HELICAL BUCKLING OF SLENDER BEAM STRUCTURES SURROUNDED BY AN ELASTIC MEDIUM

S. Harasawa

Graduate School of Engineering
Hokkaido University
Sapporo, Japan

M. Sato *

Faculty of Engineering
Hokkaido University
Sapporo, Japan

ABSTRACT

The goal of this study was to investigate analytically the buckling phenomena of slender beam structures embedded in an elastic medium. In such cases, there is a possibility that spiral-shaped buckling modes can occur due to the restraining effect of the surrounding medium. As a result of analytical investigations, helical buckling occurs radically. Furthermore the pitch of the helix does not fluctuate as the radius of the helix increases. The interesting mechanical implications of the embedding effect are demonstrated by the theoretical approach established in this study.

Keywords: Helical buckling, Helix, Strain, Elastic medium.

1. INTRODUCTION

It is well-known that simple Euler buckling modes can occurr if any axial stresses are applied on the cross section of a slender beam without any restrictions surrounding the beam. Moreover in the case of a beams on an elastic foundation, higher-order buckling modes may arise (Brush and Almroth [1]). We consider here the axially-compressed buckling deformation of the outermost part of a beam surrounded by an elastic medium; Practical examples of such structures range from pipe-in-pipe systems (Sato and Patel [2], Sato *et al*. [3]) to very small carbon nanotubes (He *et al.* [4]). Under this condition, spiral-shaped helical buckling modes can occur due to the restraining effect of the surrounding medium. Helical buckling phenomena can be widely found in nature (Silverberg [5]), as well as fields of engineering (Sun and Lukasiewicz [6], Thompson *et al.* [7], Valverde and van der Heijden [8]) and nanoscience (Huang and Pattillo [9], Foster and Harland [10], Svensek and Podgornik [11]).

The purpose of this study is to analytically investigate the helical buckling characteristics of slender beam structures embedded in an elastic medium from the viewpoint of pure structural mechanics. A variational method to calculate the ground state total strain energy of the bending beam is used to determine the shape of the helical buckling modes by setting three parameters, the radius of the helix, the pitch of the helix and the coefficient of contraction (axial strain), which is the contraction ratio of slender structure surrounded by an elastic medium. These three parameters allow us to attempt to depict the behaviour of helical buckling deformation. We did not consider torsion energy (or the other energies) because the deformation of the slender structure would be so small that torsion energy cannot be predominant element. We consider, simply put, the three energies written in the paper, strain energy of slender structure (contraction energy and bending energy) and strain energy of elastic medium, are mainly predominant for deformation of helical buckling. The elastic medium can comfine the deformation of helical buckling within the diameter of elastic layer so that the buckling condition can be seemed to be in relatively small scale.

2. ANALYTICAL MODEL

As denoted in Fig. 1, we consider the spiral-shaped buckling model to be represented by an analytical model of a slender beam surrounded by an elastic medium. The initial length of the slender structure is sufficiently long and the effect of the boundary condition is not considered in this problem. As shown in Fig. 1, the elastic medium can be modeled using a coil spring model. Here b is the radius of the helix, and represents the deformation displacement that results from the formation of the helix. s is the pitch of the helix and d is the diameter of the circular section of the slender structure.

In this model, we assume following three points; the buckling mode of the central slender beam would be that of helical buckling under the condition where a critical load acts horizontally in the axial cross section, the cross-sectional shape of the slender beam in the centre is assumed to be circular, and the shape of the slender beam after deformation becomes a helix with constant radius b and pitch s, and that the deformation will not distort the circular cross section.

* Corresponding author (tayu@eng.hokudai.ac.jp)

DOI : 10.1017/jmech.2014.92
Copyright © 2014 The Society of Theoretical and Applied Mechanics, R.O.C.
https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press

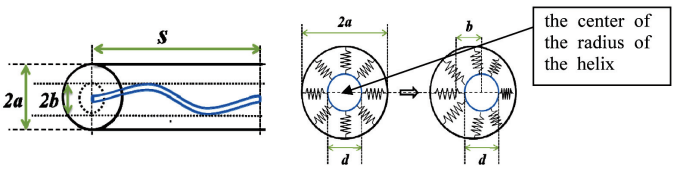


Fig. 1 Analytical model.

Note that the shear deformation is seemingly necessary to be taken into consideration, however here we decided the shear deformation is negligible as well as torsion energy because the scale of deformation is enough modest to ignore its stored energy.

We will focus on and trace the value of the three parameters, mentioned above, the radius of the helix b, the pitch of the helix s and coefficient of contraction ε. We will also consider the total strain energy, which can be divided into three segments; the strain energy due to contraction in the axial direction E_{strain}, the strain energy due to helical deformation E_{bend} and the stored strain energy due to deformation under the condition of an elastic restraint $E_{elastic.}$

2.1 Strain Energy of Slender Structure

The contractile force of the analytical model, shown in Fig. 1, acts uniformly only in the horizontal axis direction. Under this condition, the strain energy is yielded by a contractile force per unit length that can be described as follows

$$E_{strain} = \frac{1}{2} E_g S_g \varepsilon^2 \tag{1}$$

where S_g is the cross-sectional area of the slender structure, E_g is the Young's modulus of the slender structure, and ε is the coefficient of contraction of the slender structure per unit length.

The structural model mentioned above will be shortened by compression from the initial length to the length of the deformed slender structure l_h. The apparent compressive strain of the slender structure ε per unit length can be written as

$$\varepsilon = 1 - l_h \tag{2}$$

Considering the helix deformation shown in Fig. 2, the length of the deformed slender structure l_h per unit length due to contractile deformation caused by an applied critical load can be described.

Let ds be an infinitesimal distance along the helical deformed structure, the equation for ds is given by

$$ds = (dx^2 + dy^2 + dz^2)^{\frac{1}{2}} \tag{3}$$

Additionally, each parameter dx, dy, dz is defined by Fig. 2 as follows

$$dx = -b \sin\gamma \cdot d\gamma \tag{4}$$

$$dy = b \cos\gamma \cdot d\gamma \tag{5}$$

$$\frac{dz}{d\gamma} = \frac{s}{2\pi} \tag{6}$$

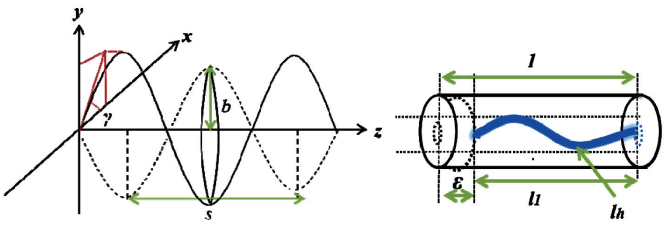


Fig. 2 Simplified diagram of helical formation of slender structure.

where γ is the polar angle, b is the radius and s is the pitch of the helix. Thus, ds in Eq. (3) becomes as follows.

$$ds = \left(b^2 + \frac{s^2}{4\pi^2} \right)^{\frac{1}{2}} d\gamma \tag{7}$$

And the length of the helix per pitch H can be written as

$$H = \int_0^{2\pi} ds = \int_0^{2\pi} \left(b^2 + \frac{s^2}{4\pi^2} \right)^{\frac{1}{2}} d\gamma = 2\pi \left(b^2 + \frac{s^2}{4\pi^2} \right)^{\frac{1}{2}} \tag{8}$$

Note that H is not the same as l_h. H is the the length of the helix that has at least one pitch. While l_h is the length of the helix per unit length that does not have the pitch (means it does not have the helix yet). Then, the helical length per pitch l_h is given by as below

$$l_h = \frac{H}{s} l_1 = \frac{H}{s}(1 - \varepsilon) \tag{9}$$

where l_1 is the length of the model after the axial compression. In Eq. (9), ε is the coefficient of contraction, l_h is the length of the helix per unit length, H is the length of the helix per pitch and s is the pitch of the helix. l_1 is the length of the model per unit length. Therefore, l_1 can be equal to Eq. (10).

$$l_1 = 1 \cdot (1 - \varepsilon) \tag{10}$$

Eventually, the stored strain energy per unit length E_{strain} can be expressed as

$$\begin{aligned} E_{strain} &= \frac{1}{2} E_g S_g \left[1 - \frac{H}{s}(1 - \varepsilon) \right]^2 \\ &= \frac{1}{2} E_g S_g \left[1 - (1 - \varepsilon) \left(1 + \frac{4\pi^2 b^2}{s^2} \right)^{\frac{1}{2}} \right]^2 \end{aligned} \tag{11}$$

where S_g is the area of the circular cross section of the slender structure, as given by

$$S_g = \frac{\pi d^2}{4} \tag{12}$$

https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press

2.2 Strain Energy of Bending

Because the spiral-shaped bending deformation of the slender structure has a constant radius of curvature, the stored strain energy in the infinitesimal element shown in Fig. 3 can be expressed using the bending moment M as follows

$$E_{bend} = \frac{1}{2} M \, d\theta \tag{13}$$

The equation for relationship between $d\theta$ and the constant radius of curvature ρ can be derived from Fig. 3.

$$d\theta = \frac{1}{\rho} dx \tag{14}$$

Additionally, there is the relative equation between the applied bending moment M and the radius of curvature ρ.

$$\frac{1}{\rho} = \frac{M}{E_g I_g} \tag{15}$$

where E_g is the Young's modulus of the slender structure and I_g is the area moment of inertia of the slender structure given by

$$I_g = \frac{\pi d^4}{64} \tag{16}$$

In the above equation, d is the diameter of the circular cross section of the slender structure. Therefore, Eq. (13) per unit length of the structure can be rewritten as

$$E_{bend} = \frac{1}{2} E_g I_g \frac{1}{\rho^2} \tag{17}$$

Assuming that the mode of helical deformation has a constant radius, the parameters of helical shape can be expressed as

$$x(\gamma) = \begin{pmatrix} b\cos\gamma \\ b\sin\gamma \\ z \end{pmatrix} \tag{18}$$

where z is the parameter of the z-axis. By substituting in Eqs. (4) ~ (6), we obtain

$$x'(\gamma) = \frac{dx(\gamma)}{d\gamma} = \begin{pmatrix} b\sin\gamma \\ -b\cos\gamma \\ \frac{s}{2\pi} \end{pmatrix} \tag{19}$$

Here $x'(\gamma)$ denotes the differentiation of Eq. (18) with respect to the polar angle γ. The definition of the orientation vector of the curve is given by following equation.

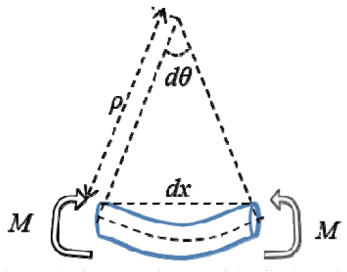


Fig. 3 Bending deformation of infinitesimal element.

$$e_1(\gamma) = \frac{x'(\gamma)}{\| x(\gamma) \|} = \frac{1}{\sqrt{b^2 + \left(\frac{s}{2\pi}\right)^2}} \begin{pmatrix} b\sin\gamma \\ -b\cos\gamma \\ \frac{s}{2\pi} \end{pmatrix} \tag{20}$$

The curvature κ is given by the norm of the curvature in the vector expression k, which can be written as

$$k = \frac{e_1'(\gamma)}{\| x'(\gamma) \|} = \frac{b}{b^2 + \left(\frac{s}{2\pi}\right)^2} \begin{pmatrix} -\cos\gamma \\ -\sin\gamma \\ 0 \end{pmatrix} \tag{21}$$

$$\kappa = \| k \| = \frac{b}{b^2 + \left(\frac{s}{2\pi}\right)^2} \tag{22}$$

The radius of curvature ρ is the reciprocal of the curvature κ. Hence, the equation for ρ becomes

$$\rho = \frac{1}{\kappa} = \frac{b^2 + \left(\frac{s}{2\pi}\right)^2}{b} = b\left(1 + \frac{s^2}{4\pi^2 b^2}\right) \tag{23}$$

Substituting Eq. (23) into Eq. (17), the stored strain energy per unit length of the model E_{bend} due to helical deformation can be described as

$$E_{bend} = \frac{1}{2} E_g I_g \frac{1}{\rho^2} = \frac{1}{2} E_g I_g \left[\frac{1}{b\left(1 + \frac{s^2}{4\pi^2 b^2}\right)} \right]^2 \tag{24}$$

2.3 Strain Energy of Elastic Medium

In accordance with helical buckling deformation, a slender beam surrounded by an elastic medium would be deformed from the initial centre position to the radius of the helix in the axial cross section. The central point of cross section is the reference point how the radius of the helix is deformed. The radius of the helix b is the distance between the central point of the initial position of the slender structure and the deformed position. Provided that it is assumed that the entire deformation is elastic and carried on sorely in the cross

https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press

section, the dynamic model of helical deformation can be illustrated such as the coil model in Fig. 1 at the first section.

The entire stored strain energy E_{coil} when a slender beam is displaced from the initial center position to the radius of the helix b would be given by

$$E_{coil} = \Sigma \frac{1}{2} k \left[(x_{initial}) - (x_{after}) \right]^2 \tag{25}$$

where k is the spring constant of the coil model. The coils are substantially distributed into the perimeter of the circular cross section, which means that we can integrate Eq. (25) to obtain.

$$E_{coil} = \int_0^{2\pi} \frac{1}{2} k \left(a_1 - \sqrt{a_1^2 + b^2 + 2a_1 b \cos\theta} \right)^2 d\theta \tag{26}$$

where a_1 is the initial length in the ground state condition, which is expressed as

$$a_1 = a - \frac{d}{2} \tag{27}$$

a_1 is the diameter of a cross section of the model a minus half the diameter of the slender beam. After the displacement shown Fig. 1 occurs, the length of the coil can be described as.

$$\sqrt{a_1^2 + b^2 + 2a_1 b \cos\theta} \tag{28}$$

The half-angle formula of cosine given by Eq. (29) is subsitituted into Eq. (28), we obtain Eq. (30).

$$\cos\theta = 1 - 2\sin^2 \frac{\theta}{2} \tag{29}$$

$$\sqrt{a_1^2 + b^2 + 2a_1 b \cos\theta} = (a_1 + b) \sqrt{1 - \frac{4a_1 b}{(a_1 + b)^2} \sin^2 \frac{\theta}{2}} \tag{30}$$

By setting $2\sqrt{a_1 b} / (a_1 + b) = \phi$ in Eq. (30), the entire stored strain energy of the coils E_{coil} can be written as

$$\begin{aligned} &\int_0^{2\pi} \frac{1}{2} k \left(2a_1 - 2a_1 (a_1 + b) \sqrt{1 - \phi^2 \sin^2 \frac{\theta}{2}} + b^2 + 2a_1 b \cos\theta \right) d\theta \\ &= \frac{1}{2} k \left(2\pi (2a_1^2 + b^2) - 2a_1 (a_1 + b) \int_0^{2\pi} \sqrt{1 - \phi^2 \sin^2 \frac{\theta}{2}} \right) d\theta \end{aligned} \tag{31}$$

The second term that includes integration can be substituted for a complete elliptic integral of the second kind $E(\varphi)$ given by

$$E(\phi) = \int_0^{\frac{\pi}{2}} \sqrt{1 - \phi^2 \sin^2 \theta} \, d\theta \tag{32}$$

If the coefficient of the second term is replaced with Eq. (32), the entire strain energy per unit length in part of an elastic medium $E_{elastic}$ can be expressed as

$$E_{elastic} = k \left[\pi (b^2 + 2a_1^2) - 4a_1 (b + a_1) E\left(\frac{2\sqrt{ba_1}}{b + a_1} \right) \right] \tag{33}$$

where k is the spring constant per unit angle in the circular direction, which can be written as

$$k = \frac{1}{2} \left(\frac{2a + d}{2a - d} \right) E_s \tag{34}$$

where E_s is the Young's modulus of the elastic medium.

2.4 Total Strain Energy

By summing up the three strain energies stored in the whole model as mentioned above, we can obtain the total strain energy E_{total} as follows.

$$\begin{aligned} E_{total} &= \frac{\pi}{8} d^2 \left[1 - (1 - \varepsilon) \sqrt{1 + \left(\frac{2\pi b}{s} \right)^2} \right]^2 E_g \\ &+ \frac{\pi}{128} \frac{d^4}{b^2} \left(\frac{2\pi b}{s} \right)^4 \frac{1}{\left[1 + \left(\frac{2\pi b}{s} \right)^2 \right]^2} E_g \\ &+ \frac{1}{2} \left(\frac{2a + d}{2a - d} \right) \left[\pi (b^2 + 2a_1^2) - 4a_1 (b + a_1) E\left(\frac{2\sqrt{ba_1}}{b + a_1} \right) \right] E_s \end{aligned} \tag{35}$$

To non-dimensionalise Eq. (35), the following non-dimensional variables are defined.

$$S = \frac{s}{d} \tag{36}$$

$$B = \frac{b}{d} \tag{37}$$

$$A = \frac{a}{d} \tag{38}$$

$$\alpha = \frac{E_s}{E_g} \tag{39}$$

Now we can substitute all four of the variables mentioned above into Eq. (35) to obtain the non-dimentionalised equation of total strain energy stored in the model. Equation (35) can therefore be written as

$$\begin{aligned} E_{total} &= \frac{\pi}{8} \left[1 - (1 - \varepsilon) \sqrt{1 + \left(\frac{2\pi B}{S} \right)^2} \right]^2 \\ &+ \frac{\pi}{128} \frac{1}{B^2} \left(\frac{2\pi B}{S} \right)^4 \frac{1}{\left[1 + \left(\frac{2\pi B}{S} \right)^2 \right]^2} \\ &+ \frac{1}{2} \left(\frac{2A + 1}{2A - 1} \right) \left[\pi (B^2 + A_1^2) - 4A_1 (B + A_1) E\left(\frac{2\sqrt{BA_1}}{B + A_1} \right) \right] \alpha \end{aligned} \tag{40}$$

https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press

We attempt to solve Eq. (40) by using the variation method with regard to the radius of the helix b, the pitch of the helix s and the coefficient of contraction ε.

3. ANALYTICAL RESULTS AND DISCUSSION

To find the total strain energy stored in the model, three parameters which can define the shape of helical buckling, the radius of the helix b, the pitch of the helix s and coefficient of contraction ε, would be solved. The reproducible helical buckling shape would be decided by solving equations with regard to b and s when the partial derivatives relative to the whole stored strain energy would be zero. Hence, the solving equation is as follows.

$$\frac{\partial E_{total}}{\partial b} = \frac{\partial E_{total}}{\partial s} = 0 \tag{41}$$

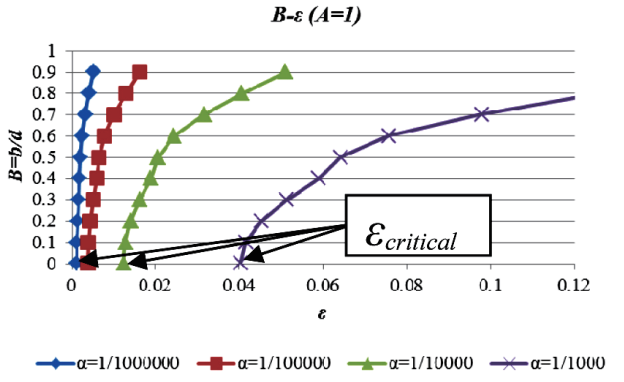
We have got two equations from Eq. (41) regarded as three parameters b, s and ε. In numerical calculations based on Eq. (41), the ratio of the Young's modulus of the outer elastic medium E_s to that of the slender beam E_g, which is expressed as α, is increased from 1/1,000,000 to 1/1000 by factors of 10 in order to investigate how the ratio of the Young's modulus a has influenced on the helical buckling deformation. The validity of our analytical model is guaranteed because the calculation result with regard to critical compressive strain (εcritical) is nearly equal to the value of ε critical measured by the experiment (Tanaka *et al*. [12]).

In addition, we depict three types of graphs that indicate the unusual behaviour between each parameter; the radius of the helix b, the pitch of the helix s and coefficient of contraction ε, to simulate the specific behaviour of the helical buckling deformation. Throughout this section, the dimensionless parameters are divided by d, the diameter of the slender structure, to neglect the scale order.

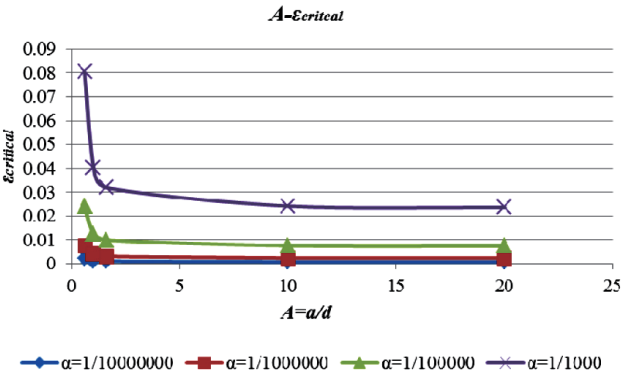
3.1 Relationships Between the Radius of the Helix *B* and Coefficient of Contraction ε

We first plot the behaviour of the radius of the helix and the coefficient of contraction where A is the ratio of the radius of the outer elastic layer a and the diameter of the inner slender structure d and the ratio of Young's modulus α. Note that the value of A is fixed at 1.

As Fig. 4(a) indicates, the main point is that a radical change in the radius of the helix B can ensue in accordance with an increasing coefficient of contraction ε. We here determine the "critical" compressive strain, $\varepsilon_{critical}$, as the moment when the beam deforms as helix. Here, $\varepsilon_{critical}$ is the critical compressive strain of forming the helical buckling deformation, and as illustrated Fig. 4(a) and 4(b), the value of $\varepsilon_{critical}$ increases in accordance with an increase in the ratio of Young's modulus α. In other words, the harder the outer layer is, the longer the period until contraction starts.



(a) A = 1



(b) A and $\varepsilon_{critical}$

Fig. 4 Relationship between (a) the radius B and the coefficient of contraction ε, and (b) A and $\varepsilon_{critical}$.

Additionally, as Fig. 5 shows, the values of $\varepsilon_{critical}$ decrease rapidly where the value of A is no more than 1. This implies that the critical point of increasing the radius of the helix B will shift to the smaller value if the value of A increases.

3.2 Relationships Between the Helix Pitch *S* and the Coefficient of Contraction ε and Helix Radius *B*

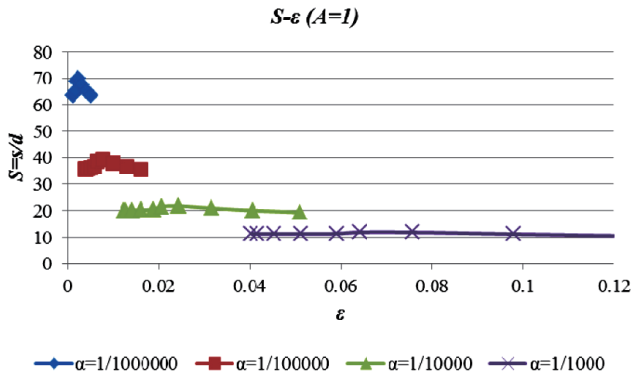
Similar to section 3.1, we plotted the behaviour of the pitch of the helix S and the coefficient of contraction with discrete changes in the ratio of Young's modulus α as shown Fig. 5.

There are two unusual points from Fig. 5; one is that the pitch of the helix S remains stable despite changes in the coefficient of contraction ε and the radius of the helix B. This means that the pitch of the helix would not be shortened due to the binding effect of the outer elastic layer, even though the outer layer is noticeably contracted.

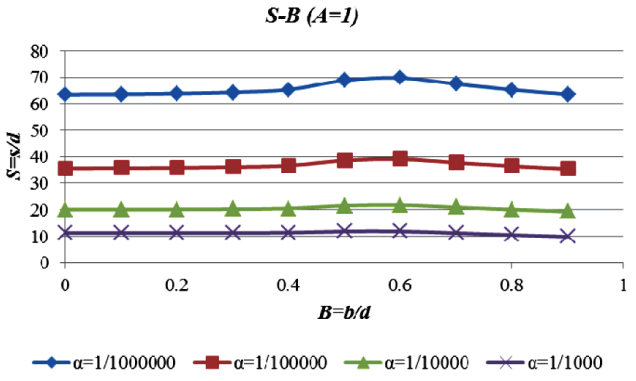
The second unusual point is that the pitch of the helix S decreases if the ratio of the Young's modulus α increases, as shown in Fig. 5(b).

These results indicate the specific deformations of helical buckling and it would be counterintuitive to the actual behaviour of the helical buckling deformation. In the process of helical buckling deformation, the pitch is stable even though the radius of the helix would be forming.

https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press

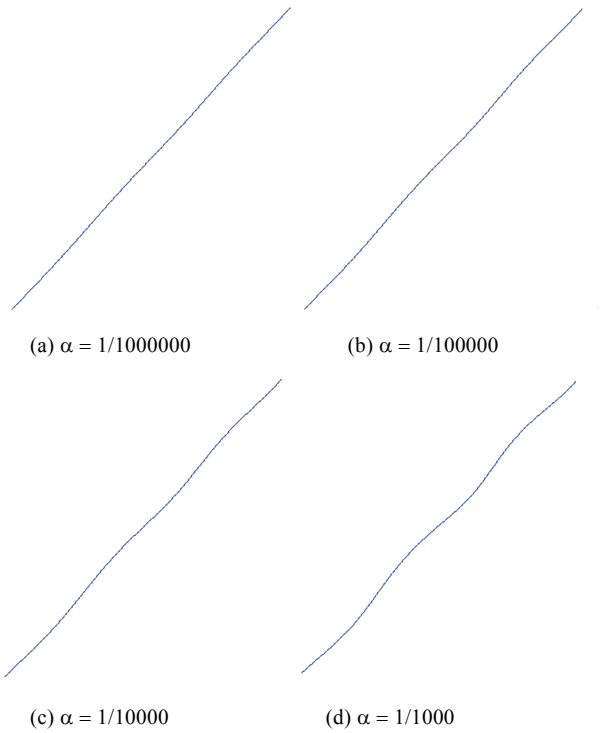


(a) S-ε (A = 1)



(b) S-B (A = 1)

Fig. 5 Relationships between (a) the pitch of the helix S and the coefficient of contraction ε, and (b) the pitch and the radius of the helix B.



(a) α = 1/1000000 (b) α = 1/100000

(c) α = 1/10000 (d) α = 1/1000

Fig. 6 Helical buckling modes (B = 1, A = 1).

3.3 Helical Buckling Mode Against Various Values of the Young's Modulus α.

As shown in Fig. 6, the helical buckling modes bellow are the buckling modes after the radius of the helix B reaches 1.

As mentioned in previous sections, the behaviour of the helical buckling modes may vary. The pitch of the helix S decreases in response to the increasing of the ratio of the Young's modulus α (which means the winding tightness increases). In addition, the radius of the helix B appears to be larger with an increase of the Young's modulus α.

4. CONCLUSIONS

The goal of this study is to investigate analytically the buckling phenomena of slender beam structures embedded in an elastic medium. In this study, when applying the load on the corss section of the model, we confirm the unusual buckling deformation occurs in simulation.

As a result of the numerical calculations, these following unexpected observations can be obtained.

- The pitch of the helix does not fluctuate during the deformation of helical buckling.
- When the ratio of the Young's modulus α decreases, the pitch of the helix S increases.
- When the ratio of the Young's modulus α decreases, the value of $\varepsilon_{critical}$ decreases.
- When the ratio of the Young's modulus α decreases, the changing rate of the radius of the helix B decreases.
- When the value of A increases, the value of $\varepsilon_{critical}$ decreases.

An increase in the value of A, the ratio of the radius of the outer elastic layer a and the diameter of the inner slender structure d, indicates that the diameter of the slender structure is broader, in other words, the binding effect of the elastic medium would become weakened.

Weakening of the binding effect also decreases the value of $\varepsilon_{critical}$ because the deformation of helical buckling would start at an earlier stage of deformation.

On balance, the influences of the value of A and the ratio of the Young's modulus α on the deformation of helical buckling are convergent on the following two points;

- When the value of A increases, the buckling helix forms earlier and is more loosely wound.
- When the ratio of the Young's modulus α decreases, the buckling helix forms earlier and is more loosely wound. Its pitch becomes longer and its radius becomes smaller.

ACKNOWLEDGMENTS

This work was supported by JSPS KAKENHI (Grant Number 24686096).

https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press

REFERENCES

1. Brush, D. O. and Almroth, B. O., *Buckling of Bars, Plates, and Shells*, McGraw-Hill, Inc., Columbus, Ohio (1975).
2. Sato, M. and Patel, M. H., "Exact and Simplified Estimations for Elastic Buckling Pressures of Structural Pipe-in-Pipe Cross-Sections under External Hydrostatic Pressure," *Journal of Marine Science and Technology*, **12**, pp. 251–262 (2007).
3. Sato, M., Patel, M. H. and Trarieux, F., "Static Displacement and Elastic Buckling Characteristics of Structural Pipe-in-Pipe Cross-Sections," *Structural Engineering & Mechanics*, **30**, pp. 263–278 (2008).
4. He, X. Q., Kitipornchai, S. and Liew, K M., "Buckling Analysis of Multi-Walled Carbon Nanotubes: A Continuum Model Accounting for Van Der Waals Interaction," *Journal of Mechanics and Physics of Solids*, **53**, pp. 303–326 (2005).
5. Silverberg, J. L., Noar, R. D., Packer, M. S., Harrison, M. J., Henley, C. L., Cohen, I. and Gerbode, S. J., "3D Imaging and Mechanical Modeling of Helical Buckling in Medicago Truncatula Plant Roots," *Proceedings of the National Academy of Sciences of the United States of America*, **109**, pp. 16794–16799 (2012).
6. Sun, C. and Lukasiewicz, S., "A New Model on the Buckling of a Rod in Tubing," *Journal of Petroleum Science and Engineering*, **50**, pp. 78–82 (2006).
7. Thompson, J. M. T., Silveira, M., van der Heijden, G. H. M. and Wiercigroch, M., "Helical Post-Buckling of a Rod in a Cylinder: With Applications to Drill-Strings," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, **468**, pp. 1591–1614 (2012).
8. Valverde, J. and van der Heijden, G. H. M., "Helical Buckling of a Whirling Conducting Rod in a Uniform Magnetic Field," *International Journal of Nonlinear Mechanics*, **47**, pp. 38–53 (2012).
9. Huang, N. C. and Pattillo, P. D., "Helical Buckling of a Tube in an Inclined Wellbore," *International Journal of Nonlinear Mechanics*, **35**, pp. 911–923 (2000).
10. Foster, D. and Harland, D., "Helical Buckling of Skyrme-Faddeev Solitons," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, **468**, pp. 3172–3190 (2012).
11. Svensek, D. and Podgornik. R., "Confined Nanorods: Jamming Due to Helical Buckling," *Physics Review E*, **77**, p. 031808 (2008).
12. Tanaka, S., Yamanishi, T., Tsuneishi, K. and Mitsunaga, Y., "Compressive Strain and Low Temperature Characteristics of Nylon Jacketed Fiber," *Journal of the Institute of Electronics, Information and Communication Engineers*, **66**, pp. 943–950 (1983). (in Japanese)

(Manuscript received March 13, 2014,
accepted for publication June 11, 2014.)

https://doi.org/10.1017/jmech.2014.92 Published online by Cambridge University Press