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CoMargin

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Abstract

We present CoMargin, a new methodology to estimate collateral requirements in derivatives central counterparties (CCPs). CoMargin depends on both the tail risk of a given market participant and its interdependence with other participants. Our approach internalizes trading externalities and enhances the stability of CCPs, thus reducing systemic risk concerns. We assess our methodology using proprietary data from the Canadian Derivatives Clearing Corporation that include daily observations of the actual trading positions of all of its members from 2003 to 2011. We show that CoMargin outperforms existing margining systems by stabilizing the probability and minimizing the shortfall of simultaneous margin-exceeding losses.

I. Introduction

In an effort to enhance market stability after the 2008 financial crisis, G20 member countries mandated the central clearing of standardized derivatives

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(U.S. Department of Treasury (2009), European Union (2012)). As a consequence, a growing fraction of the $700 trillion (notional) derivatives market (Bank for International Settlements (BIS) (2015a)) is now being channeled through central counterparties (CCPs). The resulting concentration of credit risk in CCPs has increased their systemic importance (Acharya, Engle, Figlewski, Lynch, and Subrahmanyam (2009), Pirrong (2011), Duffie, Li, and Lubke (2010), Duffie (2013a), and Menkveld (2013)), making it necessary for regulators and market participants to assess and potentially re-evaluate the risk management practices of these institutions.[1]

Precautionary measures used to protect CCPs against default include margin (collateral) requirements, minimum capital levels, and contributions to risk mutualization arrangements, such as a default fund. In addition, clearing members are required to segregate their firm (i.e., proprietary) and client accounts (Jordan and Morgan (1990)) and enter into private insurance agreements (Jones and Pérignon (2013)). Among all of these clearing house safeguards, however, margin requirements are the most widely used, the most important, and, arguably, the most effective.[2]

In this paper, we propose a new margining methodology, called CoMargin, which accounts for the variability and interdependence of the profits and losses (P&Ls) of clearing members. Indeed, other margining systems commonly used in CCPs, such as the Standard Portfolio Analysis of Risk (SPAN) or Value-at-Risk (VaR) approaches, estimate collateral requirements based on a coverage level of potential losses for an individual contract or portfolio of contracts (Figlewski (1984), Kupiec (1994), Booth, Broussard, Martikainen, and Puttonen (1997), Cotter (2001), Day and Lewis (2004), and Chicago Mercantile Exchange (CME) (2012)). However, by focusing primarily on individual exposures, these alternative systems ignore the fact that the P&Ls of clearing members are often highly interdependent and can leave the CCP exposed to simultaneous extreme losses.

To highlight the importance of P&L interdependence, consider the role of collateral in a CCP. Margin requirements are designed to protect the CCP against losses in clearing member portfolios. However, there are times when these losses exceed the value of pledged collateral, resulting in negative margin account balances. Clearing members facing these margin "exceedances" may delay payment or even default on their obligations, thus generating a shortfall in the market and forcing the CCP to compensate all winning counterparties using the default fund.[3]

[1] In a derivatives exchange, the clearing house confirms, matches, and settles all trades. Clearing houses operate with a small number of clearing members who are allowed to submit proprietary and client trades for clearing. The process of novation allows the clearing house to become the sole counterparty to every trade cleared by its members (Bliss and Steigerwald (2006), Pirrong (2011)). Thus, the clearing house concentrates credit risk but remains market risk neutral by matching/netting long and short positions.

[2] Aggregate margins exceed 90% of total default protection (i.e., margin + default fund + default insurance + other guarantees) for most leading CCPs (source: www.cmegroup.com/clearing and www.lchclearnet.com). Margins in CCPs primarily consist of Treasuries (>90%) and other high-quality liquid assets. Cash accounts for less than 1% of all collected margins.

[3] The typical default waterfall system in a CPP works as follows: i) if firm i incurs losses that it cannot cover within a predefined period, then the CCP liquidates the assets in that member's margin account to cover these losses. ii) If the proceeds are insufficient, the CCP liquidates firm i's contribution to the default fund. iii) Remaining losses are then mutualized by liquidating other firms'

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Usually, financing the shortfall of a single clearing member over a short period does not impose a hefty financial burden on the CCP. On the other hand, when two or more large clearing members experience simultaneous exceedances, due to their P&L interdependence, the resources of the CCP can be eroded to the point of financial distress or even failure. While rare, CCP failures have occurred in the past and have proven to be economically significant. Examples include Paris in 1973, Kuala Lumpur in 1983, and Hong Kong in 1987 (Bernanke (1990), Knott and Mills (2002), Duffie (2013b), and Bignon and Vuillemey (2017)).[4]

P&L interdependence increases with both trade crowdedness and underlying asset comovement. Trade crowdedness occurs when clearing members hold similar portfolio positions (i.e., portfolio weights). Hirshleifer, Subrahmanyam, and Titman (1994) argue that this situation arises among large market participants with common informational advantages that lead them to pursue similar directional trades, arbitrage opportunities, or hedging strategies.[5] Underlying asset comovement, on the other hand, occurs when asset returns move in the same direction. This is a common phenomenon during economic slowdowns or periods of high volatility.[6]

Given the potentially severe ramifications from two or more clearing members defaulting simultaneously, this paper formally incorporates P&L interdependence in the calculation of margin requirements. By doing so, we contribute to a global effort to design macroprudential regulations for systemically important institutions and market infrastructures. To the best of our knowledge, CoMargin is the first margining methodology that explicitly provides "financial resources sufficient to cover a wide range of potential stress scenarios that should include, but not be limited to, the default of the two participants and their affiliates that would potentially cause the largest aggregate credit exposure to the CCP" (Committee on Payment and Settlement Systems and International Organization of Securities Commissions (CPSS–IOSCO) (2012)).[7]

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contributions to the default fund. iv) If unfunded losses remain, they can be covered with CCP equity or v) calls for additional capped or uncapped clearing member contributions to previously unfunded default facilities (see Duffie et al. (2010), Ghamami (2014)).

[4]Default of clearing members is of course, more frequent. Recent examples in the CME include Refco in 2005, Lehman in 2008, and MF Global in 2011 (see Jones and Pérignon (2013)).

[5]Much of the proprietary trading activity on derivatives exchanges consists of arbitraging futures and over-the-counter (OTC) or cash markets (e.g., cash-futures arbitrage of the Standard & Poor's (S&P) 500 index, Eurodollar-interest rate swap arbitrage, etc.).

[6]Menkveld (2015) proposes Margin(A), a factor-based measure of trade crowdedness. By construction, however, Margin(A) bundles both components of loss dependence and cannot differentiate between the portion that arises from similar portfolio positions and that which arises from increasing asset comovement. Differentiating between these two sources is critical for regulatory and risk management purposes, as the first can be linked to individual trading decisions and the second to aggregate market conditions. Gemmill (1994), for example, highlights the importance of asset comovement, noting the diversification benefit clearing houses obtain from combining contracts on low correlated assets. Extreme dependence and contagion are discussed in Longin and Solnik (2001), Bae, Karolyi, and Stulz (2003), Longstaff (2004), Poon, Rockinger, and Tawn (2004), Boyson, Stahel, and Stulz (2010), Harris and Stahel (2011), and Christoffersen, Errunza, Jacobs, and Langlois (2012) among others. Billio, Getmansky, Lo, and Pelizzon (2012) connect extreme financial equity comovements with systemic risk.

[7]The Principles for Financial Market Infrastructures have been widely advocated and are in the process of being adopted by G20 member countries. For instance, Mark Carney, Governor of the Bank

We define CoMargin as the P&L VaR of a clearing member's portfolio, *conditional* on one or several other members being in financial distress. In our context, a clearing member is said to be in financial distress if its losses exceed the VaR of its P&L distribution. In this light, CoMargin builds on existing margin methods and reduces clearing house risk. We show that this intuitive adjustment to existing margining systems is sufficient to capture the interdependence of clearing member P&Ls. CoMargin exhibits a wide range of desirable features, including increasing margin requirements proportionally with P&L dependence, stabilizing the probability of exceedance events given financial distress, and reducing the probability that the clearing house might exhaust its funds.

Moreover, CoMargin is simple to estimate. The process starts by taking the trading positions of all clearing members at the end of the trading day as given. Then, in the spirit of stress testing, a series of 1-day-ahead scenarios, based on projected price and volatility changes in the underlying assets, is used to estimate hypothetical P&Ls, that is, potential changes in the value of member portfolios. Using these hypothetical P&Ls, we compute margin requirements that target a prespecified probability of margin exceedances conditional on the financial distress of other members. In this regard, CoMargin nests the VaR margining system, currently used by CCPs, when dependence between P&Ls is 0.

CoMargin adopts a conditional VaR setting similar to that in Adrian and Brunnermeier's (2016) CoVaR. However, it is important to note that the objective and estimation of these methodologies are radically different. While CoMargin is designed to quantify robust margin requirements from a forward-looking probability of exceedances, CoVaR assesses the contribution and exposure of financial institutions to marketwide tail risks and can be used to estimate capital surcharges based on (forward) systemic risk contribution changes. Moreover, CoMargin estimates are obtained through simulations or stress testing, whereas CoVaR measures are based on quantile regressions.

We first show, through Monte Carlo simulations, that CoMargin indeed enhances the stability of the CCP by adjusting collateral allocations such that the probability of margin exceedances, conditional on the financial distress of other firms, remains constant. Our simulations also show that CoMargin improves the resilience of the CCP relative to other methods by minimizing the average shortfall when simultaneous exceedances occur. More importantly, this stability and resiliency are robust to a variety of distributional assumptions.

In addition to simulations, we also employ actual clearing member data from the Canadian Derivatives Clearing Corporation (CDCC) to empirically assess the performance of CoMargin relative to existing margining systems.[8] Our data set, the first of its kind to be used in an academic study, includes actual daily positions for all 48 CDCC members from 2003 to 2011. Each trading day, we observe the number of long and short positions held at market close in the firm, client, and omnibus accounts of each member for the three most actively traded TMX

of England explicitly states that "It is extremely important that CCPs organize themselves to make sure they can provide the necessary resilience plan . . . [to] cover the failure of one or two major institutions" (Sourbes (2013)).

[8]The CDCC is the CCP of the TMX Montreal Exchange, the largest derivatives exchange in Canada.

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Montreal derivatives contracts; namely, the Three-Month Canadian Bankers' Acceptance Futures, the Ten-Year Government of Canada Bond Futures, and the S&P/Toronto Stock Exchange (TSX) 60 Index Standard Futures.

Confirming the results from our simulations, these real-world data show that CoMargin leads to fewer exceedances and lower expected shortfalls than SPAN and VaR margins. Moreover, CoMargin exceedances are less clustered in time and are more likely to emanate from small, less active clearing members; that is, those less likely to pose a systemic threat to the clearing house. This data set shows that the relative performance of CoMargin improves when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, our evidence demonstrably shows that CoMargin provides more protection to CCPs when needed most.[9]

This paper makes two important contributions to the literature. Our first contribution is methodological. We develop a novel margining system that internalizes market interdependencies and enhances the stability and resiliency of CCPs. Our approach aims at addressing current concerns associated with CCPs, particularly in terms of systemic risk, and offers a general framework to think about collateral allocation in a broad range of financial markets. Our second contribution is empirical. We conduct an extensive analysis of the margining systems widely used in CCPs. To the best of our knowledge, this is the first study that uses actual trading positions of clearing members. Our proprietary data set allows us to conduct our analysis with actual trader positions and without the need to simulate or assume the trading behavior of market participants.

Our paper complements the nascent literature on derivatives clearing. Duffie and Zhu (2011) show that the optimal number of CCPs needed to minimize counterparty default exposure is 1. Duffie, Scheicher, and Vuillemey (2015) provide an empirical analysis of the collateral requirements induced by broader application of central clearing. Ghamami (2014) theoretically studies the default waterfall resources of a CCP and derives optimal levels for the capital of clearing members and the CCP. Menkveld (2013) develops a model for centralized clearing focused on systemic liquidation risk due to the default of market participants.[10] Boissel, Derrien, Ors, and Thesmar (2017) show that market participants did not perceive the protection offered by the CCP for the European repo market as effective at the peak of the sovereign crisis of 2011. Loon and Zhong (2014) empirically show that centrally clearing credit default swap (CDS) contracts reduces counterparty risk, hence increasing the value of credit protection and improving liquidity (see also Arora, Gandhi, and Longstaff (2012), Slive, Witmer, and Woodman (2013), and Bernstein, Hughson, and Weidenmier (2014)). Following Gârleanu and

[9]These results are robust to equalizing aggregate margin collections across CoMargin, VaR and SPAN. When the P&Ls of clearing members are independent, CoMargin collects the same collateral as existing margining systems. When P&Ls are not independent, the additional CoMargin collected depends on the coverage level selected by the CCP. In these cases, it is possible to select CoMargin coverage rates to the *same* aggregate margin as SPAN. We compare systems using this "budget neutral" approach below.

[10]Other theoretical studies on derivatives clearing include Biais, Heider, and Hoerova (2012), (2016), Koeppl, Monnet, and Temzelides (2012), Fontaine, Pérez Saiz, Slive (2013), and Acharya and Bisin (2014).

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Pedersen (2011), Hedegaard (2013) tests the effect of margins on futures prices using data on 597 margin changes on CME futures contracts. More directly relevant to our study is Jones and Pérignon (2013) who show that the most severe losses of the largest clearing members of the CME tend to occur on the same days. These events are precisely what our CoMargin framework aims to protect against by internalizing P&L dependence.

In a similar vein, Menkveld (2015) proposes an alternative margining system called Margin(A) that accounts for similarities in trading positions across clearing members. Following Duffie and Zhu (2011), he considers the aggregate exposure of the CCP as the sum of all potential losses in clearing members' portfolios. He then derives the total amount of margin that the CCP should collect from all of its members under the assumption that losses are normally distributed and assigns margin requirements for each member using an allocation rule (see Brunnermeier and Cheridito (2014)).

CoMargin has significant practical and theoretical advantages over Margin(A). First, because it is scenario based, CoMargin is robust to a wide variety of distributions and loss dependence structures, while Margin(A) crucially relies on the normality assumption of clearing member losses and, therefore, is constrained to linear dependence. CoMargin's ability to capture different dependence structures makes it appropriate and practical for many derivative products with nonlinear payoffs, including CDS and options. This is a crucial advantage, given the push across the world, in the wake of the recent financial crisis, to require the central clearing of these products. Second, by construction, CoMargin allows risk managers to target an actual coverage probability that can be backtested according to regulatory principles, something that cannot be properly targeted with Margin(A).

The remainder of the paper is organized as follows: In Section II, we describe how margin requirements are currently estimated under the SPAN and VaR margining systems. In Section III, we present the theoretical foundations of the CoMargin system and highlight some of its properties with simulations. We discuss the implementation of CoMargin in Section IV. In Section V, we demonstrate its empirical effectiveness using data from the Canadian Derivatives Clearing Corporation. In Section VI, we study how clearing members could strategically react to an implementation of CoMargin. Last, we provide concluding remarks in Section VII.

II. Standard Margining Systems

A. Derivatives Market

Consider a derivatives exchange with N clearing members and D derivatives securities (futures, options, CDS, etc.) written on U underlying assets. In practice, each clearing member usually has two accounts with the CCP: one to manage client trades, called a client account, and one to manage proprietary trades, called a firm account. We turn our focus to firm accounts, as they are the most important source of risk within the CCP (Jones and Pérignon (2013)). Nevertheless, our methodology can be generalized to include all accounts.

Let $w_{d,i,t}$ be the number of d contracts in the derivatives portfolio of the firm account of clearing member i, for $d=1,\ldots,D$ and $i=1,\ldots,N$, at the end of day t, such that

$$w_{i,t} = \begin{bmatrix} w_{1,i,t} \\ \vdots \\ w_{D,i,t} \end{bmatrix}. \quad (1)$$

Initial margins are collected every day for each clearing member's account to guarantee the performance of their obligations and to guard the clearing house against default over a period of 1 day. Unlike margins collected for client accounts, those for firm accounts are computed at the portfolio level and allow for netting across positions. Let $B_{i,t}$ be the initial firm account margin collected by the CCP from clearing member i at the end of day t. This margin is a function of the outstanding firm account trading positions $w_{i,t}$ of member i.

We assume that current exposures are settled daily. Thus, a variation margin amount, $V_{i,t}$, equivalent to the profits or losses in the portfolio of clearing member i on day t, is deposited or withdrawn, respectively, from its margin account at the end of the trading day. By construction, the sum of the variation margins across all accounts (N firm accounts plus N client accounts $=2N$ accounts) is equal to 0, as for every buyer there is a seller. However, the sum of the variation margins across the N firm accounts can be different from 0.

Our interest focuses on cases when trading losses exceed margin requirements; that is, when $V_{i,t} \leq -B_{i,t-1}$. In these cases, we say that firm i has experienced an exceedance. Identifying firms in this state is important because they have an incentive to default on their positions or to delay payment on their obligations, which generates a shortfall in the market that needs to be covered by the CCP. Given the limited funds available to the CCP, simultaneous exceedance events can threaten its stability and survival.

B. SPAN Margin

The CME, the world's largest derivatives exchange, introduced the SPAN margining methodology in 1988. It has since become the most widely used margining system in derivatives exchanges around the world. Every day, following the market close, clearing houses, such as the CDCC, the CME, Eurex, LCH.Clearnet, Nymex, and the Options Clearing Corporation, among others, use the SPAN system to determine the margin requirements of their members.

SPAN is a scenario-based methodology that is used to assess potential changes in the value of the derivatives held by each clearing member. However, SPAN does not take a portfolio-wide approach. Instead, it divides each portfolio into contract families, defined as groups of contracts that share the same underlying asset, and estimates a charge for each family independently. Thus, for a portfolio with $d \leq D$ derivatives written on $u \leq U$ underlying assets, the SPAN system computes initial margin requirements for each of the u contract families.

To compute margin requirements for a family of derivatives, the SPAN system simulates 1-day-ahead changes in the value of each contract by using 16 scenarios that vary the price and the volatility of the underlying asset, as well as the time to expiration of the contract (see Table A1 in the Internet Appendix, available

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at www.jfqa.org). The range of the potential price changes of the underlying asset usually covers 99% of its daily price movements over a historical calibration window. A similar approach is adopted for the volatility. Extreme price changes are used to assess potential changes in deep out of the money options. The scenario analysis yields a risk array for each contract that contains sixteen 1-day-ahead potential value changes (i.e., each maturity and each strike price has its own array).[11] The scenario with the worst potential loss for the entire contract family is identified and that loss becomes the first part of the contract family charge.

The second part of the contract family charge consists of a discretionary adjustment that is needed because contracts with different expiration months are assumed to be equivalent in the scenario analysis. In other words, long and short positions written on the same underlying asset but with different expiration months offset each other. Therefore, CCP risk managers are required to add an *intracommodity spread charge* to the worst-case scenario loss to account for time-spread trading. A third component of the contract family charge consists of another discretionary adjustment to account for commodity spread trading (i.e., simultaneous long and short positions in contracts with the same expiration months but written on different but correlated underlying assets). These adjustments are known as *intercommodity spread charges.*

It is important to note that both intra- and intercommodity spread charges involve the discretion of CCP risk managers. Thus, these adjustments are rarely consistent across commodities, market conditions, or clearing houses. This situation, coupled with the fact that the SPAN system targets underlying price and volatility ranges instead of the probability of portfolio-wide margin-exceeding losses, makes the SPAN system inconsistent (in terms of its P&L coverage) across time and across markets.

C. VaR Margin

VaR is defined as a lower quantile of a P&L distribution. It is the standard measure used to assess the aggregate risk exposure of banks (Berkowitz and O'Brien (2002), Berkowitz, Christoffersen, and Pelletier (2011)), as well as their regulatory capital requirements (Adrian and Shin (2014)). VaR can also be used to set margins on a derivatives exchange. In this case, the margin requirement corresponds to a given quantile of a clearing member's 1-day-ahead P&L distribution.

Definition 1. The VaR margin of firm i, B_i, corresponds to the α quantile of its P&L distribution:

$$\Pr(V_{i,t+1} \leq -B_{i,t}) \quad = \quad \alpha. \tag{2}$$

The coverage level, determined by α, depends on the CCP's risk aversion and additional available financial resources (e.g., default fund) that could be used if a member defaults.

Like the SPAN system, the VaR margin method is applied on a firm-by-firm basis using a scenario analysis. However, the scenarios are applied to the entire portfolio. More specifically, we consider S scenarios derived from simulated

[11] The projected price changes of nonlinear contracts, such as options, are obtained by using numerical valuation methods or option pricing models.

1-day-ahead changes in the value of the price and the volatility of the underlying assets and use them to evaluate each clearing member's entire portfolio. The hypothetical P&L or variation margin of each clearing member is computed by *marking-to-model* its positions in each scenario. Thus, for each clearing member and date t, we obtain a simulated sample of daily P&Ls denoted $\{v^s_{i,t+1}\}^S_{s=1}$ that can be used to estimate the VaR margin requirement as follows:

$$\hat{B}_{i,t} = -\text{QUANTILE}(\{v^s_{i,t+1}\}^S_{s=1}, \alpha). \tag{3}$$

III. CoMargin

The VaR and SPAN collateral systems focus only on firm-specific risk; that is, the unconditional probability of an individual clearing member experiencing a margin-exceeding loss. By adopting either method, the clearing house guards itself from unique or independent exceedances, but it leaves itself exposed to simultaneous exceedance events across firms. These events, however, tend to be more economically significant because they place a more substantial burden on the resources of the clearing house.

A. Methodology

Consider the firm accounts of two of the N firms, denoted firms i and j. The probability of simultaneous exceedances is given by

$$\begin{aligned}\Pr[(V_{i,t+1} \le -B_{i,t}) \cap (V_{j,t+1} \le -B_{j,t})] \\ = \Pr(V_{i,t+1} \le -B_{i,t} | V_{j,t+1} \le -B_{j,t}) \times \Pr(V_{j,t+1} \le -B_{j,t}).\end{aligned} \tag{4}$$

Equation (4) shows that simultaneous exceedance events tend to happen more frequently not only when firm-specific risk increases (i.e., when $\Pr(V_{j,t+1} \le -B_{j,t})$ increases), but also when P&L dependence increases (i.e., when $\Pr(V_{i,t+1} \le -B_{i,t} | V_{j,t+1} \le -B_{j,t})$ increases). In the first case, firms are more likely to experience losses that exceed their collateral levels in all states of the world. In the second case, firms are more likely to experience these losses at the same time as other firms, either because they hold similar positions (i.e., trade crowdedness is high) or because underlying assets have a tendency to move together (i.e., underlying asset comovement is high). However, VaR and SPAN margins completely disregard P&L dependence and its potential effect on the stability of the CCP. In the case of the VaR system, risk managers target only unconditional exceedance probabilities by setting a coverage level $1-\alpha$ for each clearing member individually. In the case of the SPAN system, risk managers do not have direct control over the unconditional exceedance probabilities, so the clearing house is potentially left even more vulnerable to simultaneous exceedance events.

Now, consider a hypothetical situation in which two of the N firms have orthogonal risk exposures and their exceedances are independent.[12] Under the VaR system, this means

$$\Pr(V_{i,t+1} \le -B_{i,t} | V_{j,t+1} \le -B_{j,t}) = \Pr(V_{i,t+1} \le -B_{i,t}) = \alpha \quad \text{and} \tag{5}$$

[12] Obviously, *all 2N accounts* (i.e., all firm and client accounts) cannot exhibit orthogonal risk exposures, as that would violate the zero-sum property for variation margins.

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$$\Pr[(V_{i,t+1} \leq -B_{i,t}) \cap (V_{j,t+1} \leq -B_{j,t})] \quad = \quad \alpha^2. \tag{6}$$

Equation (6) has an important implication for market stability, which we define as constancy in the probability of exceedance events for i, regardless of the situation of j. Moreover, equation (6) shows that, once the risk manager selects the target α, the probabilities of simultaneous exceedance events are also fixed (i.e., α^2 for two events, α^3 for three events, etc.). This property holds true regardless of the collateral system being adopted by the clearing house. Therefore, a setting with fully orthogonal exposures provides a conceptual benchmark to assess the stability of a margining system.

With this in mind, we propose a new collateral system, called CoMargin, which enhances financial stability by taking into account the dependence of clearing members' P&Ls.

Definition 2. The CoMargin of firm i, $B_t^{i|j\neq i}$, corresponds to the α quantile of its P&L distribution conditional on an event affecting a set of firms that exclude firm i:

$$\Pr(V_{i,t+1} \leq -B_t^{i|j\neq i} | \mathbf{C}(V_{j\neq i,t+1})) \quad = \quad \alpha. \tag{7}$$

Notice that CoMargin requires us to define an event for a conditioning set of firms that exclude firm i. We define this event as the financial distress (i.e., a loss in excess of $\alpha\%$ VaR) of at least one of the firms in the conditioning set.

In the most general case, the conditioning set of firms could include all of the remaining $N-1$ firms in the CCP, excluding firm i, such that[13]

$$\mathbf{C}(V_{j\neq i,t+1}) \quad = \quad \bigcup_{j=1, j\neq i}^{N} (V_{j,t+1} \leq -B_{j,t}). \tag{8}$$

Alternatively, we can narrow the number of firms contained in the conditioning set based on their size or risk. For example, the Principles for Financial Market Infrastructures (CPSS-IOSCO (2012)) suggest selecting the two firms with the largest expected shortfall. If we denote these firms by F1 and F2, this means that the CoMargins of all other firms depend on F1 and F2. However, as shown in equation (7), the CoMargin of F1 does not depend on F1, but instead on F2 and F3, the firm with the third largest expected shortfall. As a result, CoMargin does not necessarily increase the margin requirements of conditioning firms.

Because it is defined as a conditional probability, CoMargin is related to the CoVaR measure of Adrian and Brunnermeier (2016). However, the conditioning event presented in equation (8) is different from the distress of a single institution that is used to define CoVaR. Our conditioning event, instead, is multidimensional and requires that at least one firm in the conditioning set is in financial distress. Furthermore, similar to Girardi and Ergun (2013), we use a less-than-or-equal sign in the conditioning event. This reflects the fact that we define financial distress as a range, as opposed to a single point, in the P&L distribution. Another difference

[13] The conditioning event in equation (8) could be based on CoMargin instead of VaR, $B_{j,t}$. In this case, all CoMargins would be jointly defined by solving a system of equations.

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between CoVaR and CoMargin is the fact that the former is used as an input to compute a systemic risk measure (ΔCoVaR) while the latter directly corresponds to an operational margin requirement.

Last, notice that there are other techniques that we could have used to capture P&L dependence in a margining system. These range from simple methods, such as linear correlation or quadrant dependence, to the more sophisticated extreme dependence measures discussed in Poon et al. (2004), Bae et al. (2003), and Christoffersen et al. (2012) or systemic risk measures surveyed in Benoit, Colliard, Hurlin, and Pérignon (2017). However, to compute margins, dependence measures need to be transformed to dollar values.[14] The quantile-based dependence measure embedded in CoMargin offers the distinctive advantage of directly corresponding to the dollar value that the CCP should collect as margin.

B. Properties of CoMargin

In this section, we highlight several attractive features of CoMargin. First, CoMargin achieves exceedance probabilities that are consistent with those of a fully orthogonal exposure case. For instance, take two of the N firms, i and j. We know through Bayes' theorem that

$$\Pr(V_{i,t+1} \leq -B_t^{i|j} \,|\, V_{j,t+1} \leq -B_{j,t}) = \frac{\Pr[(V_{i,t+1} \leq -B_t^{i|j}) \cap (V_{j,t+1} \leq -B_{j,t})]}{\Pr(V_{j,t+1} \leq -B_{j,t})}, \tag{9}$$

where $B_t^{i|j}$ denotes the CoMargin of firm i, conditional on the financial distress of firm j. Thus, the numerator represents the joint probability of i exceeding its CoMargin and j experiencing financial distress. From Definitions 1 and 2, we can see that

$$\Pr[(V_{i,t+1} \leq -B_t^{i|j}) \cap (V_{j,t+1} \leq -B_{j,t})] = \alpha^2. \tag{10}$$

This alternative expression shows that, with CoMargin, the probability of i and j exceeding their margin simultaneously corresponds to that obtained under fully orthogonal exposures (see equation (6)). In other words, CoMargin "orthogonalizes" the risk exposures of the CCP, thus enhancing its stability.[15]

Second, when members i and j have orthogonal positions, then

$$\Pr(V_{i,t+1} \leq -B_t^{i|j} \,|\, V_{j,t+1} \leq -B_{j,t}) = \Pr(V_{i,t+1} \leq -B_t^{i|j}) = \alpha, \tag{11}$$

and $B_t^{i|j} = B_{i,t}$; that is, CoMargin and VaR margins are equal. This property illustrates the fact that CoMargin nests the VaR margining system that is currently used by CCPs while internalizing trading externalities.

[14] For instance, one could compute the systemic expected shortfall or marginal expected shortfall proposed by Acharya, Pedersen, Philippon, and Richardson (2017), the systemic risk measure of Brownlees and Engle (2017), or any other dependence measure for all clearing members. Then, a bucketing procedure or another transformation would be required to translate risk estimates into dollar values for margin requirements.

[15] This property also holds when the number of firms is greater than 2.

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Third, if members i and j have identical positions, then

$$\begin{aligned}(12)\quad \Pr(V_{i,t+1} \le -B_t^{i|j} \,|\, V_{j,t+1} \le -B_{j,t}) &= \Pr(V_{i,t+1} \le -B_t^{i|j} \,|\, V_{i,t+1} \le -B_{i,t}) \\ &= \frac{\Pr[(V_{i,t+1} \le -B_t^{i|j}) \cap (V_{i,t+1} \le -B_{i,t})]}{\Pr(V_{i,t+1} \le -B_{i,t})} \\ &= \frac{\Pr(V_{i,t+1} \le -B_t^{i|j})}{\alpha} = \alpha,\end{aligned}$$

since $B_t^{i|j} \ge B_{i,t}$. Therefore, $\Pr(V_{i,t+1} \le -B_t^{i|j}) = \alpha^2$, which means that CoMargin(α) is equal to VaR(α^2).

As can be seen from these properties, CoMargin values range from VaR(α), when i and j have orthogonal exposures, to VaR(α^2), when they have identical exposures. Hence, margin requirements estimated with the CoMargin methodology have a well-defined lower bound and are not explosive when P&L dependence becomes very high. This result can be extended to $N-1$ firms with an upper limit for the CoMargin equal to VaR(α^{N-1}).

In Internet Appendix A, we analytically derive three additional CoMargin properties for the more restrictive case of normally distributed P&Ls. We show that the CoMargin of firm i increases with the variability of its own P&L, but it does not depend on the variability of the P&L of firm j. This result prevents potential predatory practices, where clearing members take on more individual risk to increase the margin requirements of other members. Additionally, we show that the CoMargin of a given firm increases with the dependence between its P&L and that of other firms. These results can also be derived numerically through Monte Carlo simulations for more general distributions, as shown in the next section.

C. Monte Carlo Simulations

In order to illustrate the performance of the CoMargin system, we consider the case of four clearing members, where two of them, members 1 and 2, have correlated P&Ls, such that $V \sim N(0, \Sigma)$, where

$$(13)\quad V = (V_1, V_2, V_3, V_4)' \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

We allow the correlation between the P&Ls of firms 1 and 2, ρ, to increase from 0 to 0.8. As explained above, the rising correlation between the P&Ls of these firms can reflect an increase in the similarity of their firm trading positions or an increase in the comovement of the underlying assets. Graph A of Figure 1 shows the initial margin requirements for each clearing member under both the VaR and CoMargin systems for the levels of correlation being considered. To estimate the CoMargin of a given clearing member, we define the conditioning event as at least one of the other three firms being in financial distress.

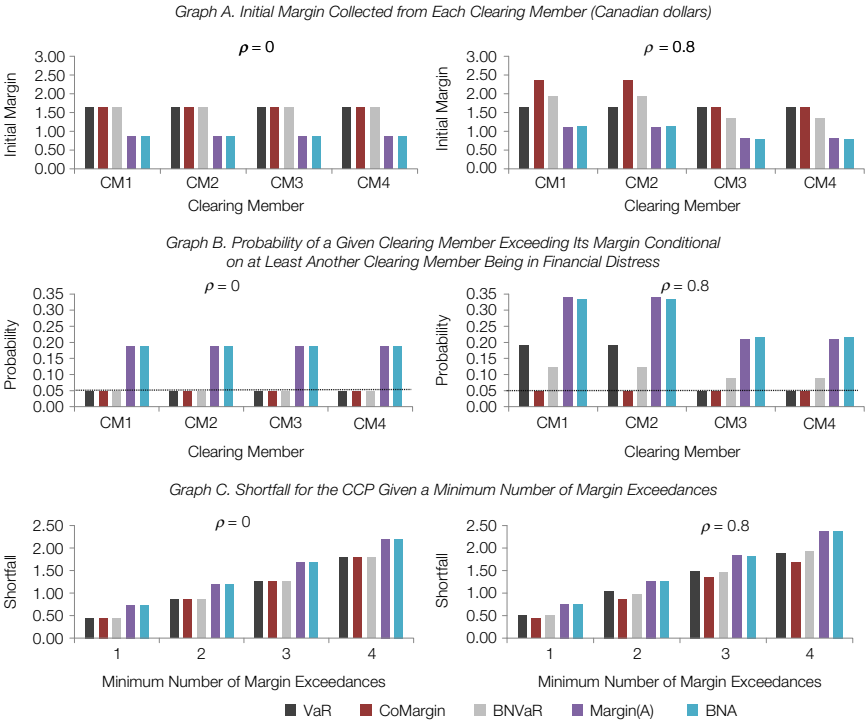
The main finding in these simulations is that CoMargin outperforms the standard VaR margining system. Graph A of Figure 1 shows that VaR margins remain constant for all firms regardless of their correlation level because this method does not take into account P&L dependence. Also, notice that CoMargin increases

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FIGURE 1

Theoretical Performance of Margining Systems Assuming Jointly Normally Distributed P&Ls

Figure 1 presents the theoretical performance of VaR (equation (2)), CoMargin (equation (7)), Budget-Neutral VaR (BNVaR), Margin(A), and Budget-Neutral Margin(A) (BNA), assuming four clearing members with jointly normally distributed P&Ls, such that $V=(V_1, V_2, V_3, V_4)' \sim N(0, \Sigma)$. The ρ parameter corresponds to the correlation between the P&Ls of the first two clearing members. All the remaining correlations are equal to 0, and all the variances are fixed to 1. We report our results for different levels of the correlation parameter, ρ, that ranges from 0 to 0.8. When computing CoMargin, we define the conditioning event as at least one of the other three firms being in financial distress.



with P&L dependence, which is an important feature for managing risk in a CCP. Moreover, Graph B of Figure 1 shows that the probability of a given clearing member exceeding its margin conditional on at least another clearing member being in financial distress is lower with CoMargin. More importantly, as illustrated by the horizontal line in these charts, this probability remains constant at a level that corresponds to that prevalent when $\rho=0$; that is, when the risk exposures of all members are orthogonal. This feature clearly shows how CoMargin enhances control and stability in the CCP from a risk management perspective. Similarly, Graph C of Figure 1 clearly shows that CoMargin leads to lower shortfalls for the CCP given a minimum number of margin exceedances.

We also document that CoMargin dominates existing margining systems at the CCP level in Table 1. The unconditional probability of having at least one exceedance, the probability of having additional exceedances given that one has occurred, and the expected shortfalls associated with these events are all lower

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TABLE 1

Theoretical Performance of VaR and CoMargin Systems

Table 1 presents the theoretical performance of the VaR (equation (2)), CoMargin (equation (7)), Budget-Neutral VaR (BNVaR), Margin(A), and Budget-Neutral Margin(A) (BNA), assuming four clearing members whose P&Ls are jointly normally or Student's t distributed. Panel A presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0, \Sigma)$. We allow the correlation between the P&Ls of firms 1 and 2, ρ, to be positive, whereas the other correlations are set to 0. Panel B shows the case where P&Ls are Student's t distributed with degrees of freedom υ, $V \sim t_\upsilon(0, \Sigma)$. The variance–covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case, we set $\rho = 0.4$ and let the degrees of freedom decrease from 30 to 5. When computing CoMargin, we define the conditioning event as at least one of the other three firms being in financial distress.

	Panel A. Jointly Normally Distributed P&Ls				*Panel B. Jointly Student's t Distributed P&Ls*			
	Unconditional		Conditional on One Exceedance		Unconditional		Conditional on One Exceedance	
Margin System	Prob. of at Least One Exceed.	Aggregate Shortfall	Prob. of Extra Exceed.	Aggregate Shortfall	Prob. of at Least One Exceed.	Aggregate Shortfall	Prob. of Extra Exceed.	Aggregate Shortfall
	$\rho=0$				$\upsilon=30$, $\rho=0.4$			
VaR	0.185	0.083	0.076	0.451	0.177	0.094	0.123	0.531
CoMargin	0.185	0.083	0.076	0.451	0.115	0.056	0.074	0.485
BNVaR	0.185	0.084	0.076	0.451	0.185	0.098	0.110	0.530
Margin(A)	0.569	0.417	0.290	0.733	0.517	0.404	0.301	0.781
BNA	0.569	0.417	0.290	0.733	0.518	0.404	0.300	0.781
	$\rho=0.4$				$\upsilon=10$, $\rho=0.4$			
VaR	0.179	0.084	0.109	0.466	0.171	0.119	0.151	0.695
CoMargin	0.138	0.060	0.069	0.433	0.081	0.053	0.091	0.650
BNVaR	0.188	0.088	0.096	0.468	0.178	0.123	0.138	0.689
Margin(A)	0.530	0.392	0.302	0.739	0.487	0.430	0.298	0.884
BNA	0.531	0.392	0.301	0.739	0.487	0.431	0.297	0.883
	$\rho=0.8$				$\upsilon=5$, $\rho=0.4$			
VaR	0.165	0.083	0.193	0.505	0.164	0.175	0.191	1.067
CoMargin	0.111	0.048	0.062	0.431	0.052	0.060	0.129	1.170
BNVaR	0.203	0.101	0.119	0.500	0.169	0.178	0.179	1.053
Margin(A)	0.491	0.370	0.315	0.753	0.430	0.475	0.292	1.104
BNA	0.495	0.373	0.311	0.753	0.430	0.475	0.292	1.104

under CoMargin. Overall, we see that, as ρ (the correlation between P&Ls) increases, CoMargin provides the best overall coverage.

At this point, it is important to note that there are two possible reasons why CoMargin is more effective than VaR margin at reducing conditional margin exceedances. First, it is possible that there is an "allocation effect" and that CoMargin provides a better distribution of collateral, which helps to stabilize risk and reduce shortfalls. Second, there might be a "level effect" and the efficiency of CoMargin could be driven simply by its larger aggregate collateral collections that result in a higher coverage level for the CCP. Therefore, in Figure 1 and Table 1, we report Budget-Neutral VaR (BNVaR) margins to show that allocations, and not aggregate margin collections, drive the relative effectiveness of CoMargin. BNVaR corresponds to CoMargin(α'), where the α' parameter is set such that the total CoMargin collected is equal to the sum of the VaR(α) margins. Thus, BNVaR effectively cancels out the level effect and allows us to assess the merits of each margining system based on the allocation effect.

Conditioned on having one exceedance, BNVaR outperforms VaR whatever the correlation level considered. For instance, in Table 1, for $\rho = 0.8$, the conditional probability of additional margin exceedance in the CCP is equal to 0.119 for BNVaR and 0.193 for VaR, whereas the aggregate shortfall is slightly smaller for BNVaR (0.500 vs. 0.505). The fact that the outperformance of CoMargin does

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not vanish when comparing it to an artificial margining scheme that corrects for the level effect is reassuring evidence that suggests that the allocation effect is the driving force of the CoMargin technology.[16]

We repeat the previous exercise for P&Ls that are jointly Student's t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$, and present the results of this exercise in Table 1. Changing the distributional assumption of the previous exercise from a normal to a Student's t multivariate distribution allows us to simulate scenarios where all clearing members have some level of tail dependence in their P&Ls, which is consistent with empirical evidence. The variance–covariance structure, Σ, is the same as that considered in the Gaussian distribution example; however, in this case, we set $\rho = 0.4$ and let the degrees of freedom decrease from 30 to 5.[17] Thus, the resulting P&L distributions have progressively fatter tails.

As shown in Table 1, these results are consistent with those obtained under the Gaussian assumption, but they highlight an important finding: CoMargin is able to capture P&L dependence structures that go beyond simple correlations. Recall that the entire P&L dependence structure is fully characterized by correlation only if P&Ls are normally distributed. However, asset prices and P&Ls, particularly those of nonlinear portfolios, rarely follow normal distributions, making CoMargin uniquely capable of incorporating this stylized fact.

For completeness, we also compare CoMargin with Margin(A). Following Menkveld (2015), we compute Margin(A) in two steps according to a top-down methodology. The first step consists in defining the aggregate margin as the delta-normal VaR of aggregate loss. The second step consists in applying the Euler's homogeneous function theorem to decompose the total margin requirement into N individual margins.

Just as with CoMargin, Margin(A) increases with P&L dependence. However, we observe in Figure 1 that Margin(A) always leads to significantly smaller margins than CoMargin and VaR alike. In practice, Margin(A) needs to be scaled up by the ratio between aggregate VaR margin and aggregate Margin(A). However, when scaling by the sum of individual VaR(5%), the implied risk level is not properly controlled for and may end up being higher than 5%.

Besides the level effect, a second difference is the fact that Margin(A) induces a "subsidy" for the clearing members who do not exhibit crowded positions, through reduced margins.[18] For instance, in Graph A of Figure 1, the Margin(A) of members 3 and 4, whose P&Ls are not related to the other members' P&Ls, decreases from 0.879 for $\rho = 0$ to 0.809 for $\rho = 0.8$. Thus, the higher the dependence between the P&Ls of members 1 and 2, the lower the margins of members 3 and 4. On the contrary, the CoMargins for members 3 and 4 remain equal to

[16] We also consider an alternative budget-neutral margining scheme in which the margin is set to VaR(α) plus a constant amount Δ for all clearing members. The Δ coefficient is equal to the difference between the sum of CoMargin(α) and the sum of VaR(α) across clearing members, divided by the number of clearing members. This alternative margining system collects the same amount of collateral as the CoMargin system and leads to similar results as those presented here.

[17] We conduct a similar experiment using $\rho = 0$, which leads to the same conclusions. These results are available from the authors.

[18] Menkveld (2015) defines "crowded positions" as correlated exposures and does not differentiate the portion of this correlation that arises from similar portfolio weights and that which arises from increasing asset comovement.

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their VaR margins, for all correlation levels between the P&Ls of the other two members. This feature seems logical since the individual risk of these firms does not change with ρ.

In order to neutralize the level effect of CoMargin, we define a budget-neutral Margin(A), called BNA. The latter corresponds to CoMargin(α'), where the α' parameter is set such that the total CoMargin collected is equal to the sum of the Margin(A). We see in Graph A of Figure 1 that BNA is slightly larger than Margin(A) for the clearing members that exhibit "crowded trades" (1 and 2) and slightly smaller for the others (3 and 4). Once we properly control for the level effect, Margin(A) and BNA exhibit similar performance in terms of conditional probability of additional financial distress and aggregate shortfall. This result suggests that, in this symmetric Monte Carlo design, the allocation schemes of the two methods are equally good.

IV. Implementing CoMargin

A. Scenario Generation

One common feature of all margining methods is that they are scenario based. As a consequence, generating meaningful scenarios is a crucial stage when setting margin requirements. Unlike the SPAN margining system, VaR margin and CoMargin use a portfolio-wide approach. This allows us to take into account the asset comovement within the portfolio of each clearing member without the need for ad hoc netting adjustments (i.e., SPAN's inter- and intracommodity spreads, explained above). In order to assess the potential P&L of the entire portfolio of each clearing member, we need to jointly simulate S vectors of 1-day-ahead changes in the underlying asset prices, $\{r^s_{u,t+1}\}^S_{s=1}$. While there are different ways to estimate multivariate probability distribution functions, we suggest using a copula to link the marginal probability distribution functions, say, $F_1(r_1)$, $F_2(r_2),\ldots,F_U(r_U)$, to form a multivariate probability distribution function, $F(r_1,r_2,\ldots,r_U)$, where r_x is the standardized return of underlying asset x and U is the number of assets underlying the derivatives cleared by the CCP.

Using copulas to model the multivariate structure of underlying asset returns has advantages in this context. First, marginal distributions do not need to be similar to each other to be linked together with a copula structure. Second, the choice of the copula or multivariate structure is not constrained by the choice of the marginal distributions. Third, copulas can be used with U marginal distributions to cover all of the underlying assets cleared by the CCP (see Oh and Patton (2017)). Last, the use of copulas allows us to model the tails of the marginal distributions and the tail dependence across underlying assets separately. This feature is particularly important in our case because the likelihood of an extreme underlying asset return might increase either because of fatter tails in their marginal distributions or because of fatter tails in their multivariate distribution. We use Student's t copulas in our modeling because, unlike their Gaussian counterparts,

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they more closely resemble some of the stylized features of asset returns, such as fat tails in the marginal distributions and multivariate tail dependence.[19]

We implement a two-stage semiparametric approach to estimate a U-dimensional copula for the underlying asset returns. The first stage consists of estimating the empirical marginal distributions of the returns of each underlying asset. The second stage consists of estimating the t copula parameters, R (correlation matrix) and υ (degrees of freedom), through maximum likelihood. This approach is commonly known as the canonical maximum likelihood estimation method (Genest, Ghoudi, and Rivest (1995)). Once the copula parameters are estimated, we use the implied multivariate structure to simulate potential changes in the price of the underlying assets.[20]

We use a fixed-length estimation window that is rolled daily to simulate new scenarios every day. Once the S potential changes in the price of the underlying assets have been simulated, we mark-to-model all of the derivatives in the portfolio of each clearing member to obtain a simulated sample of P&Ls. We then use this simulated sample $\{v^s_{i,t+1}\}^S_{s=1}$ to estimate VaR margin, as described in Section II.C. To estimate CoMargin, we use the simulated samples for all clearing members, $\{v^s_{1,t+1}, v^s_{2,t+1}, \ldots, v^s_{n,t+1}\}^S_{s=1}$, and follow the procedure described in the following section.

B. Estimation

In this section, we show how to estimate CoMargin requirements from simulated P&Ls for the case of two clearing members. Internet Appendix B describes the estimation procedure for the more general case when $N > 2$.

Given the simulated sample $\{v^s_{i,t+1}, v^s_{j,t+1}\}^S_{s=1}$, conditional on $B^{i|j}_t$, a simple estimate of the joint probability $\Pr[(V_{i,t+1} \leq -B^{i|j}_t) \cap (V_{j,t+1} \leq -B_{j,t})]$, denoted $P^{i,j}_t$, is given by

$$\hat{P}^{i,j}_t = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}(v^s_{i,t+1} \leq -B^{i|j}_t) \times \mathbf{I}(v^s_{j,t+1} \leq -B_{j,t}), \tag{14}$$

where $v^s_{i,t+1}$ and $v^s_{j,t+1}$ correspond to the sth simulated P&L of firms i and j, respectively. Given this result, we can now estimate $B^{i|j}_t$. For each time t and each firm i, we look for the value $B^{i|j}_t$ such that the distance $\hat{P}^{i,j}_t - \alpha^2$ is minimized:

$$\hat{B}^{i|j}_t = \arg\min_{\{B^{i|j}_t\}} (\hat{P}^{i,j}_t - \alpha^2)^2. \tag{15}$$

Thus, for each firm i, we end up with a time series of CoMargin requirements $\{\hat{B}^{i|j}_t\}^T_{t=1}$ for which confidence bounds can be bootstrapped.

[19] A Student's t copula corresponds to the dependence structure implied by a multivariate Student's t distribution. It is fully characterized by the variance–covariance matrix of standardized returns and the degrees of freedom, υ. The degrees of freedom define the probability mass assigned to simultaneous extreme returns (both positive and negative); the lower the degrees of freedom, the higher the probability of experiencing simultaneous extreme returns relative to the Gaussian copula. However, as $\upsilon \to \infty$, the Student's t copula converges to its Gaussian counterpart.

[20] These simulations can also be obtained for different correlation matrices. For instance, a predefined range can be obtained from 95% confidence intervals used to forecast R through the dynamic conditional correlation method proposed by Engle (2002).

C. Backtesting

One way to assess the effectiveness of a margining system is by backtesting it (Hurlin and Pérignon (2012)). Backtesting aims to identify misspecified models that lead to either excessive or insufficient coverage for the CCP relative to a target. Therefore, if a margining system cannot be backtested using formal statistical methods, we cannot identify its potential shortcomings and fine tune it to meet its objectives.

VaR margins can be easily backtested because they are defined by the quantile of a P&L distribution. As is the case with VaR margin, CoMargin allows us to test the null hypothesis of an individual member exceeding its margin requirement. More importantly, however, is the fact that we can also test the probability of exceedances conditional on the financial distress of other firms, as defined by the CoMargin of firm i, $B_t^{i|j}$. The null hypothesis in this case becomes

$$H_0 : \Pr(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}) \quad = \quad \alpha. \tag{16}$$

Since the null implies that $E[\mathbf{I}(V_{i,t+1} \leq -B_t^{i|j}) \times \mathbf{I}(V_{j,t+1} \leq -B_{j,t})] = \alpha$, a simple likelihood-ratio (LR) test can be used for the testing procedure (Christoffersen (2009)).

To assess the conditional probability of exceedances, we use the historical paths of the P&Ls for both members i and j (i.e., $\{v_{i,t+1}\}_{t=1}^{T}$ and $\{v_{j,t+1}\}_{t=1}^{T}$). The corresponding LR test statistic, denoted $\mathrm{LR}_{i|j}$, takes the form

$$LR_{i|j} \quad = \quad -2\ln[(1-\alpha)^{T-N_{i|j}} \alpha^{N_{i|j}}] + 2\ln\left[\left(1 - \frac{N_{i|j}}{T}\right)^{T-N_{i|j}} \frac{N_{i|j}}{T}^{N_{i|j}}\right], \tag{17}$$

where $N_{i|j}$ denotes the total number of joint past violations observed for both members i and j; that is, $N_{i|j} = \sum_{t=1}^{T} \mathbf{I}(v_{i,t+1} \leq -B_t^{i|j}) \times \mathbf{I}(v_{j,t+1} \leq -B_{j,t})$.

V. Empirical Analysis

A. Data and Descriptive Statistics

In this section, we compare the empirical performance of the SPAN, VaR, and CoMargin systems by using proprietary data from the CDCC. The data set includes the daily trading positions at market close for each clearing member on the Three-Month Canadian Bankers' Acceptance Futures (BAX), the Ten-Year Government of Canada Bond Futures (CGB), and the S&P/TSX 60 Index Standard Futures (SXF) for the 48 clearing members active in the CDCC between Jan. 2, 2003 and Mar. 31, 2011. To the best of our knowledge, no other academic study has ever used actual clearing member positions.

Table A2 in the Internet Appendix presents a short description of the data. In a derivatives exchange, on any given day, there are many delivery dates available on each underlying asset. Over the sample period, there were 45 delivery dates available for BAX contracts and 34 for CGB and SXF contracts. Thus, the sample includes a total of 113 futures contracts. Table A3 summarizes the specifications of these contracts. The contracts in our sample do not constitute the full set of derivatives cleared by the CDCC. However, they represent around 75% of its clearing activity. The documentation provided by the CDCC states that the BAX,

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CGB, and SXF are among the most actively traded derivatives in Canada. Furthermore, BAX and CGB are the most actively traded cleared interest rate contracts in the country (Campbell and Chung (2003), TMX Montreal Exchange (2013a), (2013b), (2013c)).

Table 2 shows the summary statistics for the contracts in the sample. Panel A shows the aggregate statistics for all 113 contracts, and Panels B–D report the summary statistics by underlying asset. On a typical day, there were approximately 20 active contracts, 12 on the BAX, 4 on the CGB, and 4 on the SXF. On average, contracts remained active for 363 trading days. However, there is a significant dispersion across underlying assets. BAX contracts remained active for 551 days, whereas CGB and SXF contracts remained active for 239 and 237 days, respectively. BAX contracts were the most actively traded, with an average daily long open interest of 137,810. The corresponding long open interests for CGB and SXF contracts were 65,260 and 55,280, respectively. The corresponding gross open interest values are twice the long open interest amounts.

CDCC members have access to three accounts to submit trades for clearing: a firm, a client, and an omnibus account. The firm account is used by clearing members to submit their own trades (i.e., conduct proprietary trading). The client account is used to submit trades on behalf of clearing members' clients. The omnibus account is used for all other clearing activities and is the least active account across all clearing members.

Our data set includes 21 firm, 23 client, and 16 omnibus accounts that were active on at least 1 day of the sample period. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Due to disclosure restrictions, we are unable to report the owners of each active account. However, Table A4 in

TABLE 2

Summary Statistics of the Contracts Included in the Empirical Analysis

Table 2 shows the summary statistics of the 113 futures contracts included in the empirical analysis. These contracts are divided according to their underlying assets as follows: The Three-Month Canadian Bankers' Acceptance (BAX), Ten-Year Government of Canada Bond (CGB), and S&P/TSX 60 Index Standard (SXF). Over the sample period (Jan. 2, 2003–Mar. 31, 2011), there were 45 delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Open interest values are reported in thousands.

Variables	Average	Median	Std. Dev.	Min	Max
Panel A. All Contracts					
Active contracts per day	19.81	20.00	0.93	13.00	21.00
Trading days per contract	362.25	253.00	221.72	6.00	756.00
Panel B. BAX Contracts					
Active contracts per day	11.99	12.00	0.14	8.00	13.00
Trading days per contract	550.58	699.00	249.25	6.00	756.00
Long open interest	137.81	131.32	49.21	48.35	310.97
Panel C. CGB Contracts					
Active contracts per day	3.93	4.00	0.49	1.00	5.00
Trading days per contract	238.91	253.00	42.73	55.00	255.00
Long open interest	65.26	60.81	27.36	16.15	176.97
Panel D. SXF Contracts					
Active contracts per day	3.89	4.00	0.43	1.00	4.00
Trading days per contract	236.32	250.00	42.58	52.00	255.00
Long open interest	55.28	55.09	14.01	24.40	98.49

the Internet Appendix provides the full list of clearing members that had at least one active client, firm, or omnibus account during the sample period. Notice that this list includes more clearing members than those currently affiliated with the CDCC because some members entered and exited the market during this period.

In our tests, we consider only firm accounts and we do so for three main reasons. First, house trading has been identified as the main source of risk on CCPs (Jones and Pérignon (2013)). Second, considering only a subset of accounts allows us to apply the CoMargin methodology with any number of firms in the conditioning set because the sum of the variation margins is not 0. Third, unlike firm margins, client margins are not set at the portfolio level but, instead, on a client-by-client basis.

In Figure A1 in the Internet Appendix, we plot the daily settlement prices (obtained from Bloomberg) for the underlying assets and the futures contracts in the sample. Graph A shows the time series of underlying asset prices, Graph B shows the underlying asset returns, and Graph C shows the settlement futures prices for all delivery dates. Lines in different colors represent different delivery dates. It is evident from Graph B that the volatility of the underlying assets increased dramatically after the onset of the financial crisis in mid-2007. In addition, Graph C shows an increase in the spread of futures prices during the same period, particularly for BAX contracts.

Figure 2 plots the daily stacked P&L values implied from the positions in active firm accounts. For each date t, $n_t^a \in N$ observations are plotted, which correspond to the P&L of the n_t^a clearing members that had an active account on that day. Notice how the volatility of the P&Ls increased dramatically at the beginning of the financial crisis as well, consistent with the patterns in the underlying and futures prices in Figure A1. Therefore, we consider 2 subperiods in our analysis: the precrisis period, from Jan. 2, 2003 to July 31, 2007, and the crisis period, from Aug. 1, 2007 to Mar. 31, 2011.

Table 3 presents the summary statistics for the firm accounts in the sample. Panel A reports the values for the full sample period, and Panels B and C present the values for the precrisis and crisis periods, respectively. On a typical day, there were approximately 12 clearing members with active firm accounts. This number remained relatively stable across subperiods. The average account was active for 56% of the days in the full sample (1,145 out of 2,066 days), with 75% during the precrisis period and 56% during the crisis period. The relatively lower activity during the crisis period was partially influenced by clearing members exiting the market. The P&L numbers reported in the table focus exclusively on active accounts. The typical active account reported an implied daily loss of $60,000 on the futures contracts listed in the sample. During the precrisis period, the average account reported daily losses of $164,000. However, during the crisis period, the average account reported a daily profit of $65,000. These profits were mostly derived from long positions in BAX contracts. Over the entire sample period, the typical account made an implied loss of $38,000, with corresponding losses of $119,000 and profits of $39,000 for the precrisis and crisis periods, respectively.[21]

[21] Notice that the P&L values reported in this paper are those implied by the positions held by the clearing members in their firm accounts on the contracts included in the sample. The actual accounts

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FIGURE 2

P&L for Active Firm Accounts

Figure 2 shows the daily stacked P&L in millions of Canadian dollars implied from the positions of the 21 active firm accounts used in the empirical analysis. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. For each date t, $n_t^a \in N$ observations are plotted, which correspond to the profits and losses (P&L) of the n_t^a clearing members with an active account. The sample period is from Jan. 2, 2003 to Mar. 31, 2011.

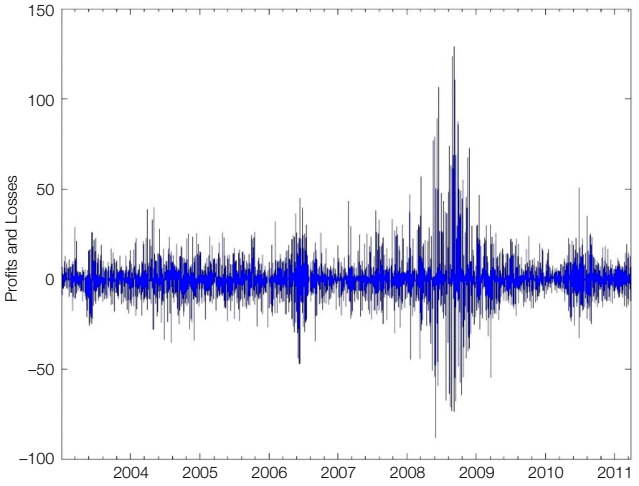


TABLE 3

Summary Statistics of the Firm Accounts Included in the Empirical Analysis

Table 3 presents the summary statistics of the 21 active firm accounts used in the empirical analysis. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from Jan. 2, 2003 to Mar. 31, 2011. The precrisis period is from Jan. 2, 2003 to July 31, 2007, and the crisis period is from Aug. 1, 2007 to Mar. 31, 2011. P&L values are reported in thousands of Canadian dollars and are estimated only for active firm accounts.

Variables	Average	Median	Std. Dev.	Min	Max
Panel A. Full Sample Period					
Active accounts per day	11.64	12.00	1.09	8.00	15.00
Active days for an account	1,145.19	1,420.00	911.72	3.00	2,066.00
Daily P&L across clearing members	−60.92	−97.80	2,659.44	−15,014.20	17,502.52
P&L over time	−37.83	0.43	160.57	−455.52	237.50
Panel B. Precrisis Period					
Active accounts per day	11.96	12.00	0.95	9.00	15.00
Active days for an account	858.00	1,148.00	431.12	3.00	1,148.00
Daily P&L across clearing members	−163.50	−156.15	2,027.37	−6,813.50	10,381.36
P&L over time	−119.12	−0.78	225.46	−671.62	39.80
Panel C. Crisis Period					
Active accounts per day	11.24	11.00	1.13	8.00	15.00
Active days for an account	516.05	684.00	418.29	3.00	918.00
Daily P&L across clearing members	65.18	−57.43	3,280.36	−15,014.20	17,502.52
P&L over time	39.83	−0.60	135.28	−110.76	484.42

of these clearing members include positions in other contracts cleared by the CDCC. In addition, our P&L values do not include trading revenues from other sources, such as noncleared OTC transactions.

B. Empirical Performance

Using the daily trading positions in each firm account, we compute the initial margin collected from each clearing member under the SPAN, VaR, and CoMargin systems. We set the risk level α to 2% for the VaR and CoMargin systems and use a rolling estimation window of 500 trading days in all cases. As mentioned in Section II.B, the SPAN system is estimated using the 16 scenarios in Table A1. In this study, as we are only dealing with futures contracts, we ignore scenarios based on volatility changes or extreme scenarios, which are targeted to deep out of the money options.[22] For the VaR and CoMargin systems, we consider $S = 100{,}000$ scenarios that are obtained using the methodology described in Section IV.A. Consistent with our earlier discussion, we set the financial distress threshold for CoMargin at the VaR margin level of the conditioning firms. The conditioning firms are the two clearing members with the largest 1-day-ahead expected shortfall, $\mathrm{ES}_{i,t+1} = \mathrm{E}(V_{i,t+1} | V_{i,t+1} \leq -B_{i,t})$.

For consistency across time periods, we ignore the ad hoc inter- and intracommodity spreads used in the SPAN system and impose a minimum margin of \$10,000 on all active accounts under all systems. This amount allows us to avoid cases where clearing members are not required to post any collateral because they have spread trades (i.e., matched long and short positions). These cases are likely to result in small exceedances as P&Ls in different contracts do not always offset each other. Thus, imposing a minimum margin amount prevents an upward bias in the number of SPAN exceedances. This amount, however, does not influence the rest of our results as it represents a constant that accounts for less than 0.2% of the average individual daily margin required under all systems.[23]

We find that accounting for P&L dependence increases margin significantly. Table 4 reports the summary statistics for the daily margin collected over the full sample period under different margining methods (Panels A and B). The average aggregate daily margin collected across all clearing members is \$112, \$101, and \$161 million for the SPAN, VaR, and CoMargin systems, respectively. The fact that CoMargin collects an extra 44% compared to SPAN margin might appear relatively costly at first sight. However, the CCP has the freedom to choose a higher risk level α to collect less margin, still benefiting from the CoMargin allocation of the collateral.

Panels C and D of Table 4 report the summary statistics for the daily margin collected over the precrisis and crisis periods, respectively. As expected given the increased volatility during the financial crisis, both aggregate and individual collateral levels are higher during the crisis period. However, the ranking of margin collections is consistent throughout the full sample period and the 2 subperiods being considered. VaR margin consistently collects the least and CoMargin con-

[22] The shocks in extreme scenarios 15 and 16 are defined by the risk managers as a percentage increase over the price range (e.g., 20% over the price range). Using an ad hoc extreme change would prevent us from keeping the same 99% confidence interval for all underlying assets. In addition, using an ad hoc extreme change only increases collateral collections (i.e., increases the SPAN coverage) but does not change the results, particularly in terms of simultaneous exceedances.

[23] We computed our results under different minimum collateral amounts ranging from \$0 to \$100,000. The results are consistent in all cases. With a minimum collateral of \$0, however, the SPAN system yields a high number of small exceedances.

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TABLE 4

Daily Margin Requirements under the SPAN, VaR, and CoMargin Systems

Table 4 presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation (2)), and CoMargin (equation (7)) systems for the 21 active firm accounts during the full sample period, from Jan. 2, 2003 to Mar. 31, 2011 (Panels A and B), during the precrisis period, from Jan. 2, 2003 to July 31, 2007 (Panel C), and during the crisis period, from Aug. 1, 2007 to Mar. 31, 2011 (Panel D). When computing CoMargin, we define the conditioning event as at least one of the two clearing members with the largest 1-day-ahead expected shortfall being in financial distress. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of Canadian dollars.

Margin System	Mean	Median	Std. Dev.	Min	Max
Panel A. Aggregate Market (CCP level)					
SPAN	112.04	105.71	38.49	49.83	328.77
VaR	101.40	95.80	36.03	42.90	301.97
CoMargin	161.31	156.20	64.93	56.89	475.27
Panel B. Per Clearing Member					
SPAN	9.76	9.16	3.54	3.83	29.86
VaR	8.83	8.26	3.28	3.30	27.45
CoMargin	14.14	13.41	6.08	4.38	43.01
Panel C. Per Clearing Member (precrisis)					
SPAN	8.49	8.32	2.38	3.83	14.98
VaR	7.72	7.47	2.15	3.30	13.79
CoMargin	11.94	11.62	4.45	4.38	25.98
Panel D. Per Clearing Member (crisis)					
SPAN	11.36	10.85	4.07	4.88	29.86
VaR	10.21	9.51	3.87	4.46	27.45
CoMargin	16.89	16.21	6.69	5.64	43.01

sistently collects the most collateral. Similarly, VaR margin consistently shows the least and CoMargin consistently shows the most dispersion of collected margin as measured by the standard deviation. This situation arises because CoMargin takes into account the variation of more factors than the other two margining methods (i.e., the factors driving P&L dependence).

Graph A of Figure 3 shows the daily stacked initial margin requirements under the SPAN, VaR, and CoMargin systems. The stacking process is the same as that used in the previous section for the P&L values of Figure 2. Notice that all three approaches produce margin requirements that are highly correlated. The average cross-sectional correlation for the full sample period is 0.99 between SPAN and VaR, 0.90 between SPAN and CoMargin, and 0.90 between VaR and CoMargin. The high correlation and low dispersion between the SPAN and VaR systems, coupled with the average collection values shown in Table 4, indicate that, at the individual clearing member level, SPAN margins behave much like VaR margins but at a higher coverage probability. However, notice that CoMargin is the least correlated of the three systems and shows the widest dispersion. This dispersion is more pronounced during the crisis period, when P&L dependence is higher. As mentioned in Section III.B, this can be explained by the fact that CoMargin converges to VaR margin as P&L dependence decreases and diverges as P&L dependence increases.

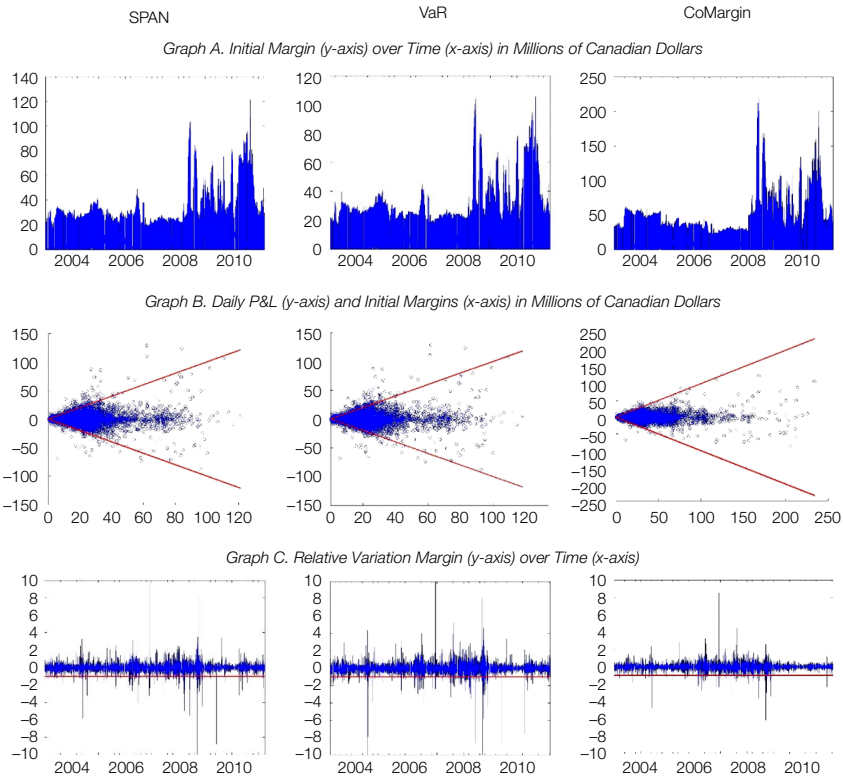
Graph B of Figure 3 plots the daily P&L of each active clearing member against its initial margin requirement for the entire time series. The 45- and −45-degree lines are indicated in red. Observations falling below the −45-degree line denote margin-exceeding losses. Notice that, of the three margining systems,

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FIGURE 3

SPAN, VaR, and CoMargin Collateral Requirements Over the Full Sample Period

Figure 3 shows the implied margin requirements from the positions in the 21 active firm accounts under the SPAN, VaR, and CoMargin systems. Graph A shows the daily stacked initial margin requirements in millions of Canadian dollars. Graph B plots the daily implied P&L against its initial margin requirement in millions of Canadian dollars. Graph C shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin. The stacking method used in Graphs A and C is as follows: for each date t, $n_t^a \in N$ observations are plotted, which correspond to the observations of the n_t^a clearing members with an active account. The sample period is from Jan. 2, 2003 to Mar. 31, 2011.



CoMargin shows the least number of margin exceedances. In addition, unlike the other systems, CoMargin tends to concentrate exceedances in low initial margin, low P&L points. These points represent clearing members with the smallest or least active portfolios; that is, those that are the least likely to pose a systemic threat to the CCP.

Graph C of Figure 3 shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin ($V_{i,t}/B_{i,t-1}$). Once again, the stacking process is the same as that used in Figure 2. Observations with a relative variation margin below -1, the level depicted with a red line, represent margin exceedances. Consistent with the study of the CME margins by Jones and Pérignon (2013), SPAN margin exceedances tend to cluster in time. In contrast, the CoMargin system exhibits the lowest number of simultaneous margin exceedances.

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Panels A and B of Table 5 summarize the performance of different margining systems over the full sample period. Panels C and D show the corresponding values for the precrisis and crisis periods, respectively. The left part of the panels measures unconditional performance in terms of the probability of experiencing at least one exceedance and the expected shortfall if at least one exceedance occurs. The right part of the panels reports the same measures but conditional on at least one member exceeding its margin.

Consistent with the simulation results presented in Section III, our empirical results show that the CoMargin system outperforms the SPAN and VaR systems in all dimensions (lower frequency of exceedances and expected shortfall), whether these are estimated unconditionally or conditionally. In addition, the relative performance of CoMargin increases when we condition on at least one exceedance event. This finding shows that CoMargin does a better job at protecting the CCP from simultaneous exceedances, even after a clearing member has surpassed its margin. Furthermore, Panels C and D indicate that the relative performance of CoMargin also improves during the crisis period, when P&L dependence is more persistent. This indicates that CoMargin tends to provide more protection to the CCP when it is needed most; that is, when simultaneous exceedance events are more likely to occur.

TABLE 5

Performance of the SPAN, VaR, and CoMargin Systems

Table 5 compares the empirical performance of the SPAN, VaR (equation (2)), and CoMargin (equation (7)) systems computed for the 21 active firm accounts over the sample period, from Jan. 2, 2003 to Mar. 31, 2011 (Panels A and B), during the precrisis period, from Jan. 2, 2003 to July 31, 2007 (Panel C), and during the crisis period, from Aug. 1, 2007 to Mar. 31, 2011 (Panel D). The left-hand side of the table reports unconditional amounts, and the right-hand side reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. When computing CoMargin, we define the conditioning event as at least one of the two clearing members with the largest 1-day-ahead expected shortfall being in financial distress. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of Canadian dollars and correspond to the losses expected if margin exceedances occur.

	Unconditional		Conditional on at Least One Exceedance	
Margin System	Frequency of Exceedances	Avg. Shortfall (CAD millions)	Frequency of Exceedances	Avg. Shortfall (CAD millions)
Panel A. Aggregate Market (CCP level)				
SPAN	0.09	0.35	0.36	3.78
VaR	0.14	0.44	0.42	3.20
CoMargin	0.07	0.13	0.28	1.85
Panel B. Per Clearing Member				
SPAN	0.01	0.03	0.14	0.34
VaR	0.02	0.04	0.15	0.29
CoMargin	0.01	0.01	0.12	0.16
Panel C. Per Clearing Member (precrisis)				
SPAN	0.01	0.01	0.11	0.08
VaR	0.02	0.01	0.13	0.09
CoMargin	0.01	0.00	0.10	0.05
Panel D. Per Clearing Member (crisis)				
SPAN	0.02	0.06	0.17	0.63
VaR	0.03	0.08	0.18	0.48
CoMargin	0.01	0.02	0.14	0.30

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The clearing member-level results in Table 5 (Panels B–D) show the probability that a typical clearing member surpasses its margin on any given day and the expected shortfall associated with this event. The conditional frequency and expected shortfalls are lower for CoMargin than for SPAN and VaR margins. In addition, notice how both the conditional and unconditional frequencies increase for the SPAN and VaR systems during the crisis period relative to the precrisis period. These results imply that when P&L dependence increases, SPAN and VaR exceedances are more likely than under CoMargin.

VI. Clearing Member Reactions to CoMargin

A. Framework

In this section, we study how clearing members could strategically react to an implementation of CoMargin. We derive a simple modeling framework, inspired by Duffie and Zhu (2011), which is flexible enough to accommodate various types of behaviors that could be spurred by CoMargin. Indeed, it is legitimate to expect that a member may engage in some form of margin arbitrage by, for instance, i) fragmenting its portfolio and clearing some positions on another CCP, ii) merging its trading portfolio with that of another member that exhibits correlated trading positions, or iii) moving all its positions off the CCP. In each case, we analyze the clearing members' incentives to switch from CoMargin to its alternative.

We consider a CCP that uses CoMargin to compute the initial margins of all its clearing members. Within this CCP, we focus on a subset of three clearing members, indexed by $j \in \{A, B, C\}$. Each clearing member can invest in four assets indexed by $i \in \{1,2,3,4\}$, which are assumed to have normally distributed returns $R = (r_1, r_2, r_3, r_4)'$:

$$R \sim N(0, \Sigma),$$
$$\Sigma = \begin{pmatrix} 1 & \rho_{12} & 0 & 0 \\ \rho_{12} & 1 & \rho_{23} & 0 \\ 0 & \rho_{23} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

with $\rho_{12} \leq 0$ and $\rho_{23} \geq 0$. For simplicity, the size of the trading portfolios of the three clearing members is normalized to 1. Let $\omega_j = (\omega_{1j}, \omega_{2j}, \omega_{3j}, \omega_{4j})'$ denote the weight vector of clearing member j, with $\sum_{i=1}^{4} \omega_{ij} = 1$, and denote by $V_j = \omega_j' R$ the corresponding portfolio P&L. In our analysis, we consider the following positions:

$$\omega_A = (1/2, 1/2, 0, 0)', \quad \omega_B = (1/2, 0, 1/2, 0)', \quad \omega_C = (0, 0, 0, 1)'.$$

Under this weighting scheme, members A and B have dependent P&Ls because they both invest in the first asset and each of them invests in specific assets (2 or 3) that are correlated. Differently, member C invests only in the fourth asset, which is uncorrelated with the other assets. We summarize the initial situation in Figure A2 in the Internet Appendix.

B. Clearing Fragmentation

When computing the CoMargin of a given clearing member, the conditioning event is a situation where at least one of the other two firms is in financial distress. The CoMargin for member A, denoted $B^{A|B,C}$, is defined by

$$\Pr(V_A \leq -B^{A|B,C} | (V_B \leq -B_B) \cup (V_C \leq -B_C)) \quad = \quad \alpha, \tag{18}$$

where B_B and B_C denote, respectively, the portfolio's VaRs of members B and C, and $V_A = (r_1 + r_2)/2$, $V_B = (r_1 + r_3)/2$, and $V_C = r_4$. While there is no analytical solution for $B^{A|B,C}$, we know that $B^{A|B,C} > B_A$ since V_A and V_B are correlated. Moreover, the higher the correlation ρ_{23}, the greater the CoMargin of member A.

One way for member A to reduce its margin requirement would be to move its outstanding position in the second asset to a different CCP (see Figure A2 in the Internet Appendix). After clearing fragmentation, its residual positions on the original CCP become $\tilde{\omega}_A = (1/2, 0, 0, 0)'$ and, as a result, the dependence between the portfolios of A and B is reduced. The postfragmentation CoMargin $B^{\tilde{A}|B,C}$ is

$$\Pr(\tilde{V}_A \leq -B^{\tilde{A}|B,C} | (V_B \leq -B_B) \cup (V_C \leq -B_C)) \quad = \quad \alpha. \tag{19}$$

We assume that, on the second CCP, margins are computed according to the VaR margining system. As a consequence, after fragmentation, the change in margin for member A is equal to

$$\Delta_f(\rho_{12}, \rho_{23}) \quad = \quad \underbrace{B^{\tilde{A}|B,C}}_{\substack{\text{CoMargin} \\ \text{after fragmentation}}} + \underbrace{B_{A1}}_{\substack{\text{VaR margin} \\ \text{for asset 1}}} - \underbrace{B^{A|B,C}}_{\substack{\text{CoMargin} \\ \text{before fragmentation}}}, \tag{20}$$

where $B_{A1} = -\Phi^{-1}(\alpha)/4$ and $\Phi(\cdot)$ denotes the standard normal cumulative density function (CDF). Clearing fragmentation has two opposite effects on total margin. First, CoMargin decreases because of the lower dependence between the portfolios of A and B. This dependence effect becomes more important as the correlation parameter ρ_{23} grows. Second, clearing fragmentation prevents the netting of member A's positions and induces an increase in total margin.[24] The closer ρ_{12} is to -1, the more diversified portfolio A is (before fragmentation), and the larger the increase in margin due to the netting effect.

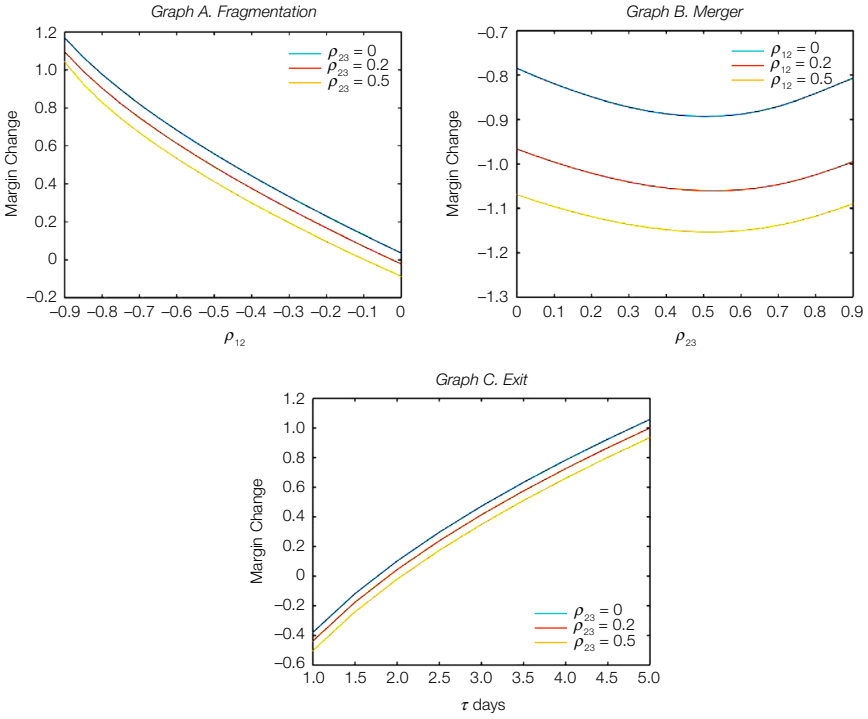
Since there is no analytical expression for CoMargin in this case, we compute $B^{\tilde{A}|B,C}$ and $B^{A|B,C}$ by numerical integration of the CDF of the multivariate normal distribution. Graph A of Figure 4 displays the margin change Δ_f as a function of ρ_{12} (netting effect) and for different values of ρ_{23} (dependence effect). If the correlation parameter ρ_{12} is small enough and the correlation parameter ρ_{23} is sufficiently large, fragmentation becomes attractive for A as it lowers total margin, $\Delta_f < 0$. However, for most of the set of parameters considered in our experiment, total margin increases after fragmentation as the diversification effect dominates the dependence effect.

[24] If the CCP uses a VaR margining system, the sum of the individual VaRs of assets 1 and 2 is larger than the VaR of the initial portfolio of member A as long as $\rho_{12} < 0$. Notice that this result is due to the subadditive property of VaR (Artzner, Delbaen, Eber, and Heath (1999)) in the case of elliptic distributions, which include the normal distribution. Similar effects are at play in the case of CoMargin.

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FIGURE 4
Change in Total Margins after Strategic Reactions

Figure 4 displays the changes in total initial margin for a clearing member that engages in some form of margin arbitrage by fragmenting its portfolio and clearing some positions in another CCP (Graph A), merging its trading portfolio with the one of another member that exhibits correlated trading positions (Graph B), or moving all of its positions off the CCP (Graph C).



In practice, transferring part of the portfolio to another clearing platform is more cumbersome than one may think. Indeed, most CCPs tend to be specialized on certain products, and there is little overlap between the set of products offered by the main CCPs, at least among those that are admissible by local regulators in G20 countries (see Financial Stability Board (2015), App. J). Furthermore, switching to another CCP located in a different currency zone exposes the clearing member to foreign exchange risk, as it needs to post acceptable collateral denominated in the local currency of the CCP.

C. Incentives to Merge Positions

As members A and B have dependent P&Ls, a potentially attractive strategy for them would be to merge into a single entity, say, $A+B$, in order to avoid posting high CoMargin (see Figure A2). Prior to the merger, the CoMargins of members A and B, $B^{A|B,C}$ and $B^{B|A,C}$, are computed by conditioning their losses on a situation in which at least one of the other two members is in financial distress (see equation (18)). After the merger, the new entity $A+B$ is long in the assets

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1, 2, and 3 and its combined portfolio weights are $\omega_{A+B}=(1,1/2,1/2,0)'$. The CoMargin of the merged entity $A+B$ is defined conditionally only on the distress of member C, and is denoted $B^{A+B|C}$. However, as the P&Ls of $A+B$ and C are independent, $B^{A+B|C}$ boils down to the VaR margin B_{A+B}. Consequently, the combined change in total margin for members A and B after the merger is equal to

$$\Delta_m(\rho_{12},\rho_{23}) = \underbrace{B_{A+B}}_{\substack{\text{CoMargin}\\ \text{after the merger}}} - \underbrace{(B^{A|B,C}+B^{B|A,C})}_{\substack{\text{CoMargins}\\ \text{before the merger}}}. \tag{21}$$

Incentives to merge crucially depend on P&L dependence. The situation is captured by the correlation between assets 2 and 3, and it has two opposite effects on the margin change Δ_m. First, the higher ρ_{23}, the larger the initial CoMargins $B^{A|B,C}$ and $B^{B|A,C}$, which increases incentives to merge. Second, a higher correlation ρ_{23} leads to more volatility (less diversification) in the combined portfolio of $A+B$. This volatility effect leads to an increase in the postmerger margin, which lowers incentives to merge. The net effect on Δ_m of these opposite forces is presented in Graph B of Figure 4. In this framework, A and B always benefit from merging, as $\Delta_m<0$ for all parameter values.

D. Incentives to Move Contracts Off CCP

We now consider the case where member A moves its entire portfolio off the CCP (see Figure A2 in the Internet Appendix). Switching from central clearing to bilateral clearing permits members to avoid paying a high CoMargin. The key quantity to determine is the level of margin requirements for noncentrally cleared derivatives contracts such that members have no incentive to move contracts off the CCP. Under current regulation, margins for noncentrally cleared derivatives have to be computed with a longer coverage horizon. As a result, the post-exit margin of member A is a τ-day VaR, which corresponds to $\sqrt{\tau}B_A$. Then, the change in margin for member A is equal to

$$\Delta_e(\tau,\rho_{23}) = \underbrace{\sqrt{\tau}B_A}_{\substack{\text{VaR Margin for}\\ \text{uncleared positions}}} - \underbrace{B^{A|B,C}}_{\substack{\text{CoMargin}\\ \text{before exit}}}. \tag{22}$$

Any member contemplating leaving the CCP faces a trade-off between P&L dependence within the CCP, which sets the initial CoMargin, and the coverage horizon of the VaR mandated for noncentrally cleared derivatives. Graph C of Figure 4 displays the margin change Δ_e as a function of the number of days τ and different values of ρ_{23} (we set ρ_{12} to 0). The main finding is that exit is an attractive strategy only for VaR horizon less than 2 days. It is interesting to notice that as ρ_{23} increases, exit becomes more attractive. Indeed, when ρ_{23} changes from 0 to 0.5, the τ value for which A is indifferent between CCP clearing and bilateral clearing shifts from 1.75 to 2 days. For instance, the final policy framework that establishes standards for margin requirements for noncentrally cleared CDS is based on a 10-day coverage horizon (BIS (2015b)).

In practice, for noncentrally cleared OTC derivatives, financial participants also have to post Basel III Capital and Liquidity Requirements on top of initial

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margins. In most cases, the capital and liquidity requirements are so high that the funding needed to satisfy them is higher than that needed for posting margin at a CCP.

E. Predation

One could think that implementing CoMargin may lead to unintended consequences, particularly since portfolio changes conducted by individual clearing members might have effects on other members' margins. For instance, by mimicking a smaller rival's trades (thus generating P&L dependence), a large, deep-pocketed clearing member might attempt to drive a smaller rival out of business. However, in practice, competitor positions are unknown and clearing members guard both their positions and trading strategies rather fiercely. In addition, and perhaps more importantly, a clearing member planning to prey on a rival would also need to accurately predict, on a daily basis, the conditioning set used by the CCP to estimate CoMargin, which is very unlikely. Therefore, we can safely conclude that clearing members do not have the information set necessary for conducting predatory practices.

VII. Conclusion

In this paper, we present a new methodology, called CoMargin, to estimate margin requirements in derivatives' CCPs. Our innovative approach explicitly takes into account both the individual risk and the interdependence of the P&Ls of market participants. As a result, CoMargin produces collateral allocations that enhance the stability and resilience of CCPs, thus reducing their systemic risk.

Using both simulated and real-world derivatives positions data, we show that CoMargin outperforms the widely popular SPAN and VaR margining approaches. CoMargin performs particularly well relative to these alternatives when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, our evidence suggests that CoMargin provides more protection to the CCP when it needs it most.

While CoMargin significantly increases aggregate collateral levels during certain periods, using budget-neutral comparisons, we show that the relative performance of CoMargin is driven by its allocation process and not the additional funds collected. Thus, increasing collateral requirements under other systems does not necessarily protect the CCP against simultaneous margin exceedance events.

Overall, we consider CoMargin as a viable alternative that might serve as the basis for further enhancements to existing margin systems. Potential improvements to the current CoMargin framework include reducing the procyclicality of collateral requirements and enriching the scenario generating process. For instance, scenarios could account for endogenous risk or feedback effects. This feature would consider shocks that reflect not only the past evolution of the underlying variables but also the reaction of clearing firms to specific shocks.

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