DYNAMIC PROBLEM CONCERNING MODE I SEMI-INFINITE CRACK PROPAGATION

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ABSTRACT

By the approaches of the theory of complex functions, dynamic problem concerning mode I semi-infinite crack propagation was studied. Analytical solutions are very easily obtained by application of the measures of self-similar functions. The problems researched can be changed into Riemann-Hilbert problems and analytical solutions to semi-infinite crack under the condition of impulse loads and increasing loads *Pt* located at the origin of the coordinates, respectively, are attained.

Keywords: Complex functions, Mode I semi-infinite crack, Dynamic problem, Self-similar functions, Analytical solutions.

1. INTRODUCTION

Dynamic problems play an important role in modern fracture mechanics. They will play a still greater role in the future, in connection with the possibility in principle of utilizing dynamic stress waves generated by internal local fracture for detection of the power and nature of a fracture focus. Only an adequate theory of such a focus, whose development is one of the missions of fracture mechanics, can help to resolve this great problem [1]. Semi-infinite crack often occurs in factual engineering structure; and many researchers studied its statics problems, but most of them only obtained numerical solutions [2-8]. However, a lot of engineering machineries work under the conditions of dynamic loadings, and the statics studies can't virtually settle a series of dynamic problems appearing in the dynamic conditions, so it is essential to investigate their fracture dynamics problems. On account of the complexity, cockamamie and difficulty in mathematics, numerical solutions and half analytical solutions was attained by fewer researchers [9-16], moreover analytical solutions were attained much less [17-20]. So it is necessary to research semi-infinite crack dynamic propagation problem. General representations of solutions are given by the methods of the theory of complex functions in this article. The problems considered can be transformed into a Riemann-Hilbert problem by application of this measure, which is very easily solved by the usual Muskhelishvili ways [21,22]. The work in this paper is how to expediently attain analytical solutions utilizing the self-similar functions under the action of impulse loads and increasing loads *Pt* located at the origin of the coordinates, respectively.

2. THE CORRELATIVE REPRESENTATIONS OF SELF-SIMILAR FUNCTIONS

In order to resolve availably fracture dynamics problems concerning semi-infinite crack, solutions will be obtained under the action of variable loads for mode I semi-infinite crack. In terms of the theorem of generalized functions, the different boundary condition problems studied will be changed into Keldysh-Sedov mixed boundary value problem by the approach of self-similar functions, and the corresponding solutions will be obtained.

Postulating at $y = 0$ there are any number of loaded sections and displacement sections along the x-axis, and the ends of these sections are running with different constant velocity. At the initial moment $t = 0$ the half-plane is at rest. In these segments the loads and displacements are arbitrary linear combination of the following functions [1,17, 19,20, 23-25]:

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$$\frac{d^k f_{k_1}(x)}{dx^k}\cdot\frac{d^s f_{s_1}(t)}{dt^s} \tag{1}$$

$$\text{where}\quad f_i(\xi)=\begin{cases}0 & \xi<0\\ \xi^i & \xi>0\end{cases} \tag{2}$$

Here k, k_1 and s, s_1 are arbitrary integer positive numbers [1,17,19,20,23-25].

An arbitrary continuous function of two variables x and t may be expressed as a linear superposition of Eq. (1), therefore resolving loads or displacements with the form of Eq. (2) will possess significance in principle. Let us introduce the linear differential operator as well as inverse:

$$L=\frac{\partial^{m+n}}{\partial x^m\partial t^n}\ ,\quad \text{inverse: } L^{-}=\frac{\partial^{-m-n}}{\partial x^{-m}\partial t^{-n}} \tag{3}$$

Here $+m+n$, $-m-n$ and 0 denote the $(m+n)$th order derivative, the $(m+n)$th order integral and function's self. It is facile to testify that there exist constants m and n, when L is put into Eq. (3), homogeneous functions of x and t of zeroth dimension (homogeneous) are obtained. The coefficients m, n will be called the indices of self-similarity [1,17,19,20,23-25].

Using corresponding expressions of elastodynamics equations of motion for an orthotropic anisotropic body [1,17,19,20,23-25]:

for the case when functions Lu and Lv are homogeneous

$$u^0=Lu\ ,\quad v^0=Lv\ ,\quad \sigma_y^0=L\sigma_y \tag{4}$$

for the case when functions $L\sigma_y$ and $L\tau_{xy}$ are homogeneous

$$u^0=\frac{\partial}{\partial t}Lu\ ,\quad v^0=\frac{\partial}{\partial t}Lv\ ,\quad \sigma_y{}^0=\frac{\partial}{\partial t}L\sigma_y\ , \tag{5}$$

The relevant self-similar functions are as follows [19,20,23-25]:

$$\sigma_y^0=(1/t)\,\mathrm{Re}\,F(\tau)\ ,\quad v^0=\mathrm{Re}\,W(\tau)\ , \tag{6}$$

where: u^0, v^0 and σ_y^0 in Eqs. (4) ~ (6) are the notation introduced [19,20,23-25], and they are correlative variables which directly resolve displacements and stresses by the course of respective computations in Eq. (7).

$$W'(\tau)=\left[D_1(\tau)/D(\tau)\right]F(\tau) \tag{7}$$

where: $\tau=x/t$, $F(\tau)$ and $W(\tau)$ are self-similar functions. The values of $D_1(\tau)/D(\tau)$ can be ascertained from Appendix 1 of literatures [19,20,23-25], indicated sole here: $D_1(\tau)/D(\tau)$ at the interval of the subsonic speeds is purely imaginary for the values which we are considering. Thus, elastodynamics problems for an orthotropic anisotropic body researched can be translated into seeking the single unknown function problems concerning $F(\tau)$ and $W(\tau)$ sufficing the boundary-value conditions. In the universal case this is Riemann-Hilbert problem in the theory of complex functions (in the simplest cases we have Keldysh-Sedov or Dirchlet problem), this kind of problem is facilely settled by the usual approaches, for example, in the books by Muskhelishvili [21,22].

Fracture dynamics problems will be investigated for an infinite orthotropic anisotropic body. Assume at the initial moment $t=0$ a crack occurs at the origin of coordinates and begins extending at constant velocity V (for the subsonic speeds) along the positive direction of x-axis; and at $t<0$, the half-plane was at rest. The surfaces of the crack are subjected to the different types of loads under the plane strain states.

3. RUDIMENTAL SHAPE OF THE SOLUTION TO SEMI-INFINITE CRACK PROPAGATION PROBLEM

Postulate at the initial moment $t=0$, a micro-crack abruptly appears at an orthotropic anisotropic body, we shall choose the Cartesian co-ordinate axes to be coincident with the axes of elastic symmetry of the body, and the problems considered are restricted to motion in the x-y-plane. The crack is running with constant velocity V in the positive direction of x-axis, the problem studied will be translated into the following boundary condition problems as:

$$\begin{aligned}&\sigma_y(x,\ 0,\ t)=f_1(x,\ t)\ ,\quad 0<x<Vt\\ &v(x,\ 0,\ t)=0\ ,\quad x>Vt\end{aligned} \tag{8}$$

Introducing the variable $\tau=x/t$. By application of the above corresponding expressions and $t\delta(x)=\delta(x/t)$ in the theory of generalized functions, here: $\delta(\ )$ represents Dirac's function in generalized functions [26-28]. Hence the previous boundary conditions can be transformed into the following boundary value issues:

$$\begin{aligned}&\mathrm{Re}\,F(\tau)=f_2\left[\tau,\ \delta(\tau)\right]\ ,\quad 0<\tau<V\\ &\mathrm{Re}\,W'(\tau)=0\ ,\quad \tau>V\end{aligned} \tag{9}$$

According to the relationship of $F(\tau)$ and $W'(\tau)$ in Eq. (7) and the above conditions, the modality of single unknown function $W'(\tau)$ can be confirmed:

$$W'(\tau)=f_3\left[(\tau,\ \xi(\tau)\right] \tag{10}$$

Then the problem researched can be come down to Keldysh-Sedov problem:

$$\begin{aligned}&\mathrm{Re}\,\xi(\tau)=0\ ,\quad \tau>V\\ &\mathrm{Im}\,\xi(\tau)=0\ ,\quad 0<\tau<V\end{aligned} \tag{11}$$

Considering synthetically asymmetry and the conditions of the infinite point of the plane corresponding to the origin of coordinates of the physical plane as well as singularities of the stress at the crack tip [29,30], the format of the solution of the above problems can be acquired:

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$$\xi(\tau) = T\left[(V-\tau),\ \tau\right] \tag{12}$$

Then utilizing Eqs. (6) or (7), we will easily deduct the stress, the displacement and the stress intensity factor under the conditions of semi-infinite crack propagation problems.

4. SOLUTIONS OF IDIOGRAPHIC PROBLEMS

In order to solve efficiently dynamics problems on an orthotropic anisotropic body, solutions will be gained under the action of the concentrated force for mode I moving semi-infinite crack. In terms of the theorem of generalized functions, the different boundary condition problems investigated will be changed into Keldysh-Sedov mixed boundary value problem by the mesures of self-similar functions, and the correlative solutions will be acquired.

1. Postulate at the initial moment $t = 0$, a semi-infinite crack suddenly occurs and begins to run with constant velocity V along the positive direction of x-axis under the conditions of an instantaneous impulse load applied at the origin of the coordinates. On the half-plane at $y = 0$, the boundary conditions will be as follows:

$$\begin{aligned} &\sigma_y(x,\ 0,\ t) = -I\delta(x)\delta(t)\ ,\quad 0 < x < Vt \\ &v(x,\ 0,\ t) = 0\ ,\quad \tau > V \end{aligned} \tag{13}$$

where: I in Eq. (13) shows the instantaneous impulse load.

In this case the displacement will evidently be homogeneous functions, in which $L = \partial^{-1}/\partial t^{-1} = L_t^{-1}$, Utilizing $\tau = x/t$, the theory of generalized function [26-28] as well as Eqs. (4) and (6), the first of Eq. (13) can be written:

$$\mathrm{Re}\,F(\tau) = -It\delta(x) = -I\delta(\tau) \quad 0 < \tau < V \tag{14}$$

In terms of Eq. (7), boundary conditions (14) will be further rewritten as:

$$\begin{aligned} &\mathrm{Re}\left[\frac{D(\tau)}{D_1(\tau)}\cdot W'(\tau)\right] = -I\delta(\tau)\ ,\quad 0 < \tau < V \\ &\mathrm{Re}\,W'(\tau) = 0\ ,\quad \tau > V \end{aligned} \tag{15}$$

Deducing from the above formulas, the solution of $W'(\tau)$ must have the following form:

$$W'(\tau) = \xi(\tau)/\tau \tag{16}$$

In the formula $\xi(\tau)$ has no singularity at the interval of $0 < \tau < V$, while $D(\tau)/D_1(\tau)$ is purely imaginary for the subsonic speeds, therefore $\xi(\tau)$ must be purely real in the region of $0 < \tau < V$. Thus, Eq. (15) becomes as:

$$\begin{aligned} &\mathrm{Re}\,\xi(\tau) = 0\ ,\quad \tau > V \\ &\mathrm{Im}\,\xi(\tau) = 0\ ,\quad 0 < \tau < V \end{aligned} \tag{17}$$

According to dissymmetry and the conditions of the infinite point of the plane corresponding to the origin of coordinates of the physical plane as well as singularities of the crack tip [29,30], the unique solution of the Keldysh-Sedov problem (17) can be obtained:

$$\xi(\tau) = \frac{A\tau^n\sqrt{(V-\tau)\,\tau}}{\tau} \tag{18}$$

where A is an unknown constant and n is an unknown index.

Then replacing Eq. (18) into Eqs. (16) and (7), we can gain:

$$W'(\tau) = \frac{A\tau^n\sqrt{(V-\tau)\,\tau}}{\tau^2} \tag{19}$$

$$F(\tau) = \frac{A\tau^n\sqrt{(V-\tau)\tau}\cdot D(\tau)/D_1(\tau)}{\tau^2} \tag{20}$$

Substituting Eq. (20) into Eq. (14), at $\tau \to 0$, $n = 1/2$ can be deduced. After that, constant A can be confirmed:

$$A = -\frac{I}{\pi\sqrt{V}\,\mathrm{Im}\left[D(0)/D_1(0)\right]} \tag{21}$$

Now putting Eq. (20) into (4), we will attain the following:

$$\sigma_y^0 = L_t^{-1}\sigma_y = \frac{1}{t}\cdot\mathrm{Re}\left[\frac{D(\tau)}{D_1(\tau)}\cdot\frac{A\sqrt{V-\tau}}{\tau}\right] \tag{22}$$

Derivating with respect to t, at the surface $y = 0$, the stress will be obtained:

$$\begin{aligned} \sigma_y &= \frac{\partial}{\partial t}\mathrm{Re}\left[\frac{1}{t}\cdot\frac{A\sqrt{V-\tau}}{\tau}\cdot\mathrm{Im}\frac{D(\tau)}{D_1(\tau)}\right] \\ &= \frac{\partial}{\partial t}\left[\frac{1}{t}\cdot\mathrm{Im}\left\{\frac{A\sqrt{V-\tau}}{\tau}\right\}\cdot\mathrm{Im}\left\{\frac{D(\tau)}{D_1(\tau)}\right\}\right] \\ &= \frac{\partial}{\partial t}\left[\frac{1}{t}\cdot\frac{-A\sqrt{V-\tau}}{\tau}\cdot\mathrm{Im}\left\{\frac{D(\tau)}{D_1(\tau)}\right\}\right]\ ,\quad x > Vt \end{aligned} \tag{23}$$

Now substituting Eq. (19) into (4), the following expression will be fetched:

$$v^0 = L_t^{-1}v = \int_0^t v\,dt = \mathrm{Re}\,W(\tau) \tag{24}$$

Differentiating with respect to t, at the surface $y = 0$, the displacement will be obtained:

$$v = \mathrm{Re}\,W'(\tau)\cdot\frac{\partial\tau}{\partial t} = -\frac{A\sqrt{(Vt-x)}}{t^{3/2}}\ ,\quad 0 < x < Vt \tag{25}$$

According to Eq. (23), we will facilely obtain stress intensity factor:

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$$K_1(t)=\lim_{x\to Vt}\sqrt{2\pi(x-Vt)}\cdot\frac{\partial}{\partial t}\left[\frac{1}{t}\cdot\frac{A\sqrt{V-\tau}}{\tau}\cdot\mathrm{Im}\frac{D(\tau)}{D_1(\tau)}\right]$$
$$=\frac{A\sqrt{\pi/2}}{t^{3/2}}\mathrm{Im}\frac{D(V)}{D_1(V)} \tag{26}$$

Known from the above, dynamic stress intensity factor $K_1(t)$ decays gently to slow and has evident singularity because it only contains one variable t in the denominator, and that the rest quantities are all real constants.

2. Supposing that all conditions are the same as those in the previous example except that the surfaces of the crack are subjected to an increasing load Pt at the origin of the coordinates. The boundary conditions will be:

$$\begin{aligned}&\sigma_y(x,\ 0,\ t)=-Pt\delta(x)\ ,\quad 0<x<Vt\\&v(x,\ 0,\ t)=0\ ,\quad x>Vt\end{aligned} \tag{27}$$

In this case the stress will apparently be homogeneous functions, in which $L = 1$. In terms of Eqs. (5) and (6), the first expression of the boundary condition (27) can be written as follows:

$$\mathrm{Re}\,F(\tau)=-Pt\delta(x)=-P\delta(\tau)\ ,\quad 0<\tau<V \tag{28}$$

According to Eq. (7), boundary conditions (28) will be further rewritten as:

$$\begin{aligned}&\mathrm{Re}\left[\frac{D(\tau)}{D_1(\tau)}\cdot W'(\tau)\right]=-P\delta(\tau)\ ,\quad 0<\tau<V\\&\mathrm{Re}\,W'(\tau)=0\ ,\quad \tau>V\end{aligned} \tag{29}$$

From the above formulas, the sole solution of $W'(\tau)$ can be easily deducted:

$$W'(\tau)=\xi(\tau)/\tau \tag{30}$$

In the formula $\xi(\tau)$ has no singularity in the section of $0 < \tau < V$, while $D(\tau)/D_1(\tau)$ is purely imaginary for the subsonic speeds, accordingly $\xi(\tau)$ must be purely real at the interval of $0 < \tau < V$. Thus, Eq. (29) becomes:

$$\begin{aligned}&\mathrm{Re}\,\xi(\tau)=0\ ,\quad \tau>V\\&\mathrm{Im}\,\xi(\tau)=0\ ,\quad 0<\tau<V\end{aligned} \tag{31}$$

In the light of asymmetry and the conditions of the infinite point of the plane corresponding to the origin of coordinates of the physical plane as well as singularities of the crack tip [29,30], the unique solution of the Keldysh-Sedov problem (31) can be obtained:

$$\xi(\tau)=\frac{A\tau^n}{\left[(V-\tau)\tau\right]^{3/2}} \tag{32}$$

where A is an unknown constant.

Then substituting Eq. (32) into Eqs. (30) and (7), we can gain:

$$W'(\tau)=\frac{A\tau^n}{\tau\left[(V-\tau)\tau\right]^{3/2}} \tag{33}$$

$$F(\tau)=\frac{D(\tau)}{D_1(\tau)}\cdot\frac{A\tau^n}{\tau\left[(V-\tau)\tau\right]^{3/2}} \tag{34}$$

Putting Eq. (34) into Eq. (29), at $\tau \to 0$, $n = 3/2$ can be deducted. From that, constant A can be determined as:

$$A=-\frac{PV^{3/2}}{\pi\,\mathrm{Im}\left[D(0)/D_1(0)\right]} \tag{35}$$

In an orthotropic isotropic body, the disturbance range of elastic wave can be expressed by the circular area of radius c_1t and c_2t. Here c_1 and c_2 are the velocities of longitudinal and transverse waves ($c_1 > c_2$) of elastic body respectively. In an orthotropic anisotropic body, the disturbance range of elastic wave is not the circular zone and can not overrun threshold value $C_d=\sqrt{C_{11}/\rho}$ of elastic body, where C_{11} is an elastic constant of materials and ρ is the density of materials. At $|x| > C_dt$, with $\mathrm{Im}[D_1(\tau)/D(\tau)] = 0$, thus the stresses and the displacements are zero; and this illustrates that disturbance of elastic wave can not surpass C_dt.

Now putting Eq. (34) into (6) and (5), at the surface $y = 0$, the stresses and the dynamic stress intensity factor are acquired:

$$\begin{aligned}\sigma_y&=\int_{\infty}^{x}\frac{1}{t}\mathrm{Re}\frac{AD(\tau)/D_1(\tau)}{\tau(\tau-V)^{3/2}}\,dt\\&=-\mathrm{Re}\int_{C_d}^{\frac{x}{t}}\frac{A\,\mathrm{Im}\left[D(\tau)/D_1(\tau)\right]}{\tau^2(\tau-V)^{3/2}}\,d\tau\ ,\\&\quad x>Vt\end{aligned} \tag{36}$$

$$\begin{aligned}K_1(t)&=\lim_{x\to Vt}\sqrt{2\pi(x-Vt)}\cdot\mathrm{Re}\int_{C_d}^{\frac{x}{t}}\frac{-A\,\mathrm{Im}\left[D(\tau)/D_1(\tau)\right]}{\tau^2(\tau-V)^{3/2}}\,d\tau\\&=2\sqrt{2\pi t}\cdot\frac{A\cdot\mathrm{Im}\left[D(V)/D_1(V)\right]}{V^2}\end{aligned} \tag{37}$$

The limit of Eq. (37) belongs to the modality $0\cdot\infty$, which should be only changed into the format of ∞/∞, then its result can be calculated by means of L'Hospital theorem [31]. Dynamic stress intensity factor $K_1(t)$ gradually increases from zero and even reaches or overruns fracture toughness of this material with the increase of time, because sole variable t lies in the numerator of the above, moreover the rest quantities are also regarded as real constants.

Integrating Eq. (33), we will obtain $W(\tau)$:

$$\begin{aligned}W(\tau)&=\int W'(\tau)\,d\tau=\int\frac{A}{\tau(V-\tau)^{3/2}}\,d\tau\\&=\int\frac{2AV\sin\theta\cos\theta}{V\sin^2\theta\cdot V^{3/2}\cos^3\theta}\,d\theta\\&=\frac{2A}{V^{3/2}}\int\frac{1}{\sin\theta\cdot\cos^2\theta}\,d\theta\\&=\frac{2A}{V^{3/2}}\left[\ln(\csc\theta-ctg\theta)+\sec\theta\right]\\&=\frac{2A}{V^{3/2}}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}}-\frac{2A}{V\sqrt{V-\tau}}+C\end{aligned} \tag{38}$$

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Equation (38) is deducted by this relationship: $\tau = V\sin^2\theta$. Its integral is gained by application of integral formulae of literature [32].

The crack runs along the x-axis, so $W(\tau)$ can be computed in the definite integral, we take constant $C = 0$. Then putting Eq. (38) into Eqs. (5), (6), the displacement v is attained:

$$
\begin{aligned}
v &= \int_0^t \mathrm{Re}W(\tau)\,dt = \mathrm{Re}\int_\infty^{x/t} -\frac{x}{\tau^2}W(\tau)\,d\tau = \mathrm{Re}\int_\infty^{x/t} -\frac{2A}{V^{3/2}}\frac{x}{\tau^2}\left[\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}} - \sqrt{\frac{V}{V-\tau}}\right]d\tau \\
&= \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\int_\infty^{x/t}\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}}\;d\left(\frac{1}{\tau}\right) + \int_\infty^{x/t}\frac{1}{\tau^2}\sqrt{\frac{V}{V-\tau}}\,d\tau\right\} = \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}}\right. \\
&\quad -\int_\infty^{x/t}\frac{1}{\tau}\cdot\frac{\sqrt{\tau}}{\sqrt{V}-\sqrt{V-\tau}}\cdot\left[\frac{-\frac{-1}{2\sqrt{V-\tau}}\cdot\sqrt{\tau}-\frac{1}{2\sqrt{\tau}}(\sqrt{V}-\sqrt{V-\tau})}{(\sqrt{\tau})^2}\right]d\tau \\
&\quad \left.+\int_\infty^{x/t}\frac{1}{\tau^2}\sqrt{\frac{V}{V-\tau}}\,d\tau\right\} = \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}}\right. \\
&\quad \left.-\int_\infty^{x/t}\frac{1}{\tau}\cdot\frac{\sqrt{\tau}}{\sqrt{V}-\sqrt{V-\tau}}\cdot\left[\frac{\tau-\sqrt{(V-\tau)V}+(V-\tau)}{2\sqrt{(V-\tau)\tau}\cdot\tau}\right]d\tau+\int_\infty^{x/t}\frac{1}{\tau^2}\sqrt{\frac{V}{V-\tau}}\,d\tau\right\} \\
&= \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}} - \int_\infty^{x/t}\frac{1}{2\tau^2}\cdot\sqrt{\frac{V}{V-\tau}}\,d\tau + \int_\infty^{x/t}\frac{1}{\tau^2}\sqrt{\frac{V}{V-\tau}}\,d\tau\right\} \\
&= \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}} + \int_\infty^{x/t}\frac{1}{2\tau^2}\sqrt{\frac{V}{V-\tau}}\,d\tau\right\} \\
&= \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}} + \int_\infty^{x/t}\frac{1}{2\tau^2}\sqrt{\frac{V}{V-\tau}}\,d\tau\right\} \\
&= \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}} - \frac{\sqrt{V}}{2}\left(-\frac{\sqrt{V-\tau}}{V\tau} + \frac{1}{2V\sqrt{V}}\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{V}+\sqrt{V-\tau}}\right)\right\}\Bigg|_{C_d}^{x/t} \\
&= \frac{2Ax}{V^{3/2}}\mathrm{Re}\left\{\frac{1}{\tau}\cdot\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}} + \frac{\sqrt{V-\tau}}{2\tau\sqrt{V}} - \frac{1}{2V}\ln\frac{\sqrt{V}-\sqrt{V-\tau}}{\sqrt{\tau}}\right\}\Bigg|_{C_d}^{x/t} \\
&= \frac{2A}{V^{3/2}}\left[\left(t-\frac{x}{2V}\right)\cdot\ln\frac{\sqrt{Vt}-\sqrt{Vt-x}}{\sqrt{x}} + \frac{t}{2}\sqrt{\frac{Vt-x}{Vt}}\right], \quad 0<x<Vt
\end{aligned}
\tag{39}
$$

5. INTERPRETATION OF DYNAMIC STRESS INTENSITY FACTOR

In the light of practical situations of concrete problems, mutative law of dynamic stress intensity factor should be interpreted better. The corresponding parameters are put into Eqs. (26), (37) to plot $K_1(t)$ as a function of time t, and their numerical solution is facilely obtained. The following constants are as follows [8,19,20,25,33,34]:

$C_{11} = 19.24\mathrm{Gpa}$; $C_{12} = 1.25\mathrm{Gpa}$; $C_{22} = 17.83\mathrm{Gpa}$;
$C_{66} = 1.00\mathrm{Gpa}$
$V = 300\mathrm{m}\times\mathrm{s}^{-1}$; $\rho = 0.5\times 9.8\times 10^3\,\mathrm{N}\times\mathrm{m}^{-3}$;
$I = P = 200\mathrm{N}$;

Known from Eq. (26), dynamic stress intensity factor $K_1(t)$ decreases tardily to slow and has evident singularity in virtue of unique variable t in the denominator, and the rest quantities are regarded as real constants. Such a trend is displayed by the curve in Fig. 1. This variational current is similar to the aftermath of Literatures [6,19,20,25,33-36] ·

It is known from Eq. (37) that dynamic stress intensity factor $K_1(t)$ enhances progressively from zero and even reaches or exceeds fracture toughness of this material, because sole variable t locates in its numerator, while the rest units are also referred to as real constants. This result must conduce the structural instability, as discribed in Fig. 2. This tendency is similar to the result in references [6,19,20,25,33-34,37-41], consequently it is right.

The relative numerical values between $K_1(t)$ and time t are represented in Tables 1, 2 in terms of curves in Figs. 1, 2, respectively.

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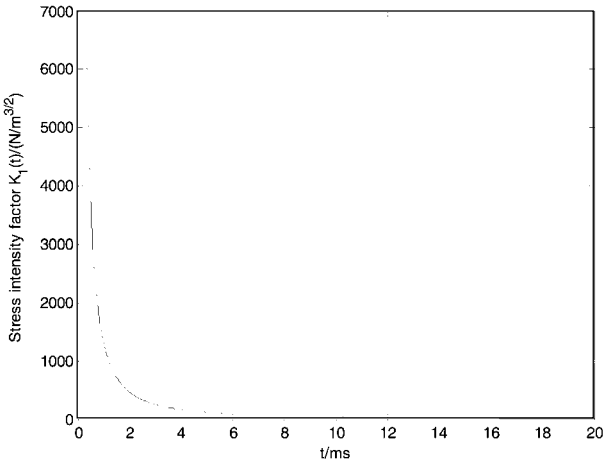


Fig. 1 Dynamic stress intensity factor $K_1(t)$ versus time t

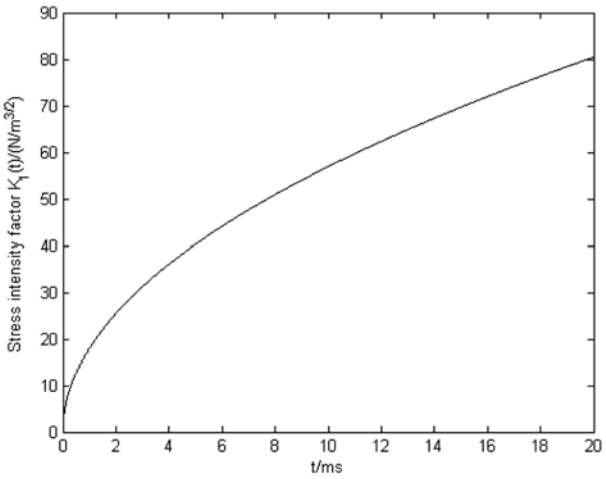


Fig. 2 Dynamic stress intensity factor $K_1(t)$ versus time t

Table 1 Relevant numerical values between dynamic stress intensity factor $K_1(t)$ versus time t

t / ms	2	4	6	8	10
$K_1(t)$ / N·m$^{-3/2}$	476.841	168.590	91.768	59.605	42.651
t / ms	12	14	16	18	20
$K_1(t)$ / N·m$^{-3/2}$	32.445	25.747	21.074	17.661	15.079

Table 2 Relative numerical values between dynamic stress intensity factor $K_1(t)$ versus time t

t / ms	2	4	6	8	10
$K_1(t)$ / N·m$^{-3/2}$	25.431	35.966	44.049	50.863	56.866
t / ms	12	14	16	18	20
$K_1(t)$ / N·m$^{-3/2}$	62.294	67.285	71.931	76.294	80.421

6. CONCLUSIONS

Under the actions of an instantaneous impulse load I and an increasing load Pt located at the origin of the coordinates, respectively, analytical solutions of mode I dynamic semi-infinite crack extension problem were attained by the measures of the theory of a complex variable function. The approach based on the methods of the self-similar functions makes it conceivable to gain the concrete solution of the dynamic semi-infinite crack propagation problem. The radical solution of mode I semi-infinite crack expansion problem can be easily derived based on the self-similar functions, which is regarded as the analogous class of dynamic problem of the elasticity theory. In the light of the concrete boundary conditions, self-similar function $W'(\tau)$ can be facilely deducted by means of corresponding to variable τ, accordingly analytical solutions of stresses, displacements and stress intensity factors will be readily calculated. However, the present solution occurs to be the most straightforward and intuitive of all alternative approaches appeared up to now. Indeed, we have succeeded in a mixed Keldysh-Sedov boundary value problem on a half-plane. The problem is of adequate real interest, since all of the members of structures in which fractures may run are of finite dimensions and are frequently in the modality of long strips. The measure of solution is based exclusively on techniques of analytical-function theory and is straightforward and compendious. By making some observations regarding the solution of the mixed boundary value problem we have reasonably decreased the amount of the computational work needed to resolve such a semi-infinite crack expansion problem. The changeable rule of dynamic stress intensity factor $K_1(t)$ versus time is similar to that of unlike boundary conditions in Literatures [6,19,20,25,33-41], consequently it is proved to be right.

Applying correlative representation: $f(x, y, t) = t^n f(x/t, y/t)$, where n is an integral number, and the problem considered can be facilely transformed into homogeneous function of x and t of zeroth dimension, namely self-similar function. Analytical solutions can be acquired by means of Eqs. (4), (5), (6) and (7) with homologous function of variable τ. This technique can be used not only in elastodynamics [19,20,22-25, 33,34], but also in elastostatics [21,30,42,43] and even in other domains [21,43,44]. The measures of self-similar functions are still applicable in researches of mode III crack [45-51] and mode III interface crack [52-59] as well as axially symmetrical crack, detailed in [1,6,21,60].

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APPENDIX 1

The algebra involved for finding D and D_1 for orthotropic anisotropy, and also for isotropy will be given. The notation follows those used by Broberg [61]. Denote $D_1 \equiv 0$ when $T_1 = -T_2$ as mentioned in [19,20,23-25,33], and also that D / D_1 is purely imaginary for the possible crack velocities involved in the problem.

Calculation of D, D_1 for orthotropy (for the subsonic speeds)

Now, in order to illustrate these representations for universal orthotropy, refer to the Eq. (7) in [19,20,23-25, 33], where η is substituted by τ, and hence

$$MT^4 + NT^2 + P = 0 \tag{A.1}$$

where $M = C_{66}\, C_{22}$

$$P = (C_{11} - \rho\tau^2)(C_{66} - \rho\tau^2)$$
$$N = C_{66}(C_{66} - \rho\tau^2) + C_{22}(C_{11} - \rho\tau^2) - (C_{12} + C_{66})^2 \tag{A.2}$$

write down certain properties of the roots, *i.e.* the sums and products, *etc*. From (A.1) there results

$$T_1^2 + T_2^2 = -N/M \ , \ (T_1 - T_1)^2 = -(N - 2\sqrt{PM})/M$$
$$T_1^2 T_2^2 = P/M \ , \ (T_1^2 - T_2^2)^2 = (N^2 - 4MP)/M^2 \tag{A.3}$$

write $C_r = \sqrt{C_{66}/\rho}$, $C_d = \sqrt{C_{11}/\rho}$, $a = (C_{66} - \rho\tau^2)$, $b = (C_{11} - \rho\tau^2)$

At $\tau^2 > C_d^2$, when $a < 0$, $b < 0$; presumed $C_{66} < C_{11}$, from the Eq. (A.2) the following results is obtained:

$$N^2 - 4PM = (C_{66}a + C_{22}b)^2 - 2(C_{66}a + C_{22}b)(C_{12} + C_{66})^2 + (C_{12} + C_{66})^4 - 4C_{66}\, C_{22}\, ab \tag{A.4}$$

Apparently $0 < N^2 - 4PM < N^2$, and $N < 0$, reduce $-N \pm \sqrt{N^2 - 4PM} > 0$, this represents T_1^2 , T_2^2 are both positive real, therefore all of the four roots of (A.1) are real. This tests that for $\tau^2 > C_d^2$ such that $\mathrm{Im}[D(\tau)/D_1(\tau)] = 0$, which indicates that the disturbance of elastic wave can not overrun $C_d t$.

Then putting Eqs. (6), (8) into (13) in literature [19,20,23-25,33], there results

$$D / D_1 = (T_1 - T_2) S(T_1, T_2, \tau) / \left[(T_2^2 - T_1^2)(C_{12} + C_{66})(C_{11} + \rho\tau^2) \right]$$
$$S = T_1 T_2 (C_{12} + C_{66}) \left[C_{12}^2 - C_{22}(C_{11} - \rho\tau^2) \right] - C_{66}(C_{11} - \rho\tau^2)(T_1^2 + T_2^2) + C_{12}\, C_{22}\, C_{66}\, T_1^2 T_2^2 + (C_{11} - \rho\tau^2) \left[C_{12}^2 + C_{12} C_{66} - C_{22}(C_{11} - \rho\tau^2) \right] \tag{A.5}$$

It is not difficult to attest that for $|\tau| < C_r$, D / D_1 is purely imaginary for the subsonic speeds. It follows that

Case (1) For $C_r < |\tau| < C_d$, remembering that we have taken the positive square root, it follows that

$$\mathrm{Im}[D / D_1] = \frac{Mg\sqrt{P'/M}\,(C_{12} + C_{66}) \left[C_{12}^2 - C_{22}(C_{11} - \rho\tau^2) \right] + MhS'}{\sqrt{N^2 - 4PM} \cdot (C_{12} + C_{66})(C_{11} - \rho\tau^2)} \tag{A.6}$$

where $P' = (C_{11} - \rho\tau^2)(-C_{66} + \rho\tau^2)$

$$g = \left(\frac{-N + \sqrt{N^2 + 4PM}}{2M} \right)^{1/2} \ , \ h = P' \left(\frac{-N + \sqrt{N^2 - 4PM}}{2} \right)^{1/2}$$
$$S' = C_{12}P + (C_{11} - \rho\tau^2) \left[C_{12}^2 + C_{12}C_{66} - C_{22}(C_{11} - \rho\tau^2) \right] + (C_{11} - \rho\tau^2) N / C_{22}$$

Case (2) For $|\tau| < C_r < C_d$, then it is also found that

$$\mathrm{Im}[D / D_1] = \left(\frac{M}{N + 2\sqrt{PM}} \right)^{1/2} \frac{S''}{(C_{12} + C_{66})(C_{11} - \rho\tau^2)} \tag{A.7}$$

$$S'' = (P/M)^{1/2}(C_{12} + C_{66}) \left[C_{12}^2 - C_{22}(C_{11} - \rho\tau^2) \right] - (C_{11} - \rho\tau^2) N / C_{22} + C_{12}P + (C_{11} - \rho\tau^2) \left[C_{12}^2 + C_{12}C_{66} - C_{22}(C_{11} - \rho\tau^2) \right]$$

where

$$M = C_{66}C_{22}$$
$$P = (C_{11} - \rho\tau^2)(C_{66} - \rho\tau^2)$$
$$N = C_{66}(C_{66} - \rho\tau^2) + C_{22}(C_{11} - \rho\tau^2) - (C_{12} + C_{66})^2 \tag{A.2}$$

Case (3) For isotropy

Isotropy is regarded a special ensample as orthotropy, from isotropy, we will have

$$C_{11} = C_{22} = \rho C_1^2 \ , \ C_{66} = 0.5(C_{11} - C_{12}) = \rho C_2^2 \tag{A.8}$$

where C_1, C_2 are the wave velocities in the isotropic medium, simply gives D/D_1 as a function of τ, then substituting them into Eq. (5) in literature [19,20,23-25, 33] and (A.6), there results

$$\frac{D(\tau)}{D_1(\tau)} = \frac{i\rho C_1 \left[(2C_2^2 - \tau^2)^2 - 4C_2^3 C_1^{-1} \sqrt{(C_1^2 - \tau^2)(C_2^2 - \tau^2)} \right]}{\tau^2 \sqrt{C_1^2 - \tau^2}} \tag{A.9}$$

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