\theta)$, $\theta \in \{\alpha, \beta, \gamma, \delta\}$, describe the composite $S_1 + S_2$ system on the hyperplane θ. Suppose that between the hyperplanes α and β (which we can regard as surfaces of constant time for some inertial observer), the composite $S_1 + S_2$ system undergoes a unitary evolution governed by $U_1 \otimes U_2$. Then $\psi(\beta) = (U_1 \otimes U_2)\psi(\alpha)$. It follows that $\psi(\gamma) = (U_1 \otimes I)\psi(\alpha)$, because lying at the intersection of the hyperplanes α and γ, S_2 must be in the same state on each. Likewise $\psi(\delta) = (I \otimes U_2)\psi(\alpha)$. Thus, the initial state $\psi(\alpha)$ and the evolution operator

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$U_1 \otimes U_2$ fixes the set of states $\psi(\theta)$. The modal interpretation can be applied to determine what determinate observables pertain to S_1 at x_1 and x_2, and S_2 at y_1 and y_2. But notice that for each point, there are two ways—one for each of the two hyperplanes passing through that point—to apply the modal interpretation to identify, and assign a probability distribution to, the observables determinate on the system at the point.

When a hyperplane θ passes through points x and y on which observables $A = \sum a_i E_{|a_i\rangle}$ and $B = \sum b_i E_{|b_i\rangle}$ for S_1 and S_2 respectively are determinate according to the modal interpretation, Myrvold requires their joint probabilities to be just the Born Rule probabilities determined by $\psi(\theta)$, e.g., $\Pr((A = a_i) \,\&\, (B = b_j)) = \langle \psi(\theta) | E_{|a_i\rangle} \otimes E_{|b_j\rangle} | \psi(\theta) \rangle$.[3] Calling this the *relativistic Born Rule*, Myrvold holds the modal interpretation to be consistent with relativity only if the relativistic Born Rule probabilities are mutually consistent. He then shows that there are initial states $\psi(\alpha)$ and evolution operators $U_1 \otimes U_2$ such that the mutual consistency requirement fails. This No-Go result can be seen as a version of Bell-type argumentation: the modal interpretation identifies a quartet of observables (the ones determinate at the four points at which the hyperplanes intersect) that cannot be assigned a joint probability distribution that yields the Born Rule probabilities assigned by each $\psi(\theta)$ as marginals.

There are various loopholes in the attempted demonstrations of an incompatibility between the modal interpretation and relativity theory. For example, Berkovitz and Hemmo (2005a, 2005b) contend that a modal interpretation whose property attributions are relational survives Myrvold's argument. Hyperplane-dependent observables of the sort posited by Fleming (1996, 2000, 2003) would avert the No-Go result as well. But we are not interested in exploring such loopholes. We wish rather to observe that the target of Myrvold's No-Go result is ordinary QM, relieved of a privileged foliation of time slices, and subject to the relativistic Born Rule. This is not a full-blown manifestly relativistic quantum theory, but a mixture of bits and pieces of ordinary nonrelativistic quantum mechanics and relativity theory. The targets of other extant results indicating a tension between modal interpretations and relativity are similarly chimerical. A more fruitful way to investigate the alleged tension is to chart the fortunes of the modal interpretation for well-defined quantum theories that are indisputably relativistic.

2. Really Relativistic Quantum Theories. We want to explore the prospects for a modal interpretation of a manifestly relativistically invariant quantum theory. For such a theory, we turn to axiomatic and algebraic ap-

3. With the reference to hyperplanes removed, this is a commitment of most modal interpretations.

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proaches to local relativistic quantum field theory (QFT), which are especially self-conscious about fulfilling the demands of relativity theory (see, for example, Haag 1996). Moreover, these approaches furnish more than mere axiomatics, for there are proofs that the axioms expressing locality and relativistic invariance are instantiated for the free scalar field on (3 + 1)-dimensional Minkowski spacetime and for several cases of nontrivial interacting fields on (1 + 1) and (2 + 1)–dimensional Minkowski spacetime.

For our purposes a second aspect of the algebraic approach is more important. As has to be emphasized to undergraduates who are trying to learn relativity theory, the first step to understanding is to leave behind the notion that relativity theory means that everything is relative. On the contrary, relativity theory shows that there is an intrinsic reality independent of coordinate systems, observers, reference frames, etc. And so it is in local relativistic QFT. The intrinsic situation is characterized by two elements. The first element is an algebra $\mathcal{A}$ of quasi-local observables and an association $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}$ between open-bounded regions $\mathcal{O} \subset \mathcal{M}$ of Minkowski spacetime $\mathcal{M}$ and *local subalgebras* $\mathcal{A}(\mathcal{O})$. Each algebra in question is a *-algebra (cf. Appendix A) and so equipped with an adjoint operation under which it is closed. The self-adjoint part of the subalgebra $\mathcal{A}(\mathcal{O})$ is supposed to contain the observables corresponding to local operations in $\mathcal{O}$. (It is a terminological imperfection that algebras like $\mathcal{A}(\mathcal{O})$ are called observable algebras, even though it isn't the case that all of their elements are observables in this sense.) The association between open bounded spacetime regions and local subalgebras is assumed to give a net structure, i.e. if $\mathcal{O}_1 \subseteq \mathcal{O}_2$ then $\mathcal{A}(\mathcal{O}_1) \subseteq \mathcal{A}(\mathcal{O}_2)$, and it is assumed that quasi-local algebra $\mathcal{A}$ is obtained by taking an appropriate closure of $\cup_{\mathcal{O}}\mathcal{A}(\mathcal{O})$. (In the 'abstract' C*-algebra approach described below in Section 3, the norm closure is used.) Locality is enforced by requiring that $\mathcal{A}(\mathcal{O}_1)$ and $\mathcal{A}(\mathcal{O}_2)$ commute when regions $\mathcal{O}_1$ and $\mathcal{O}_2$ are relatively spacelike.

The second element characterizing the intrinsic situation is the state, specified by a normed, positive linear functional ω on $\mathcal{A}$, the interpretation being that $\omega(A)$ is the expectation value of self-adjoint $A \in \mathcal{A}$. Relativistic covariance of observables is enforced by demanding that to the group of Poincaré transformations Λ there corresponds an automorphism group α_Λ of $\mathcal{A}$ such that for any $\lambda \in \Lambda$ and any local subalgebra $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}$, $\alpha_\lambda(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\mathcal{O}_\lambda)$ where $\mathcal{O}_\lambda$ denotes the Poincaré transform by λ of $\mathcal{O}$. In addition it is demanded that the vacuum state ω_0 (when it exists) is invariant under Poincaré transformations, i.e. $\omega_0 \circ \alpha_\Lambda = \omega_0$.

Comparing this algebraic approach to relativistic QFT to the hybrid of QM and relativity targeted by the Myrvold No-Go result suggests a puzzle about how there could possibly be an inconsistency between

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relativistic invariance and the modal interpretation. The algebraic state ω is a state on the quasi-local algebra $\mathcal{A}$, a state for all time. By contrast the states $\psi(\theta)$ that fail to dovetail in Myrvold's No-Go result are states at a time. The expectation values that ω assigns to self-adjoint elements of the algebra are manifestly consistent and relativistically covariant. In Myrvold's No-Go argument, applying the Born Rule to states associated with different hyperplanes issues in conflicting probability assignments. But it doesn't seem that a modal interpretation offered in terms of intrinsic objects—the quasi-local algebra $\mathcal{A}$, the net of local algebras, and the state ω—can possibly go haywire in this way. But (as Myrvold 2005 stresses), to declare interpretations deployed in the context of the algebraic approach to relativistic QFT immune to No-Go results like Myrvold's is to miss the generality of the consistency concern those results express. Recast in terms of the really relativistic context, Myrvold's concern is something like this: Suppose that the open bounded regions $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, and $\mathcal{O}_4$ contain, respectively, Figure 1's points x_1, y_1, x_2, and y_2, and that each of the following pairs of regions is relatively spacelike: $(\mathcal{O}_1, \mathcal{O}_2)$, $(\mathcal{O}_1, \mathcal{O}_4)$, and $(\mathcal{O}_3, \mathcal{O}_2)$. Suppose also that the modal interpretation, applied to regions $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, and $\mathcal{O}_4$ respectively, declares observables P_1, P_2, P_3 and P_4 determinate. Finally, suppose (as seems reasonable) that the products of commuting determinate observables (e.g., assuming $P_i \in \mathcal{A}(\mathcal{O}_i)$, P_1P_3) are also determinate. Then there could be global states ω on the quasilocal algebra $\mathcal{A}$ such that no joint distribution over P_1, P_2, P_3, and P_4 returns $\omega(X)$, for all determinate X, as marginals. Nothing about the setup of algebraic relativistic QFT inoculates a modal interpretation of that theory, applied to different local algebras and subject to a 'natural' compositionality requirement, from declaring too many observables determinate to accord with some global state ω's expectation value assignments.

While we acknowledge the generality of this concern, we suspect that counterexamples of the specific sort Myrvold develops won't spread easily to Clifton's QFT-adapted modal interpretation (see Section 3). And we want to advertise a more subtle way in which the modal interpretation may fail in the relativistic context. When the modal interpretation is appropriately generalized to QFT, it may turn out that there are important physical situations for which the relevant algebra of observables $\mathcal{A}$ and state ω are such that the modal interpretation is handcuffed because it cannot assign any nontrivial (i.e. different from a multiple of the identity) observable a definite value. If the point of an interpretation of a theory is to tell us what the world would be like if the theory were true, a modal interpretation thus handcuffed would be pointless, for it would say of each system only that it was what it was—something we don't need an

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interpretation to tell us.[4] The malady usually at issue in No-Go results is one of excess, of privileging too many observables. The malady we try to diagnose here is one of deficiency, or privileging too few observables. If No-Go results of this type hold and if their source can be traced to the requirement of Poincaré invariance, then the modal interpretation is shown to be incompatible with relativistic invariance. But the reasons are rather different from the ones suggested by results mentioned in the preceding section.

Clearly, in order to investigate this issue we need a way to generalize the modal interpretation to local QFT. Rob Clifton (2000) provided one. We review it in the next section and identify one sort of triviality to which it is prone. Section 4 musters the resources of modular theory to characterize other routes to triviality for Clifton's modal interpretation. Section 5 demonstrates that certain systems of physical interest take these routes. The modal interpretation under Clifton's prescription has nothing to say about physical situations regarding which it seems it should say a lot. Section 6 discusses ways out of this predicament for the modal interpreter. In the concluding Section 7, we return to the question of whether relativistic invariance is responsible for the troubles the modal interpretation encounters in QFT.

3. Clifton's Proposal for Modal Interpretations of QFT. Before considering Clifton's proposal we need to say more about two versions of the algebraic approach introduced in the preceding section. In the 'abstract' version, the quasi-local algebra and the local algebras are C^*-algebras conceived in abstraction from any concrete Hilbert space realization. (See Appendix A for an informal introduction to some basic algebraic notions.) In the 'concrete' version, the global and local algebras are von Neumann algebras, algebras of bounded Hilbert space operators. One can pass from the abstract to the concrete version by considering a representation $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$, a structure-preserving map from an abstract C^*-algebra $\mathcal{A}$ into the bounded operators $\mathcal{B}(\mathcal{H})$ on a Hilbert space $\mathcal{H}$. One expects that in physically relevant representations the automorphisms α_Λ on $\mathcal{A}$ carrying the action of the Poincaré group are unitarily implementable in the sense that there is a unitary operator $U(\lambda)$ acting on $\mathcal{H}$ such that $U(\lambda)\pi(A)U^{-1}(\lambda) = \pi(\alpha_\lambda(A))$ for all $A \in \mathcal{A}$ and all $\lambda \in \Lambda$. The von Neumann algebra $\mathcal{R}_\pi(\mathcal{A})$ affiliated with the representation π is the weak closure of $\pi(\mathcal{A})$ or, equivalently, the double commutant $\pi(\mathcal{A})''$. Because $\pi(\mathcal{A})$ is

4. A modal advocate might maintain that, provided the interpretation applied to apparatus systems reveals sufficiently many determinate observables to account for measurement outccomes, we need not fret if the interpretation comes out trivial when applied to other systems. We discuss this maneuver in Section 6.

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only norm-closed, and because norm convergence implies but is not implied by weak convergence, it can happen that $\pi(\mathcal{A})''$ is larger than $\pi(\mathcal{A})$. It can also happen that different representations produce different, i.e. non-*-isomorphic, von Neumann algebras. This representation dependence of the von Neumann algebras can sometimes be removed by restricting attention to an appropriate class of states. As explained in Appendix A, a theorem of Gelfand, Naimark, and Segal (GNS) shows that any state ω on a C^*-algebra $\mathcal{A}$ determines a unique (up to unitary equivalence) cyclic representation π_ω. If states ω_1 and ω_2 determine quasi-equivalent GNS representations (see Appendix A), then the affiliated von Neumann algebras $\mathcal{R}_{\omega_1}(\mathcal{A})$ and $\mathcal{R}_{\omega_2}(\mathcal{A})$ are the same, i.e., are *-isomorphic. (The converse is also true.) Thus, if the physically relevant representations are those that arise from the GNS representations of a class of states that determine quasi-equivalent GNS representations, their von Neumann algebras achieve a sort of representation-free status. The Hadamard states, which are candidates for the physically realizable states of free quantum fields (see Wald 1994), are known to determine quasi-equivalent representations with respect to the local algebras $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}$ associated with open-bounded regions $\mathcal{O}$.[5]

Clifton's (2000) proposal for extending the modal interpretation to QFT, a proposal on which we will rely below, is articulated for von Neumann algebras; it is neutral as to whether these algebras come from the abstract or the concrete version. However, since the results from the physics literature we will combine with Clifton's proposal were derived in the context of the abstract version, one has to question the representation-dependence of the von Neumann algebras in this approach. The short answer is that in the physical applications we consider in Section 5, the

5. There is a potential ambiguity in the notions of local von Neumann algebras and local quasi-equivalence. Given a state ω on the quasi-local algebra $\mathcal{A}$, one could obtain a representation for a local subalgebra $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}$ by first constructing the GNS representation π_ω of $\mathcal{A}$ determined by ω and then taking the restriction $\pi_\omega|\mathcal{A}(\mathcal{O})$. The weak closure produces a von Neumann algebra associated with $\mathcal{O}$. Given two states, one can ask whether the representations of $\mathcal{A}(\mathcal{O})$ determined by the states in said fashion are quasi-equivalent, making the affiliated von Neumann algebras isomorphic. Alternatively, one could obtain a representation for $\mathcal{A}(\mathcal{O})$ by first taking the restriction $\omega|\mathcal{A}(\mathcal{O})$ of ω to $\mathcal{A}(\mathcal{O})$ and then constructing the GNS representation $\pi_\omega|\mathcal{A}(\mathcal{O})$ determined by $\omega|\mathcal{A}(\mathcal{O})$. Again, one can ask whether the representations obtained in this fashion from two states are quasi-equivalent, making the von Neumann algebras affiliated with the two representations isomorphic. The answers that apply to these two methods of obtaining local representations need not coincide. However, if the local von Neumann algebras are factors, this difference disappears. This will be the case for regions $\mathcal{O}$ with nonnull causal complement $\mathcal{O}^c$ if the so-called *duality property* holds, namely, $\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}^c)$. Duality is known to hold for the cases discussed in Section 5.

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representation-dependence is not bothersome because the representations considered are privileged in that they arise from privileged states, e.g. vacuum states.

In order to understand why some creative effort is needed to extend the modal interpretation to relativistic QFT, start from the fact that the modal interpretation was originally developed for the environment of ordinary QM in which the observables pertaining to a quantum system belong to a particular type of von Neumann algebra, the algebra $\mathcal{B}(\mathcal{H})$ of bounded operators on that system's Hilbert space $\mathcal{H}$. Von Neumann algebras are classified (roughly speaking) with respect to the dimensions of the subspaces onto which the projection operators they contain project. (For an introduction to the typology of von Neumann algebras, see Kadison and Ringrose 1983, chapter 5; 1986, chapter 6.) In terms of this classification, the algebras of ordinary QM, which have one-dimensional projection operators, are Type I von Neumann algebras. Normal (that is, countably additive) states on these algebras are density matrices. Both features of Type I algebras are exploited by the modal interpretation. Assuming that quantum states are density matrix states, the modal interpretation identifies the observables determinate on quantum systems by appeal to the one-dimensional projection operators giving the spectral resolution of those density matrices; each such one-dimensional projection determines a truth valuation on the set of determinate propositions corresponding to the set of determinate observables.

In QFT, however, the algebra of observables pertaining to a local quantum system is typically *not* a Type I algebra but a Type III factor von Neumann algebra. The modal interpretation is not well-adapted to algebras of this type. Type III factor algebras contain only infinite projection operators. Again speaking roughly, a infinite projection operator is one associated with a subspace $\mathcal{V} \subset \mathcal{H}$ that itself has a proper subspace $\mathcal{W}$ such that $\mathcal{V}$ and $\mathcal{W}$ are isometrically embeddable into one another. The pure states of a Type I von Neumann algebra are determined by its one-dimensional projectors. Type III factor algebras, admitting no finite projectors, admit no pure normal states either. Nor does a Type III factor algebra admit normal density matrix states. So when it comes to Type III factor algebras, the modal interpretation lacks both its raw material—the density matrix describing the system to which it would attribute determinate observable values—and its final product—the projection operators by which to identify, and assign probabilities to, those determinate values.

Unfortunately for the modal interpretation, Type III factor algebras are the norm, not the exception, in local QFT. Consider, for example, the Klein-Gordon field on Minkowski spacetime. For the Minkowski vacuum state or, more generally, for any state of bounded energy, the von Neu-

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mann algebra associated with open, bounded spacetime regions are Type III factors. The same conclusion holds for an open unbounded region $\mathcal{O}$ if it has nonnull causal complement and there is a spacelike vector $\vec{a}$ such that $\mathcal{O} = \mathcal{O} + \mathbb{R}\vec{a}$ (see Størmer 1967, Theorem 2.4). An example of such a region is a Rindler wedge of Minkowski spacetime (see Figure 2 below and Section 5).

In spite of the considerable hurdle thrown up by Type III factors, Clifton (2000) found a well-motivated way to generalize the modal interpretation to cover QFT. The approach assumes that the algebra of observables pertaining to a system is a von Neumann algebra $\mathcal{R}$. What it takes to have a modal interpretation for a state ω, Clifton suggests, is a specification of a subalgebra $\mathcal{S} \subseteq \mathcal{R}$ such that ω defines a mixture of dispersion free states on $\mathcal{S}$, where a state ω_δ is dispersion free just in case $\omega_\delta(A^2) = \omega_\delta(A)^2$ for all self-adjoint $A \in \mathcal{S}$. The state ω can be said to define a mixture of dispersion free states on $\mathcal{S}$ just in case there is a probability measure μ on the space of dispersion free states Δ on $\mathcal{S}$ such that $\omega(A) = \int_\Delta \omega_\delta(A) d\mu(\delta)$ for all $A \in \mathcal{S}$. The elements of $\mathcal{S}$ are then candidates for being assigned definite values in the state ω. If ω is faithful—that is, if $\omega(A) = 0$ implies that $A = 0$ for each positive A in $\mathcal{R}$—the only subalgebras for which this is possible are Abelian subalgebras. Thus Clifton's QFT-adapted modal interpretation proceeds by (i) "using only ω and the algebraic operations available within $\mathcal{R}$" (2000, 172) to identify an Abelian subalgebra $\mathcal{S}$ of $\mathcal{R}$, and then (ii) proving, for a faithful normal state ω on $\mathcal{R}$, that the subalgebra so specified is the maximal algebra, identified using only ω and algebraic operations within $\mathcal{R}$, over which ω defines a mixture of dispersion free states.[6]

This brief sketch is enough to provide an appreciation of the capacity of Clifton's proposal to resist Myrvold's No-Go result. Recall the algebraic reformulation of the situation depicted in Figure 1 in terms of modally determinate observables $P_i \in \mathcal{A}(\mathcal{O}_i)$. Myrvold's worry—that there exist states ω of the quasilocal algebra such that no joint distribution over $\{P_i\}$ returns $\omega(X)$ for all modally determinate X as marginals—evaporates if $[P_i, P_j] = 0$ for all i, j. For then a joint distribution over $\{P_i\}$ can come directly from the self-consistent global state ω. Because of spacelike separation, we get $[P_1, P_2] = [P_1, P_4] = [P_4, P_3] = [P_3, P_2] = 0$ straight from the axioms. To get $[P_1, P_3] = [P_2, P_4] = 0$, we transfer Myrvold's assumption that $\psi(\beta) = (U_1 \otimes U_2)\psi(\alpha)$ from the hybrid context to this one as follows: Let $\mathcal{A}(\mathcal{O}_1)$ be the really relativistic analog of the observables pertaining to S_1 at x_1 in Myrvold's original example, and let $\mathcal{A}(\mathcal{O}_3)$ be the really relativistic analog of the observables pertaining to S_1 at x_2 (see

6. Kitajima (2004) extends the result to states that are not faithful. The apparatus of modular theory, sketched in Section 4, does not apply to such states.

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Figure 1). Then the assumption that S_1's state evolves via U_1 between x_1 and x_2 implies that there is some time evolution automorphism α of $\mathcal{A}(\mathcal{O}_1)$ (schematically, $\alpha(X) = UXU^{-1}$ for all $X \in \mathcal{A}(\mathcal{O}_1)$) such that $\mathcal{A}(\mathcal{O}_3)$ is the image of $\mathcal{A}(\mathcal{O}_1)$ under that automorphism. But the image of an algebra $\mathcal{A}$ under an automorphism of $\mathcal{A}$ is just $\mathcal{A}$. Thus for Clifton, the observables determinate on region $\mathcal{O}_3$ are the same as the observables determinate on region $\mathcal{O}_1$ (and mutatis mutandis for regions $\mathcal{O}_2$ and $\mathcal{O}_4$): they're the observables identified by applying Clifton's prescription to the global state ω and the algebra those regions share. (Notice that if the automorphism α implementing the time evolution is different from the identity, determinate $P \in \mathcal{A}(\mathcal{O}_3)$ isn't the 'evolute' of determinate $P \in \mathcal{A}(\mathcal{O}_1)$, that is, its image under the automorphism. So this feature of Clifton's approach is consistent with nontrivial time evolution.) Thus $[P_1, P_3] = [P_2, P_4] = 0$, and the noncommutativity needed for things to go wrong in the manner Myrvold envisions doesn't arise for Clifton's proposal.

Now, we can state Clifton's proposal in detail. When ω is a faithful normal state on a von Neumann algebra $\mathcal{R}$, Clifton's modal interpretation identifies the subalgebra of observables determinate on a system in state ω with the center of ω's centralizer. ω's *centralizer* (sometimes also called a *normalizer*) is $\mathfrak{C}_\omega := \{A \in \mathcal{R} : \omega([A, B]) = 0 \text{ for all } B \in \mathcal{R}\}$. Since $\mathfrak{C}_\omega$ is a von Neumann algebra, its center is $\mathcal{Z}(\mathfrak{C}_\omega) := \mathfrak{C}_\omega \cap \mathfrak{C}'_\omega$. The specification of the algebra of determinate observables for unfaithful states is somewhat more involved. Thankfully its details need not concern us, because the states at issue in QFT are generally faithful.

When $\mathcal{R}$ is a Type I algebra, Clifton's prescription coincides with the familiar modal interpretation of ordinary QM. If $\mathcal{R}$ is Type I, ω is a density matrix state W, assumed for simplicity to be faithful *and nondegenerate*. The familiar modal criterion is that A is determinate on a system in W if and only if $[W, A] = 0$. This condition is equivalent to Clifton's requirement that A belong to the center of W's centralizer. To show this, we will show that $[W, A] = 0$ if and only if $A \in \mathfrak{C}_\omega$. The equivalence of Clifton's requirement and the familiar one follows because, for nondegenerate W, $\{A \in \mathfrak{N} : [W, A] = 0\}$ is Abelian, so that $\mathfrak{Z}(\mathfrak{C}_\omega) = \mathfrak{C}_\omega$. That $A \in \mathfrak{C}_\omega$ follows easily from $[W, A] = 0$. For any B,

$$\omega(AB - BA) = \mathrm{Tr}(WAB) - \mathrm{Tr}(WBA) = \mathrm{Tr}(WAB) - \mathrm{Tr}(AWB)$$

(where the invariance of the trace under cyclic permutation is used in the second step). If W commutes with A, this expression reduces to 0, and

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$A \in \mathfrak{C}_\omega$. The converse is also easy to derive. Assume, for contradiction, that $X := [W, A] \neq 0$. If $A \in \mathfrak{C}_\omega$,

$$0 = \omega(AB) - \omega(BA) = \mathrm{Tr}(WAB) - \mathrm{Tr}(WBA)$$
$$= \mathrm{Tr}(WAB) - \mathrm{Tr}(AWB) = \mathrm{Tr}(XB)$$

for all B. But letting $B = X^*$, this implies that $\mathrm{Tr}(X^*X) = 0$, which implies that $X = 0$, which is a contradiction. Thus $A \in \mathfrak{C}_\omega$ implies that $[W, A] = 0$. So $[W, A] = 0$ if and only if $A \in \mathfrak{C}_\omega$, and in the setting of ordinary QM, Clifton's revamped modal interpretation coincides with the usual one.

It is instructive to see what befalls Clifton's prescription (and ergo the familiar modal interpretation) in this setting when W is degenerate. Take the worst case, that $W = (1/n)I$, where I is the identity operator on an n-dimensional $\mathcal{H}$. Then for all $B \in \mathcal{B}(\mathcal{H})$,

$$\mathrm{Tr}(WAB) - \mathrm{Tr}(WBA) = \frac{1}{n}[\mathrm{Tr}(AB) - \mathrm{Tr}(BA)] = 0$$

(appealing in the last step to the invariance of the trace under cyclic permutation). And $\mathfrak{C}_\omega = \{A \in \mathcal{R} : \mathrm{Tr}(WAB) - \mathrm{Tr}(WBA) = 0\}$ just is $\mathcal{B}(\mathcal{H})$. Because $\mathcal{B}(\mathcal{H})$ is irreducible, the center of $\mathfrak{C}_\omega$ is trivial. According to Clifton's prescription, *no (nontrivial) observables are determinate* on an ordinary QM system whose state is given by a maximally degenerate density matrix.

More familiar formulations of the modal interpretation for ordinary QM have this problem as well: when W is highly degenerate, continuously many different sets of pairwise orthogonal projections conspire in its spectral resolution; for Hilbert spaces of dimension higher than 2, the modal interpretation assigns all those projections determinate values on pain of reinviting Kochen-Specker type contradictions. One response to this problem is to assign only trivial observables determinate values when W is maximally degenerate, and take comfort in the fact that maximally degenerate states form sets of measure 0 in the state space.[7] Clifton's QFT-adapted modal prescription duplicates the first part of this response. But the second part may not be available. To see why, we need a sharper

7. Another response, one available when the degeneracy of the density operator is a fleeting condition out of which it evolves, is to appeal to considerations of dynamical continuity to single one eigenbasis out from the infinite collection of eigenbases that diagonalize W. See Bacciagaluppi, Donald, and Vermaas 1995. The discussion of Clifton's proposal's resistance to Myrvold's result suggests that this strategy won't rescue Clifton's proposal from triviality in the case of systems (such as those mustered in Section 5) whose dynamics are encoded as automorphisms of their observable algebra.

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characterization of the circumstances under which Clifton's prescription comes up trivial.

4. Clifton's Proposal in Light of Modular Theory. ω is a *tracial state* on the von Neumann algebra $\mathcal{R}$ if and only if $\omega(AB) = \omega(BA)$ for all A, $B \in \mathcal{R}$. The state $W = (1/n)I$ discussed in the last section is an example. The centralizer $\mathfrak{C}_\omega$ of a tracial state ω is $\mathcal{R}$ itself, and the centers of $\mathfrak{C}_\omega$ and $\mathcal{R}$ coincide: $\mathcal{Z}(\mathfrak{C}_\omega(\mathcal{R})) = \mathcal{Z}(\mathcal{R})$. If $\mathcal{R}$ is a factor algebra, it follows from Clifton's proposal that no nontrivial observable is modally definite in a tracial state ω.

Now it might seem that, as inhospitable as Type III factor algebras might otherwise be, they offer the modal interpretation at least this much comfort: they admit no tracial states (see Bratteli and Robinson 1997, Theorem 2.7.17), and so cancel the possibility of triviality due to traciality. Type III factor algebras, however, institute a new route to triviality for Clifton's modal interpretation, mapped out in this section. Tomita-Takesaki theory provides a generalization of the tracial property in terms of the modular condition and a qualitative measure of how near a state is to being tracial in terms of the modular automorphism group (see Kadison and Ringrose 1986, 592). And it turns out that for an interesting class of far from tracial states, the set of modally definite observables is again trivial.

The Tomita-Takesaki theorem shows that if ω is a faithful normal state on a von Neumann algebra $\mathcal{R}$, there is a unique one-parameter group σ_t^ω, $t \in \mathbb{R}$, of *-automorphisms of $\mathcal{R}$—the *modular group*—that satisfies a condition intimately linked to thermodynamic considerations. Let $\mathcal{R}$ be a von Neumann algebra acting on a separable Hilbert space $\mathcal{H}$. A vector $|\psi\rangle \in \mathcal{H}$ is said to be *cyclic* for $\mathcal{R}$ just in case $\{\mathcal{R}|\psi\rangle\}$ is dense in $\mathcal{H}$. A vector $|\psi\rangle \in \mathcal{H}$ is said to be *separating* for $\mathcal{R}$ just in case $A|\psi\rangle = 0$ implies $A = 0$ for any $A \in \mathcal{R}$. If ω is a faithful normal state on a von Neumann algebra $\mathcal{R}$, then there is a cyclic and separating vector $|\Omega\rangle$ in the Hilbert space on which $\mathcal{R}$ acts such that $\langle\Omega|A|\Omega\rangle = \omega(A)$ for all $A \in \mathcal{R}$. (In order to facilitate exposition, we are suppressing some complications. $|\Omega\rangle$ is the cyclic vector of the GNS representation π_ω of $\mathcal{R}$ determined by the state ω. It is the affiliated von Neumann algebra $\pi_\omega(\mathcal{R})''$ with respect to which $|\Omega\rangle$ is cyclic and separating. Because $\mathcal{R}$, $\pi_\omega(\mathcal{R})$, and $\pi_\omega(\mathcal{R})''$ are all *-isomorphic, we omit reference to the representation π_ω.) Consider a *conjugation operator* S such that $SA|\Omega\rangle = A^*|\Omega\rangle$ for all $A \in \mathcal{R}$. The closure of S in the strong topology—which we will continue to call S—has a unique polar decomposition $S = J\Delta^{1/2}$. Here J is anti-unitary and the *modular operator* Δ is a self adjoint, positive (generally unbounded) operator. The modular group σ_t^ω of automorphisms of $\mathcal{R}$ associated with ω is given by $\sigma_t^\omega(A) = \Delta^{it}A\Delta^{-it}$ for all $A \in \mathcal{R}$. It is the unique group satisfying

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the *modular condition*, requiring that for all $A, B \in \mathcal{R}$, there is a complex valued function f, continuous and bounded on $\{z \in \mathbb{C} : 0 \leq \mathrm{Im}z \leq \beta\}$, with $\beta \in (0, +\infty)$, such that $f(t) = \omega(\sigma_t^\omega(A)B)$ and $f(t + i\beta) = \omega(B\sigma_t^\omega(A))$ for all $t \in \mathbb{R}$. Intuitively, the modular group indicates how far the behavior of the algebra $\mathcal{R}$ under the state ω is from being commutative. When $\mathcal{R}$ is Abelian, or when ω is a tracial state, σ_t^ω is the identity automorphism for all t (Kadison and Ringrose 1986, 592).

The modular condition is also called the *KMS condition*. It is used to characterize the class of states in quantum statistical mechanics that are generalizations of the Gibbs equilibrium states of classical statistical mechanics: an equilibrium state with respect to a dynamics implemented by an automorphism group σ_t^ω satisfies the modular or KMS condition with respect to that group, and β corresponds to the inverse temperature of the system. (It turns out that ω is a tracial state just in case it is a KMS state at infinite temperature, i.e. $\beta = 0$.) Note, however, that the modular group determined by an arbitrary faithful normal state on a von Neumann algebra may lack a natural dynamical interpretation, in which case scare quotes should be understood when referring to β as the inverse temperature.[8] In Section 5 we will give an example from QFT where the modular group is identical with the dynamical automorphism group. Note also that a σ_t^ω-KMS state is invariant under the action of σ_t^ω, but a σ_t^ω-invariant state is not necessarily a KMS state.

The relevance of all this to Clifton's modal interpretation is that the centralizer of ω—that object whose center gives the modally determinate observables—can be characterized in terms of its corresponding modular automorphism group:

> **Lemma** (Kadison and Ringrose 1986, 617). Let ω be a faithful normal state on a von Neumann algebra $\mathcal{R}$. Then $\mathfrak{C}(\mathcal{R}) := \{A \in \mathcal{R} : \sigma_t^\omega(A) = A \text{ for all } t\}$, i.e. the elements of the centralizer, are precisely the modular invariants.

For a tracial state ω, the modular group is trivial and all the elements of $\mathcal{R}$ are modular invariants. Intuitively, at the opposite extreme are the *ergodic states* defined by the property that the only modular invariants are trivial, i.e. of the form $\mathbb{C}I$. For such states, $\mathcal{Z}(\mathfrak{C}_\omega(\mathcal{R})) = \{\mathbb{C}I\}$ and,

8. Here we should mention Rovelli's 'thermal time hypothesis' (see Rovelli 1993 and Connes and Rovelli 1993). In a generally covariant theory, there is no external time parameter in whose terms to express system evolution, and all generally covariant quantities are constants of the motion. The thermal time hypothesis recovers a state-dependent notion of time for generally covariant quantum theories. Rovelli's idea is that with each thermal state (supposed to be a faithful normal state over a von Neumann algebra) is associated a flow of time given by that state's modular automorphism group.

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thus, on Clifton's proposal no nontrivial observable is modally definite in such a state. Clifton's (2000) challenge to the modal interpretation as applied to algebraic QFT is that ergodic states, unlike maximally degenerate states in ordinary QM, amount to a set of states dense in the state space for a Type III factor.

Like other density claims, this one does not suffice to show that ergodic states arise in physically reasonable situations. And if one wanted to denigrate the result that modal assignments are trivial for ergodic states, one could emphasize just how 'extremal' ergodic states are. Suppose that ω is a σ_t^ω-KMS on $\mathcal{R}$. In general, ω will neither be the unique σ_t^ω-KMS state (for unless $\mathcal{R}$ is a factor, it can sustain other σ_t^ω-KMS states) nor will ω be the unique σ_t^ω-invariant state (for states can be invariant under an automorphism group without being KMS with respect to it). If, however, ω is ergodic, it is not only the unique σ_t^ω-KMS state but also the unique σ_t^ω-invariant state on $\mathcal{R}$. (These claims follow from a pair of lemmas outlined in Appendix B.) In cases where the modular group corresponding to ω coincides with the dynamical evolution group, this latter means that ω is the only normal state that is stationary under dynamical evolution. Citing these features of ergodic states, the modal proponent might attempt to excuse her interpretation from confronting them.

It is not clear to us that this 'extremal' nature of ergodic states is a reason not to take them seriously. After all, there is a proud tradition of investing with physical significance states, such as the Minkowski vacuum state, singled out by invariance requirements. But we will concede the point, in order to emphasize the opposite side of the coin. Ergodic states may be, in some mathematical senses, 'out of the ordinary,' but they also occur in physically important applications. And in these applications, the embarrassment for the modal theorist is real.

5. Some Concrete Cases of Trivialization. In this section, we exhibit two physical applications of QFT for which the modal interpretation under Clifton's prescription is trivial. This first is the $m \geq 0$ Klein-Gordon field, where the modal interpretation assigns determinate values to no nontrivial observables associated with Rindler wedges. The second is the $m = 0$ Klein-Gordon field, where the modal interpretation assigns determinate values to no nontrivial observables associated with open diamond-shaped regions of spacetime, with forward or backward light cones, or with double cones.

We will follow the standard local QFT presentation in which one starts with a net of local C^*-algebras $\mathcal{A}(\mathcal{O})$ associated with open bounded regions $\mathcal{O} \subset \mathcal{M}$ of Minkowski spacetime $\mathcal{M}$ and satisfying the standard assumptions of locality, relativistic invariance, etc., and then forms the

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quasi-local algebra $\mathcal{A}$ by taking the norm closure of the union of these local algebras. Problems for the modal interpretation follow from

> **Reeh-Schleider theorem**. Let $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ be a representation, and suppose that there is a vector $|\Omega\rangle \in \mathcal{H}$ that is cyclic for $\pi(\mathcal{A})$ and analytic in the energy. Let $\mathcal{X} \subset \mathcal{M}$ be a region of spacetime with nontrivial causal complement. Then $|\Omega\rangle$ is cyclic and separating for the von Neumann algebra $\mathcal{R}_\pi(\mathcal{X}) := \pi(\mathcal{A}(\mathcal{X}))''$.

General application: Consider the case of the Klein-Gordon field with a mass $m \geq 0$. The algebras $\mathcal{A}_{\mathrm{KG}}(\mathcal{O})$ generated by smearing the field with test function of support confined to $\mathcal{O}$ are known to satisfy the axioms of local relativistic QFT. Let ω_0 be the Minkowski vacuum state on $\mathcal{A}_{\mathrm{KG}}$. In the GNS representation $(\pi_{\omega_0}, \mathcal{H}_{\omega_0}, |\Omega_{\omega_0}\rangle)$ determined by ω_0, the GNS vector $|\Omega_{\omega_0}\rangle$ is cyclic and analytic in the energy. Thus, by the Reeh-Schleider theorem, if $\mathcal{X} \subset \mathcal{M}$ is any region with nontrivial causal complement, $|\Omega_{\omega_0}\rangle$ is a cyclic and separating vector for $\mathcal{R}_{\omega_0}(\mathcal{X}) := \pi_{\omega_0}(\mathcal{A}(\mathcal{X}))''$. Note that if $|\Omega\rangle \in \mathcal{H}$ is a cyclic and separating vector for a von Neumann algebra $\mathcal{R}$ acting on a separable Hilbert space $\mathcal{H}$, then $\omega(\cdot) := \langle\Omega| \cdot |\Omega\rangle$ defines a faithful normal state on $\mathcal{R}$. Thus, we can apply all of the apparatus of modular theory to $\mathcal{R}_{\omega_0}(\mathcal{X})$. Furthermore, as already noted above, it is known that $\mathcal{R}_{\omega_0}(\mathcal{X})$ is Type III for a variety of open spacetime regions $\mathcal{X} \subset \mathcal{M}$ with nonnull causal complements. It remains to be shown that for some of these types of regions the restriction to them of the Minkowski vacuum state produces an ergodic state. This we do for two cases.

Particular Application 1. The Klein-Gordon field of mass $m \geq 0$.

Let $\mathcal{X}$ be a Rindler wedge $\mathcal{W}$, e.g. the Rindler wedge to the right of the origin of the Lorentz coordinate system (x_1, x_2, x_3, t) and defined by $|t| < |x_1|$, $x_1 > 0$ (see Figure 2). In this case the modular automorphism group has a nice physical interpretation, for the Bisognano-Wichmann theorem (Bisognano and Wichmann 1975) shows that the modular operators coincide with the unitary operators $U(\Lambda_{2\pi t})$ that implement the Lorentz boosts $\Lambda_{2\pi t}$ on $\mathcal{W}$, where

$$\Lambda_{2\pi t} = \begin{pmatrix} \cosh(2\pi t) & \sinh(2\pi t) & 0 & 0 \\ \sinh(2\pi t) & \cosh(2\pi t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Since for any open bounded region $\mathcal{O} \subset \mathcal{M}$, $U(\Lambda_{2\pi t})\mathcal{A}(\mathcal{O})U(\Lambda_{2\pi t}) = \mathcal{A}(\mathcal{O}_{\Lambda_{2\pi t}})$ where $\mathcal{O}_{\Lambda_{2\pi t}}$ is the image of $\mathcal{O}$ under the Lorentz boost $\Lambda_{2\pi t}$, it might seem obvious that the modular invariants for the Rindler wedge consist of multiples of the identity. However, it is nontrivial to show there are no nontrivial modular invariants in the von Neumann algebra

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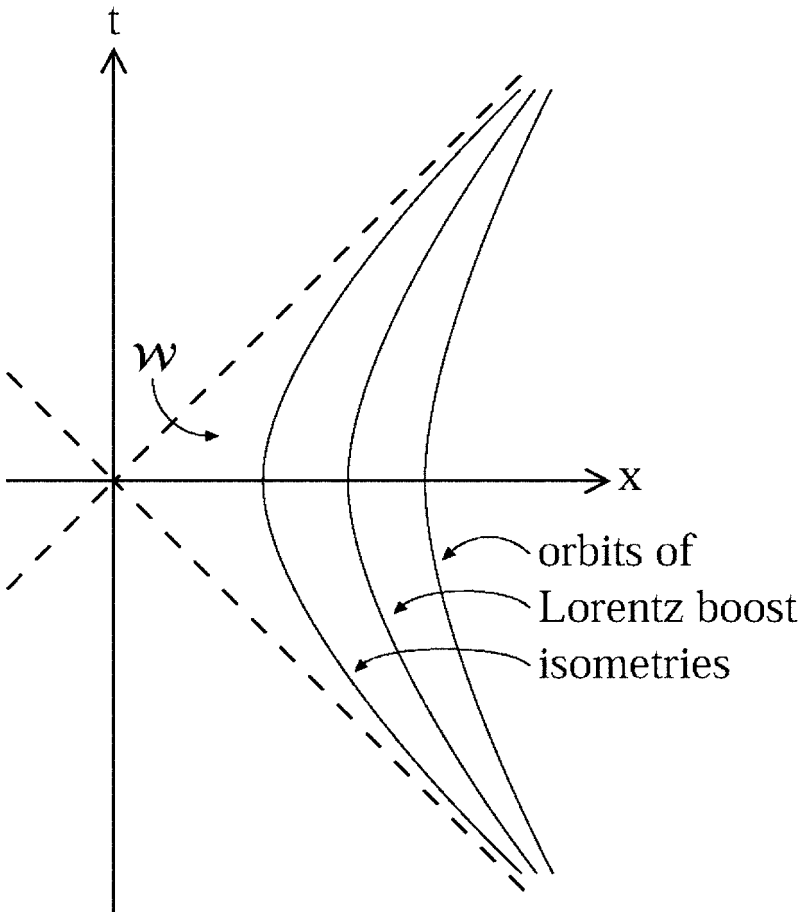


Figure 2. The right Rindler wedge.

$\mathcal{R}_{\omega_0}(\mathcal{W})$, and we simply refer the reader to Driessler (1975, Theorem 2.2) for the details.

Now the Minkowski vacuum state, when restricted to the right Rindler wedge, seems a prime candidate for treatment by the modal interpretation. For it is a statistical state par excellance—a thermal state at a finite temperature in the sense that it is a KMS state with respect to the Lorentz boost isometries. So using either classical statistical mechanics or ordinary QM as a model, one would expect this state to have an interpretation as a mixture of dispersion free states. Clifton's prescription disappoints this expectation, since the state's centralizer is trivial. (Notice that this puzzle

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about the Rindler case has nothing to do with what is usually—and, we claim, misleadingly (see Arageorgis et al. 2002)—marketed as *the* puzzle about the Rindler case, which is the challenge it poses to particle interpretations of QFT.)

Particular Application 2. The Klein-Gordon field of mass $m = 0$.

(a) Let $\mathcal{X}$ be a diamond region $\mathcal{D}$, e.g. the unit diamond $|\mathbf{x}| + |t| < 1$ centered at the origin of a Lorentz coordinate system $(\mathbf{x}, t)$. The modular automorphisms of $\mathcal{R}_{\omega_0}(\mathcal{D})$ are the conformal transformations of Minkowski spacetime that leave $\mathcal{D}$ invariant. Again it is intuitive that no nontrivial elements of $\mathcal{R}_{\omega_0}(\mathcal{D})$ are preserved by these transformations and, thus, that the centralizer is trivial. But again the proof is rather involved. It exploits the fact that there is a conformal transformation of $\mathcal{M}$ that takes the diamond $\mathcal{D}$ to the right wedge $\mathcal{W}$ considered in Application 1. Using this fact, Hislop and Longo (1982) show that $\mathcal{R}_{\omega_0}(\mathcal{W})$ and $\mathcal{R}_{\omega_0}(\mathcal{D})$ are unitarily equivalent, which allows the results for the centralizer for the Rindler wedge to be transferred to the centralizer for the diamond. These results also extend to a thin lens shaped region $\mathcal{L}$ surrounding the local spacelike hypersurface S that forms the base of the diamond (see Figure 3).[9]

According to the primitive causality axiom of axiomatic QFT, the algebra associated with a spacetime region coincides with the algebra associated with that region's domain of determinacy, the set of spacetime points every inextendible causal curve through which intersects said region. And $\mathcal{L}$'s domain of determinacy is the diamond that surrounds it.

(b) Hislop and Longo (1982) also show that similar results hold when $\mathcal{X}$ is a forward, a backward, or a double lightcone (see also Longo 1982).

The results in Applications 2a and 2b presumably extend to other conformally invariant field theories. It is thought that at short distances all relativistic quantum field theories will exhibit conformal invariance (see Fredenhagen 1987).

6. Possible Responses for the Modal Theorist. We will consider four ways the modal theorist can respond to the triviality results discussed in the preceding section.

(1) Reject Clifton's proposal for extending the modal interpretation to local QFT. For example, one way to open up more possibilities is to allow the modal assignments for a subalgebra $\mathcal{S} \subseteq \mathcal{R}$ to depend not just on $\mathcal{S}$ and the state ω on $\mathcal{S}$ but on other factors.

Rejoinder: (a) One implementation of this escape would make the modal

9. A four-dimensional region such as $\mathcal{L}$ rather than the three-dimensional region S (see Figure 3) is needed because the field algebras have been constructed by smearing field operators with test functions of compact support on open regions.

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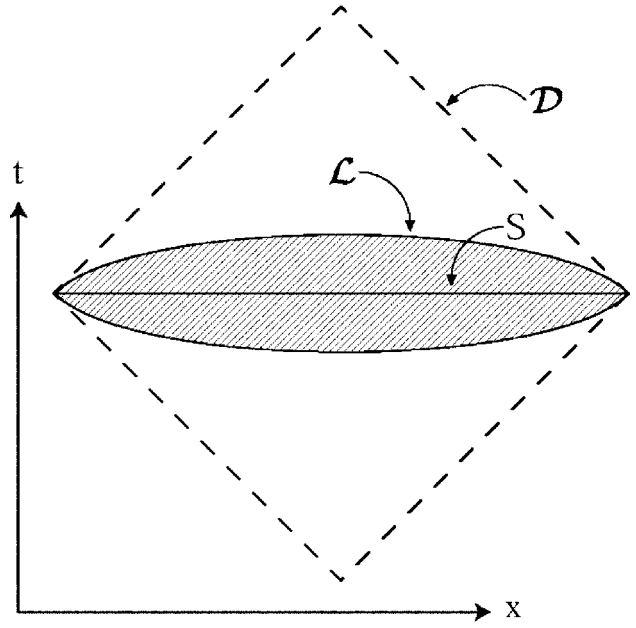


Figure 3. A region for Application 2(a).

assignment of values dependent on a choice of spacelike hyperplane of Minkowski spacetime. But the hyperplane formalism, at least as it is developed by Fleming (1996, 2000, 2003), clashes with the approach of local QFT presupposed here. Of course, it is not written across the sky that this approach is sacrosanct. But if the abandonment of local relativistic QFT is the price for reconciling the modal interpretation with relativistic invariance, then for us the prima facie presumption is that the modal interpretation is in trouble. (b) Another implementation of the escape would be to make the modal assignment of values depend not only on $\mathcal{S}$ and the state ω but also on additional physical considerations. If these considerations take the form of 'hidden variables' not supplied by either QFT or by the spacetime structure, their introduction signals (to us, at least) a move to a new game in which the distinctive and attractive features of the modal interpretation are lost. If they take the form of overt observables already in the local algebra and singled out for privilege (a strategy Bub 1997 charts for nonrelativistic QM), we would like to hear more about the criteria by which the privileged observables are to be singled out.

(2) To get observables with definite values, go to a larger von Neumann

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algebra. The idea can be implemented with the help of Combes' (1967) theorem, which concerns the von Neumann algebra associated with the *universal representation* of a C^*-algebra $\mathcal{A}$, obtained by taking the direct sum, over the complete set of algebraic states on $\mathcal{A}$, of their GNS representations (see Kadison and Ringrose 1983, 281). Combes shows that if φ is a factor state on a C^*-algebra $\mathcal{A}$, then the unique normal extension $\tilde{\varphi}$ of φ to the universal von Neumann algebra $\mathcal{U}(\mathcal{A})$ enveloping $\mathcal{A}$ is dispersion free on the center $\mathcal{Z}(\mathcal{U}(\mathcal{A}))$ of $\mathcal{U}(\mathcal{A})$. Thus, the members of $\mathcal{Z}(\mathcal{U}(\mathcal{A}))$, some of which are nontrivial, can be assigned definite values.

Rejoinder: (a) The original problem was to assign definite values to some of the nontrivial observables belonging to $\mathcal{A}$ or to the von Neumann algebra $\pi_\varphi(\mathcal{A})''$ affiliated with the state φ of interest, and it is no solution to this problem to be told that values can be assigned to observables in a larger algebra. (b) Furthermore, there is nothing specifically modal about the interpretative strategy at issue because there is nothing state-dependent about the set of determinate observables $\mathcal{Z}(\mathcal{U}(\mathcal{A}))$. And for that matter there is nothing specifically quantum about the set of determinate observables since $\mathcal{Z}(\mathcal{U}(\mathcal{A}))$ is usually identified with the set of classical observables.

(3) The third response relies on the 'split property': for any two concentric diamond shaped regions $\mathcal{D}_r$, $\mathcal{D}_{r+\epsilon}$ of Minkowski spacetime with respective radii r and $r + \epsilon$, there is a Type I factor $\mathcal{N}_{r+\epsilon}$ such that $\mathcal{R}(\mathcal{D}_r) \subset \mathcal{N}_{r+\epsilon} \subset \mathcal{R}(\mathcal{D}_{r+\epsilon})$, where $\mathcal{R}(\mathcal{D}_r)$ and $\mathcal{R}(\mathcal{D}_{r+\epsilon})$ are the von Neumann algebras associated with the regions $\mathcal{D}_r$ and $\mathcal{D}_{r+\epsilon}$ respectively. The idea then is to apply to $\mathcal{N}_{r+\epsilon}$ the usual interpretational moves developed for ordinary QM.

Rejoinder: The split property has been verified for a number of important cases in QFT, but it is known to fail in others (see Horuzhy 1990, 22–23). Clifton (2000) has also criticized this maneuver on the grounds of arbitrariness; in particular, there is arbitrariness in choosing the sizes of the diamond regions, and having chosen them, arbitrariness in choosing the interpolating Type I factors.[10] We would add that the status of the interpolating Type I factors is somewhat mysterious. In the standard approaches to local QFT Minkowski spacetime, the local von Neumann algebra $\mathcal{R}(\mathcal{O})$ associated with an open bounded region $\mathcal{O}$ with nonempty

10. Hans Halvorson has conjectured that the latter sort of arbitrariness can be avoided. The key is to relax Clifton's requirement that $\mathcal{N}_{r+\epsilon}$ be invariant under automorphisms that leave the state on $\mathcal{R}(\mathcal{D}_{r+\epsilon})$ invariant. Instead, require that $\mathcal{N}_{r+\epsilon}$ be invariant under automorphisms that preserve the structure of the *net of algebras* that the algebraic approach associates with the larger diamond $\mathcal{D}_{r+\epsilon}$ and its subregions. *If* any such 'net automorphism' that maps $\mathcal{R}(\mathcal{D}_{r+\epsilon})$ to itself also maps $\mathcal{R}(\mathcal{D}_r)$ to itself, then it follows from a result of Doplicher and Longo (1984) that there is a unique Type I factor that interpolates between $\mathcal{R}(\mathcal{D}_{r+\epsilon})$ and $\mathcal{R}(\mathcal{D}_r)$ and is invariant under net automorphisms.

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spacelike complement is a Type III factor. The basic interpretational premise of these approaches is that elements of this algebra correspond to operations that can be performed locally in $\mathcal{O}$. For diamonds of finite extent, every open bounded region $\mathcal{O}$ that 'interpolates' between $\mathcal{D}_r$ and $\mathcal{D}_{r+\epsilon}$, in the sense that $\mathcal{D}_r \subset \mathcal{O} \subset \mathcal{D}_{r+\epsilon}$, will therefore be associated with a Type III factor algebra. The interpolating Type I factor $\mathcal{N}_{r+\epsilon}$ is, therefore, the local algebra of no interpolating region. Although it can be mathematically associated with local spacetime regions, the association lacks operational significance unless additional interpretational premises are introduced.

(4) Grant that it is counterintuitive that, according to the modal interpretation, no nontrivial observables are assigned definite values in the cases described in the preceding section. But count the modal interpretation as a success if it succeeds in solving the measurement problem in the sense that it shows how measurements have definite outcomes.

Rejoinder: This is not a response but a program for a response. Suppose for sake of argument that one goes along with the minimalist modal program of merely trying to account for definite outcomes of measurements. We insist on some conditions for the successful implementation of this program. First, 'measuring instrument' is not to be treated as a primitive notion, nor is it to be analyzed in terms of notions imported from classical physics or elementary QM, e.g., 'particle'; rather it must be fully analyzed in terms of local quantum fields. With this understanding, the challenge to the modal theorist now comes to this: show that when the analysis is applied to what is pre-analytically taken to be a typical measurement situation, and when the state for the quasi-local algebra is restricted to that local algebra of field observables that the analysis takes as characterizing the measurement apparatus in the appropriate spacetime region, the centralizer is nontrivial and contains sufficiently many nontrivial elements to account for measurement outcomes.

Meeting the challenge of completing the programmatic minimalist modal interpretation is, we claim, what has to be done in order to establish that the modal interpretation can be successfully extended to relativistic QFT. We do not say that the challenge cannot be met. But we confess to some skepticism: what special features of measurement interactions wreck the properties of algebras and states that result in the trivialization of modal assignments?

7. Conclusion. To what extent can the troubles for the modal interpretation that were documented above be laid at the doorstep of relativistic invariance? We suspect that these troubles are due less to the Poincaré invariance of relativistic QFT vs. the Galilean invariance of ordinary nonrelativistic QM than to the infinite number of degrees of freedom of

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the former vs. the finite number of degrees of freedom of the latter. Evidence for this suspicion comes from the facts that Type III factor algebras naturally occur in the thermodynamic limit of nonrelativistic quantum statistical mechanics (see Hugenholtz 1967) and that, as noted by Emch, "Type III factors occur repeatedly in general local theories and even more generally when the system considered possesses a symmetry group with certain asymptotic properties" (1972, 141). But firm support for the suspicion can be provided only by showing that it is fulfilled in a number of genuinely different models. This is a project for another paper. (In particular, it would be interesting to know whether Type III algebras arise in Galilean QFT; for examples of Galilean QFT see Lévy-Leblond 1967.)

Whether or not the troubles for the modal interpretation are traceable to relativistic invariance, they are real troubles. The troubles arise not only for applications in QFT but also for physically important applications in quantum statistical mechanics. For example, as mentioned above, it is assumed in quantum statistical mechanics that an equilibrium state has the KMS property, and it is common to assume also that the equilibrium state is asymptotically Abelian; such a state is ergodic. (See Kastler and Robinson 1966, Ruelle 1966, and Emch 1972.)

We doubt that the troubles discussed here are fatal, since the modal interpretation has shown itself to be quite resilient. But they do need to be addressed if the modal interpretation is to be regarded as a live option.

Appendix A: Some Notions Basic to the Algebraic Approach

An algebra over the field $\mathbb{C}$ of complex numbers is a set $\mathcal{A}$ of elements $(A, B, \ldots)$ on which are defined operations (binary addition and binary multiplication, not necessarily commutative) under which it is closed. A *-algebra is in addition closed with respect to an involution * satisfying $(A^*)^* = A$, $(A + B)^* = A^* + B^*$, $(\lambda A)^* = \bar{\lambda} A^*$, and $(AB)^* = B^* A^*$ for all $A, B \in \mathcal{A}$ and all complex λ (where the overbar denotes the complex conjugate). A C^*-algebra is a *-algebra equipped with a norm, satisfying $\|A^*A\| = \|A\|^2$ and $\|AB\| \leq \|A\|\|B\|$ for all $A, B \in A$, and is complete in the topology induced by that norm. It will be assumed that all of the algebras discussed herein contain the identity operator I.

One example of a C^*-algebra familiar from ordinary QM is the set $\mathcal{B}(\mathcal{H})$ of bounded operators on a separable Hilbert space $\mathcal{H}$. The adjoint operation on $\mathcal{B}(\mathcal{H})$ provides the involution, *, and the Hilbert space operator norm provides the C^*-algebraic norm. In familiar formulations of ordinary QM, observables are identified with self-adjoint elements of such an algebra, i.e., elements A such that $A^* = A$. Notice that $\mathcal{B}(\mathcal{H})$ has structure in excess of its C^*-algebraic structure—it has for instance an

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inner product and a vector norm. While every $\mathcal{B}(\mathcal{H})$ is a C^*-algebra, the converse is not the case: a C^*-algebra need not have the full structure enjoyed by a set of bounded operators on a separable Hilbert space.

Abstract C^*-algebras admit Hilbert space representations. A *representation* of a C^*-algebra $\mathcal{A}$ is a structure-preserving map or *-morphism $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ from the abstract algebra into the concrete algebra $\mathcal{B}(\mathcal{H})$ of bounded linear operators on a Hilbert space $\mathcal{H}$. This representation map π may be into rather than onto $\mathcal{B}(\mathcal{H})$—that is, $\pi(\mathcal{A})$ may be a proper subset of $\mathcal{B}(\mathcal{H})$. A representation is *faithful* if $\pi(A) = 0$ implies $A = 0$. A representation $(\pi, \mathcal{H})$ of $\mathcal{A}$ is *irreducible* just in case the only closed subspaces of $\mathcal{H}$ that are invariant under $\pi(\mathcal{A})$ are the null space and $\mathcal{H}$. Two representations $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ of a C^*-algebra $\mathcal{A}$ are *unitarily equivalent* just in case there is an one-to-one invertible norm-preserving linear map $U : \mathcal{H}_1 \to \mathcal{H}_2$ such that $U\pi_1(A)U^{-1} = \pi_2(A)$ for all $A \in \mathcal{A}$.

A *von Neumann algebra* is a *-algebra of bounded operators that is strong operator-closed in its action on some Hilbert space. The algebra $\mathcal{B}(\mathcal{H})$ of all bounded operators on a separable Hilbert space $\mathcal{H}$ is a von Neumann algebra. But not every von Neumann algebra is isomorphic to some $\mathcal{B}(\mathcal{H})$. According to von Neumann's celebrated double commutant theorem, the strong and weak closures of a *-algebra $\mathcal{A}$ of operators acting on a Hilbert space coincide—and coincide as well with $\mathcal{A}$'s *double commutant*. (Given an algebra $\mathcal{A}$ of bounded operators on a Hilbert space $\mathcal{H}$, its *commutant* $\mathcal{A}'$ is the set of all bounded operators on $\mathcal{H}$ that commute with every element of $\mathcal{A}$. $\mathcal{A}$'s *double commutant* $\mathcal{A}''$ is then $\mathcal{A}'$'s commutant. A *factor algebra* is one whose intersection with its commutant is trivial.) Thus $\pi(\mathcal{A})''$ is a von Neumann algebra, known as the von Neumann algebra affiliated with the representation π. Closed in a weaker topology than the norm topology in which the C^*-algebra $\pi(\mathcal{A})$ is closed (Bratteli and Robinson 1997, Proposition 2.3.1), $\pi(\mathcal{A})''$ can contain bounded Hilbert space operators that do not make it into $\pi(\mathcal{A})$. Projection operators provide an important example. A C^*-algebra of bounded Hilbert space operators need only contain the projections 0 and I (the additive and multiplicative identities, respectively, for the algebraic operations), whereas von Neumann algebras are rich with projections. Where π_1 and π_2 are representations of the same C* algebra $\mathcal{A}$, the affiliated von Neumann algebras $\pi_1(\mathcal{A})''$ and $\pi_2(\mathcal{A})''$ need not be isomorphic. π_1 and π_2 are *quasi-equivalent* if and only if there is a *-isomorphism α from $\pi_1(\mathcal{A})''$ to $\pi_2(\mathcal{A})''$ such that $\alpha[\pi_1(A)] = \pi_2(A)$ for all $A \in \mathcal{A}$.

A state ω on a C*-algebra $\mathcal{A}$ is a positive linear map $\omega : \mathcal{A} \to \mathbb{C}$ such that $\omega(I) = 1$. A state is said to be *pure* just in case it cannot be written as a nontrivial convex combination of distinct states; otherwise it is said to be *mixed*. For a representation $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ of $\mathcal{A}$, a state ω is said to be π-*normal* just in case there is a density matrix ρ on $\mathcal{H}$ such that

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$\omega(A) = \mathrm{Tr}(\rho\pi(A))$ for all $A \in \mathcal{A}$. The representations π_1 and π_2 are quasi-equivalent if and only if the π_1-normal states and the π_2-normal states coincide. A fundamental link between algebraic states and Hilbert space representations is forged by the Gel'fand-Naimark-Segal (GNS) theorem:

> **Theorem** (GNS). For any state ω on a C^*-algebra $\mathcal{A}$ there is a representation $(\pi_\omega, \mathcal{H}_\omega)$ of $\mathcal{A}$ and a cyclic vector $|\Omega_\omega\rangle \in \mathcal{H}_\omega$ (i.e. $\{\pi_\omega(\mathcal{A})|\Omega_\omega\rangle\}$ is dense in $\mathcal{H}_\omega$) such that $\omega(A) = \langle\Omega_\omega|\pi_\omega(A)|\Omega_\omega\rangle$ for all $A \in \mathcal{A}$. This GNS representation is, moreover, the unique, up to unitary equivalence, cyclic representation $(\pi_\omega, \mathcal{H}_\omega, |\Omega_\omega\rangle)$ of $\mathcal{A}$ such that $\omega(A) = \langle\Omega_\omega|\pi_\omega(A)|\Omega_\omega\rangle$ for all $A \in \mathcal{A}$.

A state is pure if and only if its GNS representation is irreducible. For the GNS representations π_{ω_1} and π_{ω_2} corresponding respectively to pure states ω_1 and ω_2, unitary equivalence and quasi-equivalence coincide.

Appendix B: Ergodic States Are 'Extremal'

> **Lemma 1.** Let ω be a faithful normal state on a von Neumann algebra $\mathcal{R}$. If the invariants of the modular group σ_t^ω are $\{\mathbb{C}I\}$, then ω is the unique normal σ_t^ω-invariant state on $\mathcal{R}$.[11]
>
> **Proof.** By the Pedersen-Takesaki form of the Radon-Nikodým theorem (Pedersen and Takesaki 1973), any normal σ_t^ω-invariant semifinite weight φ is of the form $\varphi(\cdot) = \omega(T^{1/2} \cdot T^{1/2})$ for some positive self-adjoint $T \in \mathcal{R}$ affiliated with $\mathfrak{S}_\omega(\mathcal{R})$.[12] Therefore, if $\mathfrak{S}_\omega(\mathcal{R}) = \{\mathbb{C}I\}$, T must be a multiple of I and thus any normal σ_t^ω-invariant semifinite weight φ on $\mathcal{R}$ is proportional to ω. Let ψ be a normal σ_t^ω-invariant state on $\mathcal{R}$. Since its restriction to $\mathcal{R}_+$ defines a normal σ_t^ω-invariant semifinite weight, ψ is proportional to ω on $\mathcal{R}_+$. But two normal states proportional on $\mathcal{R}_+$ are proportional on $\mathcal{R}$, and since both are normalized the proportionality factor is unity.

An alternative proof of Lemma 1 can be obtained from Theorem 11

11. A similar result is stated in Driessler 1975. No proof is given presumably because it is so obvious. We give the proof for sake of completeness.

12. A *weight* φ on $\mathcal{R}$ is a mapping from the positive elements $\mathcal{R}_+$ of $\mathcal{R}$ to $[0, \infty]$ such that $\varphi(A + B) = \varphi(A) + \varphi(B)$ and $\varphi(\alpha A) = \alpha\varphi(A)$ for $A, B \in \mathcal{R}_+$ and $\alpha \in \mathbb{R}_+$. A weight φ is *normal* if and only if there are bounded normal positive linear functionals ω_i on $\mathcal{R}$ such that $\varphi(A) = \sup_i \omega_i(A)$ for all $A \in \mathcal{R}_+$. A weight φ is *semifinite* if and only if the span of $\{A \in \mathcal{R}_+ : \varphi(A) < \infty\}$ is weakly dense in $\mathcal{R}$.

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of Emch (1984, 440), which can also be used to prove the converse of Lemma 1: If ω is a faithful normal state on a von Neumann algebra $\mathcal{R}$ and if ω is the unique normal σ_t^ω-invariant state on $\mathcal{R}$, then the invariants of the modular group σ_t^ω are $\{\mathbb{C}I\}$.

> **Lemma 2** (Bratteli and Robinson, 1997, Proposition 5.3.29). If ω is a faithful normal state on a factor von Neumann algebra $\mathcal{R}$, then ω is the unique σ_t^ω-KMS state on $\mathcal{R}$.

A σ_t^ω-KMS state is σ_t^ω-invariant, but the converse is not necessarily true. And if ω is a faithful normal state on a factor von Neumann algebra $\mathcal{R}$, it can be the case that there are normal σ_t^ω-invariant states distinct from ω. This is just what is ruled out if ω is an ergodic state. So if the faithful normal state ω is ergodic, then by Lemma 2, ω is the unique σ_t^ω-KMS state on $\mathcal{R}$ (since $\mathcal{R} \cap \mathcal{R}' \subseteq \mathfrak{C}_\omega(\mathcal{R})$); and by Lemma 1 ω is the unique σ_t^ω-invariant state.

Scare quotes have been put around the word extremal in the title of this Appendix to avoid confusion with another commonly used sense of the word. Let $\mathcal{R}$ be a von Neumann algebra and let $\mathcal{P}$ be some convex set of states on $\mathcal{R}$. Call a $\mathcal{P}$-state *extremal* if it cannot be decomposed into a nontrivial convex combination of $\mathcal{P}$-states. Of course, if ω is the unique $\mathcal{P}$-state, then it is an extremal $\mathcal{P}$-state. What we have seen is that if ω is a faithful normal state that is ergodic with respect to the automorphism group σ_t^ω, then ω is the unique normal σ_t^ω-invariant state. Thus, it is an extremal normal σ_t^ω-invariant state. It is also worth mentioning that 'ergodic' is sometimes used in a more general sense than the one used here; namely, a state ω on $\mathcal{R}$ is said to be ($\mathcal{G}$-)ergodic with respect to an automorphism group $\mathcal{G}$ just in case ω is an extremal $\mathcal{G}$-invariant state. Under some conditions (e.g. asymptotic Abelianness) it can be proved that such a state is the unique normal $\mathcal{G}$-invariant state. Then when $\mathcal{G}$ is the modular group associated with ω, such a state will be ergodic in the sense used here.

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