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New technique in plasma polarimetry: Evolution equations for angular parameters 'amplitude ratio–phase difference' of polarization ellipse

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Abstract. New technique is suggested in plasma polarimetry: Differential equations for angular parameters of polarization ellipse, characterizing the amplitude ratio and the phase difference between orthogonal components of the wave field. Equations for angular variables 'amplitude ratio–phase difference' are derived, which allow direct calculation of the parameters of polarization ellipse, omitting solutions for the Stokes vector. The simplest analytical solutions are presented for the pure Faraday and the pure Cotton–Mouton effects. Behavior of angular parameters in the homogeneous and inhomogeneous plasmas is illustrated by numerical modeling in conditions when the Faraday and Cotton–Mouton effects are large enough and comparable in strength.

1. Introduction

Traditional plasma polarimetry deals with two sets of angular parameters of polarization ellipse. The first set (ψ, χ) consists of polarization (azimuthal) angle ψ and ellipticity angle χ, shown in Fig. 1 (left). Azimuthal angle ψ determines the tilt of the major axis of polarization ellipse in the frame of the orthogonal coordinate system $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{l})$, where $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors, transverse to the ray and $\mathbf{l}$ is a unit vector tangent to the ray. Angle χ, characterizing ellipticity e of polarization ellipse, is involved as [1]

$$\tan\chi = \frac{b}{a} = e. \tag{1}$$

Here a and b respectively are the major and minor axes of polarization ellipse (Fig. 1).

The second set of angular variables (α, δ) consists of the phase difference $\delta = \delta_2 - \delta_1$ between orthogonal components $\Gamma_1 = a_1 \exp(-i\omega t + i\delta_1)$ and $\Gamma_2 = a_2 \exp(-i\omega t + \delta_2)$ of the polarization vector $\Gamma = (\Gamma_1, \Gamma_2)$ and auxiliary angle α, defined through the amplitude ratio [1]:

$$\tan\alpha = \left|\frac{\Gamma_2}{\Gamma_1}\right| = \frac{a_2}{a_1}. \tag{2}$$

As a result, the complex amplitude ratio (CAR), $\zeta = \Gamma_2/\Gamma_1$ takes the form [2]

$$\zeta = \frac{\Gamma_2}{\Gamma_1} = \frac{a_2}{a_1}\exp(i\delta) = \tan\alpha \cdot \exp(i\delta). \tag{3}$$

Besides traditional angular parameters, (ψ, χ) and (α, δ), a third set of angular variables consisting of real and imaginary parts of complex polarization angle (CPA), $\gamma = \mathrm{Re}\,\gamma + i\mathrm{Im}\,\gamma$, has been suggested. The latter was involved in [3] by the relation,

$$\tan\gamma \equiv \zeta. \tag{4}$$

(Also see [4–11].) As was shown in [8], the real part of CPA equals to polarization (azimuthal) angle ψ,

$$\mathrm{Re}\,\gamma = \psi, \tag{5}$$

whereas the hyperbolic tangent of the imaginary part, $\mathrm{Im}\,\gamma \equiv \beta$, coincides with the tangent of the ellipticity angle (1):

$$\tanh(\mathrm{Im}\,\gamma) \equiv \tanh\beta = \tan\chi = e. \tag{6}$$

Equations for the evolution of angular parameters (ψ, χ), (α, δ), and $(\mathrm{Re}\,\gamma, \mathrm{Im}\,\gamma) \equiv (\psi, \beta)$ in magnetized plasma are of significant practical and theoretical interest for plasma polarimetry. Chronologically, the first equations for angular parameters were suggested in [3] for CPA, $\gamma = \mathrm{Re}\,\gamma + i\mathrm{Im}\,\gamma$. Equation for CPA, derived in [3], was rewritten in [8] in the following form:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\sigma} = \frac{1}{2}(\Omega_3 - i(\Omega_1 \sin 2\gamma - \Omega_2 \cos 2\gamma)), \tag{7}$$

where σ is the distance along the ray. Here, Ω_1, Ω_2, and Ω_3 are parameters involved in plasma polarimetry as suggested by Segre [12–14]. These parameters are reproduced below (10).

Simultaneously, Czyz et al. [8] derived the following system of coupled differential equations for azimuthal and ellipticity angles (ψ, χ):

$$\begin{aligned} \frac{\mathrm{d}\psi}{\mathrm{d}\sigma} &= \frac{1}{2}\Omega_3 + \frac{1}{2}(\Omega_1 \cos 2\psi + \Omega_2 \sin 2\psi)\tan 2\chi, \\ \frac{\mathrm{d}\chi}{\mathrm{d}\sigma} &= -\frac{1}{2}(\Omega_1 \sin 2\psi - \Omega_2 \cos 2\psi). \end{aligned} \tag{8}$$

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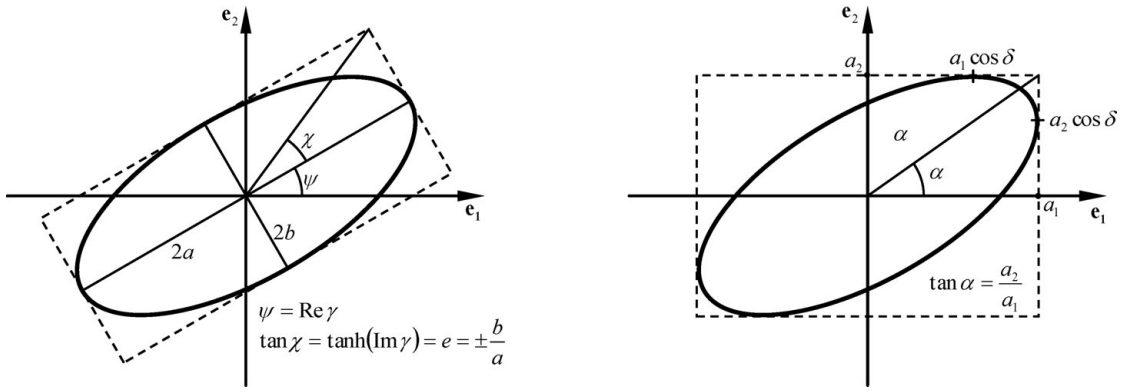


Figure 1. Two sets of angular parameters of polarization ellipse used in polarimetry: (left) 'azimuth-ellipticity' parameters (ψ, χ), and (right) 'amplitude ratio–phase difference' parameters (α, δ).

In this paper we intend to derive the evolution equation for angular parameters (α, δ), thereby accomplishing the derivation of equations for all three sets of angular parameters, $(\mathrm{Re}\,\gamma, \mathrm{Im}\,\gamma) \equiv (\psi, \beta)$, (ψ, χ), and (α, δ), mentioned above. Besides, we illustrate the behavior of angular parameters (α, δ) by the results of numerical modeling in the conditions of simultaneously strong Faraday and Cotton–Mouton effects.

The paper is organized as follows. Section 2 derives the evolution equations for angular parameters (α, δ). Section 3 outlines the simplest analytical solutions of these equations. Finally, Sec. 4 presents the results of numerical modeling of angular parameters (α, δ).

2. Derivation of equations for angular parameters α and δ

According to the quasi-isotropic approximation (QIA) of the geometrical optics method, which describes electromagnetic waves in weakly anisotropic media [3–11], differential equation for CAR, $\zeta = \Gamma_2/\Gamma_1$, has a form

$$\frac{\mathrm{d}\zeta}{\mathrm{d}\sigma} = \frac{1}{2}((\Omega_3 + i\Omega_2) - 2i\Omega_1\zeta + (\Omega_3 - i\Omega_2)\zeta^2), \quad (9)$$

Plasma parameters $\Omega_1, \Omega_2, \Omega_3$, and $\Omega_\perp$ are involved in plasma polarimetry as mentioned by Segre [12–14]:

$$\begin{cases} \Omega_1 = \frac{1}{2}k_0 X Y^2 \sin^2\alpha_{||}\cos 2\alpha_\perp = \Omega_\perp \cos 2\alpha_\perp, \\ \Omega_2 = \frac{1}{2}k_0 X Y^2 \sin^2\alpha_{||}\sin 2\alpha_\perp = \Omega_\perp \sin 2\alpha_\perp, \\ \Omega_3 = k_0 X Y \cos\alpha_{||}, \\ \Omega_\perp = \sqrt{\Omega_1^2 + \Omega_2^2} = \Omega_{\perp 0}\sin^2\alpha_{||}, \quad \Omega_{\perp 0} = \frac{1}{2}k_0 X Y^2. \end{cases} \quad (10)$$

Parameter Ω_3, which is linear in magnetic field B_0, corresponds to the Faraday effect, while the terms with parameters Ω_1, Ω_2, and $\Omega_\perp$, which are quadratic in B_0, describe the Cotton–Mouton effect. In turn, parameters $\Omega_1, \Omega_2, \Omega_3$, and $\Omega_\perp$ can be expressed through standard plasma parameters X and Y, defined as [15, 16]

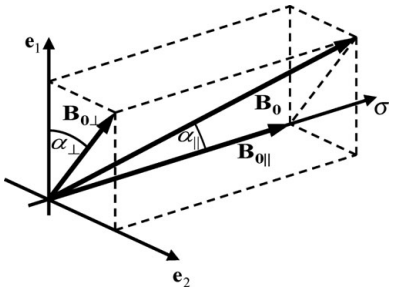


Figure 2. Magnetic vector $\mathbf{B}_0$ in frame of the coordinate system $\mathbf{e}_1, \mathbf{e}_2, \mathbf{l}$, which performs parallel transport of the vector wave field $\mathbf{E}$ along the ray.

$$X = \frac{\omega_p^2}{\omega^2} = \frac{e^2 N_e}{\varepsilon_0 m\omega^2}, \quad Y = \frac{\omega_c}{\omega} = \frac{eB_0}{m\omega}. \quad (11)$$

Parameter X characterizes electron density N_e. In particular, it determines the isotropic part of dielectric permittivity, $\varepsilon_0 = 1 - X$. In turn, parameter Y characterizes static magnetic field $\mathbf{B}_0$. In above equations, $\alpha_{||}$ is an angle between the ray and magnetic vector $\mathbf{B}_0$, $\alpha_\perp$ is an angle between the unit vector $\mathbf{e}_1$ and transverse component of the magnetic vector $\mathbf{B}_{0\perp}$, as shown at Fig. 2. It is assumed that unit vectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{l})$ belong to parallel transport coordinate system [17, 18], which is quite helpful in plasma polarimetry [10–12].

We will make use of (9) for the derivation of the equations governing angular parameters, α and δ. Substituting (3) into (9) and separating the real and imaginary parts, we arrive at the following system of differential equations for angular parameters, α and δ:

$$\begin{aligned} \frac{\mathrm{d}\alpha}{\mathrm{d}\sigma} &= \frac{1}{2}(\Omega_2 \sin\delta + \Omega_3 \cos\delta), \\ \frac{\mathrm{d}\delta}{\mathrm{d}\sigma} &= -\Omega_1 + (\Omega_2 \cos\delta - \Omega_3 \sin\delta)\cot 2\alpha. \end{aligned} \quad (12)$$

Derivation of (12) is the main result of this paper.

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3. Simple analytical solutions

Equations (12) can be solved analytically for the simplest situations, corresponding to the pure Faraday and pure Cotton–Mouton effects.

(1) *Pure Faraday effect*. In the case of the pure Faraday effect, the Cotton–Mouton terms, Ω_1 and Ω_2, are small as compared with the Faraday term, Ω_3, so that (12) takes the form

$$\frac{d\alpha}{d\sigma} = \frac{1}{2}\Omega_3 \cos\delta, \quad \frac{d\delta}{d\sigma} = -\Omega_3 \sin\delta \cot 2\alpha. \tag{13}$$

In the partial case of *circular polarization*, when the initial phase difference equals $\delta_0 = \pm\pi/2$ and $\alpha = \pi/4$, the right-hand side of (13) turns out to be zero:

$$\frac{d\alpha}{d\sigma} = 0, \quad \frac{d\delta}{d\sigma} = 0. \tag{14}$$

Solution of (14),

$$\alpha = \text{const} = \pi/4, \delta = \text{const} = \pm\pi/2, \tag{15}$$

corresponds to the wave of right- or left-hand circular polarization.

For *linearly polarized wave*, the phase difference δ is zero, and (12) takes the form

$$\frac{d\alpha}{d\sigma} = \frac{1}{2}\Omega_3, \quad \frac{d\delta}{d\sigma} = 0. \tag{16}$$

Solution to (16) is

$$\alpha = \alpha_0 + \tfrac{1}{2}\int \Omega_3 d\sigma = \alpha_0 + \tfrac{1}{2}W_3, \quad \delta = \delta_0 = 0. \tag{17}$$

The amplitude ratio, $\tan\alpha$, changes periodically along the ray according to

$$\tan\alpha = \tan\left(\alpha_0 + \tfrac{1}{2}W_3\right). \tag{18}$$

(2) *Pure Cotton–Mouton effect*. In this case parameter Ω_3 is negligibly small as compared with parameters Ω_1 and Ω_2: $\Omega_3 \ll \Omega_1, \Omega_2$. In these conditions, (12) becomes

$$\frac{d\alpha}{d\sigma} = \frac{1}{2}\Omega_2 \sin\delta, \quad \frac{d\delta}{d\sigma} = -\Omega_1 + \Omega_2 \cos\delta \cdot \cot 2\alpha. \tag{19}$$

Partial solution of these equations, when coordinate vector $\boldsymbol{e}_1$ is directed along magnetic field and $\alpha_\perp = 0$, $\Omega_2 = 0$, $\Omega_1 = \Omega_\perp$, takes a very simple form,

$$\frac{d\alpha}{d\sigma} = 0, \quad \frac{d\delta}{d\sigma} = -\Omega_\perp. \tag{20}$$

Solution of this system is

$$\alpha = \text{const}, \quad \delta = \delta_0 - \int \Omega_\perp d\sigma = \delta_0 - W_\perp, \tag{21}$$

which corresponds to the superposition of two linearly polarized normal waves, whereas propagation constants differ in $\Omega_\perp$.

(3) *Normal waves in plasma*. Equation (12) admits exact solution for normal waves in *the homogeneous* plasma. In this case the parameters α and δ obey the equations

$$\frac{d\alpha}{d\sigma} = 0, \quad \frac{d\delta}{d\sigma} = 0, \tag{22}$$

which are satisfied by

$$\tan\delta = -\Omega_3/\Omega_2 \quad \tan\alpha = (-\Omega_1 \pm \Omega)/\sqrt{\Omega_2^2 + \Omega_3^2} \tag{23}$$

with $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2 + \Omega_3^2}$. These results are in agreement with the CAR $\zeta = (-\Omega_1 \pm \Omega)/(\Omega_2 + i\Omega_3)$ derived in [11].

(4) *The limit accuracy of the Cotton–Mouton measurements*. As is well known, in presence of poloidal magnetic field, magnetic lines in tokamak acquire spiral form. In conditions when coordinate vector $\boldsymbol{e}_1$ is directed along toroidal magnetic field, dominating in tokamaks, the toroidal component of shear angle $\alpha_\perp$ is taken to be zero: $\alpha_{\perp\text{Toroidal}} = 0$. Then due to poloidal field the shear angle $\alpha_\perp$ will experience only small variations:

$$\alpha_{\perp\text{Poloidal}} = \tilde{\alpha}_\perp. \tag{24}$$

In these conditions parameter Ω_1 can be presented as

$$\Omega_1 = \Omega_\perp \cos 2\tilde{\alpha}_\perp \cong \Omega_\perp(1 - 2\sin^2\tilde{\alpha}_\perp). \tag{25}$$

Considering the value $(-2\Omega_\perp \sin^2\tilde{\alpha}_\perp)$ as small perturbation, solution of (12) can be presented as

$$\alpha \cong \text{const}, \quad \delta = \delta_0 - \int \Omega_\perp d\sigma + \int \Omega_\perp \sin^2\tilde{\alpha}_\perp d\sigma. \tag{26}$$

The last term in (26) describes small deviation from the pure Cotton–Mouton phase difference, $\delta_{CM} = \delta_0 - \int \Omega_\perp d\sigma$, and coincides with the value of deviation,

$$\delta_1 = \int \Omega_\perp \sin^2\tilde{\alpha}_\perp d\sigma, \tag{27}$$

which was derived in [19] on the basis of perturbative approach, applied to (9) for CAR. Kravtsov et al. [19] have used a simple model of spiral magnetic lines in toroidal plasma, suggested in [20]. According to Kravtsov et al. [19], deviation (27) for toroidal plasma with parameters, similar to the ITER project, does not exceed 6% of the basic Cotton–Mouton effect, $\delta_{CM} = \delta_0 - \int \Omega_\perp d\sigma$. Though the value δ_1 is small as compared with the Cotton–Mouton phase difference δ_{CM}, it plays an important role in plasma polarimetry, determining the limit accuracy of the Cotton–Mouton measurements [19].

4. Numerical modeling in conditions of the strong Faraday and Cotton–Mouton effects

Here we will consider evolution of angular parameters α and δ in conditions of the sufficiently strong Faraday and Cotton–Mouton effects for toroidal plasma, with sounding ray forward through the center of the plasma, upright, with inclination $\alpha_{||}$ to the magnetic axis, close to $\pi/2$ (Fig. 3). As in toroidal plasma, the toroidal magnetic field value depends only on major radius R and does not depend on the vertical coordinate ξ, B_{tor} is constant along the assumed probing beam. In addition, we will assume that density profile, $N_e(\xi)$, depends only on a normalized distance $\rho = \xi/a$ from a plasma axis and takes the quadratic form

$$N_e = N_0(1 - \rho^2). \tag{28}$$

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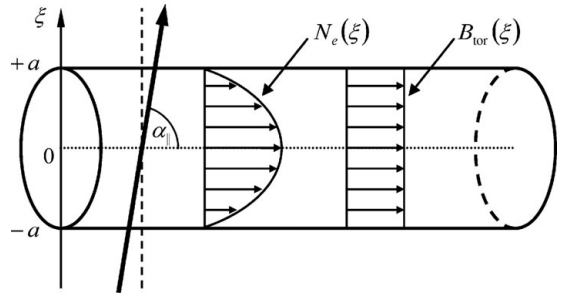


Figure 3. Probing beam path at cylindrical plasma, characterized by density profile $N_e(\xi)$ and toroidal magnetic field profile $B_{\rm tor}(\xi)$.

For such an experimental configuration, the Faraday effect rises only from the parallel to the beam component of toroidal magnetic field and the Cotton–Mouton effect mainly rises from the transverse to the beam component of that field. The poloidal field, as a much weaker field than the toroidal field and therefore giving negligible access to the Cotton–Mouton effect ($\alpha_\perp \approx 0$), will be omitted in our calculations. The components of Ω vector could be written as

$$\begin{cases} \Omega_1 = \frac{e^4}{16\pi^3\varepsilon_0 m^3 c^4}\lambda^3 N_e B_{tor}^2 \sin^2\alpha_{||}, \\ \Omega_2 \approx 0, \\ \Omega_3 = \frac{e^3}{4\pi^2\varepsilon_0 m^2 c^3}\lambda^2 N_e B_{tor}\cos\alpha_{||}. \end{cases} \tag{29}$$

It is worth to notice that the relative strength

$$R = \frac{\Omega_1}{\Omega_3} = \frac{e}{4\pi mc}\lambda_e B_{\rm tor}\frac{\sin^2\alpha_{||}}{\cos\alpha_{||}} \tag{30}$$

of the Faraday and Cotton–Mouton effects remains constant along the whole path and for a given wavelength it depends only on inclination angle, $\alpha_{||}$. In particular, when

$$\alpha_{||1} \approx \arccos\left(\frac{e}{4\pi mc}\lambda_e B_{\rm tor}\right), \tag{31}$$

parameters Ω_1 and Ω_3 are equal to each other and consequently the strengths of the Faraday and Cotton–Mouton effects are the same. Angle $\alpha_{||1}$ separates the domain of quasi-transverse propagation, where $\Omega_1 > \Omega_3$ ($R > 1$) and the Cotton–Mouton effect prevails from the domain of quasi-longitudinal propagation, where $\Omega_1 < \Omega_3$ ($R < 1$), and the Faraday effect plays a leading role.

In this entire section numerical modeling will be performed for the following plasma parameters: electron density, $N_e = 1\cdot 10^{20}\mathrm{m}^{-3}$; toroidal magnetic field, $B_{\rm tor} = 4\,\mathrm{T}$; wavelength, $\lambda = 195\,\mu\mathrm{m}$; and plasma cylinder diameter, $D = 2a = 6\,\mathrm{m}$. Initial polarization $\alpha_0 = \alpha(-a) = \pi/4$ and $\delta_0 = \delta(-a) = 0$ correspond to the linear initial polarization of the wave field, when vector **E** with respect to magnetic field $\mathbf{B}_{\rm tor}$ is inclined at 45°. In what follows, we take for analysis three different values of angle $\alpha_{||}$:

$$\alpha_{||R} \approx \arccos\left(\frac{e}{4\pi mc}\frac{\lambda_e B_{\rm tor}}{R}\right) \tag{32}$$

corresponds to three values of parameter, R: $R = 1/2$, $R = 1$, and $R = 2$. Since all angles $\alpha_{||R}$ are close to $\pi/2$, in all three cases we deal with comparable strengths of the Cotton–Mouton effect,

$$\Omega_{1R} \approx \frac{e^4}{16\pi^3\varepsilon_0 m^3 c^4}\lambda^3 N_e B_{\rm tor}^2, \tag{33}$$

whereas the strength of the Faraday is changeable and proportional to $\alpha_{||R}$,

$$\Omega_{3R} \approx \frac{e^3}{4\pi^2\varepsilon_0 m^2 c^3}\lambda^2 N_e B_{\rm tor}\left(\frac{\pi}{2} - \alpha_{||R}\right). \tag{34}$$

The results of numerical solution for (12) along the ray path, compared with solutions on amplitude angle, α_F, for the pure Faraday ($\Omega_{3R} \equiv 0$) and on phase difference, δ, for the pure Cotton–Mouton ($\Omega_{1R} \equiv 0$) transformations, are presented at Fig. 4.

According to Fig. 4, the interaction between the Faraday and Cotton–Mouton effects is negligible for small values of α and δ angles. However, at some point this interaction brings on the suppression of the amplitude angle α and amplification of the phase difference δ, compared with the pure Faraday and Cotton–Mouton effects. It is clear from (12) that in the case when $\Omega_2 \approx 0$, α suppression is weak as long as $\cos\delta \approx 1$ – that is change in δ caused by the Cotton–Mouton effect is small. In practice, this condition could be rewritten in the form

$$\cos\delta \geqslant 0.9 \text{ or } \delta < 0.45. \tag{35}$$

At the same time the phase difference amplification is weak as long as α and δ remain so small that $\sin\delta\cot 2\alpha \gg \Omega_1/\Omega_3$. Last condition could be rewritten as

$$\sin\delta\cot 2\alpha < 0.1\frac{\Omega_1}{\Omega_3} = 0.1R. \tag{36}$$

Equations (35) and (36) determine conditions when the effect of interaction between the Faraday and Cotton–Mouton effects is not sensible. Then, within an approximation of 10%, the amplitude angle α could be calculated for the pure Faraday effect – $\alpha = \alpha_0 + \frac{1}{2}\int\Omega_3 d\sigma = \alpha_0 + \frac{1}{2}W_3$ and the phase difference δ as for the pure Cotton–Mouton effect, $\delta = \delta_0 - \int\Omega_1 d\sigma = \delta_0 - W_1$.

5. Conclusions

This paper suggests new instrument in plasma polarimetry: A system of two coupled differential equations of first order for angular parameters 'amplitude ratio–phase difference'. These equations round out differential equations for all angular variables of polarization ellipse suggested until now: ζ, γ, (ψ,χ), and (α,δ). In the presented form these are able to describe variations of angular variables (α,δ) in all weakly anisotropic media. In particular, the introduced equations allow to determine condition when the Faraday and Cotton–Mouton effects interaction is negligible and present analytical solutions in such a case.

Significant advantage of the technique under consideration is that it allows omitting the solution of the

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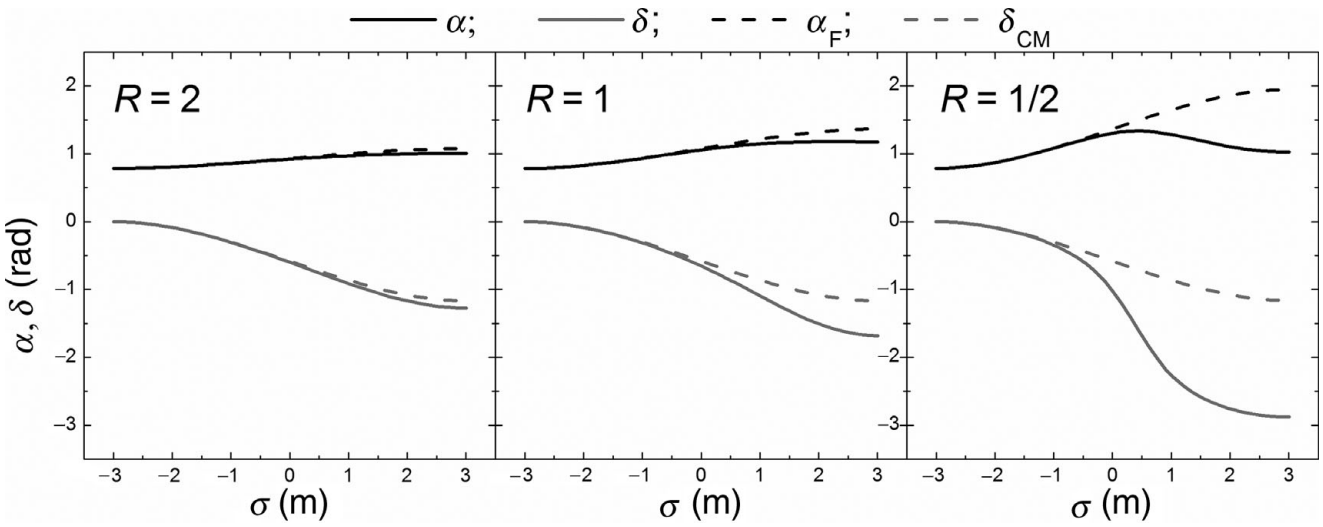


Figure 4. Variations of amplitude angle α (black) and phase difference δ (gray) along the ray in plasma under joint action (solid curves) and the 'pure' (dashed curves) Faraday and Cotton–Mouton effects.

equation for the Stokes vector, dealing only with angular variables.

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