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Approximation of a Great Circle by using a Circular Arc on a Mercator Chart

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This paper describes George Biddell Airy's almost completely unknown method of approximating an orthodromic arc (great circle arc) using a circular arc in the normal aspect Mercator projection of a sphere. In addition, it is demonstrated that the centre of the circle can be defined in at least two different ways, which yields slightly different results. Airy's approach is built upon in this paper. The method of computing coordinates of Airy's circle arc centre is described. The formulae derived in the paper can be used to calculate the length of Airy's approximation of the orthodromic arc connecting two points on the sphere and on the Mercator chart. Moreover, the actual length of the orthodromic arc on the sphere and on the Mercator chart can be computed using the formulae derived in this paper. The purpose of the paper is not to suggest an application of Airy's method in navigation, but to analyse Airy's proposal and to show that a great circle arc on a Mercator chart is close to a circular arc for distances which are not too great. This property can be useful in education, having in mind that the stereographic projection is the only one that maps any circle on a sphere onto a circle in the projection plane.

KEYWORDS

1. George Biddell Airy. 2. Orthodrome. 3. Mercator projection.

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1. INTRODUCTION. Sir George Biddell Airy was an astronomer, mathematician and physicist especially interested in geodesy and geophysics. Many of his achievements are associated with planet orbits, the Earth's density and solving two-dimensional problems in mechanics. In 1830, he determined dimensions of an ellipsoid based on surveying arcs of 14 parallels and four meridians. The ellipsoid was named after him. Airy established the Prime Meridian (Greenwich) in 1851, which was accepted internationally in 1884. Airy's most significant paper is "Figure of the Earth" which was published in Cambridge (Airy, 1826) and London (Airy, 1845).

Airy's contribution to science is best illustrated with a series of terms named after him: Airy's disk, Airy's point, Airy's hypothesis, Airy's ellipsoid 1830, Airy's map projection, Airy's isostasy theory, Airy's wave theory, Airy's functions Ai(x) and Bi(x) and the differential equation based on them, Airy-Heiskanen's gravity anomaly, Airy-Heiskanen's gravity reduction. Craters on Mars and the Moon were also named after Airy (Rambaut, 1911).

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According to his autobiography, Airy published 518 papers (Airy, 1896). His proposal of a new azimuthal projection is exceptional and it attracted a lot of attention from cartographic scientists (Snyder, 1993). A different paper, about approximating an orthodromic arc using an arc of a circle on a map produced in the normal aspect Mercator projection, is almost completely unknown and will be addressed in this paper. We are going to make Airy's original method more contemporary. Rather than use an ordinary pair of compasses to draw a circle, we can use a computer program. Furthermore, we can construct a circle using three points rather than the circle's centre and its radius. Finally, we are going to provide formulae which can be used to compare lengths of Airy's arc of a circle in the normal aspect Mercator projection with the actual length of the orthodromic arc on the Earth's sphere.

2. AIRY'S APPROXIMATION OF AN ORTHODROME ON A MERCATOR CHART.

Airy's paper published in 1858 (Airy, 1858) discusses approximating an orthodrome (geodetic line or a great circle on a sphere) with a circular arc on a map produced in the normal aspect Mercator projection of a sphere. His paper is rather unknown; it is not even mentioned in Snyder and Steward's (1988) famous bibliography on map projections. Airy had an idea about finding a method of approximating an orthodromic arc which could be used on a map using a ruler and an ordinary pair of compasses. In his paper, Airy (1858) explained:

> "1. Join the first and last points of the course (or the point of departure and the point of arrival) by a straight line; find its middle; erect there a perpendicular to that line, on the side next the equator; and produce it, if necessary, beyond the equator. The centre of the sweep will be on this perpendicular.
>
> 2. With middle latitude, enter the Table 1, and take out the corresponding parallel. (It will be remarked that the four first and the last can never come into use).
>
> 3. The centre of the sweep will be the intersection of the perpendicular (drawn in conformity with the precept in article 1) with the parallel thus found.
>
> 4. Fix one point of the compasses in this intersection, and with the other point sweep through the point of departure and the point of arrival; this sweep is the curve required."

Airy calculated the latitude of the parallel necessary to construct an orthodromic approximation in the Mercator projection using the following formula (by using Airy's original notation)

$$\log\tan\left(\frac{\varphi_S}{2}+45^\circ\right)=\log\tan\left(\frac{\varphi_M}{2}+45^\circ\right)-0{\cdot}43429448\times\text{nat.cosec}\varphi_m \qquad (1)$$

where φ_S is the required latitude, φ_M is the given middle latitude, 0·4342448 is the factor for transitioning to decimal logarithms and nat.cosec is the natural trigonometric function cosecant.

Airy lived during a period without computers and when people used various tables in order to facilitate calculations. That is why he provided necessary values in the table in his paper, in addition to formulae. Furthermore, there were tables with decimal logarithms (Briggs, 1617; sine data). Therefore, Airy provided Equation (1) with decimal logarithms.

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Table 1. Middle latitudes and corresponding parallel latitudes necessary for approximating an orthodrome in the Mercator projection (Airy, 1858). "Opposite name" means opposite sign, and "same name" means same sign.

Middle Latitude.	Corresponding Parallel.	Middle Latitude.	Corresponding Parallel.
5°	89°59′55″	48°	21°42′
10	89 34	50	16 39
15	86 52	52	11 33
20	81 13	54	6 24
22	78 16	56	1 13
24	74 59	58	4 0
26	71 26	60	9 15
28	67 38	62	14 32
30	63 37	64	19 50
32	59 25	66	25 9
34	55 5	68	30 30
36	50 36	70	35 52
38	46 0	72	41 14
40	41 18	74	46 37
42	36 31	76	52 1
44	31 38	78	57 25
46	26 42	80	62 51
		85	76 25

Opposite Name (5°–46°); Opposite Name (48°–58°); Same Name (60°–85°)

Nowadays, we do not have problems with using natural logarithms, so Airy's Equation (1) can be simplified to

$$\ln\tan\left(\frac{\varphi_S}{2}+\frac{\pi}{4}\right)=\ln\tan\left(\frac{\varphi_M}{2}+\frac{\pi}{4}\right)-\frac{1}{\sin\varphi_M}, \tag{2}$$

from which one is able to calculate the unknown value φ_S:

$$\varphi_S=2\arctan\left\{\exp\left[\ln\tan\left(\frac{\varphi_M}{2}+\frac{\pi}{4}\right)-\frac{1}{\sin\varphi_M}\right]\right\}-\frac{\pi}{2}. \tag{3}$$

Angles in Equations (2) and (3) are expressed in radians. They can be transformed into degrees in the usual way. Of course, tables such as Table 1 are no longer necessary.

3. COORDINATES OF THE CENTRE OF THE CIRCULAR ARC IN AIRY'S ORTHODROMIC APPROXIMATION. In describing his method of approximating an orthodrome using a circular arc, Airy applied a method of constructive geometry in which one coordinate of the circle's centre is obtained by calculating or reading from a table, while the other one is obtained from the intersection of two straight lines. Nowadays, it is easy to find a map in the normal aspect Mercator projection on the Internet (most navigational charts produced in national hydrographic organisations or for example, Google Maps produced in the so-called "web-Mercator" that is not the same but very similar to the conventional Mercator projection), where one is also able to mark any point based on its coordinates. The natural question is whether the other coordinate (latitude) of the centre of Airy's circle can be obtained numerically. The answer is yes.

Let us assume we have geographic coordinates of two places (φ_1,λ_1) and (φ_2,λ_2) on the Earth's sphere. Formulae for the normal aspect Mercator projection are as follows

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(Snyder, 1993):

$$x = R\lambda, \quad y = R \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right). \tag{4}$$

R is the proportionality constant, and we can take $R = 1$ without reducing generality. Therefore, equations for normal aspect Mercator projections are as follows:

$$x = \lambda, \quad y = \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right), \tag{5}$$

and we can use these formulae and geographic coordinates (φ_1, λ_1) and (φ_2, λ_2) to calculate rectangular coordinates of places / points $T_1(x_1, y_1)$ and $T_2(x_2, y_2)$ in the projection plane.

Let us mark the midpoint of segment $\overline{T_1T_2}$ with $M(x_M, y_M)$,

$$x_M = \frac{x_1 + x_2}{2}, \quad y_M = \frac{y_1 + y_2}{2}. \tag{6}$$

Let us suppose that the middle latitude φ_M in Airy's Equation (2) is the latitude of point M, i.e.

$$\varphi_M = 2 \arctan(\exp y_M) - \frac{\pi}{2} \tag{7}$$

because

$$y_M = \ln \tan\left(\frac{\pi}{4} + \frac{\varphi_M}{2}\right). \tag{8}$$

In order to determine latitude φ_S of centre S of the circular arc, we are going to apply Airy's Equation (2), which can also be written as

$$y_S = y_M - \frac{1}{\sin \varphi_M}. \tag{9}$$

Knowing y_S, we can calculate the latitude φ_S

$$\varphi_S = 2 \arctan(\exp y_S) - \frac{\pi}{2} \tag{10}$$

which is basically another form of Equation (3). If we want φ_S in degrees, we have to multiply the value from the last formula by $\frac{180^\circ}{\pi}$.

According to Airy, centre S of a circular arc, which approximates the geodetic line, is situated on the perpendicular bisector of the line which connects the two points or places in the normal aspect Mercator projection. We have the latitude φ_S of that centre, and we are going to obtain the longitude λ_S next.

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This is the equation of the straight line through points $T_1(x_1, y_1)$ and $T_2(x_2, y_2)$:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1). \tag{11}$$

The equation of the straight line perpendicular to the segment T_1T_2 and intersecting it at the middle point $M(x_M, y_M)$ given by Equation (6) is

$$y - y_M = -\frac{x_2 - x_1}{y_2 - y_1}(x - x_M). \tag{12}$$

Considering centre S is on that straight line, it has to be

$$y_S - y_M = -\frac{x_2 - x_1}{y_2 - y_1}(x_S - x_M). \tag{13}$$

Finally

$$\lambda_S = x_S = -\frac{y_2 - y_1}{x_2 - x_1}(y_S - y_M) + x_M. \tag{14}$$

If we want λ_S in degrees, the value from the last formula has to be multiplied by $\frac{180^\circ}{\pi}$.

Example 1: Let us assume we want to fly from London to Moscow. The geographic coordinates of London and Moscow are 51°30′N, 0°08′W and 55°45′N, 37°37′E, respectively. In order to construct a circular arc as an approximation of a great circle on a map in the normal aspect Mercator projection, we require the middle latitude φ_M, which in this case equals 53°41′N, obtained by Equation (7). Applying Equation (3) or (10) results in $\varphi_S = 7^\circ 14'$S. Compare this result with data in Table 1. According to Equation (15), $\lambda_S = 32^\circ 15'$E. Airy's circular arc and the image of the orthodrome connecting London and Moscow are almost the same, as represented in Figure 1.

4. ANOTHER APPROACH TO COORDINATES OF THE CENTRE OF THE CIRCULAR ARC IN AIRY'S ORTHODROMIC APPROXIMATION. In the previous section, we supposed the middle latitude φ_M in Airy's Equation (2) is the latitude of middle point M, with coordinates defined by Equation (6). Still, Airy did not explicitly determine his "middle latitude". This leads us to another possibility, which also looks natural. Let us start again with geographic coordinates (φ_1, λ_1) and (φ_2, λ_2) of two given points, and calculate rectangular coordinates of places / points $T_1(x_1, y_1)$ and $T_2(x_2, y_2)$ in the Mercator chart. Now, instead of Equation (6), we define

$$\varphi_M = \frac{\varphi_1 + \varphi_2}{2}. \tag{15}$$

Then, we can again obtain y_M by using Equation (8), y_S by using Equation (9) and φ_S by using Equation (10). Finally, $\lambda_S = x_S$ by using Equation (15), but in Equation (15) x_M and y_M are defined by Equation (6) following Airy's idea.

Example 2: Let us assume we want to fly from London to Moscow and take the same coordinates of these two places as in Example 1. By applying the approach described in this chapter, we obtained the middle latitude φ_M of 53°38′N, $\varphi_S = 7^\circ 22'$S and $\lambda_S = 32^\circ 17'$E, which are slightly different from those obtained in Example 1.

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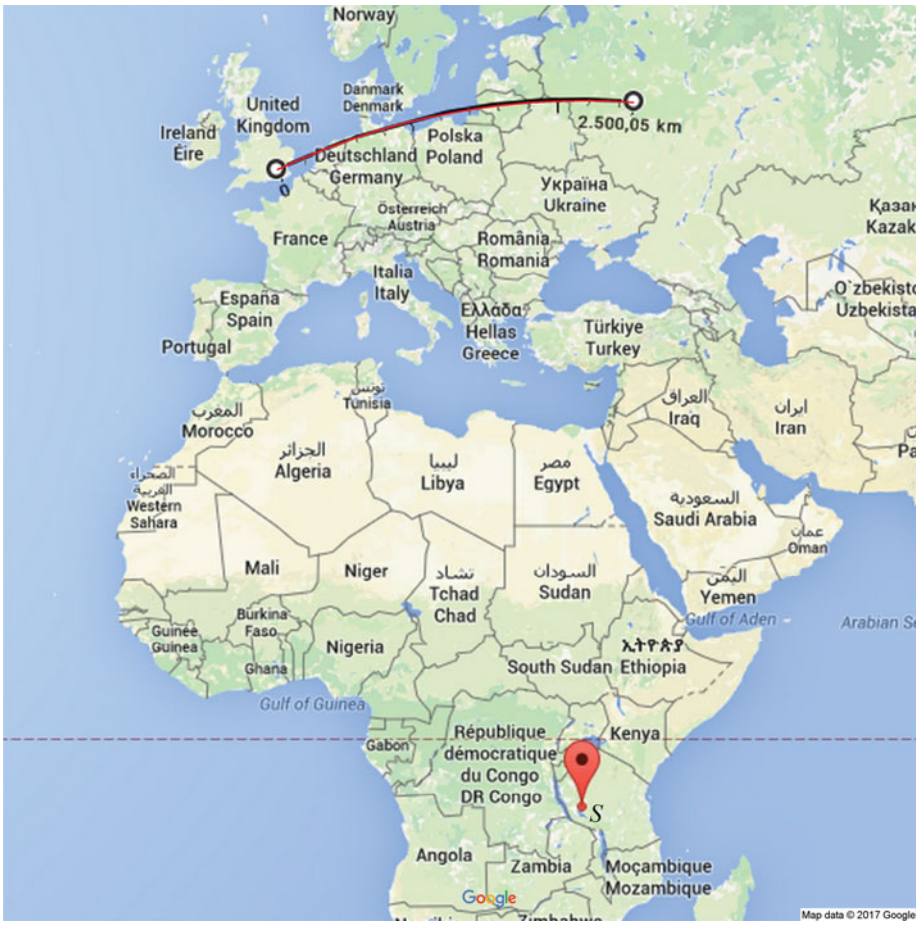


Figure 1. Airy's circular arc and the image of orthodrome connecting London and Moscow are almost the same. S is the centre of Airy's arc, all represented on a Google Maps base.

5. ARC LENGTH OF AN ORTHODROME AND ARC LENGTH OF ITS APPROXIMATION ON THE MERCATOR CHART.

5.1. *Orthodrome Arc Length.* Let us suppose the point $T_1(x_1, y_1)$ on the Mercator chart is the image of the point with coordinates (φ_1, λ_1) on the sphere, and the point $T_2(x_2, y_2)$ is the image of the point with coordinates (φ_2, λ_2). First, we would like to know the distance between the two points on the sphere measured along the orthodrome. The equation of such an orthodrome is (Lapaine, 2006):

$$\sin(\lambda - \beta) = a \tan \varphi, \tag{16}$$

where

$$\tan \beta = \frac{\tan \varphi_1 \sin \lambda_2 - \tan \varphi_2 \sin \lambda_1}{\tan \varphi_1 \cos \lambda_2 - \tan \varphi_2 \cos \lambda_1}, \tag{17}$$

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and

$$a = \frac{\sin(\lambda_1 - \lambda_2)}{\sqrt{\tan^2 \varphi_1 + \tan^2 \varphi_2 - 2 \tan \varphi_1 \tan \varphi_2 \cos(\lambda_1 - \lambda_2)}}. \tag{18}$$

From Equation (16), we can get

$$\cos(\lambda - \beta) d\lambda = a \frac{d\varphi}{\cos^2 \varphi}, \tag{19}$$

and then by using Equation (16)

$$d\lambda^2 = \frac{a^2 d\varphi^2}{\cos^2 \varphi \left[1 - (1 + a^2) \sin^2 \varphi\right]}. \tag{20}$$

The first differential form for the sphere ($R = 1$) parameterised by the geographic parameterisation reads

$$ds^2 = d\varphi^2 + \cos^2 \varphi d\lambda^2. \tag{21}$$

Taking into account Equation (20), Equation (21) can be transformed into

$$ds^2 = \frac{1 + a^2}{a^2} \cos^4 \varphi d\lambda^2, \tag{22}$$

from where we have

$$s = \int_{\lambda_1}^{\lambda_2} \frac{\sqrt{1 + a^2}}{a} \cos^2 \varphi d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{a\sqrt{1 + a^2}}{a^2 + \sin^2(\lambda - \beta)} d\lambda. \tag{23}$$

The last integral Equation (23) can be solved

$$\begin{aligned} s &= \arccos \frac{a \cos(\lambda - \beta)}{\sqrt{a^2 + \sin^2(\lambda - \beta)}} \Bigg|_{\lambda_1}^{\lambda_2} \\ &= \arccos \left[\cos \varphi_2 \cos(\lambda_2 - \beta)\right] - \arccos \left[\cos \varphi_1 \cos(\lambda_1 - \beta)\right], \end{aligned} \tag{24}$$

which, taking into account Equation (17) and after some transformations, leads to the solution of Equation (23) in the form

$$s = \arccos \left[\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos(\lambda_1 - \lambda_2)\right]. \tag{25}$$

Let s' denote the length of the orthodromic arc image on the Mercator chart. The question is how to compute it. According to the map projection theory, the linear scale c of a map projection in a point is defined by

$$c = \frac{ds'}{ds}, \tag{26}$$

where ds is a differential of an arc length on the surface to be projected (e.g. sphere), while ds' is its image in the projection plane. Furthermore, the linear scale of the normal aspect

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Mercator projection of the sphere with radius $R = 1$ and the equator as a standard parallel

$$c = \frac{1}{\cos\varphi}. \tag{27}$$

Combining Equations (26) and (27) by using Equation (22), we get

$$s' = \int_{\lambda_1}^{\lambda_2} \frac{\sqrt{1+a^2}}{a} \cos\varphi d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{\sqrt{1+a^2}}{\sqrt{a^2+\sin^2(\lambda-\beta)}} d\lambda. \tag{28}$$

The last integral Equation (28) is an elliptical integral and can be integrated numerically.

Example 3:

a) Let geographic coordinates of London and Moscow be 51°30′N, 0°08′W and 55°45′N, 37°37′E, respectively (as we know from the Examples 1 and 2). By using Equations (25) and (28) and multiplying obtained values with $R = 6,370$ km, the orthodromic distance from London to Moscow equals $s = 2,501$ km, while the distance s' of 4,325 km was obtained without taking into account distortions on a Mercator chart.
b) Let geographic coordinates of Atlanta and Frankfurt be 33°45′N, 84°23′W and 50°07′N, 8°41′E, respectively. By using Equations (25) and (28) and multiplying the obtained value with $R = 6,370$ km, the orthodromic distance from Atlanta to Frankfurt equals $s = 7,400$ km, while the distance s' equals 11,423 km, without taking into account distortions on a Mercator chart.

Example 3 reminds us that distances measured on or calculated from a map have to be corrected due to the inevitable distortions of the implemented map projection.

5.2. *Airy's Circle Arc Length.* Let us suppose again that point $T_1(x_1, y_1)$ on the Mercator chart is the image of the point with coordinates (φ_1, λ_1) on the sphere, and point $T_2(x_2, y_2)$ is the image of the point with coordinates (φ_2, λ_2). Now, we would like to know the distance between the two points on the Mercator chart measured along Airy's circle. The equation of the circle in the projection plane is going to be

$$(x - x_S)^2 + (y - y_S)^2 = r^2, \tag{29}$$

where (x_S, y_S) are coordinates of the circle centre determined by Equations (9) and (15), and r is the circle radius which can be easily computed as an ordinary distance between the circle centre and point $T_1(x_1, y_1)$ or point $T_2(x_2, y_2)$. Equation (29) can be represented in the

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following parametric form

$$x = r\cos t + x_S, \quad y = r\sin t + y_S \tag{30}$$

from where we obtain the circle arc length l' between points $T_1(x_1, y_1)$ and $T_2(x_2, y_2)$

$$l' = r(t_2 - t_1), \tag{31}$$

where

$$\tan t_1 = \frac{y_1 - y_S}{x_1 - x_S}, \quad \tan t_2 = \frac{y_2 - y_S}{x_2 - x_S}. \tag{32}$$

By using Equations (31) and (32), we can calculate the length of Airy's circle arc on a Mercator chart. The last question is: what is the length of the curve on the sphere which is going to be mapped onto Airy's circle in the projection plane? We are going to use Equations (29) and (5) to answer this question. The equation of the curve on the sphere is

$$(\lambda - x_S)^2 + \left[\ln\tan\left(\frac{\varphi}{2} + \frac{\pi}{4}\right) - y_S\right]^2 = r^2. \tag{33}$$

From Equations (5) and (30), we can write

$$y = \ln\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = r\sin t + y_S \tag{34}$$

and then derive

$$\sin\varphi = \tanh(r\sin\ t + y_S), \quad \cos\varphi = \frac{1}{\cosh(r\sin t + y_S)}. \tag{35}$$

According to the map projection theory, the linear scale c of a map projection in a point is going to be

$$c = \frac{dl'}{dl} \tag{36}$$

where dl is a differential of an arc length on the surface to be projected (e.g. sphere). The linear scale of the normal aspect Mercator projection of the sphere with radius $R = 1$ and with the equator as a standard parallel is given by Equation (27). It follows that

$$dl = \frac{dl'}{c} = \cos\varphi dl' = \cos\varphi r dt \tag{37}$$

and finally

$$l = \int_{t_1}^{t_2} \frac{r dt}{\cosh(r\sin t + y_S)}. \tag{38}$$

The last integral Equation (38) can be integrated numerically. In this way, we are able to calculate the length of Airy's approximation of the orthodromic arc connecting two points on the sphere Equation (38) and on the Mercator chart Equation (31), as well as the actual length of the orthodromic arc on the sphere Equation (25) and on the Mercator chart Equation (28). If one requires numerical values in units of length (e.g. kilometres), it

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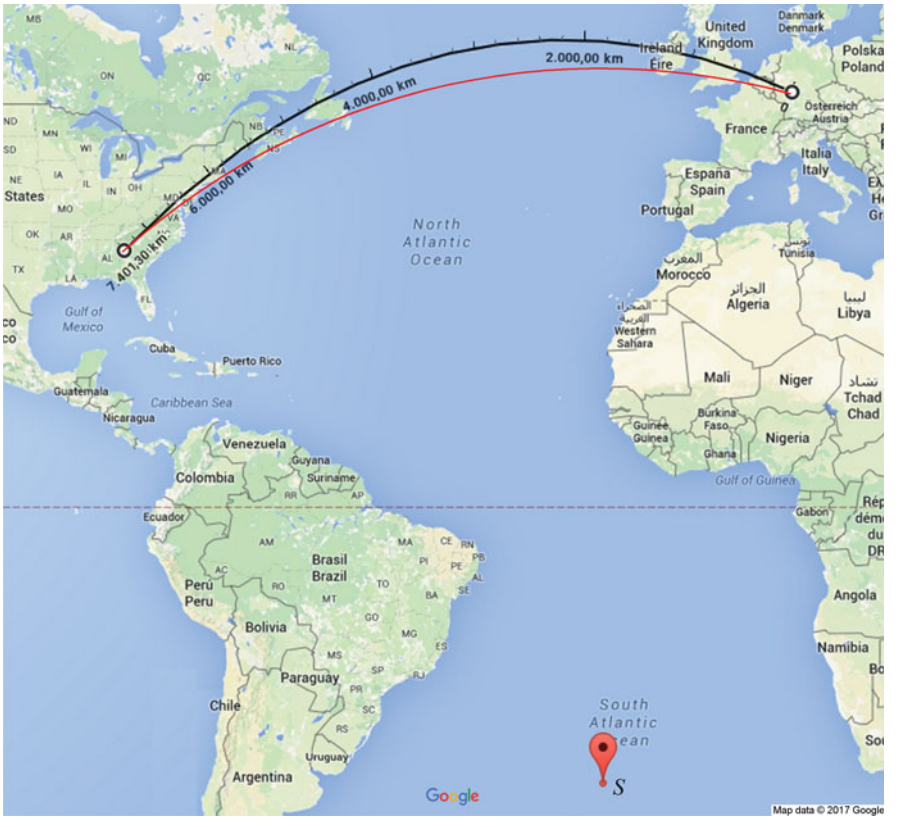


Figure 2. Airy's circular arc (line without ticks) and the image of orthodrome (line with ticks) connecting Frankfurt and Atlanta. S is the centre of Airy's arc, all represented on a Google Maps base.

is necessary to set the radius of sphere R (e.g. $R = 6,370$ km) and multiply that value with results of the formulae obtained with the assumption that $R = 1$.

Example 4:

a) Let geographic coordinates of London and of Moscow be the same as in Examples 1, 2 and 3a. By using Equations (31) and (38) and multiplying the obtained values with $R = 6,370$ km, we obtain Airy's circular distance from London to Moscow on a Mercator chart $l' = 4,320$ km without taking distortions into account, while the corresponding distance l on a sphere equals 2,501 km.
b) Let geographic coordinates of Atlanta and Frankfurt be the same as in Example 3(b). By using Equations (31) and (38) and multiplying the obtained value with $R = 6,370$ km, we obtain Airy's circular distance from Atlanta to Frankfurt on a Mercator chart $l' = 11,082$ km without taking distortions into account, while the corresponding distance l on a sphere equals 7,425 km.

By comparing the results of Example 3 to those of Example 4, we can conclude that Airy's method of approximating an image of a great circle arc by a circle arc in the projection plane provides useful values.

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5.3. *Visualisation of Airy's Approximation.* Instead of computing numerical examples using derived formulae, we can visually illustrate properties of Airy's approach. In Figure 1, we can see that Airy's circular arc and the image of the orthodrome connecting London and Moscow are almost the same. *S* is the centre of Airy's arc. Figure 2 is different. One can easily distinguish Airy's circular arc represented without ticks and the image of the orthodrome represented with ticks, connecting Frankfurt and Atlanta. The reason is obvious: Airy's method is an approximation, which is better for small distances.

6. CONCLUSION. This paper is meant to illuminate a part of cartographic, maritime and geodetic heritage which was unknown for a very long time. Airy explained the practical side of his method in his paper's introduction. In discussions held in the Scientific Office of the Board of Trade on how to facilitate navigation by drawing an orthodrome on a Mercator chart, it was proposed to draw a circular arc using a pair of compasses. This motivated Airy to do research and propose his own method, which is explained and built upon in this paper.

This paper presents the first ever method of computing coordinates of Airy's circle arc centre, defining the "middle latitude" in two ways. Furthermore, the paper explains how to calculate the length of Airy's approximation of the orthodromic arc connecting two points on the Mercator chart and on the sphere. Moreover, the actual length of the orthodromic arc on the sphere and on the Mercator chart can be computed by using formulae derived in the paper.

Finally, let us emphasise the method described is rough (as Airy put it, *very approximately*). In deriving his method, Airy observed deviation of an orthodrome from the rhumb line and applied the development into Maclaurin's series. This means an approximation is going to be better if the distance between the initial and final points is smaller. The method is described in detail in Airy's original work (Airy, 1858). It would be interesting to explore in more details how Airy's method depends on latitudes of given points and their distances.

It is not recommended to apply Airy's method to navigation, because we have new technology based on exact methods. Nevertheless, the method has historical value. It can be useful for teaching navigation and map projections, keeping in mind that the stereographic projection is the only one which maps any circle on a sphere onto a circle in the projection plane.

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