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COUNTERFACTUALS AND PROPOSITIONAL CONTINGENTISM

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Abstract. This article explores the connection between two theses: the principle of conditional excluded middle for the counterfactual conditional, and the claim that it is a contingent matter which (coarse grained) propositions there are. Both theses enjoy wide support, and have been defended at length by Robert Stalnaker. We will argue that, given plausible background assumptions, these two principles are incompatible, provided that conditional excluded middle is understood in a certain modalized way. We then show that some (although not all) arguments for conditional excluded middle can in fact be extended to motivate this modalized version of the principle.

Stalnaker (2012) defends *propositional contingentism*, the thesis that it is a contingent matter what propositions there are. Stalnaker's idea is that there are only those propositions which don't distinguish between possibilities which can't be distinguished using materials which are actually available. For example, it seems that there could have been two duplicate coins x and y such that neither they nor the materials from which they were made exist in the actual world. There seems to be no way of distinguishing such merely possible coins using any combination of qualitative distinctions and reference to actually existing things. So Stalnaker denies that there actually are any propositions that distinguish between them, such as the proposition that both are flipped and only x lands heads. A similar picture is developed in Fine (1977b), see Fritz and Goodman (2016) for an extended investigation of such views.

Stalnaker (1968, 1981) has also been a prominent defender of the principle of conditional excluded middle for natural language conditionals, according to which negating a nonvacuous conditional is equivalent to negating its consequent. In particular, he defends the validity of the schema: either (had it been the case that φ, it would have been the case that ψ) or (had it been the case that φ, it would have been the case that not -ψ). Although conditional excluded middle is highly contested, having been most famously rejected by Lewis (1973), it nevertheless enjoys wide support.

But propositional contingentism and conditional excluded middle are in tension. Recall the purportedly actually indistinguishable possible coins x and y. According to conditional excluded middle, either, had they been flipped and only one of them come up heads, x would have come up heads, or, had they been flipped and only one of them come up heads, y would have come up heads. But then x and y would seem to be actually distinguishable after all, in terms of which of them would have landed heads had exactly

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one of them done so. This argument threatens to generalize: might it turn out that, given conditional excluded middle, necessarily, all possible modal distinctions can be drawn in terms of counterfactuals, in which case Stalnaker's motivation for propositional contingentism would be undermined?

This article explores the extent to which propositional contingentism really is incompatible with conditional excluded middle. §1 introduces Stalnaker's possible-world models for contingency in what propositions there are. §2 extends this model theory to counterfactuals in a very general way, making minimal assumptions about both contingency in what propositions there are and about the semantics of counterfactuals. §3 uses these models to substantiate the informal tension between propositional contingentism and conditional excluded middle, by showing that the latter rules out certain intuitive patterns of contingency in what propositions there are. §4 and §5 prove that, given plausible auxiliary axioms, the models developed in §2 validate conditional excluded middle only if they invalidate propositional contingentism. As we show in an appendix, it is essential for these proofs that conditional excluded middle holds not just for any propositions p and q that there *are*, but also for any such propositions there *could be*. §6 therefore considers whether one might endorse conditional excluded middle only for propositions there are, and raises doubts about the tenability of such a split decision, as some of the most important arguments for conditional excluded middle can be used to motivate the modalized version of the principle. §7 concludes, noting that propositional contingentism cannot be reconciled with conditional excluded middle by appealing to indeterminacy in which counterfactuals are true. An appendix answers some technical questions concerning the strength of the assumptions required to destabilize the combination of conditional excluded middle and propositional contingentism.

§1. Propositional contingentism. Stalnaker (2012, Appendix A) sketches a way of formally modeling contingency in what propositions there are. As is familiar from possible-worlds model theory, these models are based on a set W representing possible worlds, and propositions are represented by subsets of W. There is no accessibility relation, so a proposition is understood to be necessary just in case it is identical to W. Contingency in what propositions there are is modeled by mapping each world w to an equivalence relation $\approx_w$; the idea is that the propositions at w are the subsets of W which contain either both or neither of any two worlds related by $\approx_w$. Equivalently, the propositions at w can be taken to be the unions of sets of equivalence classes under $\approx_w$. Formally, the model theory can be defined as follows:

DEFINITION 1.1. *An* equivalence system on a set W *is a function* $\approx$ *mapping every* $w \in W$ *to an equivalence relation* $\approx_w$ *on* W. *The* domain function of $\approx$, *written* $D^{\approx}$, *is the function mapping each* $w \in W$ *to*

$$D^{\approx}_w = \{P \subseteq W : \textit{for all } v, u \in W, \textit{ if } v \approx_w u \textit{ then } v \in P \textit{ iff } u \in P\}.$$

Equivalence systems are naturally used to interpret a propositional modal language with propositional quantifiers. Consider such a language, using $p, q, r, \ldots$ as propositional variables, $\neg$ for negation, $\wedge$ for conjunction, $\Box$ for necessity and $\forall$ for the universal quantifier binding propositional variables. Writing $[\![\varphi]\!]_{M,a}$ for the proposition expressed by a formula φ interpreted using an equivalence system $M = \approx$ on W and an assignment function a mapping each propositional variable p to $a(p) \subseteq W$, we define this interpretation function as follows:

$$[\![p]\!]_{M,a} = a(p),$$
$$[\![\neg\varphi]\!]_{M,a} = W \setminus [\![\varphi]\!]_{M,a},$$

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$$[\![\varphi \wedge \psi]\!]_{M,a} = [\![\varphi]\!]_{M,a} \cap [\![\psi]\!]_{M,a},$$
$$[\![\Box\varphi]\!]_{M,a} = \{w \in W : v \in [\![\varphi]\!]_{M,a} \text{ for all } v \in W\},$$
$$[\![\forall p\varphi]\!]_{M,a} = \{w \in W : w \in [\![\varphi]\!]_{M,a[P/p]} \text{ for all } P \in D_w^{\approx}\}.$$

Here, $a[P/p]$ is the assignment function which maps p to P and any propositional variable $q \neq p$ to $a(q)$.

Note that $w \in [\![\varphi]\!]_{M,a}$ can be understood as φ being true in w according to M on the assignment function a. As usual, other Boolean operators as well as $\Diamond$ and $\exists$ will be treated as syntactic abbreviations. We call a formula a *sentence* if it has no free variables. Since the evaluation of a sentence φ does not depend on the assignment function, we omit it, writing simply $[\![\varphi]\!]_M$.

It is natural to assume that necessarily, for any proposition, there is its negation. More generally: necessarily, there is every Boolean combination of propositions there are. In equivalence systems, this is guaranteed by the fact that the domain of propositions of every world is a (complete and atomic) Boolean algebra. It is natural to extend this constraint beyond Boolean combinations to combinations obtained using the necessity operator and the universal quantifier. That is, it is natural to require the domain of a given world w to contain $[\![\varphi]\!]_{M,a}$ for any formula φ of the language specified above and assignment function a which maps every free variable of φ to a proposition in the domain of w. This constraint can equivalently be formulated by requiring each instance of the following comprehension schema to be true in every world:

(Comp_C) $\forall\bar{p}\exists q\Box(q \leftrightarrow \varphi(\bar{p}))$.

Here, $\forall\bar{p}$ indicates a finite sequence of the form $\forall p_1 \ldots \forall p_n$, and $\varphi(\bar{p})$ indicates that φ is a formula of the above-specified language in which the only free variables are $p_1, \ldots, p_n$. Note that 'p_1', ..., 'p_n' and 'q' are object language variables themselves, rather than metavariables ranging over object language variables. (As usual, quotation marks around object language expressions are normally omitted.) It is therefore guaranteed that 'q' is distinct from 'p_1', ..., 'p_n', and so the paradoxical $\forall q\exists q\Box(q \leftrightarrow \neg q)$ is not an instance of Comp_C. For further discussion and motivation of Comp_C, see Williamson (2013, sec. 6.4).

It turns out that Comp_C is not valid on the class of all equivalence systems. However, it is valid on the subclass of equivalence systems that are *coherent* in the sense described in §8.1. This coherence constraint is developed in Fritz (2016) on the basis of work in Stalnaker (2012, Appendix A); as shown in Fritz (forthcoming), the resulting class of structures is equivalent to the propositional fragment of the more general model theory of Fine (1977b), as well as several variants discussed in Fritz and Goodman (2016).

§2. Semantics for counterfactuals. Many wellknown ways of constructing a possible-worlds semantics for counterfactuals are easily adapted to the setting of equivalence systems. For example, one formulation of Lewis's (1973) theory associates each world with an order of comparative similarity among worlds. Adapting this framework to equivalence systems, we can associate each world w with an order of comparative similarity among atomic propositions in w (equivalence classes of the equivalence relation $\approx_w$). A different approach is the selection function semantics of Stalnaker (1968); instead of associating worlds with selection functions mapping propositions to worlds, we can take them to map propositions to atomic propositions ($\approx_w$-equivalence-classes of worlds).[1]

[1] A note on terminology: our use of 'world' and 'atomic proposition' correspond, respectively, to the use of 'point of logical space' and 'world' in Stalnaker (2012).

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Adapting Lewis's or Stalnaker's truth-conditions, the above evaluation clauses can then be extended to a language containing an additional binary operator $\boxrightarrow$ for the counterfactual conditional. In order to obtain the validity of Comp_C for formulas containing $\boxrightarrow$, the coherence condition alluded to in the previous section must be extended to cover the additional semantic structure used to interpret this operator; see §8.1. Indeed, so long as we restrict our attention to structures that are coherent in the relevant sense, it makes no difference whether we interpret counterfactuals using a comparative similarity order/selection function defined on worlds or defined on the atomic propositions of a given world.

Rather than exploring the fine details of such approaches, we will instead present an extremely general strategy for interpreting counterfactuals on equivalence systems, demonstrating that our subsequent results do not depend on any such details. In the present setting, the semantic contribution of a dyadic sentential operator like $\boxrightarrow$ corresponds to a function from pairs of propositions to propositions (in this case, to the proposition that the second of the pair of propositions would have been the case had the first been the case). Model theoretically, then, all we will require of $\boxrightarrow$ is that its semantic contribution is determined compositionally by some such function from pairs of sets of worlds to sets of worlds. We do not impose any further coherence requirements, other than demanding that every world be able to distinguish itself from all others. This yields the following definition of a model:

DEFINITION 2.1. *A* model *is a tuple* $\langle W, \approx, C\rangle$*, where* W *is a set,* $\approx$ *is an equivalence system on* W *such that* $\{w\} \in D^{\approx}_{w}$ *for all* $w \in W$*, and* C *is a function* $C : \mathcal{P}(W) \times \mathcal{P}(W) \to \mathcal{P}(W)$.

The evaluation clauses for connectives other than $\boxrightarrow$ are unchanged from the previous section, to which we add the clause:

$$[\![\varphi \boxrightarrow \psi]\!]_{M,a} = C([\![\varphi]\!]_{M,a}, [\![\psi]\!]_{M,a}).$$

For any class of models X, we define a consequence relation and a property of validity among sentences as usual: $\Gamma \vDash_X \varphi$ iff $\bigcap_{\gamma \in \Gamma} [\![\gamma]\!]_M \subseteq [\![\varphi]\!]_M$ for all M in X (taking the intersection to be W for empty Γ); $\vDash_X \varphi$ iff $\emptyset \vDash_X \varphi$. If X is the class of all models, we drop the index.

Note that allowing models to use an arbitrary two-place function on sets of worlds to interpret the counterfactual conditional does not mean that in a fuller higher-order logic, quantifiers binding variables in the position of binary sentential connectives should be modeled as ranging over all such functions at every world. This idea would be disastrous for propositional contingentism, as such models only validate the instances of Comp_C in such a higher-order language if the propositional domain of each world contains every set of worlds. Rather, the interpretation of the counterfactual conditional is left completely unconstrained in order to show that our results require no further model-theoretic assumptions. For explorations of models of higher-order logic with variable domains for all syntactic types, see Fritz and Goodman (2016).

The aim of this article is to demonstrate the tension between propositional contingentism and conditional excluded middle. We illustrating this tension in the next section, and give a general argument in the following two sections, showing that the falsity of propositional contingentism follows from Comp_C, a version of conditional excluded middle, and some auxiliary premises. In order to forestall any misunderstandings, let us clarify a couple of aspects of the language and the models introduced here, and the role they play in the argument. First, we will take as premises any instances of the schematic principle Comp_C in the language under consideration here—i.e., the propositionally quantified

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modal language introduced in the previous section, expanded by the addition of the binary sentential operator $\boxrightarrow$. We make no claim about the truth of instances of Comp_C in more expressive languages. Second, we will use the model theory in an instrumental capacity to argue that propositional contingentism is false if the aforementioned premises are true. The language under consideration contains no nonlogical constants; furthermore, all of the principles we will consider as premises and conclusions are sentences, rather than open formulas. Each such sentence is therefore true or false *simpliciter* on the intended interpretation of the logical constants. Thus, all we need to assume about the model-theoretic consequence relation is that it is truth-preserving, on the intended interpretation of the language. Although we don't take this to be trivial, it seems at least *prima facie* plausible.

§3. An illustration. For reasons that will become clear, in the present context we need to be particularly careful about what we mean by 'conditional excluded middle'. In the debate between Lewis and Stalnaker, it is formulated as the schema $(\varphi \boxrightarrow \psi) \vee (\varphi \boxrightarrow \neg\psi)$. However, in the present setting in which our language contains propositional quantifiers, it is natural, at least initially, to formulate the principle using the following sentence of our formal language:

$(CEM\varepsilon\varepsilon)\ \ \forall p \forall q((p \boxrightarrow q) \vee (p \boxrightarrow \neg q))$.

(A general naming convention which explains the label '$\varepsilon\varepsilon$' will be introduced shortly.)

Before illustrating the tension between $CEM\varepsilon\varepsilon$ and propositional contingentism, we need to discuss three relatively uncontroversial principles concerning counterfactuals. The first is a principle of agglomeration which says that if two propositions q_1 and q_2 would each be the case had p been the case, then so too would be their conjunction $q_1 \wedge q_2$. Using propositional quantifiers, we can formulate this principle thus:

$(\mathrm{Agg}\varepsilon\varepsilon)\ \ \forall p \forall q_1 \forall q_2(((p \boxrightarrow q_1) \wedge (p \boxrightarrow q_2)) \rightarrow (p \boxrightarrow (q_1 \wedge q_2)))$.

The second and third principles are that possibility is closed under counterfactual implication, and that every proposition counterfactually implies itself; formally:

$(B1)\ \ \forall p \forall q((\Diamond p \wedge (p \boxrightarrow q)) \rightarrow \Diamond q)$,
$(B2)\ \ \forall p(p \boxrightarrow p)$.

We will now illustrate the tension between $CEM\varepsilon\varepsilon$ and propositional contingentism by showing that a simple equivalence system fitting Stalnaker's motivation of propositional contingentism cannot be extended to a model in which $CEM\varepsilon\varepsilon$ is true in every world. The equivalence system is based on four worlds, representing the following toy model of modal space. Assume that the universe of worlds is constructed by freely recombining two electrons a and b, taking only into account what individuals there are. That is, there are four worlds, one for every subset X of $\{a, b\}$. This naturally induces an equivalence system if one postulates that from the perspective of any world w, worlds are related by $\approx_w$ just in case they can't be distinguished in terms of the elementary particles that there are at w. (A generalization of this idea is central to Fine (1977b) and Fritz and Goodman (2016).) Call the four worlds ab, a, b and e, where e is the empty world, a is the world containing only a, etc. So all pairs of distinct worlds are discernible from the perspective of every world, with the crucial exception that a and b are indiscernible from the perspective of e.

More formally, let $W = \{ab, a, b, e\}$ (where these are four distinct things), and let $\approx$ be the equivalence system on W mapping ab, a, b to the identity relation on W and mapping e

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to the equivalence relation on W that relates a and b to each other while relating e and ab only to themselves. (This system is coherent, in the sense of Definition 8.1 in §8.1.) We show that there is no model $M = \langle W, \approx, C\rangle$ such that Comp_C, $CEM\varepsilon\varepsilon$, $\text{Agg}\varepsilon\varepsilon$, $B1$, and $B2$ are true in all worlds.

Assume for contradiction that M is such a model. At ab, there are (in the sense of being in $D^{\approx}_{ab}$) both $\{a\}$ and $\{b\}$, so by $CEM\varepsilon\varepsilon$, at ab, either, had $\{a, b\}$ been the case, $\{a\}$ would have been the case, or, had $\{a, b\}$ been the case, $\{ab, b, e\}$ would have been the case—in which case $\{b\}$ would have been the case, by $\text{Agg}\varepsilon\varepsilon$, since, by $B2$, $\{a, b\}$ would have been the case had $\{a, b\}$ been the case. By $\text{Agg}\varepsilon\varepsilon$ and $B1$, one disjunct is false: since $\{a, b\}$ is not impossible, it cannot counterfactually imply both $\{a\}$ and $\{b\}$, since their conjunction is impossible. But this means that, at e, exactly one of $\{a\}$ and $\{b\}$ is a proposition that, had $\{ab\}$ been the case, would have been the case had $\{a, b\}$ been the case. Now consider the following instance of Comp_C: $\forall p_1 \forall p_2 \exists q \Box(q \leftrightarrow \forall r(r \leftrightarrow (p_1 \boxdot\!\!\rightarrow (p_2 \boxdot\!\!\rightarrow r))))$. Since at e there are both $\{ab\}$ and $\{a, b\}$, this sentence is true at e only if, for some proposition X that there is at e, $\Box(q \leftrightarrow \forall r(r \leftrightarrow (p_1 \boxdot\!\!\rightarrow (p_2 \boxdot\!\!\rightarrow r))))$ is true at e relative to an assignment of $\{ab\}$ to p_1, $\{a, b\}$ to p_2, and X to q. By our previous observations, this formula must be satisfied at e either only by $\{a\}$ or by $\{b\}$, since it is satisfied only by the proposition that things are all and only the ways that, at ab, they would have been had it been the case that $\{a, b\}$. Since at e there are neither of these two propositions, this instance of Comp_C cannot be true at e, completing our reductio.

§4. The finite case. In one respect, $CEM\varepsilon\varepsilon$ expresses a restricted version of conditional excluded middle—it applies only to the propositions that there actually are. To obtain a general argument that conditional excluded middle is incompatible with propositional contingentism, a stronger version of the principle is needed which also applies to all propositions that there could have been. Whether such a principle is motivated will be discussed in §6. To formulate it, we need a way of simulating quantification over all possible propositions. We can do so by adapting a strategy going back to Fine (1977a). Fine's idea is that we can express universal quantification over possible propositions using the phrase 'necessarily, all propositions are such that…'. For this strategy to work within modal contexts, we must find a way of undoing the effect of 'necessarily' for the evaluation of the complement clause. This can be achieved as follows.

First, we need ways of talking about 'world propositions'. Recall that in our models the propositional domain of any world includes the singleton of that world. So relative to this class of models, φ being a world proposition is definable as it being possible that φ is true and it necessitates every proposition or its negation:

$$\text{WP}(\varphi) := \Diamond(\varphi \wedge \forall s(\Box(\varphi \rightarrow s) \vee \Box(\varphi \rightarrow \neg s))) \qquad (s \text{ not free in } \varphi).$$

It will be useful to have the following abbreviations for binding a free variable p in a formula with the definite description 'the true world proposition' having a particular scope, and for something's being 'true at' a given world proposition:

$$\downarrow p\varphi := \forall p((p \wedge \text{WP}(p)) \rightarrow \varphi),$$
$$@\, p\varphi := \Box(p \rightarrow \varphi).$$

We can now simulate universal quantification over all propositions there could have been as follows:

$$\Pi p\varphi := \downarrow q \Box \forall p @ q \varphi \qquad (q \text{ not free in } \varphi).$$

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Using these 'modalized' quantifiers, we can formulate a strengthened version of *CEM* by replacing $\forall$ with Π. In fact, we only require a version in which the second quantifier is modalized:

$(CEM\varepsilon\pi)\ \ \forall p \Pi q((p \boxright q) \vee (p \boxright \neg q))$.

As with *CEM*, we need to strengthen agglomeration so that it applies to the consequents of counterfactuals even when they are merely possible propositions:

$(\mathrm{Agg}\varepsilon\pi)\ \ \forall p \Pi q_1 \Pi q_2(((p \boxright q_1) \wedge (p \boxright q_2)) \rightarrow (p \boxright (q_1 \wedge q_2)))$.

In general, our naming scheme for *CEM* and Agg will add two tags corresponding to the quantifiers binding p and q (or q_1 and q_2, in the case of Agg), with ε indicating $\forall$ and π indicating Π.

With these modalized principles, we can give a more general argument against the compatibility of conditional excluded middle with propositional contingentism. This argument is not yet fully general; with $\mathrm{Agg}\varepsilon\pi$, we can only show that $CEM\varepsilon\pi$ rules out propositional contingentism in *finite* models. The argument can be extended to apply to all models if $\mathrm{Agg}\varepsilon\pi$ is replaced by an infinitary analog, but since this complicates the argument somewhat, we first give a simpler version of the argument for finite models based on $\mathrm{Agg}\varepsilon\pi$, postponing the infinite case until the next section.

Let U be the claim that there are all possible propositions:

$(U)\ \ \Pi p \exists q \Box(p \leftrightarrow q)$.

Two comments are in order. The first is that, strictly speaking, U says only that every possible proposition is *necessarily equivalent* to some proposition that there (actually) is. If (*pace* Stalnaker) necessary equivalence fails to suffice for propositional identity, then our initial gloss on U was an overstatement. But it is nevertheless a strong enough conclusion for present purposes, since it suffices to refute Stalnaker's contention that there are distinctions in modal space that, actually, are not drawn by any proposition. See Fritz and Goodman (forthcoming) for further discussion of these issues. Second, we should note that U is compatible with propositional contingentism, since it is compatible with the possibility that there be strictly fewer propositions than there actually are. But it is hard to see a principled view on which U is only contingently true. In any case, the necessitations of the premises of the following arguments are no less compelling than the premises themselves, and so all of those arguments can be straightforwardly adapted to apply to the necessitation of U if desired.

Let fin be the class of models $M = \langle W, \approx, C\rangle$ such that W is finite. The result to be proven can now be stated as follows:

PROPOSITION 4.1. $\{CEM\varepsilon\pi, \mathrm{Agg}\varepsilon\pi, \mathrm{Comp}_C, B1, B2\} \models_{\mathsf{fin}} U$.

Proof. Let $M = \langle W, \approx, C\rangle$ be a model in fin and $w \in W$ such that $CEM\varepsilon\pi$, $\mathrm{Agg}\varepsilon\pi$, Comp_C, $B1$, and $B2$ are true in w. We prove that U is true in w, by showing that $D_w^{\approx}$ contains every world proposition. Consider any $v \in W$, and let $A = [v]_{\approx_w}$ (the equivalence class under $\approx_w$ containing v). Since all world propositions are in the domains of the corresponding worlds, $CEM\varepsilon\pi$ entails, for all $u \in A$, that either $w \in C(A, \{u\})$ or $w \in C(A, W\backslash\{u\})$. By $B2$, $w \in C(A, A)$, so for any $u \in A$ such that $w \in C(A, W\backslash\{u\})$, by $\mathrm{Agg}\varepsilon\pi$, $w \in C(A, A\backslash\{u\})$.

First, we show that there must be at least one $u \in A$ such that $w \in C(A, \{u\})$: otherwise, for all $u \in A$, $w \in C(A, A\backslash\{u\})$, and so by $|A|$ (guaranteed to be finite) applications

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of Agg$\varepsilon\pi$, $w \in C(A, \emptyset)$, contradicting $B1$. Second, we show that there can be at most one $u \in A$ such that $w \in C(A, \{u\})$: otherwise, by Agg$\varepsilon\pi$, $w \in C(A, \emptyset)$, again contradicting $B1$.

Thus there is a unique $u \in A$ such that $w \in C(A, \{u\})$. So, for any assignment function a such that $a(q) = \{w\}$ and $a(r) = A$, $[\![\downarrow p@q(r \boxright p)]\!]_{M,a} = \{u\}$. By Comp$_C$, $\{u\} \in D^{\approx}_w$. Since $u \in A = [v]_{\approx_w}$, $\{v\} \in D^{\approx}_w$, as claimed. □

§5. The infinite case. There is no obvious way of extending the above proof to infinite models: the argument that there is at least one $u \in A$ such that $w \in C(A, \{u\})$ requires us to go through a chain of reasoning containing an application of Agg$\varepsilon\pi$ for each such u. Indeed, an infinite countermodel to the entailment in Proposition 4.1 will be given in §8.3. If A is infinite, we need a version of agglomeration which allows us to show even for infinite A that if $w \in C(A, A\backslash\{u\})$ for each $u \in A$, then $w \in C(A, \bigcap_{u \in A} A\backslash\{u\})$. Intuitively, we want a principle that allows us to say, of any plurality of (perhaps merely possible) propositions, if each of them would have been the case had φ been the case, then, had φ been the case, all of them would have been the case. Although Lewis (1973, pp. 19–21) famously rejected this kind of infinite agglomeration (corresponding in his semantics to the failure of the 'limit assumption'), this is widely seen as an implausible consequence of his system (see Herzberger (1979)). In any case, his motivations were the same as those that led him to reject conditional excluded middle, and so can be set aside for present purposes. We know of no philosopher who accepts conditional excluded middle while rejecting the idea of infinite agglomeration (although the formal tenability of such views is explored in Bacon (unpublished); see also Swanson (2008)).

One might try to give voice to the idea of infinite agglomeration by moving to an infinitary language in which we can form conjunctions of infinite sets of formulas and have universal quantifiers binding infinitely many variables simultaneously. Indeed, in such a language we could lay down the following schema:

$$\forall p \forall_{i<\kappa} q_i \left(\left(\bigwedge_{i<\kappa} (p \boxright q_i) \right) \rightarrow \left(p \boxright \bigwedge_{i<\kappa} q_i \right) \right),$$

which would have the model-theoretic effect of imposing agglomeration regarding infinite sets of propositions that there actually are. But it would not achieve the effect of infinite agglomeration regarding sets of merely possible propositions. Nor could it be modified to do so by allowing for modalized quantifiers in the initial prefix, for rather technical reasons discussed in Fritz and Goodman (forthcoming). (Briefly, in order to accommodate the case of infinite sets of propositions that are infinitely incompossible, in the sense that those propositions are not included in the union of the domains of any finite set of worlds, we would have to have infinitely many q_i bound by different modalized quantifiers, which would require an infinite nesting of necessity operators (given the definition of modalized quantifiers), which is impossible given that, even in infinitary languages, formulas are well-founded.)

Luckily, there is a different way we can give voice to an appropriately modalized principle of infinitary agglomeration within the finitary language we have already been working with. For simplicity, we will restrict our attention to sets of (perhaps merely possible) propositions q that can be picked out in terms of a condition $\varphi(p, q)$ relating them to the antecedent p of the counterfactuals whose consequents we are concerned to agglomerate. In order not to lose track of these propositions as we move from world to world, we must 'rigidify' the condition we use to pick them out. Having done so, we can formulate a schematic infinitary modalized version of agglomeration that is strong enough for present purposes:

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$(\forall \mathrm{Agg}\varepsilon\pi)\ \downarrow r \forall p(\Pi q(\varphi(p,q) \to (p \boxarrow\!\!\rightarrow q)) \to (p \boxarrow\!\!\rightarrow \Pi q(@r\varphi(p,q) \to q)))$.

Note that φ is not allowed to contain any free variables other than p and q. It would be natural to lift this restriction, but this won't be needed for the following result.

PROPOSITION 5.1. $\{CEM\varepsilon\pi, \forall \mathrm{Agg}\varepsilon\pi, \mathrm{Comp}_C, B1, B2\} \vDash U$.

Proof. It suffices to show how, using $\forall \mathrm{Agg}\varepsilon\pi$, the step in the proof of Proposition 4.1 appealing to the finiteness of the model can be carried out for an infinite model. Assume for contradiction that there is no $u \in A$ such that $w \in C(A, \{u\})$. Then for all $u \in A$, $w \in C(A, W\backslash\{u\})$. Let a be an assignment function such that $a(p) = A$ and $a(r) = \{w\}$. By $\forall \mathrm{Agg}\varepsilon\pi$,

$w \in [\![\Pi q((p \boxarrow\!\!\rightarrow q) \to (p \boxarrow\!\!\rightarrow q)) \to (p \boxarrow\!\!\rightarrow \Pi q(@r(p \boxarrow\!\!\rightarrow q) \to q))]\!]_{M,a}$,

and so, since the antecedent is evidently true in w,

$w \in [\![p \boxarrow\!\!\rightarrow \Pi q(@r(p \boxarrow\!\!\rightarrow q) \to q)]\!]_{M,a}$.

Given a's assignment of p and r, the consequent expresses the conjunction of all possible propositions which at w would have been the case had A been the case:

$[\![\Pi q(@r(p \boxarrow\!\!\rightarrow q) \to q)]\!]_{M,a} = \bigcap\{Q \in \bigcup_{x \in W} D_x^{\approx} : w \in C(A, Q)\}$.

Since $w \in C(A, W\backslash\{u\})$ for all $u \in A$, this set contains no element of A. With $B2$, it follows that it is empty, and so that $w \in C(A, \emptyset)$, which contradicts $B1$. □

The appendix shows that this result cannot be strengthened to various natural ways of weakening one of the three premises $CEM\varepsilon\pi$, $\forall \mathrm{Agg}\varepsilon\pi$, and Comp_C. Among a natural range of options, the result is therefore as strong as possible.

Given the widely endorsed background assumptions $\forall \mathrm{Agg}\varepsilon\pi$, Comp_C, $B1$, and $B2$, propositional contingentism is incompatible with $CEM\varepsilon\pi$. Those who endorse conditional excluded middle and propositional contingentism might respond to this result by continuing to endorse $CEM\varepsilon\varepsilon$ but rejecting $CEM\varepsilon\pi$. We now show that this position is unstable, as most of the arguments for conditional excluded middle support not only $CEM\varepsilon\varepsilon$ but also $CEM\varepsilon\pi$.

§6. Arguments for conditional excluded middle. A variety of data have been used to argue for some form of conditional excluded middle. Some of these data concern our reactions to unembedded counterfactuals—for example, how we treat them in deliberation (Stalnaker, 1981 [1972]; Gibbard & Harper, 1981 [1978]), or how we respond to them when posed as questions (Goodman, unpublished). Since in all such cases presumably there actually are the propositions expressed by these counterfactuals' antecedents and (more importantly) consequents, it is not clear that these cases support the modalized principle $CEM\varepsilon\pi$ as opposed to merely the unmodalized $CEM\varepsilon\varepsilon$.

On the other hand, some arguments for conditional excluded middle do seem to support the stronger modalized versions. This section considers two such arguments: one concerning certain inferences involving quantified counterfactuals, and another concerning generalizations about the chances of counterfactuals. In both cases we will argue that the data in question are robust with respect to whether or not we take the relevant quantification to be modalized. So, insofar as we ought to accept $CEM\varepsilon\varepsilon$ in order to accommodate the unmodalized data, it seems that we also ought to accept $CEM\varepsilon\pi$ in order to accommodate the modalized data. In fact, both of the arguments to be considered turn out to support even the following doubly modalized version of conditional excluded middle:

$(CEM\pi\pi)\ \Pi p \Pi q((p \boxarrow\!\!\rightarrow q) \vee (p \boxarrow\!\!\rightarrow \neg q))$.

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6.1. Quantified counterfactuals. Williams (2010) argues for conditional excluded middle for counterfactuals by appeal to the validity of certain inferences involving counterfactuals in the scope of restricted quantifiers, adapting examples from Higginbotham (1986) that Klinedinst (2011), Kratzer (forthcoming, sec. 5) and others have used to argue for conditional excluded middle in the case of indicative conditionals.[2] He notes that the following inference strikes us as valid:

(1) No student would have passed if they had goofed off.

(2) Therefore, every student would have failed to pass if they had goofed off.

(1) is plausibly at least materially equivalent to the more cumbersome (3):

(3) Every student is such that it is not the case that they would have passed if they had goofed off.

The inference from (3) to (2) strikes us as valid too. But this seems to require that, in the case of any student, if they aren't such that they would have passed had they goofed off, then they would have failed to pass had they goofed off: so, either, if they had goofed off, they would have passed, or, if they had goofed off, they would have failed to pass. In other words, the validity of this inference seems to require that conditional excluded middle hold for every instance of the quantified counterfactual in question. Given the generality of the phenomenon, this strongly suggests that conditional excluded middle is generally valid in some strong sense.

Using propositional quantifiers, we can make this informal argument more systematic as follows. If the inference from (3) to (2) is valid, then this should not depend on the particular properties of being a student, goofing off or passing. So, in an extension of our language with first-order quantifiers, the following sentence should be valid, where x is a first-order variable:

(4) $\forall x(Sx \rightarrow \neg(Gx \mathrel{\Box\!\!\rightarrow} Px)) \rightarrow \forall x(Sx \rightarrow (Gx \mathrel{\Box\!\!\rightarrow} \neg Px))$.

The validity of (4) is unlikely to be specific to the use of first-order quantifiers, and so an analogous principle should hold for propositional quantifiers. Further, the validity of (4) is unlikely to depend on the use of atomic predications or the use of a single quantifier. This leads us to the following schematic principle:

(5) $\forall \bar{p}(\varphi \rightarrow \neg(\psi \mathrel{\Box\!\!\rightarrow} \chi)) \rightarrow \forall \bar{p}(\varphi \rightarrow (\psi \mathrel{\Box\!\!\rightarrow} \neg\chi))$.

Modulo relabeling bound variables, the following is an instance of that schema:

(6) $\forall p\forall q(\neg(p \mathrel{\Box\!\!\rightarrow} q) \rightarrow \neg(p \mathrel{\Box\!\!\rightarrow} q)) \rightarrow \forall p\forall q(\neg(p \mathrel{\Box\!\!\rightarrow} q) \rightarrow (p \mathrel{\Box\!\!\rightarrow} \neg q))$.

The antecedent of this formula is clearly true, which leads to the following principle:

(7) $\forall p\forall q(\neg(p \mathrel{\Box\!\!\rightarrow} q) \rightarrow (p \mathrel{\Box\!\!\rightarrow} \neg q))$.

Modulo truth-functional equivalence within the scope of quantifiers, this is equivalent to $CEM\varepsilon\varepsilon$.

An analogous argument can be given for $CEM\pi\pi$. Consider the result of replacing quantificational phrases like 'all students' by their modalizations 'all possible students' in the inference from (1) to (2):

[2] Williams cites von Fintel and Iatridou (2002) in this connection, but this attribution is complicated by the role played by presupposition in their account. See Leslie (2008) for a dissenting voice in the case of indicatives and Huitink (2010) for a reply.

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(1π) No possible student would have passed if they had goofed off.

(2π) Therefore, every possible student would have failed to pass if they had goofed off.

The appearance of validity remains. We will assume that, if contingentism is true, then uses of 'all possible' typified by (2π) can be adequately formalized using the modalized quantifier Π defined previously; we defend this assumption in Fritz and Goodman (forthcoming). The apparent validity of the inference from (1π) to (2π) therefore motivates the following modalized analogue of (4):

(4π) $\Pi x(Sx \to \neg(Gx \mathrel{\Box\!\!\to} Px)) \to \Pi x(Sx \to (Gx \mathrel{\Box\!\!\to} \neg Px))$.

Now recall that on the propositional contingentist views we are concerned with, contingency in what things there are is suitably reflected in contingency in what propositions there are. So for many merely possible students, the propositions that they are a student, that they goof off and that they pass are merely possible as well. Speaking loosely, this means that there is a collection of triples of possible propositions p_i, q_i, and r_i such that the truth of $p_i \to \neg(q_i \mathrel{\Box\!\!\to} r_i)$, for all i, entails the truth of $p_i \to (q_i \mathrel{\Box\!\!\to} \neg r_i)$, for all i. As before, this fact doesn't seem to depend on these merely possible propositions predicating being a student, goofing off, or passing. Likewise, it would be surprising if it depended on the p_i, q_i, and r_i being possible propositions, rather than being expressed by formulas built up from proposition letters expressing possible propositions. This leads us to the following schematic principle using propositional quantifiers:

(5π) $\Pi\bar{p}(\varphi \to \neg(\psi \mathrel{\Box\!\!\to} \chi)) \to \Pi\bar{p}(\varphi \to (\psi \mathrel{\Box\!\!\to} \neg\chi))$.

Analogous to (6), we obtain (6π), whose antecedent is again clearly true, leading to (7π):

(6π) $\Pi p\Pi q(\neg(p \mathrel{\Box\!\!\to} q) \to \neg(p \mathrel{\Box\!\!\to} q)) \to \Pi p\Pi q(\neg(p \mathrel{\Box\!\!\to} q) \to (p \mathrel{\Box\!\!\to} \neg q))$,

(7π) $\Pi p\Pi q(\neg(p \mathrel{\Box\!\!\to} q) \to (p \mathrel{\Box\!\!\to} \neg q))$.

Finally, we note that modulo truth-functional equivalence within the scope of Π, this is equivalent to $CEM\pi\pi$.

6.2. Chances of counterfactuals. Another argument for conditional excluded middle concerns counterfactuals' chances; see Skyrms (1981 [1980]), Williams (2012), and Moss (2013). The argument is based on the observation that, in many cases, the chances of counterfactuals pattern with the conditional chances, at some salient time, of their consequents given their antecedents. For example, the chance that this fair coin would land heads if it were flipped is approximately .5, and the chance that it would not land heads is 1 minus that chance. So the corresponding instance of conditional excluded middle has chance 1, since the two counterfactuals are incompatible. More generally, the principle linking counterfactuals' chances and conditional chances implies, by the probability calculus, that whenever the relevant conditional chances are defined, the corresponding instances of conditional excluded middle will have chance 1, and hence (presumably) are true. Since chancy processes are often taken to pose the most pressing challenge to conditional excluded middle (see Lewis (1979)), an argument for the principle in such cases does much to strengthen the appeal of the principle in general, and hence strongly supports $CEM\varepsilon\varepsilon$.

We will now argue that this connection between counterfactuals' chances and conditional chances has nontrivial application even to counterfactuals concerning the very propositions that, according to the sort of propositional contingentism under consideration,

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pose a direct challenge to $CEM\pi\pi$. Recall our two merely possible fair coins x and y that, according to a propositional contingentist, are indiscernible from the perspective of actuality. This indiscernibility entails that neither possible coin has the property of being the one of them that would have landed heads had they both been flipped and exactly one of them landed heads: since clearly x and y cannot both be the one that would have landed heads had exactly one of them done so, it must be that neither of them is. So propositional contingentism seems to lead to a failure of $CEM\pi\pi$, since neither the (merely possible proposition) that x lands heads nor its negation can be counterfactually implied by the (merely possible) proposition that x and y are fair, each tossed, and exactly one lands heads. But the same considerations linking counterfactuals' chances to conditional chances seem to apply to counterfactuals embedded under modalized quantifiers, and hence suggest that the relevant instances of conditional excluded middle in fact have chance 1. For it seems that, for any such possible fair coins x and y, the chance that x lands heads conditional on both being fair and flipped and only one landing heads is .5. And this seems to go along with the (independently plausible) claim that, for any such x and y, the chance that, had both been flipped and only one landed heads, it would have been x, is .5. The corresponding instance of modally quantified conditional excluded middle then has chance 1. Since propositional contingentism seems to require it to be false, the principle linking chances and counterfactuals threatens to destabilize propositional contingentism.

In response to this argument, one might object that we shouldn't be in the business of forming pre-theoretical judgments about claims that embed counterfactuals under chance operators embedded under modalized quantifiers. We grant that such claims are rather involved, and so we should proceed with caution. We note, however, that the same pattern of judgments can be elicited without modalized quantification, by instead considering counterfactual circumstances involving the contingent nonexistence of actual coins. Consider two actual fair coins, Penny and Dimey, that are never flipped together. Now consider the counterfactual: had Penny and Dimey never existed, then it would be as likely as not that, if they had both existed and been fair and been flipped and only one landed heads, then Penny would have landed heads. This judgment strikes us as true, and it is enough for the present argument.

§7. Conclusion. Proposition 5.1 shows that, given plausible background assumptions, $CEM\varepsilon\pi$ is inconsistent with propositional contingentism. As we have argued in the previous section, some of the most important considerations in favor of some form of conditional excluded middle can be used to support $CEM\pi\pi$, and so the weaker $CEM\varepsilon\pi$. Together, these observations constitute an argument against propositional contingentism. (We are not making this argument ourselves, since we are not here endorsing the aforementioned considerations in favor of $CEM\varepsilon\varepsilon$.)

It is worth noting that, unlike other unwelcome consequences of conditional excluded middle, invoking indeterminacy in the manner of Stalnaker (1981, pp. 89–91) does nothing to resolve the tension with propositional contingentism. Stalnaker's idea is that, in cases where we balk at conditional excluded middle (such as those concerning the possible outcomes of merely possible coin flips), neither disjunct of the relevant instance of conditional excluded middle is (determinately) true, despite the fact that the disjunction is (determinately) true. Stalnaker advocates a supervaluationist theory of indeterminacy: there are many ways of resolving the indeterminacy of the counterfactual, and a statement is determinately true just in case it is true on each such resolution. So according to his view, each disjunct of one of the relevant instances of conditional excluded middle is true on some but not all resolutions; however, on all resolutions, at least one disjunct is true.

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Such an account of the indeterminacy of counterfactuals opens no room between $CEM\varepsilon\pi$ and U, the claim that there are all possible propositions. For on this account, the models used above are still adequate as models of counterfactuals given a specific resolution of their indeterminacy. Since principles like $CEM\varepsilon\pi$ and the other premises appealed to in the entailment of Proposition 5.1 are supposed to be determinately true, they are true on every resolution of the indeterminacy of counterfactuals. By the model-theoretic argument, U is therefore true on every such resolution, and—since it doesn't contain counterfactuals—true *simpliciter*. This argument could easily be formalized by replacing the single function C used to interpret counterfactuals by a set of such functions (possibly relative to the world of evaluation), adding such a function as a parameter of evaluation, and adding an object language 'determinately' operator varying this parameter. It would then be straightforward to show that the entailment of Proposition 5.1 still holds on this model theory when all premises and the conclusion are prefixed by the determinately operator. (We omit the details, since they are routine.)

The notion of consequence appealed to in Proposition 5.1 is model-theoretic, defined in terms of a class of models along familiar lines of possible world semantics. As noted above, it is plausible that this relation is truth-preserving. Of course, this assumption is nontrivial, and it would be interesting to develop proof systems in which the conclusion could be shown to be *derivable* from the premises, either concretely by specifying such a proof or abstractly by proving that there must be such derivation using a completeness result, but we will not explore such questions here. (The matter is not straightforward; e.g., results of Fritz (2017) entail that the set of formulas in the $\Box\!\!\rightarrow$-free fragment valid on the present class of models is not recursively axiomatizable.)

It is also of interest to investigate how much the premises appealed to in Proposition 5.1 must be weakened to establish consistency with propositional contingentism. We consider this question in an appendix.

§8. Appendix A: Consistency results. This appendix considers ways of weakening the premises of Proposition 5.1 to achieve consistency with propositional contingentism. In particular, the following three sections consider weakening Comp_C, $CEM\varepsilon\pi$, and $\forall\mathrm{Agg}\varepsilon\pi$. It will be shown that a number of natural ways of weakening one of these principles renders them compatible with propositional contingentism, sometimes even admitting strengthened versions of the other two principles.

8.1. Weakening comprehension. Comp_C strikes us as an extremely plausible principle. Yet, from a formal perspective, it is not hard to see that propositional contingentism is consistent with the other assumptions if Comp_C is restricted to formulas not containing the counterfactual conditional:

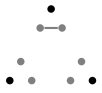
($\mathrm{Comp}_C[\Box\!\!\rightarrow]$) $\forall\bar{p}\exists q\Box(q \leftrightarrow \varphi(\bar{p}))$ (φ free of $\Box\!\!\rightarrow$).

To guarantee the validity of Comp_C, models will be required to be based on an equivalence system satisfying the coherence constraint alluded to in §1, which we can formulate as follows:

DEFINITION 8.1. *Let $\approx$ be an equivalence system on a set W. A permutation f of W is an* automorphism of $\approx$ *if for all $w, v, u \in W$, $v \approx_w u$ iff $f(v) \approx_{f(w)} f(u)$. For any $w \in W$, let* $\mathrm{aut}(\approx)_w$ *be the set of automorphisms of $\approx$ which map w to itself. Define $\approx$ to* cohere *(or to* be coherent*) if for all $w, v, u \in W$ such that $v \approx_w u$, there is an $f \in \mathrm{aut}(\approx)_w$ such that $f \subseteq \approx_w$ and $f(v) = u$.*

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Fritz (2016) describes a way of visualizing equivalence systems which will be useful here. Let $\approx$ be an equivalence system based on a finite set of worlds; for specificity, let this be the set of natural numbers from 1 to n. The worlds will be drawn in a big circle, with 1 at the top and in clockwise order. In this big circle, each world is drawn as a smaller circle of n dots, again with 1 represented by the element at the top and going in clockwise order. In the circle representing $\approx_w$, dots are connected by a path of lines just in case they are related by $\approx_w$. E.g., consider the equivalence system $\approx$ on $\{1, 2, 3\}$ in which $\approx_2$ and $\approx_3$ are the identity relation and $\approx_1$ relates 2 and 3 but neither of them to 1. This is drawn as follows:



Note that this equivalence system is coherent: the transposition mapping 2 and 3 to each other and 1 to itself is the required automorphism for $2 \approx_1 3$.

For the present consistency result, coherent equivalence systems will be expanded to models by adapting Stalnaker's selection function semantics, which associates each world with a function mapping each nonempty proposition P to a world (understood as the closest P-world):

DEFINITION 8.2. *Let W be a set, and f a function which maps every $w \in W$ to a function $f_w : \mathcal{P}(W)\backslash\{\emptyset\} \to W$. f is a* world-selection function *if for all $w \in W$ and nonempty $P, Q \in \mathcal{P}(W)$,*

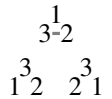
(*i*) $f_w(P) \in P$,
(*ii*) *if* $w \in P$ *then* $f_w(P) = w$, *and*
(*iii*) *if* $f_w(P) \in Q$ *and* $f_w(Q) \in P$ *then* $f_w(P) = f_w(Q)$.

Define a model $\langle W, \approx, C\rangle$ to be based on *such a function f if for all $P, Q \subseteq W$, and $w \in W$,*

$w \in C(P, Q)$ *iff* $P = \emptyset$ *or* $f_w(P) \in Q$.

Let S be the class of models based on coherent equivalence systems and world-selection functions.

As discussed in Lewis (1973, sec. 3.4), in certain cases, one can derive a selection function f from a closeness order, letting $f_w(P)$ be the closest P-world according to the order associated with w. In particular, for any finite equivalence system, any way of associating every world w with a total order on worlds starting with w determines a world-selection function. This representation of world-selection functions is useful since the above visualization of equivalence systems is naturally extended to such an assignment of total orders to worlds: for each world w, label each dot in the circle representing w with the position of its world in the sequence of worlds as ordered by the order associated with w. E.g., associate the following orders with 1, 2 and 3: $1 \leq_1 2 \leq_1 3$, $2 \leq_2 3 \leq_2 1$, and $3 \leq_3 2 \leq_3 1$; this is drawn as follows:



We can now prove that S validates the desired principles. In fact, $CEM\varepsilon\pi$ and $\forall\text{Agg}\varepsilon\pi$ can be strengthened to cover not only arbitrary propositions which are members of the

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domain of some world, but also arbitrary combinations of such propositions—note that, e.g., the conjunction of two propositions in the domains of some worlds need not itself be in the domain of any world. Further, $\forall \mathrm{Agg}\varepsilon\pi$ can be strengthened to allow for arbitrary parameters in φ. Using the tag 'σ' to signal the schematicity analogous to the earlier use of ε and π we now consider the following schematic principles:

$(CEM\sigma\sigma)$ $\Pi\bar{p}((\varphi \boxright \psi) \vee (\varphi \boxright \neg\psi))$,

$(\forall \mathrm{Agg}\sigma\sigma^*)$ $\downarrow r \Pi\bar{p}(\Pi\bar{q}(\varphi \rightarrow (\psi \boxright \chi)) \rightarrow (\psi \boxright \Pi\bar{q}(@r\varphi \rightarrow \chi)))$.

Here and in the following, we count as instances of such schematic principles only sentences, i.e., formulas without free variables.

PROPOSITION 8.3. *$CEM\sigma\sigma$, $\forall \mathrm{Agg}\sigma\sigma^*$, $B1$, and $B2$ are valid on S.*

Proof. Routine. □

By Proposition 5.1, it is clear that Comp_C is not valid on S, but it might also be instructive to establish this using a concrete counterexample:

PROPOSITION 8.4. *Comp_C is not valid on S.*

Proof. Let $M = \langle W, \approx, C\rangle$ be the model based on $\{1, 2, 3\}$ displayed above; it is easy to see that it is in S. Let a be an assignment function such that $a(p) = \{1\}$. Then $[\![\downarrow w \Box(p \rightarrow (\neg p \boxright w))]\!]_{M,a} = \{2\}$. Since $\{1\} \in D_1^{\approx}$ but $\{2\} \notin D_1^{\approx}$, the instance of Comp_C for this formula is not true in M. □

PROPOSITION 8.5. *$\mathrm{Comp}_C[\boxright]$ is valid on S.*

Proof. Analogous to the proof of Proposition 8.8. □

Since S contains nonempty models with varying propositional domains, we obtain:

COROLLARY 8.6. $\{CEM\sigma\sigma, \forall \mathrm{Agg}\sigma\sigma^*, \mathrm{Comp}_C[\boxright], B1, B2\} \not\models U$.

Furthermore, the discussion of various coherent equivalence systems in Fritz (2016) shows that there is a wide variety of countermodels to this entailment.

8.2. Weakening conditional excluded middle. We will now show that propositional contingentism can be upheld if $CEM\varepsilon\pi$ is replaced by $CEM\varepsilon\varepsilon$. To construct the relevant class of models, we first adapt the coherence constraint for equivalence systems to models.

DEFINITION 8.7. *Let $\langle W, \approx, C\rangle$ be a model. For any permutation f of W, let $\dot{f}$ be the permutation of $\mathcal{P}(W)$ mapping any $P \subseteq W$ to $\{f(w) : w \in P\}$. Let a permutation f of W be an* automorphism of C *if for all $P, Q \subseteq W$, $C(\dot{f}(P), \dot{f}(Q)) = \dot{f}(C(P, Q))$. For any $w \in W$, let $\mathrm{aut}(\approx, C)_w$ be the set of permutations which are automorphisms of $\approx$ and C and which map w to itself. Define $\langle W, \approx, C\rangle$ to* cohere *(or to* be coherent*) if for all $w, v, u \in W$ such that $v \approx_w u$, there is an $f \in \mathrm{aut}(\approx, C)_w$ such that $f \subseteq \approx_w$ and $f(v) = u$.*

PROPOSITION 8.8. *Comp_C is valid on the class of coherent models.*

Proof. Let $M = \langle W, \approx, C\rangle$ be a coherent model. For any permutation f which is an automorphism of both $\approx$ and C, an induction on the complexity of formulas φ establishes that for all assignment functions a,

$$[\![\varphi]\!]_{M,\dot{f}\circ a} = \dot{f}([\![\varphi]\!]_{M,a}).$$

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Consider any assignment function a with $\mathrm{im}(a) \subseteq D_w^{\approx}$ and $f \in \mathrm{aut}(\approx, C)_w$ such that $f \subseteq \approx_w$. Then $\dot{f} \circ a = a$. So for all φ, $\dot{f}([\![\varphi]\!]_{M,a}) = [\![\varphi]\!]_{M,a}$. Thus, if $v \approx_w u$, then there is a permutation f as required by coherence such that $f(v) = u$, and so for any such v, u, $v \in [\![\varphi]\!]_{M,a}$ iff $u \in [\![\varphi]\!]_{M,a}$. Hence $[\![\varphi]\!]_{M,a} \in D_w^{\approx}$, as required. □

In order to validate $CEM\varepsilon\varepsilon$, we again adapt Stalnaker's selection functions, although now we associate each world w with a selection function mapping each nonempty set of atomic propositions at w to an atomic proposition at w. We write $W/\approx_w$ for the set of equivalence classes of $\approx_w$.

DEFINITION 8.9. *Let $\approx$ be an equivalence system on a set W, and f a function which maps every $w \in W$ to a function $f_w : \mathcal{P}(W/\approx_w)\backslash\{\emptyset\} \to W/\approx_w$. f is an* atom-selection function *if for all $w \in W$ and nonempty $X, Y \in \mathcal{P}(W/\approx_w)$,*

(*i*) $f_w(X) \in X$,
(*ii*) *if* $w \in \bigcup X$ *then* $f_w(X) = [w]_{\approx_w}$, *and*
(*iii*) *if* $f_w(X) \in Y$ *and* $f_w(Y) \in X$ *then* $f_w(X) = f_w(Y)$.

Define a model $\langle W, \approx, C\rangle$ to be based on *such a function f if for all $P, Q \subseteq W$, and $w \in W$,*

$w \in C(P, Q)$ *iff* $P = \emptyset$ *or* $f_w(\{[v]_{\approx_w} : v \in P\}) \cap P \subseteq Q$.

Let A be the class of coherent models based on atom-selection functions.

Again, even the schematic strengthening of $\forall\mathrm{Agg}\varepsilon\pi$ is valid on this class of models:

PROPOSITION 8.10. Comp_C, $\forall\mathrm{Agg}\sigma\sigma^*$, *B*1, *and B*2 *are valid on A.*

Proof. Establishing the validity of $\forall\mathrm{Agg}\sigma\sigma^*$, *B*1, and *B*2 is routine; Comp_C follows from Proposition 8.8. □

Given Proposition 5.1, $CEM\varepsilon\pi$ cannot be valid on A. The following proof gives a concrete countermodel. As with world-selection functions on finite equivalence systems, atom-selection functions on finite equivalence systems can be specified using an assignment of total orders on atomic propositions with worlds. Again, this can be visualized by numbering dots, although now connected dots must be labeled by the same number.

PROPOSITION 8.11. $CEM\varepsilon\pi$ *is not valid on A.*

Proof. Consider the same equivalence system as in the proof of Proposition 8.4, but with the following ordering on atoms giving rise to an atom-selection function:

$$\begin{array}{ccc} & \overset{1}{2}\text{-}2 & \\ \overset{3}{1}2 & & 2\overset{3}{1} \end{array}$$

Let M be the depicted model. Consider an assignment function a such that $a(p) = \{2, 3\}$ and $a(q) = \{2\}$. Then $1 \notin [\![(p \mathrel{\Box\!\!\rightarrow} q) \vee (p \mathrel{\Box\!\!\rightarrow} \neg q)]\!]_{M,a}$. Since $a(p) \in D_1^{\approx}$ and $a(q) \in D_2^{\approx}$, $CEM\varepsilon\pi$ is not true in 1. □

The weaker principle $CEM\varepsilon\varepsilon$ is valid on S. Indeed, so is a strengthening of $CEM\varepsilon\varepsilon$ in which the consequents of the conditionals are specified using a formula not containing $\mathrel{\Box\!\!\rightarrow}$ or $\forall$, using existing parameters and the antecedent, which might itself be specified using any formula and possible parameters. To state this principle, let $\varphi[\psi/p]$ be the result of uniformly replacing p by ψ in φ. For any sequence of operators $\bar{o}$, define:

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(*CEMσ**[$\bar{o}$]) $\Pi\bar{p}\forall\bar{q}((\varphi \boxrightarrow \psi(\bar{q}, r)[\varphi/r]) \vee (\varphi \boxrightarrow \neg\psi(\bar{q}, r)[\varphi/r]))$,

(ψ free of the operators in $\bar{o}$).

PROPOSITION 8.12. *CEMσ*[∀, □→] is valid on A.*

Proof. Let $M = \langle W, \approx, C\rangle$ be a model in A based on an atom-selection function f, $w \in W$, and a an assignment function mapping each q_i to a member of $D_w^{\approx}$. Let $P = [\![\varphi]\!]_{M,a}$ and $Q = [\![\psi(\bar{q}, r)[\varphi/r]]\!]_{M,a}$. If P is empty, the relevant instance of *CEMσ**[∀, □→] is trivially true in w, so assume otherwise. Let $X = f_w(\{[v]_{\approx_w} : v \in P\})$. It suffices to prove that $X \cap P \subseteq Q$ or $X \cap P \subseteq W \setminus Q$. Note that for any formula ϑ, $[\![\Box\vartheta]\!]_{M,a} \in \{\emptyset, W\} \subseteq D_w^{\approx}$. So Q is a Boolean combination of elements of $D_w^{\approx}$ and P, from which the claim follows. □

Since A contains nonempty models with varying propositional domains, we obtain:

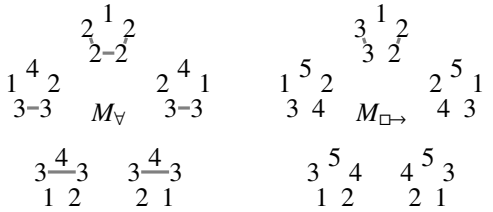
COROLLARY 8.13. {*CEMσ**[∀, □→], ∀Aggσσ*, Comp_C, *B*1, *B*2} ⊭ *U*.

As illustrated by the model constructed in the proof of Proposition 8.11, there also seems to be a nontrivial range of countermodels to this entailment.

Interestingly, both restrictions to *CEMσ**[∀, □→] in Proposition 8.12 are essential, as we now show:

PROPOSITION 8.14. *Neither CEMσ**[∀] *nor CEMσ**[□→] *is valid on A.*

Proof. Consider the following models based on atom-selection functions:



For the case of universal quantifiers, consider $M_\forall$ and let a be an assignment function such that $a(p) = \{2, 3, 4\}$ (note that at 2 and 5, there is the proposition $\{2, 3, 4\}$). Then $\forall q\Box(p \to \exists r\Box((p \wedge q) \leftrightarrow r))$ is true in 2 but not in 3 or 4, so the following formula is false in 1 under a:

$$(p \boxrightarrow \forall q\Box(p \to \exists r\Box(p \to (q \leftrightarrow r)))) \vee (p \boxrightarrow \neg\forall q\Box(p \to \exists r\Box(p \to (q \leftrightarrow r)))).$$

For the case of counterfactuals, consider $M_{\Box\to}$ and let b be an assignment function such that $b(p) = \{2, 3, 4\}$ and $b(q) = \{4, 5\}$ (note that in 1, there is the proposition $\{4, 5\}$). Then the following formula is false in 1 under b:

$$(p \boxrightarrow (q \boxrightarrow p)) \vee (p \boxrightarrow \neg(q \boxrightarrow p)).$$

As both $M_\forall$ and $M_{\Box\to}$ are coherent, the claim follows. □

Since *CEMσ**[] is not valid on A, one might wonder wether the argument against propositional contingentism can be reinstated using *CEMσ**[] instead of *CEMεπ*. The following highly restrictive model theory shows that this is not possible without additional assumptions:

DEFINITION 8.15. *For any set W, define the* minimal model on *W to be the model* ⟨*W*, ≈, *C*⟩ *such that*:

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$v \approx_w u$ *iff* $w = v = u$ *or* $w \notin \{v, u\}$, *and*

$$w \in C(P, Q) \text{ iff } \begin{cases} w \in Q & \text{if } w \in P, \\ P \subseteq Q & \text{otherwise.} \end{cases}$$

Let M be the class of minimal models.

PROPOSITION 8.16. *$B1$, $B2$, Comp_C, and $\forall \mathrm{Agg}\sigma\sigma^*$ are valid on M.*

Proof. $B1$, $B2$, and $\forall \mathrm{Agg}\sigma\sigma^*$ are routine; Comp_C follows from the fact that minimal models are coherent. □

By Proposition 5.1, $CEM\varepsilon\pi$ cannot be valid on M. It is also routine to show this using concretely using any minimal model based on a set with more than two elements.

PROPOSITION 8.17. *$CEM\sigma^*[]$ is valid on M.*

Proof. Let $M = \langle W, \approx, C\rangle$ be a minimal model, $w \in W$, and a an assignment function mapping each q_i to a member of $D^{\approx}_w$. Let $P = [\![\varphi]\!]_{M,a}$ and $Q = [\![\psi(\bar{q}, r)[\varphi/r]]\!]_{M,a}$. The claim is immediate if $w \in P$, so assume otherwise. For any $v, u \in P$, the transposition g switching v and u is an automorphism as required in the definition of coherence, so as in the proof of Proposition 8.8, $[\![\psi(\bar{q}, r)[\varphi/r]]\!]_{M,\dot{g}\circ a} = \dot{g}([\![\psi(\bar{q}, r)[\varphi/r]]\!]_{M,a})$, and as the relevant parameters are invariant under $\dot{g}$, it follows that $Q = \dot{g}(Q)$. Thus $P \subseteq Q$ or $P \subseteq W \setminus Q$ as required. □

Using minimal models on sets with more than two elements, which have varying propositional domains, we obtain a strengthening of Corollary 8.13:

COROLLARY 8.18. $\{CEM\sigma^*[], \forall \mathrm{Agg}\sigma\sigma^*, \mathrm{Comp}_C, B1, B2\} \not\models U$.

However, minimal models are so restrictive that they leave open the possibility of a *triviality* argument against propositional contingentism using $CEM\sigma^*[]$, or a principle in strength between $CEM\varepsilon\varepsilon$ and $CEM\sigma^*[]$. We leave a more detailed investigation of the diversity of models satisfying such principles for another occasion.

8.3. Weakening agglomeration. We show that Proposition 5.1 essentially relies on the infinitary version of agglomeration, by constructing a class of infinite models which validates both $CEM\varepsilon\pi$ and $\mathrm{Agg}\varepsilon\pi$ despite invalidating U. In fact, the class validates the following strengthening of these principles:

$(CEM\pi\sigma)$ $\Pi p \Pi \bar{q}((p \boxright \varphi) \vee (p \boxright \neg\varphi))$,
$(\mathrm{Agg}\sigma\sigma)$ $\Pi \bar{p}(((\varphi \boxright \psi_1) \wedge (\varphi \boxright \psi_2)) \rightarrow (\varphi \boxright (\psi_1 \wedge \psi_2)))$.

In the following definition, let a set $P \subseteq W$ be cofinite if $W \setminus P$ is finite.

DEFINITION 8.19. *For any infinite set W, define the* Fréchet model on W *to be the model $\langle W, \approx, C\rangle$ such that*:

$v \approx_w u$ *iff* $w = v = u$ *or* $w \notin \{v, u\}$, *and*

$$w \in C(P, Q) \text{ iff } \begin{cases} w \in P \text{ and } w \in Q, \text{ or} \\ w \notin P,\ P \text{ is finite and } P \subseteq Q, \text{ or} \\ w \notin P \text{ and } P \cap Q \text{ is cofinite.} \end{cases}$$

Let F be the class of Fréchet models.

PROPOSITION 8.20. *$B1$, $B2$, Comp_C, and $CEM\pi\sigma$ are valid on F.*

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Proof. $B1$ and $B2$ are routine; Comp_C follows from the fact that Fréchet models are coherent. For $CEM\pi\sigma$, we first show that for any formula $\psi(\bar{p})$, Fréchet model M and assignment function a whose range is included in $\bigcup_{w\in W} D^{\approx}_{w}$, $P = [\![\psi]\!]_{M,a}$ is finite or cofinite. Let $w_i \in W$ such that $a(p_i) \in D^{\approx}_{w_i}$ for all $i \leq n$. For any $v, u \in W\backslash\{w_i : i \leq n\}$, the transposition f switching v and u is an automorphism as required in the definition of coherence, so as in the proof of Proposition 8.8, $[\![\psi]\!]_{M,\dot{f}\circ a} = \dot{f}([\![\psi]\!]_{M,a})$. Continuing as in the proof of Proposition 8.16, all the relevant parameters are invariant under $\dot{f}$, so $\dot{f}(P) = P$. Thus P is finite or cofinite. The validity of $CEM\pi\sigma$ is now routine, using the fact just established for the third condition of C. □

By Proposition 5.1, $\forall\mathrm{Agg}\varepsilon\pi$ cannot be valid on F. It is also routine to show this using concretely using any Fréchet model.

PROPOSITION 8.21. *$\mathrm{Agg}\sigma\sigma$ is valid on F.*

Proof. Routine. □

All Fréchet models have varying propositional domains, so we obtain:

COROLLARY 8.22. $\{CEM\pi\sigma, \mathrm{Agg}\sigma\sigma, \mathrm{Comp}_C, B1, B2\} \nvDash U$.

As with minimal models, Fréchet models are so restrictive that they leave open the possibility of a triviality argument on the basis of $CEM\pi\sigma$ and $\mathrm{Agg}\sigma\sigma$. It would also be interesting to know whether a nontrivial class of models can be found which validates not only $CEM\pi\sigma$ but also $CEM\sigma\sigma$, along with $\mathrm{Agg}\sigma\sigma$ and Comp_C, but we will not pursue this question here.

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BIBLIOGRAPHY

Bacon, A. Conditional logics accommodating Stalnaker's thesis. Unpublished.

Fine, K. (1977a). *Postscript* to *Worlds, Times and Selves* (with A. N. Prior). London: Duckworth.

Fine, K. (1977b). Properties, propositions and sets. *Journal of Philosophical Logic*, **6**(1), 135–191.

Fritz, P. (2016). Propositional contingentism. *The Review of Symbolic Logic*, **9**(1), 123–142.

Fritz, P. (2017). Logics for propositional contingentism. *The Review of Symbolic Logic*, **10**(2), 203–236.

Fritz, P. (forthcoming). Higher-order contingentism, part 2: Patterns of indistinguishability. *Journal of Philosophical Logic*.

Fritz, P. & Goodman, J. (2016). Higher-order contingentism, part 1: Closure and generation. *Journal of Philosophical Logic*, **45**(6), 645–695.

Fritz, P. & Goodman, J. (forthcoming). Counting incompossibles. *Mind*.

https://doi.org/10.1017/S1755020317000144 Published online by Cambridge University Press

Gibbard, A. & Harper, W. (1981 [1978]). Counterfactuals and two kinds of expected utility. In Harper, W. L., Stalnaker, R., and Pearce, G., editors. *Ifs: Conditionals, Belief, Decision, Chance, and Time*. Dordrecht: D. Reidel, pp. 153–190.

Goodman, J. Consequences of conditional excluded middle. Unpublished.

Herzberger, H. G. (1979). Counterfactuals and consistency. *The Journal of Philosophy*, **76**(2), 83–88.

Higginbotham, J. (1986). Linguistic theory and Davidson's program in semantics. In Lepore, E., editor. *Truth and Interpretation: Perspectives on the Philosophy of Donald Davidson*. Oxford: Basil Blackwell, pp. 29–48.

Huitink, J. (2010). Quantified conditionals and compositionality. *Philosophy Compass*, **4**(1), 42–53.

Klinedinst, N. (2011). Quantified conditionals and conditional excluded middle. *Journal of Semantics*, **28**(1), 149–170.

Kratzer, A. (forthcoming). Chasing hook. In Walters, L. and Hawthorne, J., editors. *Conditionals, Probability, and Paradox: Themes from the Philosophy of Dorothy Edgington*. Oxford: Oxford University Press.

Leslie, S.-J. (2008). "If", "unless", and quantification. In Stainton, R. and Viger, C., editors. *Compositionality, Context, and Semantic Values*. Springer, Netherlands, pp. 3–30.

Lewis, D. (1973). *Counterfactuals*. Oxford: Basil Blackwell.

Lewis, D. (1979). Counterfactual dependence and time's arrow. *Noûs*, **13**(4), 455–476.

Moss, S. (2013). Subjunctive credence and semantic humility. *Philosophy and Phenomenological Research*, **87**(2), 251–278.

Skyrms, B. (1981 [1980]). The prior propensity account of subjunctive conditionals. In Harper, W. L., Stalnaker, R., and Pearce, G., editors. *Ifs: Conditionals, Belief, Decision, Chance, and Time*. Dordrecht: D. Reidel, pp. 259–265.

Stalnaker, R. (1968). A theory of conditionals. In Rescher, N., editor. *Studies in Logical Theory*. Oxford: Blackwell, pp. 98–112.

Stalnaker, R. (1981). A defense of conditional excluded middle. In Harper, W. L., Stalnaker, R., and Pearce, G., editors. *Ifs: Conditionals, Belief, Decision, Chance, and Time*. Dordrecht: D. Reidel, pp. 87–104.

Stalnaker, R. (1981 [1972]). Letter to David Lewis. In Harper, W. L., Stalnaker, R., and Pearce, G., editors. *Ifs: Conditionals, Belief, Decision, Chance, and Time*. Dordrecht: D. Reidel, pp. 151–152.

Stalnaker, R. (2012). *Mere Possibilities*. Princeton: Princeton University Press.

Swanson, E. (2008). Conditional excluded middle without the limit assumption. *Philosophy and Phenomenological Research*, **85**(2), 301–321.

von Fintel, K. & Iatridou, S. (2002). If and when 'if'-clauses can restrict (online PDF available at: web.mit.edu/fintel/fintel-iatridou-2002-ifwhen.pdf).

Williams, J. R. G. (2010). Defending conditional excluded middle. *Noûs*, **44**(4), 650–668.

Williams, J. R. G. (2012). Counterfactual triviality. *Philosophy and Phenomenological Research*, **85**(3), 648–670.

Williamson, T. (2013). *Modal Logic as Metaphysics*. Oxford: Oxford University Press.

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