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HAAVELMO'S CONTRIBUTIONS TO SIMULTANEOUS-EQUATIONS ESTIMATION

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This paper surveys Trygve Haavelmo's contributions to econometrics. A brief summary of his 1944 monograph is followed by an analysis of the six important papers he contributed during 1943–47. Four of them were devoted to macroeconomic models including estimation of the Keynesian marginal propensity to consume; the first two introduced the methodology of a system of structural equations; the third marked a milestone in econometric method by deriving the reduced form of such a system; and the fourth analyzed the contrast between time-series and cross-section analysis. The fifth (joint with M.A. Girshick) on the demand for food provided a definitive treatment of estimation of demand and supply functions; it carried out the execution of a five-equation structural model of the U.S. economy. The sixth was an interesting policy model of the interrelationship between the agricultural and the rural sector of the economy, which was fitted to U.S. data and addressed to questions of policy.

1. INTRODUCTION

One need only peruse the issues of *Econometrica* and related journals before 1943 (and even some after 1943) to verify that the subject of econometrics had been completely transformed by Haavelmo.

I shall discuss the seven significant contributions made by Haavelmo to econometrics during the eight years 1939–47 that he was in the United States: (1) the important 1944 methodological monograph *The Probability Approach to Econometrics* (which had actually been substantially completed in August 1941—cf. Bjerkholt, 2007, p. 778); (2) the four papers (1943a, 1943b, 1947a, 1947b) on estimation of the Keynesian marginal propensity to consume; (3) the 1947 joint paper with Girshick on the statistical analysis of the demand for food; and (4) the 1947 paper on agriculture and the national economy. Of these works, Schumpeter (1964, p. 1163) had this to say: "Haavelmo, . . . during his brief sojourn in the United States, without holding a teaching position, exerted an influence that would do credit to the lifetime work of a professor."

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2. THE PROBABILITY APPROACH TO ECONOMETRICS

This monograph (Haavelmo, 1944) may best be characterized as an exhortation to the econometrics profession to formulate econometric models with a clear probabilistic foundation so as to be amenable to modern statistical theory. Important works along these lines had been contributed by Frisch (1934) and Koopmans (1937), but even the last of these had been completed before the appearance of the important contributions of Neyman and Pearson (1936, 1938) which had so inspired Haavelmo (1944). I shall sketch some of the developments in this monograph that seem to me to be particularly significant.

Chapter I stresses (p. 6) that when one sets up a model, one should "have in mind some actual *experiment*, or some *design of an experiment*, which we could at least imagine arranging in order to measure those quantities in real economic life." He then set out (p. 10) the main problems in quantitative research: 1. the construction of tentative models; 2. the testing of theories; 3. the problem of estimation; and 4. the problem of prediction.

In Chapter II he introduces (pp. 31–3) a simple demand equation as a linear function of price and income, both variables expressed relatively to the cost of living, and considers the question whether it can be approximated by a function that is linear in the three variables; he shows that this may cause some unexpected problems. However, no supply equation is introduced. He also introduces concepts such as the "degree of autonomy" of the equations to be estimated.

Chapter III contains discussions of the problem of aggregation of the behavior of different individuals. Chapter IV contains a very nice exposition of the Neyman–Pearson theory of testing of hypotheses. Chapter V, under the heading of "the problem of confluent relations" or the "problem of arbitrary coefficients", discusses what Koopmans, Rubin, and Leipnik (1950, p. 70) were later to call the "identification problem". Finally, Chapter VI discusses problems of prediction.

This work was very important in its day especially in its persuasive arguments that econometric models should be formulated and analyzed stochastically. This has now become entirely accepted by the econometrics profession.

3. ESTIMATION OF THE MARGINAL PROPENSITY TO CONSUME

3.1. The Statistical Implications of a System of Simultaneous Equations

I now take up Haavelmo's seminal January 1943 paper (Haavelmo, 1943a). This paper contains two distinct parts. Part 1 discusses a purely mathematical example which might have been suggested by a model of demand and supply introduced by Marschak (1942).[1] This example is

$$Y = aX + \epsilon_1, \qquad X = bY + \epsilon_2,$$

where ϵ_1 and ϵ_2 are assumed to be independently normally distributed with zero means and variances σ_1^2 and σ_2^2, respectively. Solving the above equations one obtains

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$$Y = \frac{\epsilon_1 + a\epsilon_2}{1-ab}, \qquad X = \frac{b\epsilon_1 + \epsilon_2}{1-ab},$$

whence X and Y are jointly normally distributed with zero means. It follows that

$$E(Y|X) = \frac{b\sigma_1^2 + a\sigma_2^2}{b^2\sigma_1^2 + \sigma_2^2} X,$$

which is different from aX unless $b = 0$. This shows that in a two-equation model, it is illegitimate to follow some of the procedures that would be valid in a one-equation model.

In part 2 of this article Haavelmo took up an example of the business cycle based on the multiplier and the acceleration principle. I shall translate Haavelmo's notation (p. 4) into the probably more familiar Keynesian notation:[2]

$$\begin{aligned} C_t &= \alpha Y_t + \beta + u_{t1} \\ I_t &= \kappa(C_t - C_{t-1}) + u_{t2} \\ Y_t &= C_t + I_t, \end{aligned} \qquad \textbf{(1)}$$

where C_t is the rate of consumption, I_t is that of net investment, and Y_t is national income, at time t; u_{t1} and u_{t2} are random errors with zero means, assumed to be independently normally distributed.[3]

The first equation is of course that of the Keynesian consumption function; the second portrays the acceleration principle, whereby the stock of capital is proportionate to the rate of consumption, hence net investment is proportionate to the rate of change in consumption.[4]

Haavelmo's main point was a criticism of the usual method of fitting the first two equations of (1) to data separately, by regressing consumption on income to estimate α and β, and regressing net investment on the change in consumption to estimate κ. However, he did not provide a precise method for deriving the estimates of those three parameters. The problem is that, as he was to show four years later, to test the system (1) and estimate its coefficients one must derive its *reduced form*.[5]

This is done most easily by eliminating the national-income variable Y_t from (1), and writing the latter in the form

$$\begin{aligned} (1-\alpha)C_t - \alpha I_t &= \beta + u_{t1} \\ -\kappa C_t + I_t &= -\kappa C_{t-1} + u_{t2}. \end{aligned}$$

Solving these two equations we obtain the reduced form

$$\begin{aligned} C_t &= \frac{\beta - \alpha\kappa C_{t-1}}{1-\alpha(1+\kappa)} + \frac{u_{t1} - \alpha u_{t2}}{1-\alpha(1+\kappa)} \\ I_t &= \frac{\kappa\beta - (1-\alpha)\kappa C_{t-1}}{1-\alpha(1+\kappa)} + \frac{\kappa u_{t1} + (1-\alpha)u_{t2}}{1-\alpha(1+\kappa)}. \end{aligned}$$

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In a subsequent paper Haavelmo (1943b, p. 18) referred to this same model, and proposed testing the hypothesis that the parameters α, β, and κ be greater than zero, against annual data for the years 1930–42. However, from the above reduced form it is clear that they do not appear separately, and it is not at all clear how this could be accomplished. In particular, since in his later paper (Haavelmo, 1947a) to be discussed in Section 3.2, he abandoned the acceleration principle altogether, he would clearly have been interested in testing the hypothesis that $\kappa > 0$. This shows that there was a significant advance that still needed to be made in his methodology, which did not occur until his March 1947 paper (Haavelmo, 1947a).

3.2. Methods of Measuring the Marginal Propensity to Consume

Haavelmo returned to the problem of estimating the marginal propensity to consume in his first 1947 paper (Haavelmo, 1947a). He now abandoned the acceleration principle and gave weight to the opinions of Alvin Hansen (1941, p. 50) that "the active dynamic factor in the cycle is investment, with consumption assuming a passive, lagging role," and of Samuelson (1943, p. 41) that the behavior of investment is "sporadic, volatile, and capricious" and "almost completely independent of current statical factors (level of income, etc.)."[6] He followed this with the statement (Haavelmo, 1947a, p. 106): "We should, instead, take the regression of income on *investment*, to obtain the multiplier, and from this estimate of the multiplier we should derive the marginal propensity to consume." His argument for this proceeded as follows.

Specifically, Haavelmo started (p. 107) with the first and third relationships from (1):

$$Y_t = C_t + I_t, \qquad C_t = \alpha Y_t + \beta + u_t. \tag{2}$$

In the later Cowles Commission terminology, this was called the "structure"[7] of Haavelmo's model (here Haavelmo used the term "system")[8]

(a): u_t is a random variable with $E(u_t) = 0$ and $E\left(u_t^2\right) = \sigma_u^2$ for all $t = 1, 2, \ldots$, and

(b): the time series I_t $(t = 1, 2, \ldots)$ is "autonomous in relation to C_t and Y_t", where "autonomous" here presumably meant the same as "exogenous".[9]

He split case (*b*) into two subcases:

(b1) the sequence I_t is a sequence of given numbers, whence $E(I_t u_t) = 0$, or

(b2) each I_t is a random variable independent of u_t.

Solving equations (2), one obtains Haavelmo's equations (2.6) and (2.7):

$$C_t = \frac{\alpha}{1-\alpha} I_t + \frac{\beta}{1-\alpha} + \frac{u_t}{1-\alpha}; \tag{3}$$

$$Y_t = \frac{1}{1-\alpha} I_t + \frac{\beta}{1-\alpha} + \frac{u_t}{1-\alpha}. \tag{4}$$

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This is what came to be called the "reduced form"[10] of the above structure (2). It of course gives the formula $\partial Y_t/\partial I_t = 1/(1-\alpha)$ for the Keynesian multiplier. Writing this reduced form as

$$C_t = \pi_{11} I_t + \pi_{12} + v_{t1} \tag{5}$$

$$Y_t = \pi_{21} I_t + \pi_{22} + v_{t2}, \tag{6}$$

we see immediately from (3) and (4), as Haavelmo pointed out (1947a, p. 108), that the ordinary least-squares estimators $\hat{\pi}_{ij}$ of the π_{ij} in (5) and (6) are the Gauss–Markov ("best linear unbiased") estimators of the reduced-form parameters $\pi_{11} = \alpha/(1-\alpha)$, $\pi_{21} = 1/(1-\alpha)$, and $\pi_{12} = \pi_{22} = \beta/(1-\alpha)$ (cf. David and Neyman, 1938). He proved further (p. 109) that the corresponding estimators

$$\hat{\alpha} = \frac{\hat{\pi}_{11}}{\hat{\pi}_{21}} \qquad \text{and} \qquad \hat{\beta} = \frac{\hat{\pi}_{22}}{\hat{\pi}_{21}} \tag{7}$$

of the marginal propensity to consume α and the constant term β are consistent.

On the other hand, the conventional estimators of α and β criticized by Haavelmo were obtained by regressing C_t on Y_t, from the second relation of (2). Denoting these conventional estimators of α and β by a and b, respectively, Haavelmo (1947a, p. 110) showed that the probability limits of a and b as $N \to \infty$ are

$$\operatorname{plim} a > \alpha \text{ when } 0 < \alpha < 1, \text{ and } \operatorname{plim} b < \beta \text{ when } \sum_{t=1}^{N} \frac{I_t}{N} > -\beta, \tag{8}$$

so that the conventional method overestimates the marginal propensity to consume, α, and underestimates the constant term β. (Likewise, from (4) the conventional estimator $1/(1-a)$ overestimates the Keynesian multiplier $1/(1-\alpha)$.) Therefore they are not consistent estimators of α and β.

The historical significance of this article was emphasized by Basmann (1974) in the following words:

"Haavelmo's 1947 article [is] the first direct contribution to the subsequent development of exact finite sample distribution theory for econometric simultaneous equations systems" (p. 210). "In several very important respects Haavelmo's article constitutes the most important direct contribution to the development of the exact finite sample theory of econometric estimators and test statistics that has been published thus far, and ... is likely to remain the most important. It is the most instructive single article on the subject ..." (pp. 214–215).

Basmann went on to explain (p. 216): "Haavelmo did not present exact finite sample density functions of estimators in his 1947 article. It would be fallacious to infer from that omission, however, that Haavelmo either had not derived them or had not recognized their known mathematical forms from inspection .... In constructing an interval estimate for the marginal propensity to consume, α, Haavelmo made transparent exactly how the exact sample distribution function

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of $\hat{\alpha}$ can be derived from the marginal distribution of $\hat{\pi}_{11}$."[11] This latter refers to pp. 108–9 of Haavelmo (1947a).

It is noteworthy that Bergstrom (1962) had cited the same article by Haavelmo (1947a) in his important contribution to the exact finite-sample distribution of Haavelmo's estimator of the marginal propensity to consume, as had Basmann (1957).

Going back to the "optimal" estimators (7), we see that since the random error u_t in the intrinsically positive consumption variable C_t in formula (2) is unrestricted as to sign, and for the purpose of obtaining confidence limits is assumed to be approximately normally distributed (Haavelmo, 1947a, p. 110), the estimators $\hat{\alpha}$ and $\hat{\beta}$ of (7) are necessarily ratios of jointly normally distributed random variables, hence they have the Fieller distribution, which does not possess moments of any order. This was first pointed out by Basmann (1961, p. 621n, 1974, p. 217), who discussed Haavelmo's paper (1947a) in detail. See also Chipman (2011, p. 241).[12]

The following more detailed argument for the above has been suggested by T.W. Anderson (cf. Anderson, 2012). Rewrite (3) as

$$C_t = \frac{\alpha}{1-\alpha}(I_t - \bar{I}) + \theta + \frac{u_t}{1-\alpha}, \quad \text{where} \quad \bar{I} = \frac{1}{N}\sum_{t=1}^{N} I_t \quad \text{and} \quad \theta = \frac{\beta + \alpha \bar{I}}{1-\alpha}.$$

The least-squares estimators of $\pi_{11} = \alpha/(1-\alpha)$ and θ may be written as

$$\tilde{\pi}_{11} = \frac{\sum_{t=1}^{N} C_t (I_t - \bar{I})}{\sum_{t=1}^{N} (I_t - \bar{I})^2} \quad \text{and} \quad \tilde{\theta} = \bar{C} \equiv \frac{\sum_{t=1}^{N} C_t}{N}.$$

Then $\tilde{\pi}_{11}, \tilde{\theta}$, and the estimator of $\sigma^2/(1-\alpha)^2$ are statistically independent and sufficient.[13] All the information about the marginal propensity to consume α is in $\tilde{\pi}_{11}$.

3.3. Family Expenditure and the Marginal Propensity to Consume

I come now to what Klein (1953, p. 241) described as "a penetrating article on the relationship between estimates of the same equation from cross section and time series data." Of course, cross-section analysis does not qualify as simultaneous-equations estimation, but it is obviously of interest to compare and if possible combine the two types of information.

Haavelmo (1947b) had considered the consumer expenditure c of a family, its income y, and the prices $p_1, p_2, \ldots, p_s$ of the s goods it consumes. He assumed further that the family's total expenditure c is of the following linear type

$$c = \alpha y + \sum_{i=1}^{n} \beta_i x_i + \gamma\,(p_1, p_2, \ldots, p_s),$$

where y is family income, x_i is the ith characteristic (e.g., age of the head of the family, wealth, past income, etc.), p_i is the price of commodity i, α and the

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β_i are constants, and γ is a function of the s prices. Family income y and the n family characteristics x_i are random variables. The joint probability density function of family income y and the n characteristics x_i is defined by

$$\Phi(y, x_1, x_2, \ldots, x_n; \bar{y}),$$

where $\bar{y}$ is a parameter representing the average family income in the community. Letting $\omega(y; \bar{y})$ denote the distribution of income (the marginal probability density function of y), Haavelmo defines (in his formula (2.2)) the conditional distribution of the x_i for a given value $\bar{y}$ of y as

$$\psi(x_1, x_2, \ldots, x_n; y; \bar{y}) = \frac{\Phi(y, x_1, x_2, \ldots, x_n; \bar{y})}{\omega(y; \bar{y})}.$$

From this, he comes to the important result that

$$E(c|y; \bar{y}; p_1, p_2, \ldots, p_s) = g(y; \bar{y}) + \gamma\,(p_1, p_2, \ldots p_s),$$

"where g is some—perhaps complicated—function of family income y, and the average income, $\bar{y}$." Thus, even though the family expenditure function is linear in income, "*the apparent marginal propensity to consume, as obtained from budget data, i.e., $g(y)$, is not in general equal to α.*" This serves as a warning to avoid uncritical use of cross-section estimates in time-series models.

A slightly later study by Tobin (1950), which cited both Haavelmo (1947b) and Girshick and Haavelmo (1947), combined cross-section data and time-series data, and stated (p. 113): "The premise of this paper is that a statistical demand function should be consistent with both types of observations." He came to the conclusion (p. 138): "Thanks to the use of budget data, the statistical food demand function obtained in this study contains a higher estimate of income elasticity than functions based on time series alone (.56 compared to .3 in the Girshick-Haavelmo study)."

4. ESTIMATION OF THE DEMAND FOR FOOD

The above contributions involved studies based on simple two-equation macroeconomic models. The paper by Girshick and Haavelmo (1947) introduced a much more complex five-equation microeconomic model that has been well summarized by Klein (1953, pp. 196–198). They start, however, by discussing a simple two-equation demand-and-supply model that extends to microeconomics the same kinds of basic principles that Haavelmo had established in the previous macroeconomic models.

The first interesting problem discussed by the authors is that of estimating demand and supply equations for a commodity when the current price p_t of the commodity is the only exogenous variable in each equation, i.e. (their formulas (1.2) and (1.3a)),[14]

$$x_t = \alpha p_t + \beta + u_t, \qquad x_t = h_1 p_t + k_1 + v_t, \tag{9}$$

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where x_t and p_t are respectively the quantity and price and u_t and v_t are random variables with zero means. Solving these two equations they obtain the *reduced form* of (9) in the quantity and price

$$x_t = A + U_t, \text{ where } A = \frac{\alpha k_1 - \beta h_1}{\alpha - h_1} \text{ and } U_t = \frac{\alpha v_t - h_1 u_t}{\alpha - h_1}, \tag{10}$$

$$p_t = B + V_t, \text{ where } B = \frac{k_1 - \beta}{\alpha - h_1} \text{ and } V_t = \frac{v_1 - u_t}{\alpha - h_1}. \tag{11}$$

They showed in this example that the parameters α and β cannot be estimated (pp. 80–82). However, if the price variable in the supply equation (the second equation of (9)) is lagged by one period, i.e., if the structure (9) is replaced by

$$x_t = \alpha p_t + \beta + u_t, \qquad x_t = h_2 p_{t-1} + k_2 + w_t \tag{12}$$

(their formulas (1.2) and (1.3b)), so that the reduced-form equations (10) and (11) are replaced by

$$x_t = h_2 p_{t-1} + k_2 + w_t \quad \text{[second (supply) equation of (12)]} \tag{13}$$

$$p_t = \frac{h_2}{\alpha} p_{t-1} + \frac{k_2 - \beta}{\alpha} + \frac{w_t - u_t}{\alpha} \tag{14}$$

(their formulas (1.12) and (1.13)), respectively, then it becomes possible to estimate α and β (pp. 82–3).

In fact, this is precisely the assumption adopted by Moore (1919) in his pioneering work on estimation of demand and supply functions. In estimating demand and supply functions for cotton he had used the current price for the demand function (pp. 557–9), and for the supply function (pp. 560–1) he had used as his price variable "the percentage change in the price of cotton from the price prevailing two years before the current year to the price prevailing the year before the current year," supplying his annual data for both in two tables (pp. 558, 561). The same procedure was followed in Moore (1925, pp. 367–8), reproduced in his last work (1929, pp. 94–5), in estimating demand for and supply of potatoes.

Moore's research was followed by some methodological contributions by Holbrook Working (1925)—who provided graphs of demand curves for potatoes (1925, pp. 509–511, 513–515), based on data and methods previously described in H. Working (1922)—and by E.J. Working (1927), who provided graphs of purely hypothetical intersecting supply and demand curves, to illustrate the point that plots of these intersection points could look like plots of demand curves or of supply curves or of neither. At least, this anticipated Girshick and Haavelmo's proof of nonidentifiability. But E.J. Working apparently overlooked the fact that Moore had always used the previous year's price for the supply curve and the current year's price for the estimated demand curve (as in (13)). Thus Moore's analysis was based on a structure such as (12), which is identifiable, as Girshick and Haavelmo showed (and was noted above), whereas E.J. Working's analysis, being based on (9), in effect showed what Girshick and Haavelmo later proved, that the demand function was unidentifiable.[15]

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Marschak (1942) had posed the question (p. 138): "Suppose you fit a curve to a scatter of points representing prices and consumed quantities of a commodity. How do you know whether your curve is the demand or the supply curve?" To answer this question Marschak cited some writings of E.J. Working (1927), Frisch (1933, pp. 10–19), and Schultz (1938). Working (1927, p. 223) had noted that "In the case of agricultural commodities, ... there is likely to be a much greater shifting of the supply schedules of sellers than of demand schedules of buyers", but that with manufactures the situation "may be entirely different." Moore's student Henry Schultz had considered a number of hypotheses, one of which (1938, pp. 77–8) was that the statistician "can determine the demand curve by relating the price for any given year to the quantity for the same year, and he can determine the supply curve by relating the price for any given year to the quantity for the following year"—the method used by Moore (1919, 1925, 1929)—and the other being the second of the following in Schultz (1938, p. 81):

> Either (1) the demand curve remained fixed, and the supply curve moved; or (2) both curves moved, but the movements of the demand curve were very small compared with those of the supply curve, so that *on the average* the statistics of the quantities and prices are observations on the relatively fixed demand curve; or (3) both curves remained fixed ... but there was a fixed time interval between changes in price and corresponding changes in the quantity supplied.

Schultz never stated which of these hypotheses he actually assumed in his monumental work. Neither Moore's work nor Schultz's was referred to by Girshick and Haavelmo, but elsewhere Haavelmo (1947c, p. 910n) referred very favorably to Schultz (1938).

Marschak (1942, p. 139) had formulated the model

$$Y_t = \alpha X_t + U_t; \qquad Y_t = \beta X_t + V_t, \tag{15}$$

(which, as suggested in Section 3.1 above, probably influenced Haavelmo) where Y_t and X_t are respectively the quantity and price of a product, and the first (leftmost) equation presumably being the demand equation for this product and the second (rightmost) equation the supply. U_t and V_t are "variate parameters" by which he evidently meant random variables. Clearly they must both have positive means. In particular, Marschak derived a formula from which he concluded that the regression coefficient of quantity on price is "closer" to α (the slope of the demand curve) than to β (the slope of the supply curve) if the variance of U is smaller than the variance of V. And this seems to confirm part (2) of Schultz's above-quoted argument. But it is clear that Haavelmo could not have accepted this argument, since the estimator of α would not be consistent unless the variance of U_t was zero.[16]

It is curious that the above analysis on the part of Girshick and Haavelmo should have come under attack by Wold (1953, pp. 52, 326) in two statements.

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The first of these (p. 52) is: "Contrary to Haavelmo's statement that least-squares regression in general is biased when applied to a system of simultaneous relationships, we have seen that the regression is unbiased in the case of recursive systems." I have not found such a statement in the Girshick–Haavelmo paper or in any of Haavelmo's writings; besides, while (as we have seen) Haavelmo used best linear unbiasedness for small samples, he never championed it as a general principle of estimation as opposed to that of consistency. The second of these statements by Wold (p. 326) is: "The following quotation is typical for the wholesale dismissal of least-squares regression: '*That is, it is impossible to derive statistically the demand functions from market data without specification of the supply functions involved.*'" This latter quotation is from p. 83 of Girshick and Haavelmo. As we have just seen, specification of the supply (second) function in (12) allows derivation of the demand function, whereas specification of the supply (second) function of (9) does not. This surely proves the correctness of Girshick and Haavelmo's statement.[17] The Girshick–Haavelmo structure (12) indeed qualifies as a "recursive system" in Wold's terminology,[18] and as we have just seen, such systems go back at least to Moore (1919).

Girshick and Haavelmo (1947, p. 84) then considered the general problem in a two-equation model of whether there exists a unique structure (they used the terminology "system") that corresponds to the *reduced form*. They said (p. 85): *This is the problem of identification*.

In the applied part of their paper (Sect. III and IV, pp. 93–109) Girshick and Haavelmo (1947) provided a complete explanation of the Anderson–Rubin "reduced-form" limited-information maximum-likelihood method of estimation (also called the "least variance ratio estimate"; cf. Anderson, 1991, p. 10)—a method that was not published until 1949.

In this applied part of their paper Girshick and Haavelmo elaborated (p. 99) on the above demand and supply equations (12) by adding three explanatory variables to the demand equation, one for current and one for lagged disposable per capita income (deflated), and one for time; and to the supply equation was added the per capita production of agricultural food products and time. A third equation was added, namely the just-introduced disposable per capita income, as a linear function of net investment per capita and lagged disposable per capita income. As noted by Klein (1953, p. 197), this is just a Keynesian multiplier equation. Finally, two more equations were added, one expressing the just-introduced per capita production of agricultural food products as a linear function of current and lagged prices received by farmers for food products; and finally the deflated prices received by farmers for food products as a linear function of retail prices of food products and time. This five-equation system was fitted to annual data for the 20-year period 1922–1941—the data for which were tabulated in Girshick and Haavelmo (1947, p. 100)—using the limited-information maximum-likelihood method developed by Anderson and Rubin (1949), which at the time of their writing had not yet been published (cf. Girshick and Haavelmo, 1947, p. 94n).

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This article was not Haavelmo's only foray into the field of demand for food. When working for Frisch he had collaborated with him in a joint article on the demand for milk in Norway (Frisch and Haavelmo, 1938). According to Bjerkholt (2005, p. 511), "the overwhelming part of the joint work naturally fell on Haavelmo." Unlike a work that Frisch and Haavelmo had cited by Waite and Cox (1934), who based their arguments mostly on data summarized by tables and charts as well as maps, it went far beyond it in including a Section 5 (pp. 17–21) containing a linear regression of the logarithm of the quantity of milk on the logarithm of real income per head and a trend, and further econometric analysis in the remainder of the paper.

5. QUANTITATIVE RESEARCH IN AGRICULTURAL ECONOMICS

In this final paper, Haavelmo (1947c) constructs in the Appendix a simplified general-equilibrium model (with Keynesian components) of a two-sector economy (agricultural and nonagricultural) in which consumer preferences are implicitly assumed to be aggregable within and as between the two sectors.

x_1 and x_2 are the amounts of food and nonfood sold per year respectively, and P_1 and P_2 their respective prices; I is the annual investment of the combined sectors. $Y = x_1 P_1 + x_2 P_2 + I$ is the national income.

Defining the index of the cost of living as $P = e_1 P_1 + e_2 P_2$, where the e_i are constant weights, Haavelmo assumes that

$$\frac{x_1 P_1 + x_2 P_2}{P} = a_1 \frac{Y}{P} + a_0, \tag{16}$$

where a_1 is the marginal propensity to consume. Thus, (16) is a variant of the Keynesian consumption function. Basing himself on empirical research (presumably his joint article with Girshick), he postulated the demand for food to be of the form

$$x_1 = b_1 \frac{P_1}{P} + b_2 \frac{Y}{P} + b_0. \tag{17}$$

The endogenous variables are P_1, P_2, P, and Y. (Note that unlike Haavelmo's previous papers, this model contains no explicit random error terms.)

Now his model is prescriptive rather than descriptive, designed to answer the question: what would be the effect on P_1/P, P_2/P, and Y/P of "alternative direct decisions" regarding the variables x_1, x_2, I. By this he means (p. 919) "to have the answers if for example one were to consider introducing some new economic policy under which x_1, x_2, and I would *in fact* be subject to autonomous regulation." This means that the variables x_1, x_2, and I are treated as exogenous. The object is to evaluate derivatives such as $\partial P_1/\partial x_1$, $\partial P_2/\partial x_1$, $\partial P/\partial x_1$, and $\partial Y/\partial x_1$, when x_2 and I are kept constant.

He illustrated his method by assuming x_2 and I to be constant, and varying x_1. He then evaluated the elasticities of P_1, P_2, P, and Y with respect to x_1, resulting

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in a system of four equations in four unknowns, namely the partial derivatives of P_1, P_2, and Y with respect to x_1. For this, he employed the data used in his joint paper with Girshick. He found that if the price indices P_1, P_2, P for the base period 1935–39 are set equal to 1, and the average value of real income Y during this period is also set equal to 1, the values of the exogenous variables during 1935–39 are $x_1 = .25$, $x_2 = .65$, and $I = .10$, yielding the estimates $a_1 = .7$, $b_1 = -.06$, and $b_2 = .07$, furnishing estimates of the desired elasticities of Y, P_1, P_2, and P with respect to x_1 of .36, −3.75, 1.45, and −1.07, respectively, so that a 1% increase in the output of food would (1) increase the real income of consumers by about .36%, decrease the real price of food by about 3.75%, increase the real price of nonfood consumer goods by about 1.45%, and decrease the cost of living by about 1.10%. The empirical estimates are so reasonable as to be astonishing.

6. CONCLUSION

Edgeworth (1894, p. 544) once famously stated that "it may be doubted whether Jevons's hope of constructing demand curves by statistics is capable of realisation." This statement was frequently quoted by Moore who aimed to disprove it. Moore had estimated demand and supply functions for cotton and potatoes. To this list, Schultz (1938) later added (for demand only) sugar, corn, hay, wheat, oats, barley, and rye—all agricultural commodities. Some other products such as pig iron have been added. So despite the major progress that has been achieved, for most commodities Edgeworth's dictum has held up. As for the marginal propensity to consume, as Haavelmo showed, there have been many estimates but most of them wrong. What has mostly been proved is that econometrics is a very difficult subject.

Haavelmo was one of its few masters. He radically changed and improved the subject. It is to be hoped that by inspiring others his work may yet lead to further improvements.

NOTES

1. Marschak acknowledged (1942, p. 135n) that his paper "could not have been written without the stimulating influence of talks with T. Haavelmo." He also stated that it had been written in March 1940. His model will be discussed in Section 3. Of course, unlike Haavelmo's example under discussion here, it does not assume that ϵ_1 and ϵ_2 have zero means.

2. The model was probably influenced by that of Samuelson (1939), which differed from it by the first equation of (1) being $C_t = \alpha Y_{t-1}$. This and the second equation of (1) Samuelson called the "Hansen assumptions," presumably since Hansen (1941, p. 236) had displayed a homogeneous consumption function and had referred (p. 247) to the "consumption–income ratio." However, Hansen (p. 229, Case B) had also displayed a nonhomogeneous consumption function as in (1).

3. Note that this assumption necessarily allows consumption to take on negative values. This is a case where convenient statistical assumptions clash with economic reality. They also lead to some inconvenient statistical results, namely that some estimators may not possess means (see below).

4. Most likely, Haavelmo was influenced here by Frisch (1931), but no doubt also by Alvin Hansen, whose lectures he had attended at Harvard in 1941–42 (cf. Bjerkholt, 2007, p. 795), and whose book

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(1941, pp. 274–288) had discussed the interactions between the multiplier and the acceleration principle at length. For further background on the acceleration principle see Chipman (1993).

5. Cf. Mann and Wald (1943, pp. 201, 216); Anderson and Rubin (1949, p. 47); Marschak (1950, p. 8); Koopmans, Rubin, and Leipnik (1950, p. 189); and Anderson (1950, p. 313).

6. Here, Haavelmo inadvertently misquoted Samuelson's "statical" as "statistical". Haavelmo had also cited the articles by Smithies (1945) and Mosak (1945), both of which contained regressions of consumption on income. It may be noted that Hansen's book contained an appendix by Samuelson (1941) which referred (p. 250n) to regressions carried out by Colin Clark in 1937–38 and Michal Kalecki in 1939, and data produced by Simon Kuznets, concluding (p. 251) that "If dollar consumption figures are plotted against dollar national income figures for the nineteen years, . . . no simple relationship is apparent." Samuelson also included some statistical regressions of consumption on income, but examined them very critically. Haavelmo was certainly the first in the methodological literature to spot the error of regressing consumption on income rather than regressing income on investment to get the multiplier $\pi_{21} = 1/(1-\alpha)$ and thence the marginal propensity to consume $\alpha = 1 - 1/\pi_{21}$ (see formulas (4) and (6) below).

7. Cf. Koopmans, Rubin, and Leipnik (1950, p. 73); also Marschak (1950, p. 8, 1953, p. 3). A model that is similar in some respects to (2) was analyzed in Forchini (2006), Hillier (2006), and Phillips (2006), and one that is quite close to (2) in Phillips (2009).

8. Haavelmo (1938, p. 204) had already referred to systems such as (2) as "structural relations" (again as a "system of structural relations", p. 207), also as a system of "confluent relations" (1944, p. 29)—a term that also covered reduced forms. The term "structure" had been used in Haavelmo (1944, p. 28) to denote "a theoretical *set* of possible simultaneous sets of value or sets of time series for the economic variables."

9. Haavelmo (1950, p. 263) defined a relation to be "autonomous" "in the sense that it would hold regardless of whether or not other economic relations were fulfilled." See also Haavelmo (1944, Sect. 8, pp. 28–39), where this term is discussed at length. The term had already been used by Frisch in an unpublished 1938 memorandum referred to by Haavelmo (1944, p. 28n); see also Frisch (1948) and the valuable discussion in Aldrich (1989).

10. Cf. Marschak (1950, p. 9); Koopmans et al. (1950, p. 189); and Anderson (1950, p. 313).

11. I have substituted the above notations for Basmann's. Basmann had obtained a Fulbright grant to study under Haavelmo during the 1955–56 academic year. In an email he sent to this writer on December 8, 2011 he mentioned that he had attended Haavelmo's Saturday morning seminars, had been helped by Leif Johansen with the Norwegian (for which he had previously obtained instruction), and had obtained from Haavelmo "encouraging and useful suggestions concerning the content of" Basmann's early papers. Acknowledgements were made in Basmann (1957, 1960, 1974).

12. It should be mentioned that these examples of infinite variances occur in the special cases (such as the above) in which the parameters of an equation are "exactly identified." Precise conditions have also been obtained for other cases including the so-called two-stage least squares estimators; for a discussion see Chipman (2011, p. 243). In the case of Anderson and Rubin's (1949) limited-information maximum-likelihood (LIML) estimator, which had been shown by Mariano and Sawa (1972) in the two-equation case, and by Phillips (1985) more generally, to have no moments (cf. Chipman, 2011, p. 243), Anderson (2010) has recently shown that it has finite moments if a more suitable normalization is used; however, as pointed out to this writer by Prof. Phillips, its structure is not identifiable.

13. On sufficient statistics see Wilks (1947, p. 135): "T is said to be a *sufficient* statistic for estimating e if for any other statistic T', the conditional distribution $f(T'|T)$ of T', given T, is independent of e."

14. These are equivalent to the equations (15) below, which were introduced by Marschak (1942) and will be discussed later in this section.

15. Moore, who had studied statistics under Karl Pearson (cf. Moore, 1917, p. 17n)—and was also Paul Douglas's teacher—may be regarded as a forerunner of Haavelmo, who had studied statistics under Jerzy Neyman and Egon Pearson. But Haavelmo's principal contact was with Neyman.

16. More drastically, Marschak and Andrews (1944, p. 148n), in their fine study of the econometrics of production, specified the sufficient condition $U_t = 0$ for all t.

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17. It is therefore all the more puzzling that these statements criticized by Wold should have been removed from the 1953 abridged version of Girshick and Haavelmo's paper, pp. 96–97. No other parts of Sections I and II had been removed or altered.

18. For the nature of the controversy between Haavelmo and Wold, see the report of the meeting of the Econometric Society in Uppsala, September 1953, in Bent Hansen (1955, pp. 198–216). There was considerable discussion at that conference of recursiveness related to the paper by R. Bentzel and B. Hansen. For a general treatment of recursive models see Chipman (2011, Sect. 8.7, pp. 268–283).

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