JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 44, No. 4, Aug. 2009, pp. 987–1011
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doi:10.1017/S0022109009990184

https://doi.org/10.1017/S0022109009990184 Published online by Cambridge University Press

Term Structure, Inflation, and Real Activity

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Abstract

This paper estimates an internally consistent structural model that imposes cross-sectional restrictions on the dynamics of the term structure of interest rates, inflation, and output growth. Distinct from previous term structure settings, this model introduces both time-varying central tendencies and a stochastic conditional mean of output growth. The estimation of the model, which is based on U.S. data over a 1960 to 2005 sample period, provides reliable estimates for the implicit term structures of real interest rates, expected inflation rates, and inflation risk premia, as well as for expectations of macroeconomic variables. The model has better out-of-sample forecasting properties than a number of alternative models, and it contradicts the puzzling evidence that during the "Great Moderation" in inflation subsequent to the mid-1980s, the forecasting ability of structural models deteriorated with respect to atheoretic statistical models.

I. Introduction

The relationship between the term structure of interest rates, inflation, and real activity has long been recognized as central to both macroeconomic and finance theory, and as critical in formulating economic policy and in investment decisions. Inflation expectations and predictions of economic growth also play a key role in pricing financial instruments, as well as in determining the actions of governments and central banks in their attempts to smooth the business cycle. Conversely, changes in asset prices and economic policies influence current levels of inflation and real growth and, therefore, expectations about their future direction.

Extensive theoretical and empirical work has been devoted to the relationship between real activity, inflation, and the term structure of interest rates. Thus far, however, there is neither a commonly accepted theoretical framework nor agreement on empirical regularities.

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According to Harvey (1988), (1989) and subsequent analyses by, among others, Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Chapman (1997), Kamara (1997), and Roma and Torous (1997), the shape of the term structure reflects information about future economic growth. In particular, a positive slope in the yield curve predicts an increase in the level of real activity, whereas a flattening or a negative slope in the yield curve is associated with a future recession. Conversely, Evans and Marshall (2001), Ang and Piazzesi (2003), and Dewachter and Lyrio (2006) show that macroeconomic factors have a significant impact on the term structure of interest rates. Empirical studies on the information content of the term structure for future inflation have been undertaken by, among others, Fama (1990) and Mishkin (1990). These studies suggest that only the long end of the term structure of interest rates contains information about future inflation, whereas the short end of the term structure provides almost no such information.

All of the studies cited above relate to interest rate theory, but they do not satisfy the problem of specifying a complete model of the term structure of interest rates that fully accounts for the links between inflation, economic growth, and bond prices, and that might be usable for forecasting purposes. The inclusion of macroeconomic relationships in a general equilibrium term structure framework has its origins in the continuous time model of Cox, Ingersoll, and Ross (CIR) (1985a), (1985b), which describes a complete economy with production and a stochastic investment opportunity set and endogenously determined optimal consumption, portfolio choice, and equilibrium asset prices. Among the CIR-type term structure models that explicitly consider the role of macroeconomic variables, I make note of Breeden (1986), Pennacchi (PEN) (1991), Sun (1992), Bakshi and Chen (1996), and, more recently, Wu (2001), Ang and Piazzesi (2003), Buraschi and Jiltsov (2005), and Dewachter and Lyrio (2006). Considering data on macroeconomic variables such as inflation and output, along with data on bonds, highlights the macroeconomic underpinnings of the yield curve and makes a term structure model suitable for both investment and economic policy purposes.

This paper adopts such a macroeconomic approach and presents a structural model that allows i) investigation of the cross-sectional restrictions that link the dynamics of the term structure of interest rates to output growth and inflation; ii) reliable estimation of the unobservable term structures of real interest rates, expected inflation rates, and inflation risk premia; and iii) generation of endogenous, accurate forecasts of future inflation and gross domestic product (GDP) growth rates.

This paper relates to its field as follows. First, it builds an internally consistent structural model, which provides a bridge between fully specified equilibrium models (see, e.g., Marshall (1992) or, more recently, Wu (2001)) and nonstructural term structure models with macroeconomic factors, such as the models based on vector autoregression (VAR) of Evans and Marshall (2001), Ang and Piazzesi (2003), and Ang, Piazzesi, and Wei (APW) (2006). In particular, the model complements nonstructural models in documenting how macroeconomic factors explain the bulk of term structure movements. However, unlike Ang and Piazzesi (2003), the model does not incorporate any latent factors lacking clear

https://doi.org/10.1017/S0022109009990184 Published online by Cambridge University Press

https://doi.org/10.1017/S0022109009990184 Published online by Cambridge University Press

economic interpretation. Rather, all factors are clearly identified as components of those processes pertaining to inflation, short-term real interest rates, and returns to the production process. In this respect, my approach is close in spirit to that of Dewachter and Lyrio (2006).

Second, this paper obtains the term structures of real interest rates and expected inflation rates from observations on nominal bond prices using a more structured approach than PEN (1991) and Sun (1992), who focus only on inflation data, and Evans (2003), Goto and Torous (2003), and Ang, Bekaert, and Wei (2008), who adopt regime-switching models.

Third, I derive endogenous estimates of time-varying bond risk premia using a framework that extends the yield-only approach of Duffee (2002) and Duarte (2004), and that differs substantially from those recently proposed by Buraschi and Jiltsov (2005) and Cochrane and Piazzesi (CP) (2005). Finally, I exploit the equilibrium cross-sectional restrictions imposed by my model to obtain accurate forecasts of future inflation and GDP growth rates by making use of term structure data. The model has better out-of-sample forecasting properties than a number of alternative models, including atheoretic statistical models, and seems to contradict the puzzling evidence described by Stock and Watson (2007); that is, the fact that during the so-called "Great Moderation" in inflation subsequent to the mid-1980s, the forecasting ability of structural models deteriorated with respect to naive univariate models.

This paper proceeds as follows. Section II illustrates the structural term structure model and derives the main equations describing the relationship between interest rates and macroeconomic variables. In Section III, using U.S. data over a 1960–2005 sample period, I present the maximum likelihood–Kalman filter estimates of the model for the term structures of real interest rates, expected inflation rates, and inflation risk premia. Moreover, I provide statistical evidence of the fact that the model produces more accurate forecasts of future inflation and GDP growth than some alternative models. Section IV presents the conclusions.

II. The Model

In this section, I outline the structure of the macroeconomic term structure model.[1]

A. Production, Inflation, and State Variables

The underlying basic assumptions of the model are consistent with the CIR (1985a) general equilibrium framework,[2] in which the state variables driving the

[1] A full derivation of the model is available from the author.

[2] These assumptions are: i) a continuous time competitive economy composed of a fixed number of identical individuals with rational expectations and time-additive logarithmic utility functions; ii) a single physical good is produced, which may be allocated to consumption or investment; iii) a single endogenous production process exists, which is affected by stochastic technological changes; iv) there are markets for instantaneous borrowing and lending at the same riskless interest rate as well as markets for default-free nominal and real bonds. The solution of the individual's intertemporal problem determines the total consumption and production plan, the price level, the optimal allocation of wealth

dynamics of the economy are assumed to be the real interest rate and the expected inflation rate. In common with most term structure models including inflation (such as CIR (1985b), PEN (1991), Sun (1992), Ang et al. (2008), and Buraschi and Jiltsov (2005)), I do not explicitly model money and monetary policy and assume an exogenously given process for the price level. In other words, I assume that there exists an underlying equilibrium in the money market that supports the observed price level. However, I depart from the above-mentioned models in allowing a time-varying central tendency for both the real interest rate and the expected inflation rate variables.[3] This allows me to indirectly account for monetary policy in my framework, in that these two variables can be interpreted as proxies for Federal Reserve Board interest rate targets, as in Balduzzi, Das, and Foresi (1995) and Jegadeesh and Pennacchi (1996), for example.

As regards the real economy, I assume a single technology producing a single physical good, and that production output follows a stochastic process. In line with a CIR-type general equilibrium framework, the logarithmic utility hypothesis underlying the model implies that the expected rate of return on the production process should be equal to the sum of the real interest rate and the variance of the production process (see Breeden (1986)). However, in practice, the relationship between the real interest rate and expected output growth can change significantly over time. Chapman (1997) shows that this relationship has been quite unstable in the U.S. over the last 50 years and has also exhibited a negative sign for some periods. In fact, recursively regressing quarterly GDP growth rates against a naive estimator of the real interest rate, I observe that the slope coefficient varies significantly over time.[4] For this reason, in order to better describe the dynamics of output, I relax the CIR general equilibrium restriction by allowing the conditional mean of output growth to be modeled as an exogenous stochastic variable, which is assumed to be related to the real interest rate, both in the drift and in the innovation term.

I assume that, under the physical probability measure, the state variables, i.e., the instantaneous real interest rate r and expected inflation rate π, and their stochastic long-term means, θ and ξ, respectively, along with the conditional mean of output growth, α, follow a joint elastic random walk process,

among activities, and the equilibrium value of the riskless interest rate and nominal and real bond prices. As shown by CIR (1985a), in equilibrium all wealth should be invested in the physical production process. Therefore, equilibrium is characterized by a zero supply of bonds and no borrowing and lending.

[3]Dewachter and Lyrio (2006) also assume a stochastic central tendency for the inflation rate and the real interest rate. They adopt a rather simple specification: The central tendency of the real interest rate is assumed to be a linear function of the central tendency of the inflation rate, and the long-term means of these variables are fixed at their initial values.

[4]The regressions use a fixed 15-year window of quarterly data beginning in 1960:Q1. Proceeding recursively, I obtain parameter estimates for the period from 1975:Q1 to 2005:Q4. The estimator of the real interest rate is constructed by estimating the expected inflation rate from the one-step forecast of an ARIMA$(1, 0, 1)$ model applied to the logarithm of the GDP deflator and then subtracting this series from the observed three-month zero coupon yield. Consistent with Chapman (1997), I find that the estimated slope coefficient is, on average, around 0 and ranges between 0.68 and -0.36.

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$$ds = (\phi + \Gamma s)dt + \Sigma dz, \tag{1}$$

where
$$s \equiv \begin{pmatrix} r \\ \pi \\ \theta \\ \xi \\ \alpha \end{pmatrix}, \quad \phi \equiv \begin{pmatrix} 0 \\ 0 \\ \phi_\theta \\ \phi_\xi \\ \phi_\alpha \end{pmatrix}, \quad \text{and}$$

$$\Gamma \equiv \begin{pmatrix} \gamma_{rr} & \gamma_{r\pi} & -\gamma_{rr} & -\gamma_{r\pi} & \gamma_{r\alpha} \\ \gamma_{\pi r} & \gamma_{\pi\pi} & -\gamma_{\pi r} & -\gamma_{\pi\pi} & 0 \\ 0 & 0 & \gamma_{\theta\theta} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{\xi\xi} & 0 \\ \gamma_{\alpha r} & 0 & 0 & 0 & \gamma_{\alpha\alpha} \end{pmatrix}.$$

Here, Σ is a diagonal matrix, and z is a vector of standard Brownian motions.

I assume that both the macroeconomic variables, the production output q and the price level p, evolve according to a Gaussian process,

$$\frac{dm}{m} = \Gamma_m s dt + \Sigma_m dz_m, \tag{2}$$

where
$$\frac{dm}{m} \equiv \begin{pmatrix} \frac{dq}{q} \\ \frac{dp}{p} \end{pmatrix}, \quad \Gamma_m \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Here, Σ_m is a diagonal matrix, and z_m a vector of standard Brownian motions.

Innovations in the state variables, output and price level, could be mutually correlated and the correlation matrices $\Psi \equiv E_t\{dz, dz'\}$, $\Psi_m \equiv E_t\{dz_m, dz_m'\}$, and $\Xi \equiv E_t\{dz, dz_m'\}$ full.

From equation (2), expressions for logarithmic changes in production and price level over time interval $[t, t+\tau]$ can be obtained. For both these expressions and the stochastic differential equations (1), a closed-form solution can be derived and the dynamics of the economy summarized by the following set of equations:

$$E_t\{s(t+\tau)\} = a(\tau) + B(\tau)s(t), \tag{3}$$

$$\mathrm{Cov}_t\{s(t+\tau), s(t+\tau)'\} = C(\tau), \tag{4}$$

$$E_t\left\{\ln\left[\frac{m(t+\tau)}{m(t)}\right]\right\} = h(\tau) + J(\tau)s(t), \tag{5}$$

$$\mathrm{Cov}_t\left\{\ln\left[\frac{m(t+\tau)}{m(t)}\right], \ln\left[\frac{m(t+\tau)}{m(t)}\right]'\right\} = M(\tau), \quad \text{and} \tag{6}$$

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$$\text{(7)} \qquad \text{Cov}_t \left\{ s(t+\tau), \ln \left[\frac{m(t+\tau)}{m(t)} \right]' \right\} \quad = \quad Q(\tau),$$

where $\tau > 0$, and where $a(\tau)$, $B(\tau)$, $C(\tau)$, $h(\tau)$, $J(\tau)$, $M(\tau)$, and $Q(\tau)$ are nonlinear transformations of the original set of coefficients in equations (1)–(2).

Equations (3)–(7) completely describe the dynamics of the economy. This structure implies that cross-equation restrictions link the dynamics of all the variables in the economy, that is, the state variables, output, and price level.

B. Risk Premia and Term Structure of Interest Rates

In the CIR (1985a) framework, the volatility and correlation structures endogenously determine the market prices of risk,

$$\text{(8)} \qquad \Lambda_0 \quad \equiv \quad \Sigma^{-1} \left(\text{Cov}_t \left\{ ds, \frac{dq}{q} \right\} + \text{Cov}_t \left\{ ds, \frac{dp}{p} \right\} \right) \quad = \quad \Xi \Sigma_m \iota,$$

with $\iota' \equiv \begin{pmatrix} 1 & 1 \end{pmatrix}$.

The homoskedasticity assumption underlying the Gaussian structure of the model implies that risk premia are constant. However, following Duffee (2002) (see also Duarte (2004)), I adopt the "essentially affine" specification, which allows the market price of risk to vary independently of interest rate volatility. In this case, market prices of risk are assumed to be affine in the state variables,

$$\text{(9)} \qquad \Lambda(t) \quad \equiv \quad \Lambda_0 + \Lambda_s s(t),$$

where Λ_0 is specified as above and Λ_s is a matrix of free parameters. This specification implies that the matrix of mean reversion coefficients under the risk-adjusted probability measure is given by $\tilde{\Gamma} \equiv \Gamma - \Sigma \Lambda_s$. However, some constraints on the form of matrix Λ_s must be imposed in order to preserve a structure consistent with θ and ξ serving as the stochastic central tendencies of r and π under the risk-adjusted probability measure. Therefore, I have

$$\Lambda_s \quad \equiv \quad \begin{pmatrix} \lambda_{rr} & \lambda_{r\pi} & -\lambda_{rr} & -\lambda_{r\pi} & \lambda_{r\alpha} \\ \lambda_{\pi r} & \lambda_{\pi\pi} & -\lambda_{\pi r} & -\lambda_{\pi\pi} & 0 \\ 0 & 0 & \lambda_{\theta\theta} & 0 & 0 \\ 0 & 0 & 0 & \lambda_{\xi\xi} & 0 \\ \lambda_{\alpha r} & 0 & 0 & 0 & \lambda_{\alpha\alpha} \end{pmatrix}.$$

This model admits a closed-form solution for the equilibrium price of a nominal unit discount bond with maturity τ. The nominal term structure takes the form

$$\text{(10)} \qquad Y(t\,;t+\tau) \quad = \quad \kappa_0(\tau) + \kappa(\tau) s(t),$$

where the coefficients $\kappa_0(\tau)$ and $\kappa(\tau)$ depend on the underlying model parameters.

Instantaneous expected excess returns on nominal bonds are time-varying. In fact, by defining with $F(t\,;t+\tau)$ the price at current time t of a nominal unit

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discount bond with maturity τ and with $R(t)$ the instantaneous nominal interest rate at time t, $R(t) \equiv Y(t\,;t+0)$, I have

$$E_t\left\{\frac{dF(t\,;t+\tau)}{F(t\,;t+\tau)}\right\} - R(t)dt \quad = \quad -\tau\kappa(\tau)\Sigma\Lambda(t). \tag{11}$$

Instantaneous expected excess returns can be decomposed into their components such that the contribution of each state variable can be identified.

I can also determine the closed-form solution for the equilibrium price of a real unit discount bond with maturity τ and, therefore, an expression for the real term structure,

$$y(t\,;t+\tau) \quad = \quad \omega_0(\tau) + \omega(\tau)s(t), \tag{12}$$

where the coefficients $\omega_0(\tau)$ and $\omega(\tau)$ are nonlinear functions of the original set of parameters.

The actual[5] expected inflation rate for the maturity τ can be obtained from expression (5) above and is affine in the state variables. The difference between the τ-maturity nominal yield and the sum of the τ-maturity real interest rate and actual expected inflation rate is a variable that comprises an inflation risk premium and the Jensen's inequality term. The inflation risk premium included in the nominal term structure depends on the covariance between inflation and output growth rates, as this premium is linked to the bond's ability to hedge against a decrease in consumption.[6] The essentially affine specification implies that the inflation risk premium at time t on a bond with maturity τ is time-varying and can be calculated as

$$\begin{aligned} W(t\,;t+\tau) \quad = \quad & Y(t\,;t+\tau) - y(t\,;t+\tau) - \frac{1}{\tau}E_t\left\{\ln\frac{p(t\,;t+\tau)}{p(t)}\right\} \\ & + \frac{1}{2\tau}\mathrm{Var}_t\left\{\ln\frac{p(t\,;t+\tau)}{p(t)}\right\}. \end{aligned} \tag{13}$$

This expression shows that the greater the variability in the price level, the higher the inflation risk premium required on nominal bonds.

III. Empirical Results

In this section, I present the results of the estimation of the model for U.S. data over the 1960–2005 sample period. First, I show the model estimates of the unobservable term structures of real interest rates, expected inflation rates, and inflation risk premia. Then I evaluate the forecasting ability of the model for future inflation and GDP growth and compare its performance with that of alternative models.

[5] The term "actual" indicates that these are the inflation expectations calculated under the physical, rather than the risk-adjusted, probability measure.

[6] See, among others, Benninga and Protopapadakis (1983), Breeden (1986), and Abken (1993).

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A. Data

The empirical investigation is based on U.S. quarterly data from the first quarter of 1960 through the fourth quarter of 2005. Bond yields are end-of-quarter annualized zero coupon yields with maturities of 3 months and 1, 3, 5, 10, and 20 years. For the period between the first quarter of 1960 and the first quarter of 1991, the data are taken from McCulloch and Kwon (1993). From the second quarter of 1991 through the fourth quarter of 2005, zero coupon yields are calculated from prices of U.S. Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities).[7]

As a measure of economic activity, I use seasonally adjusted data on real GDP, expressed in constant dollars, from the U.S. Department of Commerce's Bureau of Economic Analysis. The seasonally adjusted GDP deflator is used as a proxy for the price level.

B. Estimation Method

The model is estimated using data available only at discrete observation intervals. The discrete time transformation of the continuous time model can be obtained by exploiting the solution to the stochastic differential equations that describe the dynamics of the variables. The resulting discrete time state space form is composed of state transition equations related to unobserved state variables and measurement equations related to bond yields, output growth, and inflation.

The parameters of the state space model are estimated by the maximum likelihood method with the Kalman filter algorithm used to calculate the values of the unobserved state variables.[8]

C. Estimation Results

1. Specification Tests

The model imposes several moment restrictions and encompasses more parsimonious models such as, for example, the PEN (1991) model. In order to test the adequacy of the model, I perform some preliminary specification tests following Duffee (2002) and Ang et al. (2008).

First, I apply a generalized method of moments (GMM)-type test to assess the closeness of the estimated unconditional moments to the sample moments. This test is based on the point statistic $H = (h - \bar{h})' \Sigma_h^{-1} (h - \bar{h})$, where $\bar{h}$ represents

[7] These data were kindly provided by Stephen Schaefer, who computed bond yields from prices of U.S. Treasury STRIPS supplied by Street Software.

[8] For the covariance matrices relating error terms in the state variables, output growth, and inflation, I impose all the cross-equation restrictions implied by the theoretical model, which further helps to identify the model. The deviation of actual yields from their theoretical values is assumed to be due to "observation error." In contrast, the errors in the equations for output growth and inflation are "forecasting errors," which arise from the difference between actual and expected values for output growth and inflation. In the state space form, both these error types are referred to as "measurement errors." I assume that observation errors in bond yields are mutually uncorrelated and are not correlated with forecasting errors in output growth and inflation. Moreover, in order to limit the number of parameters that must be estimated, I assume that the errors on bond yields are cross-sectionally homoskedastic.

the estimates of the unconditional moments obtained from the sample, h represents the unconditional moments estimated by the model, and Σ_h represents the covariance matrix of the sample estimates of the unconditional moments, which is estimated using GMM with the Newey-West (1987) correction for heteroskedasticity and autocorrelation. Under the null hypothesis, this statistic is distributed as $\chi^2(n)$, where n is the number of overidentifying restrictions. I test for first and second moments of bond yields and yield changes, for maturities of 3 months and 1, 3, 5, 10, and 20 years, and for GDP growth and inflation rates with time horizons of 3, 6, 9, and 12 months.

Second, I use a Lagrange multiplier (LM) statistic to test for the presence of autoregressive conditional heteroskedasticity (ARCH) effects in the estimation residuals, which might arise from the fact that I am using homoskedastic Gaussian-type processes in modeling financial and macroeconomic variables.

Third, I apply a GMM-based test for first-order serial correlation in the estimated scaled residuals of the measurement equation. I test the following null hypothesis: $\mathrm{E}(e_t e_{t-1}) = 0$.

Finally, using a Wald statistic, I test a null hypothesis involving restrictions on the model parameter vector. The restricted models are represented by: i) a two-factor model in which central tendencies are constant and the conditional mean of output growth is fixed at a constant value; ii) a three-factor model in which central tendencies are constant; iii) a four-factor model in which the conditional mean of output growth is nonstochastic; and iv) a five-factor completely affine specification in which risk premia are constant (the coefficients of matrix Λ_s in equation (9) are all equal to 0).

The results in Table 1 show that the model performs relatively well. In Panel A, I observe that the unconditional moments implied by the model estimates are within one standard error (SE) of data moments, with the exception of the variance of the GDP growth rate. The LM statistic shows that the model captures the heteroskedasticity within the bond yield data, whereas the presence of ARCH-type effects in the estimated residuals does not look negligible in the case of GDP growth and inflation rates. The model's goodness of fit is reflected in Panel B, where the χ^2 test does not allow me to reject the null hypothesis that the first and second moments of yields, yield changes, and inflation rates over various time horizons estimated by the model match the sample moments. Consistent with the previous tests, I find that the model tends to underestimate the volatility of GDP growth rates and that the null hypothesis must be rejected.

Panel C of Table 1 shows that, as predicted by the model, the estimated residuals for bond yields and quarterly GDP growth and inflation rates are not serially correlated. Moreover, I observe that the various restrictions on the model's parameters are rejected, indicating that the time-varying central tendencies, the stochastic conditional mean of output growth, and the essentially affine specification for the risk premia all play a significant role in fitting the relationship between the term structure and macroeconomic variables.

Regarding the model's goodness of fit, the estimated bond yield errors are relatively low, with no evidence of systematic under/overpricing. In fact, the errors have both positive and negative mean values at the various maturities, and the root mean squared errors (RMSEs) are below 20 basis points (bps). It is worth

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TABLE 1
Specification Tests

Table 1 shows summary statistics on specification tests based on the maximum likelihood–Kalman filter estimates of the macro term structure model over the sample period 1960:Q1–2005:Q4. Panel A compares the moments estimated by the model with the sample moments of bond yields and annualized quarterly GDP growth and inflation rates. Data are expressed in basis points (bps). The standard errors (SE) are calculated using GMM with the Newey-West (1987) correction. The last column reports the p-values of LM statistic testing for the presence of ARCH effects in the residuals. Panel B reports the p-values of a χ^2 test based on the point statistic

$$H = (h - \bar{h})' \Sigma_{\bar{h}}^{-1} (h - \bar{h}),$$

where $\bar{h}$ and h are the estimates of the unconditional moments provided by the sample and the model, respectively, and $\Sigma_{\bar{h}}$ is the covariance matrix of the sample estimates of the unconditional moments, estimated using GMM with the Newey-West correction. I test for first and second moments of bond yields and yield changes, with maturity of 3 months and 1, 3, 5, 10, and 20 years; and GDP growth and inflation rates, with time horizons of 3, 6, 9, and 12 months. Panel C contains the p-value of a GMM-based test for the null hypothesis of no serial correlation in the model-scaled residuals and the p-values of a Wald statistic testing the null hypothesis that n restrictions can be placed on the model parameter vector: i) constant central tendencies and no stochastic conditional mean of output growth (the α variable is a constant); ii) constant central tendencies; iii) no stochastic conditional mean of output growth; and iv) completely affine specification implying constant risk premia.

Panel A. Model-Estimated Sample Moments

	Mean			Variance			ARCH Test
Variable	Model	Sample	SE	Model	Sample	SE	p-Value
3-month yield	571	567	64	8.3	8.1	2.8	0.34
1-year yield	605	611	65	7.9	8.2	2.8	0.68
3-year yield	655	653	63	7.3	7.3	2.4	0.91
5-year yield	676	674	61	7.0	6.8	2.3	0.66
10-year yield	696	697	59	6.3	6.2	2.0	0.42
20-year yield	719	719	57	5.6	5.7	1.7	0.40
GDP growth rate	322	327	33	6.2	11.5	2.2	0.01
Inflation rate	370	370	54	5.1	5.8	1.6	0.06

Panel B. χ^2 Test on Unconditional Moments

	Yields		Changes in Yields		GDP Growth Rates		Inflation Rates	
	$n = 6$		$n = 6$		$n = 4$		$n = 4$	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
p-value	1.00	1.00	1.00	0.93	0.13	0.04	1.00	0.99

Panel C. χ^2 Test on Residual Correlation and Wald Test on Parameter Restrictions

	Residuals Correlation	Constant Means No α Variable	Constant Means	No α Variable	Constant Risk Premia
	$n = 8$	$n = 30$	$n = 19$	$n = 13$	$n = 9$
p-value	1.00	0.00	0.00	0.00	0.00

noting that the model also fits well the yields not included in the estimation sample (2-, 4-, and 6- to 9-year maturities).

2. Estimated Parameters and State Variables

Table 2 reports the estimated parameter values under the physical probability measure. I note that the real interest rate r exhibits significant mean reversion toward its time-varying central tendency θ, which is more persistent. In contrast, both the expected inflation rate π and its central tendency ξ are close to an RW. The estimated long-term means of the central tendencies θ and ξ are equal, respectively, to 1.7% and 3.8%.

Figure 1 shows the Kalman filter estimated series of the instantaneous real interest rate and expected inflation rate, their time-varying central tendencies,

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TABLE 2

Estimated Parameter Values

Table 2 shows the parameter values of the macro term structure model estimated by the maximum likelihood–Kalman filter method over the sample period 1960:Q1–2005:Q4. Asymptotic standard errors are in parentheses. Panel A shows the drift term coefficients of the stochastic process describing the dynamics of the state variables. Panel B contains the volatility coefficients of those processes and the coefficients which determine the market price of risk. Panel C shows the volatility coefficients of the stochastic process describing the dynamics of the macro variables and the correlation coefficients relating innovations in the state variables and innovations in the macro variables.

Panel A. Drift Term Coefficients of the State Variables

	ϕ	Γ				
Variable		r	π	θ	ξ	α
r		–0.6110 (0.0125)	–0.1301 (0.0125)	0.6110	0.1301	–0.0487 (0.0186)
π		–0.0582 (0.0072)	–0.0721 (0.0071)	0.0582	0.0721	
θ	0.0037 (0.0020)			–0.2187 (0.0059)		
ξ	0.0020 (0.0017)				–0.0515 (0.0304)	
α	0.0191 (0.0013)	–0.0391 (0.0065)				–0.5419 (0.0021)

Panel B. Volatility Coefficients of the State Variables and Risk Premium Coefficients

	Σ	Λ_S				
Variable	(diag.)	r	π	θ	ξ	α
r	0.0243 (0.0057)	4.7529 (0.1796)	5.9385 (0.1296)	–4.7529	–5.9385	—
π	0.0141 (0.0052)	2.6685 (0.3861)	12.6708 (0.1767)	–2.6685	–12.6708	
θ	0.0222 (0.0060)			–2.2262 (0.0848)		
ξ	0.0091 (0.0067)				–4.6782 (0.2331)	
α	0.0654 (0.0084)	—				1.9994 (0.4601)

Panel C. Volatility Coefficients of the Macro Variables and Correlation Coefficients

	Σ_m	Ψ_m	Ψ				Ξ	
Variable	(diag.)	P	π	θ	ξ	α		
r			–0.2842 (0.0129)	0.4172 (0.0217)	—	–0.1926 (0.0026)	0.2571 (0.0097)	—
π					0.3353 (0.0113)	–0.1959 (0.0051)	—	–0.0431 (0.0209)
θ					–0.3925 (0.0080)	—	0.0793 (0.0154)	—
ξ						—	–0.2241 (0.0155)	0.2775 (0.0098)
α							0.5009 (0.0099)	—
q	0.0036 (0.0018)	0.2339 (0.0022)						
p	0.0032 (0.0010)							

and the conditional mean of output growth. Consistent with previous estimates (see, e.g., PEN (1991), Sun (1992), and Ang et al. (2008)), the real interest rate becomes negative in the 1970s and exhibits a sharp increase in level and volatility in the 1979–1982 period, when the Federal Reserve changed its monetary policy procedures. Thereafter, especially in the second half of the sample, the real

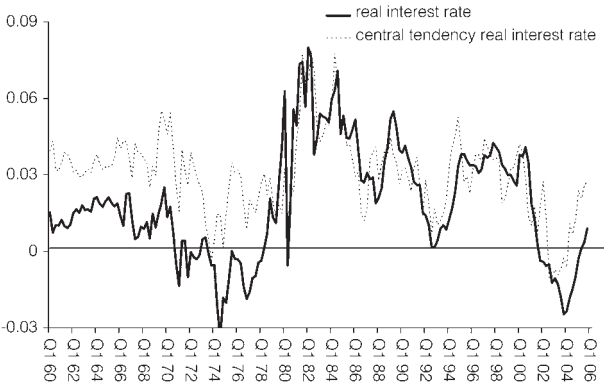
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FIGURE 1

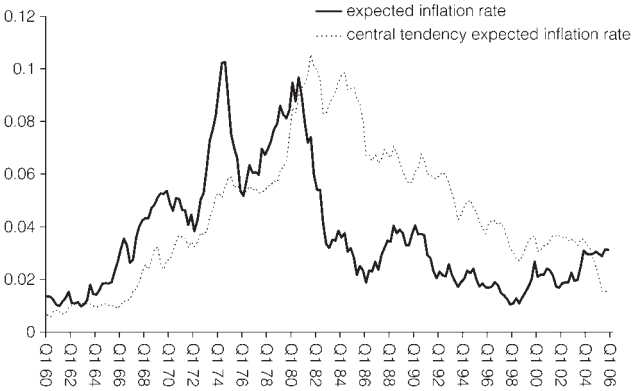
Kalman Filter Estimates of the Unobservable State Variables

Figure 1 shows the Kalman filter estimated series of the unobservable state variables over the sample period 1960:Q1–2005:Q4. Graph A shows the instantaneous real interest rate and its time-varying central tendency, Graph B the instantaneous expected inflation rate and its time-varying central tendency, and Graph C the conditional mean of output growth and, as a comparison, the cyclical component of output growth, which is estimated by applying the Hodrick-Prescott (1997) filter to the annual rate of growth of real GDP.

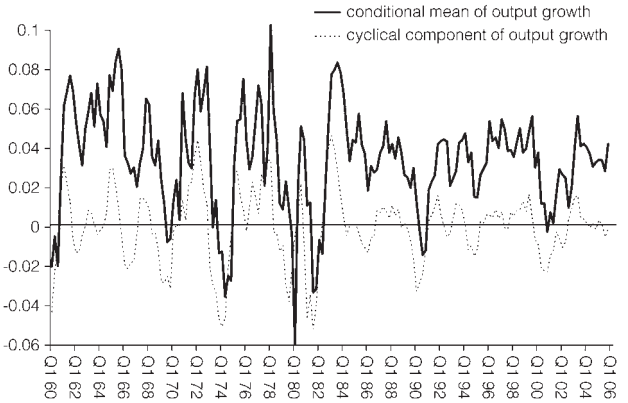
Graph A. Real Interest Rate



Graph B. Expected Inflation Rate



Graph C. Conditional Mean of Output Growth



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interest rate seems to revert quickly toward its central tendency. The expected inflation rate declines considerably after the so-called "Fed experiment." The series estimated for the stochastic central tendencies appears reasonable.[9]

The estimated series of the conditional mean of output growth exhibits a relatively high mean reversion toward a 3.5% long-term mean, and a much higher volatility than the other state variables. As shown in Figure 1, this variable seems to be highly related to the cyclical component of output growth, which is estimated by applying the Hodrick-Prescott (1997) filter to the annual rate of growth of real GDP. I note that the volatility of the conditional mean of output growth decreases sharply during the post-1984 Great Moderation in U.S. inflation. In this period, its correlation with both the real interest rate and the expected inflation rate declines considerably. This evidence seems to be consistent both with Chapman (1997), who claims that the relationship between the real interest rate and expected output growth has been quite unstable in the last few decades, and with Stock and Watson (2007), who show that some of the macroeconomic relationships implicit in models that use the Phillips curve have weakened in the post-1984 period.

D. Real and Expected Inflation Term Structures

The model allows extrapolation from nominal bond prices of the unobservable term structures of real interest rates and inflation expectations. A natural benchmark to evaluate model estimates would be represented by market values extracted from inflation-indexed bonds. However, a liquid market with a relatively long series of observations exists only in the U.K.;[10] the U.S. Treasury has only recently begun to issue inflation-indexed bonds. Estimates of the U.S. real term structure calculated from such bonds begin only in 1997. A different benchmark must be used to analyze the properties of the real and expected inflation term structures estimated by the model for the 1960–2005 sample period. In particular, I use ex post realized real yields and inflation rates. The τ-maturity real interest rate realized at time t is calculated as the difference between the time t observed τ-maturity nominal yield and the annualized inflation rate realized between time t and $t + \tau$. The latter term is used as a benchmark to test the predictive ability of the model term structure estimates for future inflation rates.

Panel A of Table 3 contains summary statistics on the estimated real and expected inflation term structures for maturities of up to five years. On average, both these term structures are upward sloping, with volatility tending to decrease with the time horizon. As in Ang et al. (2008), I find that real rates are procyclical and

[9]Following Jegadeesh and Pennacchi (1996), I might interpret the sum of the central tendencies as a proxy for the unobservable nominal target rate of Fed monetary policy. Long-run targets reflect individuals' expectations about medium- to long-term inflation and economic growth and, implicitly, individuals' beliefs regarding future monetary policy. My estimates imply that the half-life (the time it takes for the variable to revert halfway to its initial value) of the short-term nominal rate is about two years, whereas its central tendency is very persistent. This result appears to be in contrast to the evidence presented in Jegadeesh and Pennacchi (1996), where the central tendency displays higher mean reversion than the short rate.

[10]Empirical evidence on this market has been provided by, among others, Brown and Schaefer (1994), Barr and Campbell (1997), and Evans (1998).

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expected inflation is countercyclical. In fact, estimated real rates are generally higher during expansions than recessions, whereas expected inflation rates behave in the opposite way, with the volatilities of both variables increasing during recessions.[11]

TABLE 3

Real and Expected Inflation Term Structures

Table 3 shows summary statistics on the implicit unobservable real and expected inflation term structures estimated by the macro term structure model for the sample period 1960:Q1–2005:Q4. In Panel A, I compare the moments estimated by the model with the sample moments of ex post realized real yields and inflation rates with maturity from 1 to 5 years. The standard errors (SE) are calculated using GMM with the Newey-West (1987) correction. Table 3 also reports the *p*-values of a χ^2 test based on the point statistic

$$H = (h - \bar{h})' \Sigma_h^{-1} (h - \bar{h}),$$

where $\bar{h}$ and h are the estimates of the unconditional moments provided by the sample and the model, respectively, and Σ_h is the covariance matrix of the sample estimates of the unconditional moments, estimated using GMM with the Newey-West (1987) correction. The last column of the table contains the correlation between the series estimated by the model and the corresponding ex post realized values. Panel B illustrates the change in the correlation between real interest rates and expected inflation rates after the so-called "Volcker experiment." Data are expressed in basis points.

Panel A. Estimated versus Realized Sample Moments

	Mean			Variance			Corr.
	Model	Sample	SE	Model	Sample	SE	
Real Interest Rates							
1 year	222	240	54	4.4	6.1	1.7	0.97
2 years	245	268	62	3.9	7.5	2.1	0.94
3 years	261	288	64	3.3	8.1	2.3	0.92
4 years	272	305	67	2.8	8.5	2.4	0.90
5 years	276	318	70	2.4	8.8	2.4	0.87
p-value *H* statistic			0.96			0.00	
Exp. Inflation Rates							
1 year	374	371	54	5.0	5.2	1.5	0.97
2 years	379	375	54	5.0	4.9	1.4	0.91
3 years	384	379	53	4.9	4.6	1.2	0.86
4 years	389	383	53	4.9	4.3	1.1	0.81
5 years	395	387	52	4.8	4.1	1.0	0.77
p-value *H* statistic			1.00			0.97	

Panel B. Correlation between Real Rates and Expected Inflation Rates

	Pre-1979 (1960:Q1–1978:Q4)					Post-1979 (1982:Q1–2005:Q4)				
	Real Rates		Exp. Inflation			Real Rates		Exp. Inflation		
Maturity (years)	Mean	Std. Dev.	Mean	Std. Dev.	Corr.	Mean	Std. Dev.	Mean	Std. Dev.	Corr.
1	105	118	436	246	–0.76	295	214	268	90	0.37
3	157	107	434	242	–0.77	314	194	284	93	0.44
5	178	96	429	237	–0.77	309	172	296	95	0.49
7	186	86	423	233	–0.76	297	152	307	96	0.51
10	187	73	415	225	–0.76	276	127	320	98	0.54

[11]For example, I estimate that the two-year real rate is, on average, 2.46% (with the Newey-West (1987) heteroskedasticity and autocorrelation consistent SE of 0.30%) during expansions, and 2.19% (SE = 0.78%) during recessions. The corresponding two-year expected inflation rate is on average 3.51% (SE = 0.34%) during expansions and 4.92% (SE = 0.92%) during recessions. I consider expansion and recession periods as defined by the National Bureau of Economic Research (NBER).

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I compare the moments estimated by the model with the sample moments of ex post realized real yields and inflation rates with maturities from one to five years using the χ^2 test based on the point statistic H described above.

I observe that the unconditional moments implied by the model predictions for real interest rates and inflation rates are within one SE of realized data moments, with the exception of the variance of real yields. Indeed, the volatility of estimated real yields is relatively low with respect to that observed ex post. To a certain extent, this might be due to the fact that the ex post real yields include an implicit inflation risk premium. In general, however, the model seems to provide relatively accurate predictions on the dynamics of the levels of both the real and inflation term structures. In fact, the correlation between the series estimated by the model and the corresponding ex post realized values is very high.

Turning to the time series of the estimated term structures of real interest rates and expected inflation rates, I observe a change in the sign of their correlation following the 1979–1982 "Volcker experiment." Indeed, consistent with the evidence presented by Goto and Torous (2003), I show in Panel B of Table 3 that the correlation between real interest rates and expected inflation rates is significantly negative before 1979 and becomes positive after 1982. The average shape of the term structures also changes. The term structure of real rates is upward sloping before 1979 and humped at much higher levels after 1982. The term structure of expected inflation rates moves from downward sloping to upward sloping, with a sharp decline in level and volatility.

As pointed out by Goto and Torous (2003), this could have resulted from an aggressive anti-inflationary monetary policy implemented by the Federal Reserve beginning in 1982, which implies that nominal interest rates move more than one-for-one with inflation expectations, inducing a positive relationship between real interest rates and expected inflation. Indeed, by regressing τ-maturity nominal yields against τ-maturity real interest rates and expected inflation rates for this period, I obtain coefficients significantly higher than 1. This implies that the Fisher relationship, that is, the independence between real interest rates and inflation expectations, is violated. Moreover, this evidence shows that the so-called Mundell (1963)–Tobin (1965) effect, which has been documented extensively in the empirical literature (see, among others, Mishkin (1992), Boudoukh (1993)), does not hold in the face of the correlation between real interest rates and expected inflation rates in the post-1982 period. I show below that this result has a significant impact on the inflation risk premium.

E. Risk Premia

Figure 2 shows that the estimated inflation risk premia (IRP) vary significantly over time and across maturities. I note a clear change in the sign of IRP at the beginning of the 1980s (i.e., during the Volcker experiment). This result is strikingly similar to that obtained by Goto and Torous (2003) using a regime-switching approach, and it implies that a positive premium is required by investors on nominal bonds when monetary policy strictly controls inflation. As in Buraschi and Jiltsov (2005), I find that the average term structure of IRP has a positive

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slope, with values ranging between 10 bps (one-year bond) and 90 bps (10-year bond).

FIGURE 2

Inflation Risk Premia and Expected Excess Bond Returns

Figure 2 shows the estimated series of the inflation risk premia (IRP) for maturities of 1, 5, and 10 years, and of the average (across maturities) instantaneous expected excess bond returns (EER) over the sample period 1960:Q1–2005:Q4. Data are expressed in basis points.

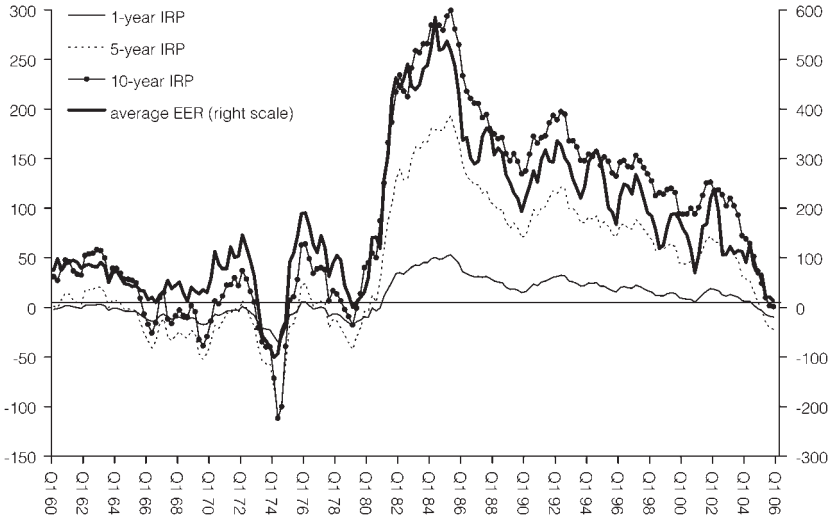


Figure 2 also shows the time-series behavior of the average (across maturities) instantaneous expected excess bond returns (EERs). I observe that these values are, on average, around 0 before 1982 and that they become significantly positive thereafter. Panel A of Table 4 presents summary statistics on the estimated instantaneous EERs for various maturities and for each state variable. I observe that the instantaneous risk premium on the real interest rate remains almost constant with maturity (around 25 bps), whereas that for expected inflation increases with bond maturity and is around 35 bps on average. The total instantaneous risk premium has an increasing structure and ranges between 42 bps (one-year bond) and 286 bps (10-year bond).

As in CP (2005), I find that EERs are linked to the business cycle and tend to be more volatile during recessions than during expansions. A similar result applies to estimated IRP.

In Panel B of Table 4, I compare the predictive power of model estimates for one-year excess bond returns with that of the CP (2005) single-factor model. The single factor in the CP (2005) model is a linear function of forward rates that is obtained by running the following regression:

$$(14)\quad \overline{\mathrm{ER}}(t+1) \;=\; \beta_0 + \sum_{i=1}^{5} \beta_i f(t;0,i) + \bar{u}(t+1) \;=\; (\beta' f(t)) + \bar{u}(t+1),$$

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TABLE 4

Instantaneous Expected Excess Bond Returns

Table 4 examines the instantaneous expected excess bond returns on τ-period nominal zero coupon bonds implicit in the estimates of the macro term structure (MTS) model for the sample period 1960:Q1–2005:Q4. Panel A contains summary statistics on estimated instantaneous expected risk premia on each state variable. Panel B compares the predictive power of the model estimates for one-year excess bond returns with those provided by the Cochrane and Piazzesi (CP) (2005) model. The following regressions are estimated in the CP (2005) case:

$$\overline{\mathrm{ER}}(t+1) = \beta_0 + \sum_{i=1}^{5} \beta_i f(t;0,i) + \bar{u}(t+1) = \left(\beta' f(t)\right) + \bar{u}(t+1) \quad \text{and}$$

$$\mathrm{ER}(t+1;T) = b_T\left(\beta' f(t)\right) + u(t+1;T), \quad T = 2, 3, 4, \text{ and } 5,$$

where $\mathrm{ER}(t+1;T) \equiv [F(t+1;T-1)/F(t;T)] - [1/F(t;1)]$ is the one-year excess return on a T-maturity bond and $\overline{\mathrm{ER}}(t+1)$ is the average of excess bond returns for $T = 2, 3, 4$, and 5. $F(t;T)$ represents the price at time t of a T-maturity unit zero coupon bond, and $f(t;T-1,T)$ is the forward rate for loans between time $t+T-1$ and $t+T$. In the MTS case, the same two regressions are estimated by using as regressors the annualized estimated T-maturity model risk premia, $T = 2, 3, 4$, and 5, instead of the forward rates. In parentheses, the Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors correcting for overlapping observations are presented.

Panel A. Estimated versus Realized Sample Moments

	r	π	θ	ξ	α	Total	
Maturity (years)	Mean	Mean	Mean	Mean	Mean	Mean	Std. Dev.
1	17	15	4	4	1	42	54
3	27	32	24	26	3	111	113
5	28	40	42	55	3	169	147
7	27	46	55	87	4	219	174
10	26	50	69	137	4	286	207

Panel B. Excess Bond Returns Forecasting Regressions

	CP		MTS	
Maturity (years)	b_T	$\bar{R}^2$	b_T	$\bar{R}^2$
2	0.49 (0.09)	0.30	0.49 (0.09)	0.30
3	0.84 (0.16)	0.29	0.83 (0.18)	0.29
4	1.18 (0.23)	0.31	1.17 (0.30)	0.30
5	1.49 (0.30)	0.30	1.51 (0.33)	0.31

where $\mathrm{ER}(t+1;T) \equiv [F(t+1;T-1)/F(t;T)] - [1/F(t;1)]$ is the one-year excess return on a T-maturity bond and $\overline{\mathrm{ER}}(t+1)$ is the average of excess bond returns for $T = 2$, 3, 4, and 5. Here, $F(t;T)$ represents the price at time t of a T-maturity unit zero coupon bond and $f(t;T-1,T)$ is the forward rate for loans between time $t+T-1$ and $t+T$.

In a second step, the following forecasting regressions are estimated:

$$\mathrm{ER}(t+1;T) = b_T\left(\beta' f(t)\right) + u(t+1;T), \qquad T = 2, 3, 4, \text{ and } 5. \tag{15}$$

I run the same two regressions by using as a predictor the annualized estimated T-maturity model risk premia, $T = 2$, 3, 4, and 5, instead of the forward rates.

I observe, consistent with CP (2005), that the normalized coefficients b_T tend to increase with the maturity of the bonds. The results show that the model's predictive power, measured in terms of $\bar{R}^2$, is comparable to that produced by the CP (2005) framework and explains about one-third of the time variation in

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one-year excess bond returns. As pointed out by CP (2005), this is considerably higher than the value obtained in the benchmark studies of Fama and Bliss (1987) and Campbell and Shiller (1991).[12] According to this evidence, it can be argued that including macroeconomic variables in a term structure model does indeed provide relevant information for the prediction of bond returns.

F. GDP Growth and Inflation Forecasts

In this section, I compare both the in- and out-of-sample accuracy of the macro term structure (MTS) model in predicting GDP growth and inflation rates with respect to other models. In particular, as competing models, I consider an essentially affine version of the PEN (1991) model, and the APW (2006) model.

The PEN (1991) two-factor model with constant long-term means and no adjustment for the conditional mean of output growth is nested within my term structure framework.[13] A comparison with the results obtained by this model allows me to evaluate the role of time-varying central tendencies and the conditional mean of output growth in the empirical properties of the model.

APW (2006) estimate the relationship between the term structure and macroeconomic variables in the context of a VAR model, in which no-arbitrage restrictions are imposed on the dynamics of bond yields. The APW model can be considered a nonstructural term structure model with macro factors (see, e.g., Evans and Marshall (2001), Ang and Piazzesi (2003)); in this respect, it represents an interesting alternative approach to my own.[14]

Moreover, as a benchmark for the evaluation of GDP growth and inflation predictions, I estimate the Harvey (HAR) (1989) term spread regression model and the RW model.[15]

[12]I estimate the Fama-Bliss (1987) regressions for my data set by regressing the one-year excess return on a T-maturity bond, $T = 2, 3, 4$, and 5, against the forward spread of the corresponding maturity. The estimated $\bar{R}^2$ ranges between 10% and 16%. For brevity, I do not report these regressions.

[13]In fact, the PEN (1991) model assumes that the state variables driving the dynamics of the economy are the instantaneous real interest rate and expected inflation rate following a joint elastic RW process with constant long-term means. The dynamics of the price level and the production output are described by Gaussian-type processes, with no stochastic conditional mean of output growth. The model is estimated using the maximum likelihood–Kalman filter method.

[14]In the original APW (2006) specification, the vector of state variables includes the short rate (three-month yield), the term spread (20-year minus three-month yield) and lagged annualized quarterly GDP growth rate. With respect to this model, I add two more factors: the "curvature" factor (three-month plus 20-year yield minus twice the five-year yield), and lagged annualized quarterly inflation rate. The five factors are assumed to follow a Gaussian VAR with one lag. Cross-equation restrictions stemming from the no-arbitrage condition are imposed and allow the model to endogenously produce expectations of future inflation and GDP growth rates over any time horizon. In estimating the parameters of the model, I adopt the consistent two-step procedure used by APW (2006). The analytical derivation of the PEN (1991) and APW (2006) models, and the estimated coefficients are available from the author.

[15]As a predictor for the annualized real GDP rate of growth between time t and $t + \tau$, the HAR (1989) model uses the slope of the term structure observed at time t. I estimate the model using the difference between the five-year and the one-year nominal yield as a measure of the slope of the term structure. Consistent with Atkeson and Ohanian (2001) and Stock and Watson (2007), the RW model uses the average of the current quarter and the preceding three quarters of inflation as its forecast for future periods.

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1. In-Sample Predictions

Table 5 shows the results of the in-sample regression of realized GDP growth (Panel A) and inflation rates (Panel B) against the corresponding expectations generated by the models.

TABLE 5

In-Sample Prediction of GDP Growth and Inflation Rates

Table 5 compares the in-sample predictive accuracy for GDP growth and inflation rates of the macro term structure (MTS) model and several competing models (i.e., the PEN (1991) two-factor model, the APW (2006) VAR model, a random walk (RW) model, and the HAR (1989) term spread regression model). Panels A and B show the estimation results of the regression of actual annualized logarithmic changes in the real GDP level and the GDP deflator, respectively, against the corresponding expectations generated by the model estimates for the sample period 1960:Q1–2005:Q4. In parentheses are the Newey-West (1987) heteroskedasticity and autocorrelation consistent standard errors. Panel C contains the root mean squared errors (RMSE), expressed in basis points, of the inflation predictions produced by the MTS and RW models for the periods from 1960:Q1 to 1983:Q4 and from 1984:Q1 to 2005:Q4.

Panel A. In-Sample Predictions of GDP Growth Rates

	MTS		PEN		APW		HAR	
Horizon	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$
3 months	1.26 (0.04)	0.85	–0.71 (0.25)	0.05	1.00 (0.21)	0.13	0.92 (0.36)	0.04
6 months	1.07 (0.02)	0.84	–0.89 (0.26)	0.13	1.23 (0.22)	0.22	1.09 (0.35)	0.10
1 year	0.80 (0.06)	0.57	–0.95 (0.25)	0.18	1.25 (0.24)	0.26	0.99 (0.31)	0.13
2 years	0.47 (0.09)	0.22	–0.91 (0.24)	0.20	1.08 (0.23)	0.23	0.86 (0.22)	0.18

Panel B. In-Sample Predictions of Inflation Rates

	MTS		PEN		APW		RW	
Horizon	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$	Slope Coeff.	$\bar{R}^2$
3 months	1.03 (0.02)	0.94	0.67 (0.15)	0.28	1.00 (0.04)	0.78	0.92 (0.08)	0.77
6 months	1.01 (0.02)	0.96	0.64 (0.15)	0.27	1.03 (0.05)	0.79	0.90 (0.09)	0.79
1 year	0.98 (0.04)	0.94	0.59 (0.16)	0.22	1.11 (0.08)	0.79	0.86 (0.10)	0.76
2 years	0.90 (0.08)	0.83	0.46 (0.16)	0.13	1.19 (0.14)	0.67	0.77 (0.11)	0.66

Panel C. Pre-1984 versus Post-1984 RMSE of Inflation Predictions

	Pre-1984		Post-1984	
Horizon	MTS	RW	MTS	RW
3 months	67	145	48	73
6 months	51	141	36	61
1 year	70	152	36	57
2 years	116	181	53	67

The output growth expectations implicit in the MTS model estimates represent relatively accurate predictions of actual GDP growth rates, although the accuracy of the fit tends to decrease as the time horizon lengthens. In fact, the $\bar{R}^2$ of the regression is 85% at the three-month horizon, and 57% at the one-year horizon. The competing models, in contrast, are poor in their ability to predict short-term GDP growth rates and to improve performance as time horizon lengthens. The

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HAR (1989) and APW (2006) estimators appear to be unbiased, whereas in the case of the PEN (1991) model I observe that the sign of the regression coefficient is wrong. This is an effect of the restriction imposed by the PEN framework (and, in general, by general equilibrium term structure models of the CIR type) on the relationship between expected output growth and real yields. In particular, it implies that output growth expectations are determined in direct relationship with the real interest rate. However, as illustrated above, the sign of this relationship can change over time because of the effect of exogenous factors, such as productivity shocks. This evidence underlines the relevance of the variable α, which captures the impact of such exogenous shocks on the dynamics of output.

The expectations produced by the MTS model represent accurate predictions of future inflation at all time horizons up to two years. Moreover, the estimator is unbiased. The inflation predictions produced by the model are more precise than those provided by alternative models. Again, the comparison with the PEN (1991) two-factor model allows me to appreciate the role of both the time-varying central tendencies and the conditional mean of output growth in improving the fit of the relationship between bond yields and macroeconomic variables.

The model's predictive properties are robust with respect to multiple time periods. According to the empirical evidence presented by Atkeson and Ohanian (2001), Fisher, Liu, and Zhou (2002), and Stock and Watson (2007), since the mid-1980s most modern macroeconometric models (for example, Stock and Watson (1999) and related models) fail to produce inflation forecasts that are more accurate than those generated by a naive RW model. Stock and Watson (2007) point out that the Great Moderation in U.S. inflation after 1984 has led to a marked decrease in inflation volatility. They observe that, on the one hand, this has lowered the size of forecasting errors; but on the other hand, it has induced a deterioration in the forecasting accuracy of structural macro models with respect to univariate naive models.

In Panel C of Table 5, I test this hypothesis by comparing the in-sample forecast RMSEs of the MTS and RW models in the pre-1984 and post-1984 periods. Consistent with Stock and Watson (2007), I observe that the MTS model yields more accurate predictions in the pre-1984 sample and that RMSE sharply decreases in the post-1984 sample. However, in contrast with the empirical findings cited above, I find that the MTS model significantly outperforms the RW model in the post-1984 period as well. This result is reinforced by the following analysis, which illustrates the out-of-sample forecasting properties of the models.

2. Out-of-Sample Forecasts

Out-of-sample forecasts are obtained by using the preceding 15 years of quarterly data to estimate the parameters needed to predict subsequent GDP growth and inflation rates over various time horizons. Proceeding recursively, I compute forecasts for all the competing models from 1975:Q1 to 2005:Q4 and compare these to realized GDP growth and inflation rates over the same period. As above, the models I consider are the MTS, PEN (1991), APW (2006), HAR (1989) (for GDP growth), and RW (for inflation). Moreover, I include the

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forecasts provided by the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (SUR).[16]

Table 6 summarizes the error properties of the estimated models over various time horizons and reports the values of the Diebold and Mariano (DM) (1995) statistic, which tests the null hypothesis of no difference in the accuracy of the forecasts generated by the alternative models with respect to those produced by the MTS model.[17]

Panel A of Table 6 shows that the GDP growth forecasts produced by the MTS model are more accurate than those generated by the alternative approaches for time horizons of up to one year. I observe that the MTS model does not systematically over/underestimate future GDP growth rates and that the mean of the forecast errors is very close to 0. The reduction in forecasting errors induced by the inclusion in the model of the stochastic central tendencies and the conditional mean of output growth is significant, as I can infer by comparing these errors with those of the PEN (1991) model. For longer time horizons, HAR's (1989) term spread appears to be the best predictor for future GDP growth.

The striking result that emerges from Panel B of Table 6 is the accuracy of the MTS model in forecasting inflation rates at time horizons of up to two years. The forecasting errors, measured in terms of RMSE, are around half those of the RW model. The DM (1995) test rejects the null hypothesis of no difference in the accuracy of MTS model forecasts with respect to competing models at all time horizons.

Consistent with Ang et al. (2007), I find that the SUR predictor outperforms the PEN (1991) and APW (2006) term structure models. However, according to the DM (1995) test, the SUR forecasts are not statistically more accurate than those generated by the RW process.

In order to test whether the forecasting properties of the MTS model are influenced by the change in the U.S. inflation regime beginning around the mid-1980s, in Panel C of Table 6 I compare the out-of-sample forecasts of the MTS and the RW models on a decade-by-decade basis. In particular, I report the ratio between the RMSE of the MTS and RW forecasts for three subperiods: 1975–1983, 1984–1994, and 1995–2005. I find that this ratio does not change

[16]This survey relies on macroeconomic forecasts supplied by several major economic research centers and financial institutions. Before 1990, this was known as the ASA/NBER survey. As in Ang et al. (2007), I consider median forecasts. However, using mean forecasts causes the results to not differ significantly.

[17]The DM (1995) statistic is calculated as the t-statistic obtained by regressing the difference in squared errors between the competing forecasts against a constant and correcting this equation for serial correlation. The statistical properties of the test can be summarized as follows. Let x_t denote the loss differential series (in my case, the difference in squared errors between the competing forecasts at time t). Under the null hypothesis that the population mean of the $\{x_t\}$ series is 0, the statistic

$$\bar{x} \Big/ \sqrt{2\pi \hat{g}_x(0)/T}$$

is asymptotically standard normal distributed, where $\bar{x}$ is the sample mean loss differential and $\hat{g}_x(0)$ is a consistent estimate of the spectral density of the loss differential at frequency 0. A consistent estimate of $2\pi g_x(0)$ is obtained by using the Newey-West (1987) correction for serial correlation. The test is valid for a wide class of loss functions and allows the forecast errors to be non-Gaussian, nonzero mean, serially correlated, and contemporaneously correlated.

https://doi.org/10.1017/S0022109009990184 Published online by Cambridge University Press

TABLE 6

Out-of-Sample Forecasts of GDP Growth and Inflation Rates

Table 6 summarizes the error properties in forecasting GDP growth and inflation rates over various time horizons of the macro term structure (MTS) model and several alternative models: the PEN (1991) two-factor model; the APW (2006) VAR model; an RW model; the HAR (1989) term spread regression model; and the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (SUR). The forecasts in Panels A and B are obtained by using 15-year windows of quarterly data to estimate the parameters needed to forecast subsequent GDP growth and inflation rates over the different horizons. Proceeding recursively, I compute forecasts for all the competing models for the period from 1975:Q1 to 2005:Q4. Root mean squared errors (RMSE) are expressed in basis points. In parentheses are the asymptotic *p*-values of the DM (1995) statistic, which tests the null hypothesis of no difference in the accuracy of the forecasts provided by the competing models with respect to those of the MTS model. Panel C shows the ratio between the RMSEs of the forecasts estimated by the MTS model and the RW model in different subperiods. In parentheses are reported the asymptotic *p*-values of the DM statistic testing the null hypothesis of no difference in the accuracy of the forecasts produced by the two models.

Panel A. RMSE of GDP Growth Forecasts

Horizon	MTS	PEN		APW		HAR		SUR	
3 months	137	375	(0.00)	329	(0.00)	296	(0.00)	300	(0.00)
6 months	107	329	(0.00)	260	(0.00)	231	(0.00)	230	(0.00)
1 year	154	280	(0.00)	224	(0.00)	197	(0.02)	181	(0.30)
2 years	187	230	(0.07)	186	(0.96)	141	(0.10)		

Panel B. RMSE of Inflation Forecasts

Horizon	MTS	PEN		APW		RW		SUR	
3 months	57	210	(0.00)	123	(0.00)	111	(0.00)	93	(0.00)
6 months	43	208	(0.00)	124	(0.00)	103	(0.00)	82	(0.00)
1 year	52	210	(0.00)	141	(0.00)	110	(0.01)	83	(0.00)
2 years	82	224	(0.00)	192	(0.00)	130	(0.02)		

Panel C. Ratio between RMSE of MTS and RW Inflation Forecasts

Horizon	1975–1983		1984–1994		1995–2005	
3 months	0.44	(0.00)	0.64	(0.00)	0.68	(0.00)
6 months	0.34	(0.00)	0.61	(0.00)	0.57	(0.00)
1 year	0.43	(0.01)	0.65	(0.02)	0.60	(0.00)
2 years	0.59	(0.02)	0.85	(0.16)	0.65	(0.00)

significantly across the three periods and is always below 1, meaning that the MTS model outperforms the RW model at all forecasting horizons.

Figure 3 shows the dynamic behavior of the ratio between the annual average of absolute inflation forecasting errors generated by the MTS and RW models, respectively, for various time horizons. I observe that this ratio is above 1 only in the case of the two-year forecasting horizon for the 1992–1995 sample period. Furthermore, in the last decade, the ratio has become relatively low (below 50%) at all forecasting horizons.

The fact that the MTS model performs better than the RW model at forecasting horizons of one and two years contradicts the evidence presented by Atkeson and Ohanian (2001) and Stock and Watson (2007), which shows that during the post-1984 Great Moderation in inflation, the forecasting ability of structural models deteriorated with respect to naive univariate models.

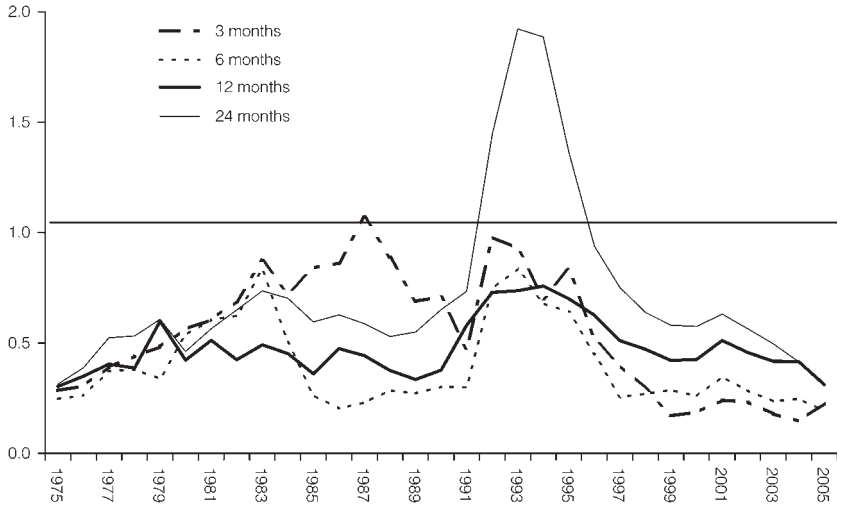
In my view, the superior performance of the MTS model in forecasting inflation in recent years can be explained by the fact that, in the post-1984 sample period, i) the size of the correlation between actual inflation and other macroeconomic variables, such as GDP growth and the unemployment rate, has declined, whereas ii) the tightness of the correlation between actual inflation rates and the level and slope of both the nominal and real term structures of interest rates has increased, especially in the last decade.

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FIGURE 3

Ratio between MTS and RW Absolute Inflation Forecasting Errors

Figure 3 shows the dynamic behavior of the ratio between the annual average of absolute inflation forecasting errors generated by the macro term structure (MTS) and random walk (RW) models, respectively, for various time horizons. Out-of-sample forecasts are obtained by using the preceding 15 years of quarterly data to estimate the parameters needed to predict subsequent inflation rates. Proceeding recursively, I compute forecasts for the two models from 1975:Q1 to 2005:Q4.



On the one hand, this evidence may explain the fact that the predictive power of backward-looking Phillips curve models that rely only on macroeconomic variables in forecasting future inflation has diminished. On the other hand, this may explain why the MTS model, which accounts for the forward-looking cross-sectional relationships between inflation, output growth, and nominal and real interest rates generates relatively accurate forecasts. In this respect, my results seem to support the multicountry empirical analysis presented in Stock and Watson (2003), where a positive role in forecasting output and inflation is assigned to asset prices because of their forward-looking nature.

IV. Conclusion

In this paper, I build and estimate an internally consistent structural model that analyzes the relationship between the term structure of interest rates, inflation, and output growth. The equilibrium aspects of the model impose nontrivial restrictions on the joint dynamics of bond yields and macroeconomic variables and imply that real and monetary variables of the economy are interrelated and influence the shape of the yield curve, while bond yields convey information that is useful for forecasting economic fundamentals.

The results indicate that by considering data on inflation and GDP, along with data on yields in an affine term structure setting, one may obtain reliable estimates of the implicit term structures of real interest rates, expected inflation rates, and inflation risk premia, and derive sensible predictions for bond returns.

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Moreover, the empirical evidence shows that the model generates forecasts of future inflation and GDP growth rates that are considerably more accurate than those produced by several well-known alternative approaches, and it contradicts the notion that the Great Moderation in inflation subsequent to the mid-1980s has induced a deterioration in the forecasting ability of structural models with respect to naive univariate models.

References

Abken, P. A. "Inflation and the Yield Curve." Federal Reserve Bank of Atlanta, *Economic Review*, 78 (1993), 13–31.

Ang, A.; G. Bekaert; and M. Wei. "Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?" *Journal of Monetary Economics*, 54 (2007), 1163–1212.

Ang, A.; G. Bekaert; and M. Wei. "The Term Structure of Real Rates and Expected Inflation." *Journal of Finance,* 63 (2008), 797–849.

Ang, A., and M. Piazzesi. "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics*, 50 (2003), 745–787.

Ang, A.; M. Piazzesi; and M. Wei. "What Does the Yield Curve Tell Us about GDP Growth?" *Journal of Econometrics*, 131 (2006), 359–403.

Atkeson, A., and L. E. Ohanian. "Are Phillips Curves Useful for Forecasting Inflation?" Federal Reserve Bank of Minneapolis, *Quarterly Review*, 25 (2001), 2–11.

Bakshi, G. S., and Z. Chen. "Inflation, Asset Prices, and the Term Structure of Interest Rates in Monetary Economies." *Review of Financial Studies*, 9 (1996), 241–275.

Balduzzi, P.; S. R. Das; and S. Foresi. "The Central Tendency: A Second Factor in Bond Yields." Working Paper, New York University (1995).

Barr, D. G., and J. Y. Campbell. "Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices." *Journal of Monetary Economics*, 39 (1997), 361–383.

Benninga, S., and A. Protopapadakis. "Real and Nominal Interest Rates Under Uncertainty: The Fisher Theorem and the Term Structure." *Journal of Political Economy*, 91 (1983), 856–867.

Boudoukh, J. "An Equilibrium Model of Nominal Bond Prices with Inflation-Output Correlation and Stochastic Volatility." *Journal of Money, Credit, and Banking*, 25 (1993), 636–665.

Breeden, D. T. "Consumption, Production, Inflation and Interest Rates: A Synthesis." *Journal of Financial Economics*, 16 (1986), 3–39.

Brown, R. H., and S. M. Schaefer. "The Term Structure of Real Interest Rates and the Cox, Ingersoll, and Ross Model." *Journal of Financial Economics*, 35 (1994), 3–42.

Buraschi, A., and A. Jiltsov. "Inflation Risk Premia and the Expectations Hypothesis." *Journal of Financial Economics*, 75 (2005), 429–490.

Campbell, J. Y., and R. J. Shiller. "Yield Spreads and Interest Rate Movements: A Bird's Eye View." *Review of Economic Studies*, 58 (1991), 495–514.

Chapman, D. A. "The Cyclical Properties of Consumption Growth and the Real Term Structure." *Journal of Monetary Economics*, 39 (1997), 145–172.

Cochrane, J. H., and M. Piazzesi. "Bond Risk Premia." *American Economic Review*, 95 (2005), 138–160.

Cox, J. C.; J. E. Ingersoll; and S. A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, 53 (1985a), 363–384.

Cox, J. C.; J. E. Ingersoll; and S. A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (1985b), 385–407.

Dewachter, H., and M. Lyrio. "Macro Factors and the Term Structure of Interest Rates." *Journal of Money, Credit, and Banking*, 38 (2006), 119–140.

Diebold, F. X., and R. S. Mariano. "Comparing Predictive Accuracy." *Journal of Business and Economic Statistics*, 13 (1995), 253–263.

Duarte, J. "Evaluating Alternative Risk Preferences in Affine Term Structure Models." *Review of Financial Studies*, 17 (2004), 379–404.

Duffee, G. R. "Term Premia and Interest Rate Forecasts in Affine Models." *Journal of Finance*, 57 (2002), 405–443.

Estrella, A., and G. A. Hardouvelis. "The Term Structure as a Predictor of Real Economic Activity." *Journal of Finance*, 46 (1991), 555–576.

Evans, C. L., and D. A. Marshall. "Economic Determinants of the Nominal Treasury Yield Curve." Working Paper, Federal Reserve Bank of Chicago (2001).

https://doi.org/10.1017/S0022109009990184 Published online by Cambridge University Press

Evans, M. D. D. "Real Rates, Expected Inflation, and Inflation Risk Premia." *Journal of Finance*, 53 (1998), 187–218.
Evans, M. D. D. "Real Risk, Inflation Risk, and the Term Structure." *Economic Journal,* 113 (2003), 345–389.
Fama, E. F. "Term Structure Forecasts of Interest Rates, Inflation, and Real Returns." *Journal of Monetary Economics*, 25 (1990), 59–76.
Fama, E. F., and R. R. Bliss. "The Information in Long-Maturity Forward Rates." *American Economic Review*, 77 (1987), 680–692.
Fisher, J. D. M.; C. T. Liu; and R. Zhou. "When Can We Forecast Inflation?" Federal Reserve Bank of Chicago, *Economic Perspectives*, 26 (2002), 30–42.
Goto, S., and W. Torous. "The Conquest of U.S. Inflation: Its Implications for the Fisher Hypothesis and the Term Structure of Nominal Interest Rates." Working Paper, UCLA (2003).
Harvey, C. R. "The Real Term Structure and Consumption Growth." *Journal of Financial Economics*, 22 (1988), 305–333.
Harvey, C. R. "Forecasts of Economic Growth from the Bond and Stock Markets." *Financial Analysts Journal*, 45 (1989), 38–45.
Hodrick, R., and E. C. Prescott. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit, and Banking,* 29 (1997), 1–16.
Jegadeesh, N., and G. G. Pennacchi. "The Behavior of Interest Rates Implied by the Term Structure of Eurodollar Futures." *Journal of Money, Credit and Banking*, 28 (1996), 426–446.
Kamara, A. "The Relation between Default-Free Interest Rates and Expected Economic Growth Is Stronger Than You Think." *Journal of Finance*, 52 (1997), 1681–1694.
Marshall, D. A. "Inflation and Asset Returns in a Monetary Economy." *Journal of Finance*, 47 (1992), 1315–1342.
McCulloch, J. H., and H.-C. Kwon. "U.S. Term Structure Data, 1947–1991." Working Paper, Ohio State University (1993).
Mishkin, F. S. "The Information in the Longer Maturity Term Structure about Future Inflation." *Quarterly Journal of Economics*, 55 (1990), 815–828.
Mishkin, F. S. "Is the Fisher Effect for Real? A Reexamination of the Relationship between Inflation and Interest Rates." *Journal of Monetary Economics*, 30 (1992), 195–215.
Mundell, R. "Inflation and Real Interest." *Journal of Political Economy*, 71 (1963), 280–283.
Newey, W. K., and K. D. West. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica,* 55 (1987), 703–708.
Pennacchi, G. G. "Identifying the Dynamics of Real Interest Rates and Inflation: Evidence Using Survey Data." *Review of Financial Studies*, 4 (1991), 53–86.
Plosser, C. I., and K. G. Rouwenhorst. "International Term Structures and Real Economic Growth." *Journal of Monetary Economics*, 33 (1994), 133–155.
Roma, A., and W. Torous. "The Cyclical Behavior of Interest Rates." *Journal of Finance*, 52 (1997), 1519–1542.
Stock, J. H., and M. W. Watson. "Forecasting Inflation." *Journal of Monetary Economics*, 44 (1999), 293–335.
Stock, J. H., and M. W. Watson. "Forecasting Output and Inflation: The Role of Asset Prices." *Journal of Economic Literature*, 41 (2003), 788–829.
Stock, J. H., and M. W. Watson. "Why Has U.S. Inflation Become Harder to Forecast?" *Journal of Money, Credit and Banking*, 39 (2007), 3–33.
Sun, T.-S. "Real and Nominal Interest Rates: A Discrete-Time Model and Its Continuous-Time Limit." *Review of Financial Studies*, 5 (1992), 581–611.
Tobin, J. "Money and Economic Growth." *Econometrica*, 33 (1965), 671–684.
Wu, T. "Macro Factors and the Affine Term Structure of Interest Rates." Working Paper, Federal Reserve Bank of San Francisco (2001).

https://doi.org/10.1017/S0022109009990184 Published online by Cambridge University Press