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Cross-field plasma acceleration and potential formation induced by electromagnetic waves in a magnetized plasma

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Abstract. A single-particle theory is developed to investigate particle acceleration along and across a magnetic field and the generation of an electric field transverse to the magnetic field induced by electromagnetic waves in a magnetized plasma. The almost perpendicularly propagating waves accelerate particles via their Landau and cyclotron damping, and the ratio of parallel and perpendicular drift velocities $v_{s\parallel}/v_d$ can be proved to be proportional to $k_\parallel/k_\perp$. Simultaneously, an intense cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_d/c$ is generated via the dynamo effect owing to perpendicular particle acceleration to satisfy the generalized Ohm's law. This means that this cross-field particle drift in a collisionless plasma is identical to $\mathbf{E} \times \mathbf{B}$ drift. It is verified that the transport equations obtained are exactly equivalent to those derived from the θ-dependent quasilinear velocity-space diffusion equation obtained from the Vlasov–Maxwell equations.

1. Introduction

It is well known that the transport of energy and particles across the magnetic flux surface in controlled nuclear fusion experiments is anomalously large. This transport is thought to be driven by electrostatic turbulence, and much theoretical works to understand fluctuation-induced transport has been done in recent years. The recent experimental and theoretical research on cross-field transport in sawtooth crashes or edge plasmas in tokamaks has shown its potential importance for understanding of fluctuation-induced anomalous transport (Helander et al. 1996; Schats et al. 1997; Wesson et al. 1997). Furthermore, the importance of the role of a radial electric field accompanied with anomalous transport in tokamak plasmas has been pointed out in the context of $\mathbf{E}_r \times \mathbf{B}$ velocity shear stabilization of turbulence (Antoni et al. 1997). In particular, perpendicular ion acceleration by localized high-frequency waves is an important aspect of both tokamak plasma experiments (Nemoto et al. 1991; Evance et al. 1992) and space plasma experiments, where the energy of accelerated ions is much larger than the ambient thermal energy (Kintner et al. 1992; Kintner 1992).

In recent years, the present author has verified theoretically, on the basis of quasilinear transport equations, that relativistic and non-relativistic particle acceleration along and across a magnetic field and the electric field transverse to the

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magnetic field can be induced simultaneously by Landau and cyclotron damping of almost perpendicularly propagating electrostatic waves in a relativistic and non-relativistic magnetized plasma (Sugaya 1997, 1998, 2000). The transport equations are provided by the θ-dependent quasilinear velocity-space or momentum-space diffusion equation derived from the relativistic or non-relativistic Vlasov–Maxwell equations (Sugaya 1991a, b, 1994a, b, 1996; Sugaya et al. 1996). Moreover, the transport equations can also be obtained from a single-particle theory, with which it is easy to understand the physical mechanism of particle acceleration and the generation of cross-field electric field induced by electrostatic waves (Sugaya 1999a). This cross-field transport is capable of explaining the fluctuation-induced anomalous transport and the electric potential formation occurring in tokamaks and the perpendicular ion acceleration in tokamaks and space plasmas. In addition, this process explains well the experimental observation of electron transport in an electron-beam–plasma system (Sugaya et al. 1995; Maehara et al. 1996; Sugaya et al. 1998).

In the present work, a single-particle theory that makes it possible to understand easily and straightforwardly the physical mechanism is developed for the investigation of plasma acceleration along and across a magnetic field and the generation of an electric field transverse to the magnetic field induced by electromagnetic waves. It is verified theoretically on the basis of the kinetic equation for a cold or warm particle beam that plasma particles can be accelerated in the $\mathbf{k}$ direction by electromagnetic waves propagating almost perpendicularly in a homogeneous magnetized plasma immersed in a uniform cross-field electric field. The same approach is used as in previous work on particle acceleration by electrostatic waves (Sugaya 1999a) and electron-beam acceleration by Compton scattering of extraordinary waves (Sugaya 1999b). Assuming that the generalized Ohm's law in a collisionless plasma $\mathbf{E}_0 + \mathbf{v}_d \times \mathbf{B}_0/c = 0$ holds simultaneously, it is shown that an intense cross-field electric field $\mathbf{E}_0$ can be generated by the dynamo effect of the perpendicular particle drift with the velocity $\mathbf{v}_d$. Parallel and perpendicular particle acceleration results from Landau or cyclotron damping of electromagnetic waves interacting quasilinearly with particles. In addition, it is predicted that the same transport equations can be also derived from the θ-dependent quasilinear velocity-space diffusion equation obtained based on the perturbation theory of the Vlasov–Maxwell equations.

In a magnetized plasma immersed in an electric field, the equation called the generalized Ohm's law, $\mathbf{E}_0 + \mathbf{v}_d \times \mathbf{B}_0/c = \eta\mathbf{J}$, holds, where $\mathbf{J}$ is the electric current density and η is the electrical resistivity. This equation means that the average ion velocity is determined by $\mathbf{E}_0$, $\mathbf{B}_0$ and $\eta\mathbf{J}$. Conversely, it can be also stated that the electric field $\mathbf{E}_0$ is enhanced or suppressed when a mechanism for acceleration or deceleration of plasma particles exists. In other words, this equation expresses the electric field $\mathbf{E}_0$ caused by the dynamo effect of the cross-field particle drift $\mathbf{v}_d$. Consequently, we find that the cross-field electric field $\mathbf{E}_0$ can be generated by the particle acceleration due to electromagnetic waves. In the present work, this phenomenon is investigated for a collisionless plasma ($\eta = 0$).

In Sec. 2, a single-particle theory for a cold particle beam is presented. In Sec. 3, a single-particle theory for a warm particle beam is developed. In Sec. 4, the θ-dependent quasilinear velocity-space diffusion equation is described. Finally, a summary is given in Sec. 5.

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2. Single-particle theory for a cold particle beam

A single-particle theory is developed for investigating and understanding fluctuation-induced particle acceleration along and across a magnetic field and the generation of an electric field transverse to the magnetic field. First, we treat a cold ion or electron beam influenced by electromagnetic waves propagating almost perpendicularly in a homogeneous magnetized plasma immersed in a uniform electric field. The kinetic equation for a cold particle beam moving obliquely to the magnetic field with velocity $\mathbf{v}$ becomes

$$m_s \frac{d\mathbf{v}}{dt} = e_s \left(\mathbf{E}_0 + \sum_{\mathbf{k}} \mathbf{E}_{\mathbf{k}} \right) + \frac{e_s}{c} \mathbf{v} \times \left(\mathbf{B}_0 + \sum_{\mathbf{k}} \mathbf{B}_{\mathbf{k}} \right), \tag{1}$$

$$\mathbf{k} \times \mathbf{E}_{\mathbf{k}} = \frac{\omega_{\mathbf{k}}}{c} \mathbf{B}_{\mathbf{k}}. \tag{2}$$

In addition, it is assumed that the following generalized Ohm's law for a collisionless plasma is satisfied simultaneously:

$$\mathbf{E}_0 + \frac{1}{c} \mathbf{v}_\perp \times \mathbf{B}_0 = 0. \tag{3}$$

Here

$$\mathbf{v} = \mathbf{v}_\perp + \mathbf{v}_\parallel + \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}},$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_\perp t + \mathbf{v}_\parallel t + \sum_{\mathbf{k}} \mathbf{r}_{\mathbf{k}},$$

$$\mathbf{E}_{\mathbf{k}}, \mathbf{B}_{\mathbf{k}}, \mathbf{v}_{\mathbf{k}}, \mathbf{r}_{\mathbf{k}} \propto \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega_{\mathbf{k}} t)],$$

$\mathbf{v}_\perp = c\mathbf{E}_0 \times \mathbf{B}_0 / B_0^2 = (v_\perp, 0, 0)$, $\mathbf{v}_\parallel = (0, 0, v_\parallel)$, $E_0 = (v_\perp / c) B_0$, and $\mathbf{k} = (k_\perp, 0, k_\parallel)$ is in the (x, z) plane. The subscript s designates the species. The Larmor radius of a particle beam is assumed to be zero; that is, a cold particle beam is considered. The background uniform stationary electric and magnetic fields $\mathbf{E}_0 = (0, E_0, 0)$ and $\mathbf{B}_0 = (0, 0, B_0)$ are in the y and z directions respectively. $\mathbf{E}_{\mathbf{k}}$ and $\mathbf{B}_{\mathbf{k}}$ are the electric and magnetic fields of electromagnetic waves respectively, and $\mathbf{v}_{\mathbf{k}}$ and $\mathbf{r}_{\mathbf{k}}$ are the oscillating terms of a particle beam. It is noted that $\mathbf{v}_\perp$ is identical to the $\mathbf{E} \times \mathbf{B}$ drift velocity. The rest of the notation is standard.

The kinetic equation at first order is given by

$$m_s \frac{d\mathbf{v}_{\mathbf{k}}}{dt} = e_s \mathbf{E}_{\mathbf{k}} + \frac{e_s}{c} (\mathbf{v}_\perp + \mathbf{v}_\parallel) \times \mathbf{B}_{\mathbf{k}} + \frac{e_s}{c} \mathbf{v}_{\mathbf{k}} \times \mathbf{B}_0. \tag{4}$$

The solutions of this equation are

$$v_{x\mathbf{k}} = i \frac{e_s \tilde{\omega}_{\mathbf{k}}}{m_s \omega_{\mathbf{k}} (\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2)} [(\omega_{\mathbf{k}} - k_\parallel v_\parallel) E_{\mathbf{k}}^x + i \sigma_s \omega_{cs} E_{\mathbf{k}}^y + k_\perp v_\parallel E_{\mathbf{k}}^z], \tag{5a}$$

$$v_{y\mathbf{k}} = -i \frac{|e_s| \omega_{cs}}{m_s \omega_{\mathbf{k}} (\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2)} [(\omega_{\mathbf{k}} - k_\parallel v_\parallel) E_{\mathbf{k}}^x + i \sigma_s \omega_{cs} E_{\mathbf{k}}^y + k_\perp v_\parallel E_{\mathbf{k}}^z] + i \frac{e_s}{m_s \omega_{\mathbf{k}}} E_{\mathbf{k}}^y, \tag{5b}$$

$$v_{z\mathbf{k}} = i \frac{e_s}{m_s \omega_{\mathbf{k}} \tilde{\omega}_{\mathbf{k}}} [k_\parallel v_\perp E_{\mathbf{k}}^x + (\omega_{\mathbf{k}} - k_\perp v_\perp) E_{\mathbf{k}}^z], \tag{5c}$$

where $\omega_{cs} = |e_s| B_0 / m_s c$ is the cyclotron frequency of species s, $\sigma_s = e_s / |e_s|$, and $\tilde{\omega}_{\mathbf{k}} = \omega_{\mathbf{k}} - k_\perp v_\perp - k_\parallel v_\parallel$ is the wave frequency, which is Doppler-shifted by both

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parallel and perpendicular drifts of the particle beam. We can also give the kinetic equation at second order:

$$m_s \frac{d\mathbf{v}^{(2)}}{dt} = e_s \mathbf{E}_{\mathbf{k}}^{(2)} + \sum_{\mathbf{k}} \left[e_s (\mathbf{r}_{\mathbf{k}} \cdot \boldsymbol{\nabla}) \mathbf{E}_{\mathbf{k}}^* + \frac{e_s}{c} \mathbf{v}_{\mathbf{k}} \times \mathbf{B}_{\mathbf{k}}^* + \frac{e_s}{c} (\mathbf{v}_\perp + \mathbf{v}_\parallel) \times (\mathbf{r}_{\mathbf{k}} \cdot \boldsymbol{\nabla}) \mathbf{B}_{\mathbf{k}}^* \right] + \frac{e_s}{c} \mathbf{v}^{(2)} \times \mathbf{B}_0. \quad (6)$$

Here, the asterisk * indicates the complex conjugate. Then the solutions of (6) are obtained as

$$m_s \frac{d\mathbf{v}^{(2)}}{dt} = \sum_{\mathbf{k}} i \frac{e_s^2 \mathbf{k}}{m_s \omega_{\mathbf{k}}^2} \mathbf{E}_{\mathbf{k}}^* \cdot \boldsymbol{\chi}_{\mathbf{k}} \cdot \mathbf{E}_{\mathbf{k}}, \quad (7)$$

$$\mathbf{E}_0^{(2)} + \frac{1}{c} \mathbf{v}^{(2)} \times \mathbf{B}_0 = 0, \quad (8)$$

where the components of the tensor $\boldsymbol{\chi}_{\mathbf{k}}$ are given by

$$\chi_{\mathbf{k}}^{xx} = \frac{(\omega_{\mathbf{k}} - k_\parallel v_\parallel)^2}{\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2} + \frac{k_\parallel^2 v_\perp^2}{\tilde{\omega}_{\mathbf{k}}^2}, \quad (9a)$$

$$\chi_{\mathbf{k}}^{yy} = 1 + \frac{\omega_{cs}^2}{\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2}, \quad (9b)$$

$$\chi_{\mathbf{k}}^{zz} = \frac{k_\perp^2 v_\parallel^2}{\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2} + \frac{(\omega_{\mathbf{k}} - k_\perp v_\perp)^2}{\tilde{\omega}_{\mathbf{k}}^2}, \quad (9c)$$

$$\chi_{\mathbf{k}}^{xy} = -\chi_{\mathbf{k}}^{yx} = i \frac{\sigma_s \omega_{cs} (\omega_{\mathbf{k}} - k_\parallel v_\parallel)}{\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2}, \quad (9d)$$

$$\chi_{\mathbf{k}}^{yz} = -\chi_{\mathbf{k}}^{zy} = -i \frac{\sigma_s \omega_{cs} k_\perp v_\parallel}{\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2}, \quad (9e)$$

$$\chi_{\mathbf{k}}^{xz} = \chi_{\mathbf{k}}^{zx} = \frac{k_\perp v_\parallel (\omega_{\mathbf{k}} - k_\parallel v_\parallel)}{\tilde{\omega}_{\mathbf{k}}^2 - \omega_{cs}^2} + \frac{k_\parallel v_\perp (\omega_{\mathbf{k}} - k_\perp v_\perp)}{\tilde{\omega}_{\mathbf{k}}^2}. \quad (9f)$$

Equation (7) shows the temporal evolution of the second-order drift velocity of the particle beam, and it demonstrates that acceleration or deceleration of the particle beam occurs in the $\mathbf{k}$ direction. Equation (8) shows the second-order cross-field electric field caused by the change in the cross-field drift velocity ($E_0^{(2)} = (v_\perp^{(2)}/c)B_0$); that is, $\mathbf{E}_0^{(2)}$ is created by the dynamo effect of $\mathbf{v}_\perp^{(2)}$. In addition, the second-order temporal evolution of the kinetic energy is given by (5) and (7) as follows:

$$\tfrac{1}{2} m_s \frac{dv^2}{dt} = m_s \left[v_\perp \frac{dv_\perp^{(2)}}{dt} + v_\parallel \frac{dv_\parallel^{(2)}}{dt} + \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \cdot \frac{d\mathbf{v}_{\mathbf{k}}^*}{dt} \right] = \sum_{\mathbf{k}} i \frac{e_s^2}{m_s \omega_{\mathbf{k}}} \mathbf{E}_{\mathbf{k}}^* \cdot \boldsymbol{\chi}_{\mathbf{k}} \cdot \mathbf{E}_{\mathbf{k}}. \quad (10)$$

Equations (7) and (10) predict that the anti-Hermitian parts of $\boldsymbol{\chi}_{\mathbf{k}}$ resulting from the linear singularities of $\tilde{\omega}_{\mathbf{k}} = 0$ and $\tilde{\omega}_{\mathbf{k}} \pm \omega_{cs} = 0$ contribute to the acceleration and heating of the particle beam.

On integrating (7) and (10) multiplied by the velocity distribution function of the particle beam in velocity space, we can derive the transport equations indicating

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the temporal evolution of the energy and momentum densities of the particle beams of species s, and they become

$$\frac{dU_s}{dt} = -\sum_{\mathbf{k}} 2\gamma_{\mathbf{k}}^{(s)} U_{\mathbf{k}}, \tag{11}$$

$$\frac{d\mathbf{P}_s}{dt} = -\sum_{\mathbf{k}} \frac{2\mathbf{k}}{\omega_{\mathbf{k}}} \gamma_{\mathbf{k}}^{(s)} U_{\mathbf{k}}. \tag{12}$$

Here

$$U_s = \int d^3\mathbf{v}\, w_s g_s, \qquad \mathbf{P}_s = \int d^3\mathbf{v}\, \mathbf{p}_s g_s = (P_{s\perp}, 0, P_{s\parallel})$$

(with $w_s = \frac{1}{2} n_s m_s v^2$, $\mathbf{p}_s = n_s m_s \mathbf{v}$, $P_{s\perp} = n_s m_s v_d$, $P_{s\parallel} = n_s m_s v_{s\parallel}$, and $v_d = cE_0/B_0$) are respectively the energy and momentum densities of the particle beams of species s,

$$U_{\mathbf{k}} = \frac{1}{8\pi} \mathbf{E}_{\mathbf{k}}^* \cdot \left\{ \frac{\partial}{\partial \omega_{\mathbf{k}}} [(\boldsymbol{\varepsilon}_{\mathbf{k}}' - \mathbf{N}_{\mathbf{k}})\omega_{\mathbf{k}}] \right\} \cdot \mathbf{E}_{\mathbf{k}}$$

is the wave energy density, $\mathbf{k}U_{\mathbf{k}}/\omega_{\mathbf{k}}$ is the wave momentum density,

$$\gamma_{\mathbf{k}}^{(s)} = -\frac{\omega_{\mathbf{k}}}{8\pi U_{\mathbf{k}}} \mathbf{E}_{\mathbf{k}}^* \cdot \boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime\prime(s)} \cdot \mathbf{E}_{\mathbf{k}}$$

is the linear Landau and cyclotron damping rate of the electromagnetic waves ascribed to the particle beams of species s, $g_s = g_s(v_\perp, v_\parallel, t)$ is the velocity distribution function of the particle beams of species s, and the tensor $\mathbf{N}_{\mathbf{k}}$ is defined by

$$N_{\mathbf{k}}^{jl} = \frac{c^2}{\omega_{\mathbf{k}}^2}(k^2\delta_{jl} - k_j k_l) \qquad (j, l = x, y, z, \quad k_x = k_\perp,\ k_y = 0,\ k_z = k_\parallel).$$

The dielectric tensor $\boldsymbol{\varepsilon}_{\mathbf{k}}$ is expressed as

$$\boldsymbol{\varepsilon}_{\mathbf{k}} = \mathbf{I} + \sum_s \boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)} = \boldsymbol{\varepsilon}_{\mathbf{k}}' + i\boldsymbol{\varepsilon}_{\mathbf{k}}'',$$

$$\boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)} = -\frac{\omega_{ps}^2}{\omega_{\mathbf{k}}^2} \int d^3\mathbf{v}\, \boldsymbol{\chi}_{\mathbf{k}} g_s = \boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime(s)} + i\boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime\prime(s)}. \tag{13}$$

Here, $I_{jl} = \delta_{jl}$, $\omega_{ps}^2 = 4\pi n_s e_s^2/m_s$, $\boldsymbol{\varepsilon}_{\mathbf{k}}'$ and $\boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime(s)}$ are the Hermitian parts of $\boldsymbol{\varepsilon}_{\mathbf{k}}$ and $\boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)}$ respectively, and $\boldsymbol{\varepsilon}_{\mathbf{k}}''$ and $\boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime\prime(s)}$ are the anti-Hermitian parts of $\boldsymbol{\varepsilon}_{\mathbf{k}}$ and $\boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)}$ respectively. The transport equations (11) and (12) predict clearly that the electromagnetic waves generate strong particle acceleration along and across the magnetic field. The relation $P_{s\parallel}/P_{s\perp} = k_\parallel/k_\perp$ ($\mathbf{P}_s \parallel \mathbf{k}$) found from (12) with $\mathbf{P}_s(0) = 0$ shows that small parallel and large perpendicular particle accelerations appears simultaneously. At the same time, we can demonstrate that the cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_d/c$ is produced by the dynamo effect of the cross-field particle drift arising from acceleration due to electromagnetic waves. These results are the same as those for electrostatic waves that were obtained previously from the Vlasov–Maxwell equations and single-particle theory (Sugaya 1997, 1998, 1999a, 2000).

In the absence of nonlinear wave–wave and wave–particle interaction, the electromagnetic waves are governed by the following kinetic wave equation:

$$\frac{dU_{\mathbf{k}}}{dt} = 2\gamma_{\mathbf{k}} U_{\mathbf{k}}, \tag{14}$$

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where

$$\gamma_{\mathbf{k}} = \sum_s \gamma_{\mathbf{k}}^{(s)} = -\frac{\omega_{\mathbf{k}}}{8\pi U_{\mathbf{k}}} \mathbf{E}_{\mathbf{k}}^* \cdot \boldsymbol{\varepsilon}_{\mathbf{k}}'' \cdot \mathbf{E}_{\mathbf{k}}.$$

This equation, (11), and (12) yield the conservation laws for the total energy and momentum densities of waves and particle beams, and they are represented as

$$\frac{d}{dt}\left(\sum_{\mathbf{k}} U_{\mathbf{k}} + \sum_s U_s\right) = 0, \tag{15}$$

$$\frac{d}{dt}\left(\sum_{\mathbf{k}} \frac{\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} + \sum_s \mathbf{P}_s\right) = 0. \tag{16}$$

We can understand easily the physical mechanism of particle acceleration and generation of the electric field. Namely, the acceleration and deceleration of the particle beam result from Landau and cyclotron damping of electromagnetic waves due to quasilinear interaction with the particle beams. The linear singularities of $\tilde{\omega}_{\mathbf{k}} - r\omega_{cs} = 0$ ($r = 0$ and ± 1) should contribute to those processes. Thus we employ the following relation:

$$\mathrm{Im}\left\{\frac{1}{\tilde{\omega}_{\mathbf{k}} - r\omega_{cs}}\right\} = -\pi\delta(k_\parallel v_\parallel + k_\perp v_\perp - \omega_{\mathbf{k}} + r\omega_{cs}). \tag{17}$$

Assuming that the velocity distribution function of the particle beam is given by

$$g_s = \frac{1}{\pi^{1/2} v_{ts}} \delta(v_\perp - v_d) \exp\left[-\frac{(v_\parallel - v_{s\parallel})^2}{v_{ts}^2}\right], \tag{18}$$

the linear damping rate is found to be expressed by

$$\begin{aligned}\gamma_{\mathbf{k}}^{(s)} = &-\frac{\omega_{ps}^2 \zeta_{0\mathbf{k}} \exp(-\zeta_{0\mathbf{k}}^2)}{4\pi^{1/2} k_\parallel |k_\parallel| v_{ts}^2 \omega_{\mathbf{k}} U_{\mathbf{k}}} |k_\parallel v_d E_{\mathbf{k}}^x + (\omega_{\mathbf{k}} - k_\perp v_d) E_{\mathbf{k}}^z|^2 \\ &- \sum_{r=\pm 1} \frac{r\omega_{cs}^2 \exp(-\zeta_{r\mathbf{k}}^2)}{16\pi^{1/2} \omega_{cs} \omega_{\mathbf{k}} |k_\parallel| v_{ts} U_{\mathbf{k}}} |(\omega_{\mathbf{k}} - k_\parallel v_{ts} \zeta_{r\mathbf{k}}) E_{\mathbf{k}}^x + i\sigma_s \omega_{cs} E_{\mathbf{k}}^y + k_\perp v_{ts} \zeta_{r\mathbf{k}} E_{\mathbf{k}}^z|^2,\end{aligned} \tag{19}$$

where

$$\zeta_{r\mathbf{k}} = \frac{\omega_{\mathbf{k}} - k_\perp v_d - k_\parallel v_{s\parallel} - r\omega_{cs}}{k_\parallel v_{ts}}.$$

3. Single-particle theory for a warm particle beam

Next we investigate the acceleration and deceleration of a warm particle beam moving obliquely to the magnetic field with a finite Larmor radius. The kinetic equations are also given by (1)–(3), and $\mathbf{v}$ and $\mathbf{r}$ are assumed as follows:

$$\mathbf{v} = \mathbf{v}_d + \mathbf{v}_\parallel + \mathbf{v}_L + \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}},$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_d t + \mathbf{v}_\parallel t + \mathbf{r}_L + \sum_{\mathbf{k}} \mathbf{r}_{\mathbf{k}},$$

$$\mathbf{v}_L = v_\perp(\cos\,\omega_{cs}t, -\sigma_s\,\sin\,\omega_{cs}t, 0),$$

$$\mathbf{r}_L = \frac{v_\perp}{\omega_{cs}}(\sin\,\omega_{cs}t, \sigma_s\,\cos\,\omega_{cs}t, 0),$$

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where, $\mathbf{v}_\perp$ in (3) should be replaced by $\mathbf{v}_d = c\mathbf{E}_0 \times \mathbf{B}_0/B_0^2 = (v_d, 0, 0)$ and $E_0 = (v_d/c)B_0$.

Accordingly the generalized Ohm's law for a collisionless plasma becomes

$$\mathbf{E}_0 + \frac{1}{c}\mathbf{v}_d \times \mathbf{B}_0 = 0, \tag{20}$$

that is, $\mathbf{v}_d$ is identical to the $\mathbf{E} \times \mathbf{B}$ drift velocity.

The kinetic equation at first order becomes

$$m_s \frac{d\mathbf{v}_\mathbf{k}}{dt} = e_s \mathbf{E}_\mathbf{k} + \frac{e_s}{c}(\mathbf{v}_d + \mathbf{v}_\parallel + \mathbf{v}_L) \times \mathbf{B}_\mathbf{k} + \frac{e_s}{c}\mathbf{v}_\mathbf{k} \times \mathbf{B}_0. \tag{21}$$

Setting

$$\mathbf{E}_\mathbf{k} = \tilde{\mathbf{E}}_\mathbf{k} \sum_{r=-\infty}^{\infty} J_r(\mu_\mathbf{k}) \exp[-i(\tilde{\omega}_\mathbf{k} - r\omega_{cs})t], \tag{22}$$

the solutions of (21) can be obtained as follows:

$$v_{x\mathbf{k}} = i\frac{e_s}{m_s\omega_\mathbf{k}} \sum_{r=-\infty}^{\infty} X_{r\mathbf{k}} \exp[-i(\tilde{\omega}_\mathbf{k} - r\omega_{cs})t], \tag{23a}$$

$$v_{yk} = -i\frac{e_s}{m_s\omega_\mathbf{k}} \sum_{r=-\infty}^{\infty} \left[\frac{i\sigma_s\omega_{cs}}{\tilde{\omega}_\mathbf{k} - r\omega_{cs}} X_{r\mathbf{k}} - J_r(\mu_\mathbf{k})\tilde{E}_\mathbf{k}^y\right] \exp[-i(\tilde{\omega}_\mathbf{k} - r\omega_{cs})t], \tag{23b}$$

$$v_{z\mathbf{k}} = i\frac{e_s}{m_s\omega_\mathbf{k}} \sum_{r=-\infty}^{\infty} Z_{r\mathbf{k}} \exp[-i(\tilde{\omega}_\mathbf{k} - r\omega_{cs})t], \tag{23c}$$

where J_r is the Bessel function of the rth order, $\mu_\mathbf{k} = k_\perp v_\perp/\omega_{cs}$, $\tilde{\omega}_\mathbf{k} = \omega_\mathbf{k} - k_\perp v_d - k_\parallel v_\parallel$, and $X_{r\mathbf{k}}$ and $Z_{r\mathbf{k}}$ are given by

$$X_{r\mathbf{k}} = \frac{\tilde{\omega}_\mathbf{k} - r\omega_{cs}}{(\tilde{\omega}_\mathbf{k} - r\omega_{cs})^2 - \omega_{cs}^2} \sum_{j=x,y,z} [b_\mathbf{k}^j J_r(\mu_\mathbf{k})]\tilde{E}_\mathbf{k}^j,$$

$$Z_{r\mathbf{k}} = \frac{1}{\tilde{\omega}_\mathbf{k} - r\omega_{cs}} \sum_{j=x,y,z} [a_{\mathbf{k},r}^j J_r(\mu_\mathbf{k})]\tilde{E}_\mathbf{k}^j, \tag{24}$$

where

$$a_{\mathbf{k},r}^x = \frac{k_\parallel}{k_\perp}(k_\perp v_d + r\omega_{cs}), \qquad a_{\mathbf{k},r}^y = i\sigma_s k_\parallel v_\perp \frac{\partial}{\partial\mu_\mathbf{k}},$$

$$a_{\mathbf{k},r}^z = \omega_\mathbf{k} - k_\perp v_d - r\omega_{cs}, \qquad b_\mathbf{k}^x = \omega_\mathbf{k} - k_\parallel v_\parallel,$$

$$b_\mathbf{k}^y = i\sigma_s\omega_{cs}\frac{\partial}{\partial\mu_\mathbf{k}}\mu_\mathbf{k}, \qquad b_\mathbf{k}^z = k_\perp v_\parallel.$$

The second-order kinetic equation is given by

$$m_s \frac{d\mathbf{v}^{(2)}}{dt} = e_s\mathbf{E}_\mathbf{0}^{(2)} + \sum_\mathbf{k} \Big[e_s(\mathbf{r}_\mathbf{k} \cdot \boldsymbol{\nabla})\mathbf{E}_\mathbf{k}^* + \frac{e_s}{c}\mathbf{v}_\mathbf{k} \times \mathbf{B}_\mathbf{k}^* + \frac{e_s}{c}(\mathbf{v}_d + \mathbf{v}_\parallel + \mathbf{v}_L) \times (\mathbf{r}_\mathbf{k} \cdot \boldsymbol{\nabla})\mathbf{B}_\mathbf{k}^*\Big] + \frac{e_s}{c}\mathbf{v}^{(2)} \times \mathbf{B}_0, \tag{25}$$

and (8) is satisfied simultaneously. Then we can derive the solutions of (25) using

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(23) and (24), and they become

$$m_s \frac{d\mathbf{v}^{(2)}}{dt} = \sum_{\mathbf{k}} i \frac{e_s^2 \mathbf{k}}{m_s \omega_{\mathbf{k}}^2} \mathbf{E}_{\mathbf{k}}^* \cdot \boldsymbol{\chi}_{\mathbf{k}}^{(s)} \cdot \mathbf{E}_{\mathbf{k}}, \tag{26}$$

$$\begin{aligned} \tfrac{1}{2} m_s \frac{dv^2}{dt} &= m_s \left[v_d \left(\frac{dv_d^{(2)}}{dt} + \frac{dv_\perp^{(2)}}{dt} \cos \omega_{cs} t \right) + \left(\frac{dv_d^{(2)}}{dt} \cos \omega_{cs} t + \frac{dv_\perp^{(2)}}{dt} \right) v_\perp \right. \\ &\qquad \left. + v_\parallel \frac{dv_\parallel^{(2)}}{dt} + \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \cdot \frac{d\mathbf{v}_{\mathbf{k}}^*}{dt} \right] \\ &= \sum_{\mathbf{k}} i \frac{e_s^2}{m_s \omega_{\mathbf{k}}} \mathbf{E}_{\mathbf{k}}^* \cdot \boldsymbol{\chi}_{\mathbf{k}}^{(s)} \cdot \mathbf{E}_{\mathbf{k}}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} \boldsymbol{\chi}_{\mathbf{k}}^{(s)} &= \sum_{r=-\infty}^{\infty} \boldsymbol{\chi}_{\mathbf{k},r}^{(s)}, \\ \chi_{\mathbf{k},r}^{(s)jl} &= \eta_j \eta_l + \frac{[a_{\mathbf{k},r}^{*j} J_r(\mu_{\mathbf{k}})][a_{\mathbf{k},r}^{l} J_r(\mu_{\mathbf{k}})]}{(\tilde{\omega}_{\mathbf{k}} - r\omega_{cs})^2} + \frac{[b_{\mathbf{k}}^{*j} J_r(\mu_{\mathbf{k}})][b_{\mathbf{k}}^{l} J_r(\mu_{\mathbf{k}})]}{(\tilde{\omega}_{\mathbf{k}} - r\omega_{cs})^2 - \omega_{cs}^2}, \end{aligned} \tag{28}$$

and $\eta_x = \eta_z = 0$, $\eta_y = 1$. It is easily confirmed that the expression (28) is reduced to (9) by setting $v_\perp = 0$ and $v_d = v_\perp$. Equation (26) shows also that acceleration and deceleration of the particle beam occur in the $\mathbf{k}$ direction. It is also seen from (26) and (27) that the singularities of $\tilde{\omega}_{\mathbf{k}} - r\omega_{cs} = 0$ in $\boldsymbol{\chi}_{\mathbf{k}}^{(s)}$ contribute to the acceleration and heating of the particle beam. Furthermore, it can be stated similarly that $\mathbf{E}_0^{(2)}$ is created by the dynamo effect of $\mathbf{v}_\perp^{(2)}$.

Carrying out the velocity-space integration of (26) and (27) multiplied by the velocity distribution function of the particle beam, we obtain the transport equations given by (11) and (12), with the dielectric tensor represented as

$$\boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)} = -\frac{\omega_{ps}^2}{\omega_{\mathbf{k}}^2} \int d^3\mathbf{v}\, \boldsymbol{\chi}_{\mathbf{k}}^{(s)} g_{s0}. \tag{29}$$

The velocity-space integration in (29) is performed in the cylindrical coordinate system $\mathbf{v} = (v_\perp, \theta, v_\parallel)$, which is displaced by $\mathbf{v}_d$ in the x direction, Accordingly, $g_{s0} = g_{s0}(v_\perp, v_\parallel, t)$, which is symmetric with respect to the magnetic field, appears in the integrand. The velocity distribution function in the stationary coordinates is given by

$$g_s = g_{s0}([(v_x - v_d)^2 + v_y^2]^{1/2}, v_\parallel, t). \tag{30}$$

The dielectric tensor $\boldsymbol{\varepsilon}_{\mathbf{k}}$ of (29) can be rewritten in the alternative form

$$\begin{aligned} \boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)} &= -\frac{\omega_{ps}^2}{\omega_{\mathbf{k}}^2} \left[\mathbf{I} + \sum_{r=-\infty}^{\infty} \int d^3\mathbf{v} \, \frac{\mathbf{T}_s}{k_\parallel v_\parallel + k_\perp v_d - \omega_{\mathbf{k}} + r\omega_{cs}} U_r(\mathbf{k}) g_{s0} \right], \\ T_s^{jl} &= [u_{\mathbf{k}}^{*j}(\mathbf{v}) J_r(\mu_{\mathbf{k}})][u_{\mathbf{k}}^{l}(\mathbf{v}) J_r(\mu_{\mathbf{k}})], \\ U_r(\mathbf{k}) &= k_\parallel \frac{\partial}{\partial v_\parallel} + \frac{r\omega_{cs}}{v_\perp} \frac{\partial}{\partial v_\perp}, \end{aligned} \tag{31}$$

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where

$$u^x_{\mathbf{k}}(\mathbf{v})J_r(\mu_{\mathbf{k}}) = \left(\frac{r\omega_{cs}}{k_\perp} + v_d\right) J_r(\mu_{\mathbf{k}}),$$

$$u^y_{\mathbf{k}}(\mathbf{v}) = i\sigma_s v_\perp \frac{\partial}{\partial \mu_{\mathbf{k}}}, \qquad u^z_{\mathbf{k}}(\mathbf{v}) = v_\parallel.$$

Equations (29) and (31) with $v_d = 0$ are identical to the dielectric tensor obtained previously (Sugaya 1989, 1994a). It is seen easily that the conservation laws (15) and (16) are satisfied similarly. Furthermore, the transport equations imposed by the electrostatic approximation of $\mathbf{k}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}}\cdot\mathbf{k}/k^2 = \varepsilon_{\mathbf{k}}$ can be proved to be reduced to those for electrostatic waves that were derived previously (Sugaya 1997, 1998, 1999a), where $\varepsilon_{\mathbf{k}}$ is the dielectric constant for the electrostatic waves.

Landau and cyclotron damping of the electromagnetic waves arising from the linear singularity of $\tilde{\omega}_{\mathbf{k}} - r\omega_{cs} = 0$ should contribute to the acceleration and deceleration of particle beams. Employing the relation

$$\mathrm{Im}\left\{\frac{1}{\tilde{\omega}_{\mathbf{k}} - r\omega_{cs}}\right\} = -\pi\delta(k_\parallel v_\parallel + k_\perp v_d - \omega_{\mathbf{k}} + r\omega_{cs}), \tag{32}$$

we can obtain the following anti-Hermitian part of the dielectric tensor:

$$\boldsymbol{\varepsilon}^{(s)\prime\prime}_{\mathbf{k}} = -\frac{\pi\omega^2_{ps}}{\omega^2_{\mathbf{k}}} \sum_{r=-\infty}^{\infty} \int d^3\mathbf{v}\, \delta(k_\parallel v_\parallel + k_\perp v_d - \omega_{\mathbf{k}} + r\omega_{cs})\mathbf{T}_s U_r(\mathbf{k}) g_{s0}. \tag{33}$$

Assuming the two-dimensional drifted Maxwellian velocity distribution function

$$g_s = \frac{1}{\pi^{3/2}v^3_{ts}} \exp\left[-\frac{(v_x - v_d)^2 + v_y^2 + (v_z - v_{s\parallel})^2}{v^2_{ts}}\right],$$

$$g_{s0} = \frac{1}{\pi^{3/2}v^3_{ts}} \exp\left[-\frac{v_\perp^2 + (v_\parallel - v_{s\parallel})^2}{v^2_{ts}}\right], \tag{34}$$

$\boldsymbol{\varepsilon}^{\prime\prime(s)}_{\mathbf{k}}$, which gives the linear Landau and cyclotron damping rate, is derived and its components are expressed as

$$\varepsilon^{\prime\prime(s)xx}_{\mathbf{k}} = \frac{2\pi^{1/2}\omega^2_{ps}|k_\parallel|\zeta_{0\mathbf{k}}}{\omega^2_{\mathbf{k}}k_\parallel} \sum_{r=-\infty}^{\infty} \left[\frac{r}{(2\lambda_s)^{1/2}} + \frac{v_d}{v_{ts}}\right]^2 \exp(-\zeta^2_{r\mathbf{k}} - \lambda_s)\, I_r(\lambda_s), \tag{35a}$$

$$\varepsilon^{\prime\prime(s)yy}_{\mathbf{k}} = \frac{\pi^{1/2}\omega^2_{ps}|k_\parallel|\zeta_{0\mathbf{k}}}{\omega^2_{\mathbf{k}}k_\parallel} \sum_{r=-\infty}^{\infty} \frac{1}{\lambda_s} \exp(-\zeta^2_{r\mathbf{k}} - \lambda_s) \times\{r^2 I_r(\lambda_s) + 2\lambda_s^2[I_r(\lambda_s) - I'_r(\lambda_s)]\}, \tag{35b}$$

$$\varepsilon^{\prime\prime(s)zz}_{\mathbf{k}} = \frac{2\pi^{1/2}\omega^2_{ps}|k_\parallel|\zeta_{0\mathbf{k}}}{\omega^2_{\mathbf{k}}k_\parallel} \sum_{r=-\infty}^{\infty} \frac{(\omega_{\mathbf{k}} - k_\perp v_d - r\omega_{cs})^2}{k^2_\parallel v^2_{ts}} \exp(-\zeta^2_{r\mathbf{k}} - \lambda_s)\, I_r(\lambda_s), \tag{35c}$$

$$\varepsilon^{\prime\prime(s)xy}_{\mathbf{k}} = -\varepsilon^{\prime\prime(s)yx}_{\mathbf{k}} = -i\frac{\sigma_s\pi^{1/2}\omega^2_{ps}|k_\parallel|\zeta_{0\mathbf{k}}}{\omega^2_{\mathbf{k}}k_\parallel} \sum_{r=-\infty}^{\infty} \left[r + \frac{(2\lambda_s)^{1/2}v_d}{v_{ts}}\right] \times \exp(-\zeta^2_{r\mathbf{k}} - \lambda_s)\,[I_r(\lambda_s) - I'_r(\lambda_s)] \tag{35d}$$

$$\varepsilon^{\prime\prime(s)yz}_{\mathbf{k}} = -\varepsilon^{\prime\prime(s)zy}_{\mathbf{k}} = i\frac{\sigma_s\pi^{1/2}\omega^2_{ps}|k_\parallel|\zeta_{0\mathbf{k}}}{\omega^2_{\mathbf{k}}k_\parallel} \sum_{r=-\infty}^{\infty} \frac{\omega_{\mathbf{k}} - k_\perp v_d - r\omega_{cs}}{k_\parallel v_{ts}}(2\lambda_s)^{1/2} \times \exp(-\zeta^2_{r\mathbf{k}} - \lambda_s)\,[I_r(\lambda_s) - I'_r(\lambda_s)], \tag{35e}$$

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$$\varepsilon''^{(s)xz}_{\mathbf{k}} = \varepsilon''^{(s)zx}_{\mathbf{k}} = \frac{2\pi^{1/2}\omega_{ps}^2|k_\parallel|\zeta_{0\mathbf{k}}}{\omega_{\mathbf{k}}^2 k_\parallel} \sum_{r=-\infty}^{\infty} \frac{\omega_{\mathbf{k}} - k_\perp v_d - r\omega_{cs}}{k_\parallel v_{ts}} \left[\frac{r}{(2\lambda_s)^{1/2}} + \frac{v_d}{v_{ts}}\right] \times \exp(-\zeta_{r\mathbf{k}}^2 - \lambda_s)\, I_r(\lambda_s), \quad (35f)$$

where I_r is the modified Bessel function of the rth order and $\lambda_s = k_\perp^2 v_{ts}^2/2\omega_{cs}^2$. It is confirmed that they are reduced to the usual forms on setting $v_d = 0$.

Similarly, it is also verified that the electromagnetic waves can accelerate or decelerate strongly warm particle beams along and across the magnetic field, and they simultaneously create a cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_d/c$ via the dynamo effect of the cross-field particle acceleration. The physical mechanism of this phenomenon is understood well in terms of the present single-particle theory. In particular, the physical mechanism of the dynamo effect can be understood easily as follows. The Lorentz force $e_s\mathbf{v}_d \times \mathbf{B}_0/c$ due to the cross-field particle drift or acceleration causes electrical charge separation and an electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_d/c$ that is balanced with the Lorentz force. Considering a magnetized plasma of finite size in the y direction, the physical mechanism of the generation of the cross-field electric field $\mathbf{E}_0$ can be easily understood. Accumulation of electrical charge at both ends of the plasma in the y direction is caused by the Lorentz force $e_s\mathbf{v}_d \times \mathbf{B}_0/c$, and hence a resulting $\mathbf{E}_0$ is created. The charge accumulation continues until the electrical force is balanced with the Lorentz force. This means that the cross-field electric $\mathbf{E}_0$ is produced by the dynamo effect of the cross-field particle drift or acceleration due to electromagnetic waves.

4. θ-dependent quasilinear velocity-space diffusion equation

In this section, we describe the θ-dependent quasilinear velocity-space diffusion equation, which leads to the same transport equations (11) and (12). It can be derived from Vlasov–Maxwell equations in the same manner as in the previous work (Sugaya 1997, 1998, 2000), and is expressed by the following equation:

$$\frac{\partial g_s}{\partial t} = \sum_{k\neq 0} \sum_{j,l} E_{\mathbf{k}}^{*j} E_{\mathbf{k}}^{l} Q_{\mathbf{k}}^{jl} g_{s0}, \quad (36)$$

with the θ-dependent velocity-space diffusion coefficient

$$Q_{\mathbf{k}}^{jl} = P_{AH}\left[\frac{1}{\omega_{\mathbf{k}}^2}\left(\frac{e_s}{m_s}\right)^2 \sum_{n,r=-\infty}^{\infty} e^{i(n-r)\theta}[H_{\mathbf{k},n}^{j}(\mathbf{v})[w_{\mathbf{k}}^{*j}(\mathbf{v})J_n(\mu_{\mathbf{k}})] + K_{n,r}^{j}(\mathbf{v})]Z_{\mathbf{k},r}^{l}(\mathbf{v})\right], \quad (37)$$

$$Z_{\mathbf{k},r}^{l}(\mathbf{v}) = \frac{[w_{\mathbf{k}}^{l}(\mathbf{v})J_r(\mu_{\mathbf{k}})]Y_{\mathbf{k},r}^{l}(\mathbf{v}) + s_l v_d J_r(\mu_{\mathbf{k}})U_r(\mathbf{k})}{k_\parallel v_\parallel + k_\perp v_d - \omega_{\mathbf{k}} + r\omega_{cs}}, \quad (38a)$$

$$Y_{\mathbf{k},r}^{x}(\mathbf{v}) = Y_{\mathbf{k},r}^{y}(\mathbf{v}) = k_\parallel v_\perp \frac{\partial}{\partial v_\parallel} + (\omega_{\mathbf{k}} - k_\perp v_d - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp}, \quad (38b)$$

$$Y_{\mathbf{k},r}^{z}(\mathbf{v}) = (\omega_{\mathbf{k}} - k_\perp v_d - r\omega_{cs})\frac{\partial}{\partial v_\parallel} + r\omega_{cs}\frac{v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp}, \quad (38c)$$

$$H_{\mathbf{k},n}^{x}(\mathbf{v}) = H_{\mathbf{k},n}^{y}(\mathbf{v}) = k_\parallel v_\perp \frac{\partial}{\partial v_\parallel} + \frac{1}{v_\perp}(\omega_{\mathbf{k}} - k_\perp v_d - k_\parallel v_\parallel)\frac{\partial}{\partial v_\perp}v_\perp, \quad (38d)$$

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$$H^z_{\mathbf{k},n}(\mathbf{v}) = Y^z_{\mathbf{k},n}(\mathbf{v}), \tag{38e}$$

$$K^x_{n,r}(\mathbf{v}) = v_d U_n(\mathbf{k}) J_n(\mu_\mathbf{k}) - \frac{1}{v_\perp}(n-r)(\omega_\mathbf{k} - k_\parallel v_\parallel) J'_n(\mu_\mathbf{k}), \tag{38f}$$

$$K^y_{n,r}(\mathbf{v}) = \frac{i}{v_\perp}(n-r)\left[\frac{n}{\mu_\mathbf{k}}(\omega_\mathbf{k} - k_\perp v_d - k_\parallel v_\parallel) - k_\perp v_\perp\right] J_n(\mu_\mathbf{k}), \tag{38g}$$

$$K^z_{n,r}(\mathbf{v}) = -\frac{k_\perp v_\parallel}{v_\perp}(n-r) J'_n(\mu_\mathbf{k}), \tag{38h}$$

$$w^x_\mathbf{k}(\mathbf{v}) J_r(\mu_\mathbf{k}) = \frac{r}{\mu_\mathbf{k}} J_r(\mu_\mathbf{k}), \qquad w^y_\mathbf{k}(\mathbf{v}) = i\sigma_s \frac{\partial}{\partial \mu_\mathbf{k}}, \qquad w^z_\mathbf{k}(\mathbf{v}) = 1, \tag{38i–k}$$

and $s_x = 1$, $s_y = s_z = 0$. Here, P_{AH} indicates the anti-Hermitian part of the tensor ($P_{AH}K^{jl} = (1/2i)(K^{jl} - K^{*lj})$) and J'_n is the the first derivative of J_n with respect to the argument. The background velocity distribution function $g_s(\mathbf{v}, t)$, which includes the fluctuation-induced cross-field particle drift $v_d = \int d^3\mathbf{v}\, v_x g_s$, is obtained from the unperturbed Vlasov equation

$$\left(\mathbf{E}_0 + \frac{1}{c}\mathbf{v} \times \mathbf{B}_0\right) \cdot \frac{\partial g_s}{\partial \mathbf{v}} = 0, \tag{39}$$

and is expressed as

$$\begin{aligned} g_s &= a_s \sum_{m=0}^{\infty} \frac{1}{m!} v_x^m \left(-\frac{v_d}{v_\perp}\frac{\partial}{\partial v_\perp}\right)^m g_{s0}(v_\perp, v_\parallel, t) \\ &= a_s g_{s0}((v_\perp^2 - 2v_d v_x)^{1/2}, v_\parallel, t), \end{aligned} \tag{40}$$

with normalization constant a_s, which is determined such that $\int d^3\mathbf{v}\, g_s = \int d^3\mathbf{v}\, g_{s0} = 1$. g_s is expressed in the displaced cylindrical coordinates in velocity space ($v_x = v_\perp \cos\theta + v_d, v_y = v_\perp \sin\theta, v_z = v_\parallel$) and g_{s0} is the background velocity distribution function, which is symmetric with respect to the magnetic field. When $\mathbf{v}_d = 0$, g_s is reduced to g_{s0}. The diffusion coefficient $Q^{jl}_\mathbf{k}$ is represented in the displaced cylindrical coordinates, and thereby g_{s0} appears on the right-hand side of (36). The azimuthal dependence of $Q^{jl}_\mathbf{k}$ represents anisotropy of the velocity-space diffusion around the magnetic field and the resulting cross-field particle acceleration or transport in the x direction. In the previous theory (Kennel and Engelmann 1966), the azimuthal dependence of the quasilinear velocity-space diffusion equation vanishes because symmetry of the unperturbed velocity distribution function around the magnetic field is assumed ($\partial g_s/\partial\theta = 0$). In the present theory, the azimuthal dependence of the velocity-space diffusion equation should be retained, since the unperturbed velocity distribution function includes the cross-field particle drift ($\partial g_s/\partial\theta \neq 0$). In the case of the electrostatic approximation of $\sum_{jl} k_j k_l Q^{jl}_\mathbf{k}/k^2 = Q_\mathbf{k}$, the quasilinear velocity-space diffusion equation (36) can be proved to be reduced to that for electrostatic waves that was derived previously (Sugaya 1997, 1998). It can be confirmed easily that the velocity-space integration of the unperturbed Vlasov equation multiplied by $\mathbf{v}$ leads to the generalized Ohm's law of (20) for a collisionless plasma.

The velocity-space integration of $Q^{jl}_\mathbf{k}$ multiplied by w_s or $\mathbf{p}_s$ provides the quasi-

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linear transport equations, since the following simple relations hold:

$$\int d^3\mathbf{v}\, w_s Q^{jl}_{\mathbf{k}} g_{s0} = \frac{\omega_{\mathbf{k}}}{4\pi} P_{AH} \varepsilon^{(s)jl}_{\mathbf{k}}, \tag{41}$$

$$\int d^3\mathbf{v}\, \mathbf{p}_s Q^{jl}_{\mathbf{k}} g_{s0} = \frac{\mathbf{k}}{4\pi} P_{AH} \varepsilon^{(s)jl}_{\mathbf{k}}, \tag{42}$$

Thus, on performing the velocity-space integration of (36) multiplied by w_s or $\mathbf{p}_s$, the quasilinear transport equations given by (11) and (12) in which d/dt is replaced by $\partial/\partial t$ can be derived, as was described in the previous work (Sugaya 1997, 1998, 2000). Moreover, the conservation laws for the total energy and momentum densities of waves and particles given by (15) and (16) (with $d/dt \to \partial/\partial t$) can be also obtained. In conclusion, the same quasilinear transport equations and the generalized Ohm's law for a collisionless plasma are derived from the θ-dependent quasilinear velocity-space diffusion equation and the unperturbed Vlasov equation.

5. Summary

A single-particle theory giving the particle acceleration along and across a magnetic field and the generation of an electric field transverse to the magnetic field has been developed. The almost perpendicularly propagating electromagnetic waves in the magnetized plasma accelerate particles in the $\mathbf{k}$ direction via their Landau and cyclotron damping. Simultaneously, an intense cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_d/c$ is generated via the dynamo effect due to the perpendicular particle acceleration to satisfy the generalized Ohm's law. It is easy and straightforward to understand the physical mechanism of this phenomenon by means of the single-particle theory. The obtained transport equations predict that the ratio of the parallel and perpendicular drift velocities $v_{s\parallel}/v_d$ is proportional to $k_\parallel/k_\perp$. It has been shown that the same transport equations can be derived from the θ-dependent quasilinear velocity-space diffusion equation obtained by means of the perturbation theory of the Vlasov–Maxwell equations. Further, it has also been demonstrated that the generalized Ohm's law for a collisionless plasma can be provided by the unperturbed Vlasov equation. These equations can be applied usefully to the theoretical investigation and explanation of the particle acceleration or transport and the electric potential formation induced by electromagnetic and electrostatic waves in tokamaks and space plasmas.

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