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Registration of a hybrid robot using the Degradation-Kronecker method and a purely nonlinear method

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SUMMARY

Although the registration of a robot is crucial in order to identify its pose with respect to a tracking system, there is no reported solution to address this issue for a hybrid robot. Different from classical registration, the registration of a hybrid robot requires the need to solve an equation with three unknowns where two of these unknowns are coupled together. This property makes it difficult to obtain a closed-form solution. This paper is a first attempt to solve the registration of a hybrid robot. The Degradation-Kronecker (D-K) method is proposed as an optimal closed-form solution for the registration of a hybrid robot in this paper. Since closed-form methods generally suffer from limited accuracy, a purely nonlinear (PN) method is proposed to complement the D-K method. With simulation and experiment results, it has been found that both methods are robust. The PN method is more accurate but slower as compared to the D-K method. The fast computation property of the D-K method makes it appropriate to be applied in real-time circumstances, while the PN method is suitable to be applied where good accuracy is preferred.

KEYWORDS: Hybrid robot; Registration; Robot-world calibration; Hand-eye calibration; Degradation-Kronecker (D-K) method; Nonlinear error function.

Nomenclature

HEC	Hand-eye calibration
RWHEC	Robot-world and hand-eye calibration
SVD	Singular value decomposition
L-M	Levenberg–Marquardt
D-K	Degradation-Kronecker
PN	Purely nonlinear
X, Y, Z, X_t, Y_t	Unknown homogenous transformation matrix
A, B, C	Known homogenous transformation matrix
R	Rotation component of a transformation matrix
T	Translation component of a transformation matrix
q	A quaternion
$R(q)$	Conversion from a quaternion to a rotation matrix
n	Number of measurements

1. Introduction

The registration of an industrial robot is crucial whenever its interaction with objects has to be detected by a tracking system. If a tracking system is used to guide the movements of a robot, the pose of its end-effector with respect to its origin has to be connected with its pose with respect to a global coordinate

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frame. This connection can be achieved with a registration procedure. In the 1980s, this registration was simplified as a hand-eye calibration (HEC),[1,2] and solved using several approaches. Shiu and Ahmad[1] proposed a general closed-form solution where the transformation matrix was separated into its rotation and translation components. It was stated that a unique solution could be obtained with at least two sets of data.[1,2] In the work reported by Richter *et al.*,[3] a non-orthogonal method was used to obtain the calibration solution. This method was proven to be more accurate than the method proposed by Tsai and Lenz.[2] To guarantee the orthogonality of the rotation component, Andreff *et al.*[4] proposed an online calibration using a two-step method for the estimation of the rotation component; however, the method produced larger errors as compared with several other methods. Different from the previous methods, Chou and Kamel[5] used quaternions as equivalent forms of rotation and translation. Without using the least-square method to solve the over-determined equation system, a criterion was developed to choose three linear equations and one nonlinear equation to obtain the rotation and two linear equations to obtain the translation. Horaud and Dornaika[6] proposed a nonlinear technique based on the quaternion method to solve the rotation and translation simultaneously. Dual quaternion[7] is another approach to solve the calibration in order to obtain a simultaneous solution for rotation and translation. Chen[8] made the first attempt to address this registration with the screw theory. This approach was adopted by Zhao and Liu,[9] where the rotation and translation were solved simultaneously using the singular value decomposition (SVD) analysis. The HEC was merged with camera calibration in a few reported works.[10–12] The combined calibration approach can determine the hand-eye parameters and the camera intrinsic parameters. However, combined calibration becomes unnecessary when the camera has been calibrated using self-calibration techniques. All these mentioned approaches require the HEC data to have a known correspondence and they cannot solve the HEC if prior knowledge of the correspondence is not available. Ackerman and Chirikjian[13] proposed a probabilistic solution for the HEC without the requirement of the correspondence. Although their approach assumed that the relative motions between consecutive reference frames were small during the calibration, this assumption was relaxed by the authors in ref. [14] using a cost function of the Kullback–Leibler divergence.

For HEC, the camera should be rigidly attached on the robot's end-effector. Nevertheless, this attachment is not always necessary. For convenience, the camera can be fixed at a specific location in the environment. In this case, the registration has additional unknown parameters than the original HEC problem. The registration is also referred to as the robot-world and hand-eye calibration (RWHEC)[15–17] or the robot-world and tool-flange calibration.[18,19] Most of the solutions fall into two categories, namely, closed-form and iterative form. A linear closed-form solution was proposed[18] to obtain the unknowns of the RWHEC with the quaternion and the screw theory. The authors claimed that this method is fast and robust. In this method, each rigid transformation was separated into a rotation and a translation. This separation was adopted by Dornaika and Horaud[15] and Shah[17] in their closed-form methods. Shah[17] formulated a solution using the Kronecker product method. After comparing with the quaternion method,[15] the Kronecker product method was proven to be reliable and accurate. To avoid accumulative errors caused by the separation, Li *et al.*[16] combined the rotation and translation together and solved them using the Kronecker product method. However, the Kronecker product method fails to provide orthogonal rotation matrices. Ernst *et al.*[19] used a similar method to solve the same equation, where the same problem has occurred. In general, closed-form methods are fast and robust although they have limited accuracy. Different from closed-form methods, iterative methods usually have better performance but require longer computation time. In the work reported by Dornaika and Horaud,[15] the rotation and translation residuals at each configuration were combined into an error function. With simulation and experimental results, it was found that the proposed nonlinear methods have better accuracy. Without a minimization function, an iterative estimation method[20] was used to solve the rotation and translation unknowns. This method was stated to be robust against noise and convergent within a reasonable tolerance. However, the authors did not compare their method with other different methods. In the work reported by Strobl and Hirzinger,[21] an optimal model was built based on the minimization of the sum of prediction errors with normal distribution. After a comparison with several other methods, it was found that the proposed method presented superior performance. This method assumed that the errors and noise in the registration complied with normal distribution. In order to coordinate multiple robots used in a task, an iterative method was adopted by Wang *et al.*[22] to address the calibration for cooperative robots. This method solved simultaneously the RWHEC and robot-robot calibration.

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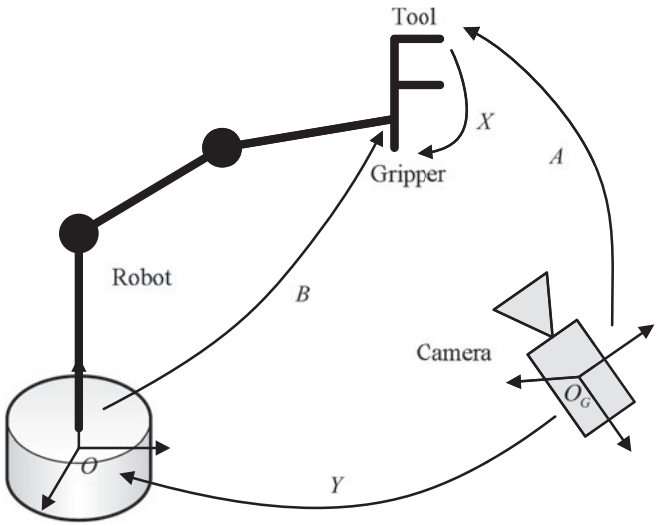


Fig. 1. RWHEC setup of a serial robot.

All the closed-form solutions mentioned above are proposed for specific registration equations; if the equations are modified, their applicability is lost. The various solutions discussed above are not proposed for HEC or RWHEC of a hybrid robot, which consists of a serial robot and a parallel manipulator. A serial robot has greater flexibility and a larger workspace but low payload-weight ratio and accumulative inaccuracy, while a parallel manipulator has limited workspace but high payload-weight ratio and high accuracy. Thus, a hybrid robot could combine the advantages of these robots and complement each other. Different hybrid robots have been proposed for surgical tasks,[23–25] and they were proven to have good performance. However, these hybrid robots have not been used clinically and the registration methods were not reported. Different from the classical registration, the registration of a hybrid robot has three unknowns to be determined, i.e., two additional unknowns as compared to the HEC problem, and one additional unknown as compared to the RWHEC problem. In addition, the registration equation of a hybrid robot has two unknowns coupled together, which makes it difficult to be solved using the methods discussed. To determine these unknowns, this paper proposes the D-K method, which provides a closed-form solution for the registration of a hybrid robot. Since closed-form methods suffer from low accuracy under perturbance, a PN method, which uses an iterative algorithm, is proposed in this paper to overcome this problem. The proposed methods are compared to analyze their performance with respect to computation time, accuracy and robustness to noise. This is to the best of the authors' knowledge the first attempt to use these methods for the registration of a hybrid robot.

This paper is organized as follows. Section 2 introduces the equations for the HEC, RWHEC, and registration of a hybrid robot. The proposed D-K method and PN method are described in Section 3 and Section 4 respectively. Simulation and experimental results are presented with the corresponding discussions in Section 5 and Section 6. Finally, conclusions are summarized in Section 7.

2. Registration Formulation of a Robot

For the HEC, the registration is formulated to solve one equation given in Eq. (1), where X is the transformation matrix of the relative pose of a camera with respect to the gripper of a robot. The camera is mounted rigidly with respect to the gripper.

$$AX = XB. \tag{1}$$

Different from HEC, the RWHEC can be formulated as in Eq. (2), where X and Y are two unknown transformations. For the experimental setup in Fig. 1, X denotes the transformation from the tool to the gripper of the robot, and Y is a transformation from a global coordinate frame to the base coordinate frame of the robot. These two transformations are constant after grasping the tool with the

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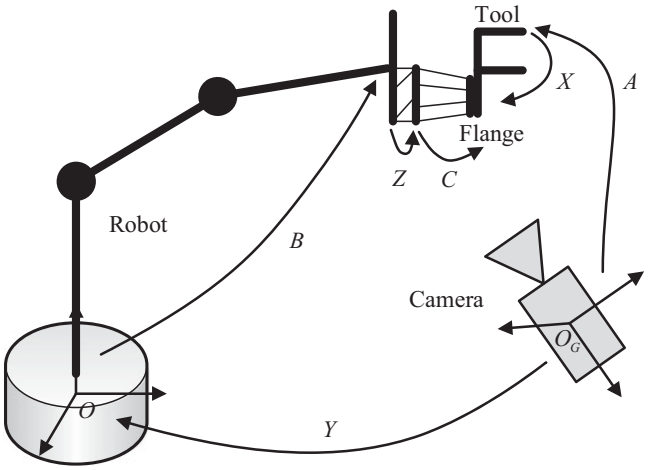


Fig. 2. Registration setup of a hybrid robot.

gripper of the robot and fixing the position of the camera.

$$AX = YB. \tag{2}$$

Both of the HEC and RWHEC can be considered as a classical registration problem. For the registration of a hybrid robot, it can be represented by an equation with three unknowns, where Z is an unknown constant that denotes the transformation from the flange of the serial robot to the base of the parallel manipulator, and C is the transformation from the base of the parallel manipulator to its flange as shown in Fig. 2.

$$AX = YBZC. \tag{3}$$

As shown in Eq. (3), there are two unknowns coupled together. This coupling increases the difficulties of solving the equation. The D-K method is proposed to obtain a closed-form solution.

3. The D-K Method

The D-K method separates the hybrid robot into two components, namely, the serial and parallel components. This separation allows the hybrid robot to be registered with respect to a tracking system using three steps. The serial component and the parallel component are determined individually in the first two steps, and the results are used to complete the registration of the hybrid robot in the last step. Figure 3 presents the flowchart of the D-K method.

First, the parallel manipulator is locked for the registration of the serial component. The transformation from the tool to the flange of the serial robot described in Fig. 2, and is denoted by X_t, $X_t = XC^{-1}Z^{-1}$. Since the parallel manipulator is locked, X_t is a constant due to the constant C. Hence, Eq. (3) is degraded to an equation that is similar to Eq. (2),

$$AX_t = YB. \tag{4}$$

In the second step, the serial robot is locked to allow the registration of the parallel component. The transformation from the global coordinate to the base of the parallel manipulator is denoted by Y_t, $Y_t = YBZ$. Since Y_t is a constant, similar to the first step, Eq. (3) is degraded to

$$AX = Y_tC. \tag{5}$$

As introduced in Section 1, many solutions have been proposed for Eqs. (4) and (5). Since the Kronecker product method is claimed to be fast and accurate, this method is used to obtain X_t,

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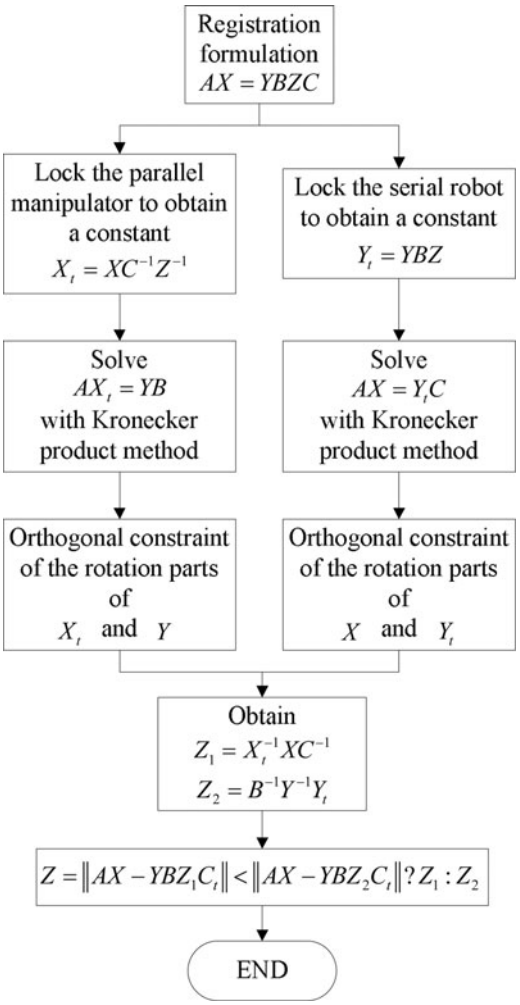


Fig. 3. Flowchart of the D-K method.

Y_t, X and Y. For arbitrary matrix A and matrix B, their Kronecker product is denoted as $A \otimes B$. It should be noted that $A \otimes B$ and $B \otimes A$ are usually different. A homogenous transformation matrix A

$$A = \begin{bmatrix} R_A & t_A \\ 0 & 1 \end{bmatrix},$$

has rotation and translation components,

$$R_A = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}, \quad t_A = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T,$$

with the representation $\mathrm{Vec}(R) = \begin{bmatrix} r_1 & r_2 & \dots & r_9 \end{bmatrix}^T$.

With the Kronecker product method, Eq. (2) can be reformulated to be a linear equation,[16]

$$\begin{bmatrix} R_A \otimes I_3 & -I_3 \otimes R_B^T & 0 & 0 \\ 0 & I_3 \otimes t_B^T & -R_A & I_3 \end{bmatrix} \begin{bmatrix} \mathrm{Vec}(R_X) \\ \mathrm{Vec}(R_Y) \\ t_X \\ t_Y \end{bmatrix} = \begin{bmatrix} 0 \\ t_A \end{bmatrix}. \tag{6}$$

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Using several measurements, this equation can be used to form an equation system to obtain a unique solution. Generally, R_X and R_Y that have been determined are not orthogonal. An orthogonal constraint is necessary to determine appropriate rotation matrices. The SVD approach is used to obtain the closest orthogonal matrices of R_X and R_Y. After obtaining X_t, Y, X and Y_t, $Z_1 = X_t^{-1} X C^{-1}$ and $Z_2 = B^{-1} Y^{-1} Y_t$ can be obtained in the third step. In theory, $Z = Z_1 = Z_2$. However, noise always causes $Z_1 \neq Z_2$ in practice. Therefore, the unknown Z is assigned the value of Z_1 or Z_2, whichever has the smaller registration residuals.

4. The PN Method

Different from the D-K method, the PN method solves all the unknowns, X, Y and Zsimultaneously. Since each rigid transformation is composed of one translation and one rotation, Eq. (3) can be decomposed into Eq. (7).

$$\begin{cases} R = R_A R_X - R_Y R_B R_Z R_C \\ T = R_A T_X - R_Y T_B - R_Y R_B T_Z - R_Y R_B R_Z T_C + T_A - T_Y \end{cases}. \tag{7}$$

Correspondingly, the error function can be defined as

$$F = \begin{bmatrix} \mathrm{Vec}(R) \\ T \end{bmatrix}, \tag{8}$$

where F is a row vector with 12 elements.

For an arbitrary rotation matrix R, it can be represented by a normalized quaternion $q = \begin{bmatrix} s & i & j & k \end{bmatrix}$. If the normalization of the quaternion is defined as $q = q/\|q\|$, and $q = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$ if $s = 0$, there is only one normalized quaternion for an arbitrary quaternion. Thus, for a given quaternion, its corresponding rotation matrix can always be found. If the conversion from the quaternion to the rotation matrix is denoted by $R(q)$, Eq. (9) can be obtained from Eq. (7). With the quaternion representations, the rotation components of X, Y and Z are always orthogonal in the iterative computation process.

$$\begin{cases} R = R_A R(q_X) - R(q_Y) R_B R(q_Z) R_C \\ T = R_A T_X - R(q_Y) T_B - R(q_Y) R_B T_Z - R(q_Y) R_B R(q_Z) T_C + T_A - T_Y \end{cases}. \tag{9}$$

Assuming that the number of measurements is n, each measurement produces a F_i with Eqs. (9) and (8). The PN method aims to find optimal solutions of q_X, q_Y, q_Z, T_X, T_Y, and T_Z to minimize Eq. (10)

$$\sum_{i=1}^{n} \sum_{j=1}^{12} f_j^2, \tag{10}$$

where f_j denotes the ith entry of the error vector F.

The registration has been transformed into a least-square problem. The Levenberg–Marquardt (L-M) algorithm can be used to obtain the optimal solutions. With the conversion $R(q)$, the final solutions of X, Y and Z can be obtained from q_X, q_Y, q_Z, T_X, T_Y, and T_Z.

5. Simulations

The D-K method and PN method are tested and compared with numerical simulations. Assuming X, Y and Z are known, the value of A can be obtained using Eq. (3) for each joint configuration of the serial robot and the parallel manipulator. Shah[17] proved that at least three different poses are necessary to obtain a unique solution to Eq. (2). Since the D-K method degrades the registration of a hybrid robot into two equations which are the same as Eq. (2), the number of poses to be measured should be $n \geq 6$. It has been found that rotation and translation errors tend to decrease with more pose measurements,[16] hence a set of data with $n = 20$ are prepared in this simulation with $n_1 = 10$

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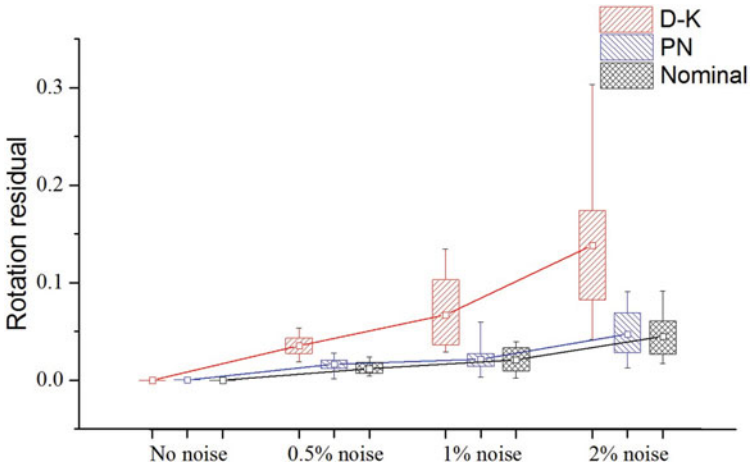


Fig. 4. Rotation residuals obtained from the D-K method and the PN method.

for the serial component and $n_2 = 10$ for the parallel component. This set of data is not perturbed by noise. Nevertheless, noise is common in practice. It is necessary to have several sets of data with noise included to investigate the performance of the proposed methods against perturbance. Hence, different magnitudes of noise are added to the nominal values. Noise can have normal distribution and its standard deviations are defined to be 0.5%, 1% and 2% of their nominal values. Since each pose contains a rotation component and a translation component, the noise generation is separated into two parts. For the rotation component, the rotation matrix is first converted to an angle-axis representation, and the noise is added to the angle and every axial value of the axis. For the translation component, the noise is added to every axial value.

To ensure that the final result is globally optimal, the methods are implemented several times with different initial start points until the variation of the norm of the residuals is within a specified tolerance. The initial starting points are generated randomly with their translation components in a range $\begin{bmatrix} -1000 & 1000 \end{bmatrix}$, and the tolerance is defined to be smaller than 1.0×10^{-6} in this paper. For visual representation, the rotation residual is defined to be $\|R\|$, and $\|T\|$ denotes the translation residual.

Figure 4 shows the rotation residuals of the nominal values and the optimal solutions obtained using the D-K method and PN method under different noise magnitudes. The box and whiskers in Fig. 4 depict the distributions of the residuals of all the 20 simulated measurements. It can be seen that the mean values of the rotation residuals increase with increase in noise. Higher noise level also increases the intervals between the minima and the maxima of the residuals. There are larger residuals when more noise is included in the measurements. This phenomenon can be observed in Fig. 5, which illustrates the translation residuals under different noise levels. Figures 4 and 5 show that the PN method can obtain smaller mean value and maximum of the residuals than the D-K method, and hence the PN method is more accurate than the D-K method. The worse accuracy of the D-K method is possibly attributed to error propagation in the degradation process and the orthogonal regulation in the Kronecker product method. Additionally, it should be noted that the rotation residuals of the PN method are comparable to that of the nominal values, while the translation residuals of the PN method are smaller than that of the nominal values. Although the D-K method cannot decrease the residuals compared to the nominal values, the difference is not significant. Therefore, it can be stated that both methods can succeed in solving the registration problem under different noise magnitudes. These two methods are robust to perturbances.

To compare the computation time of these two methods, they are implemented 100 times under each noise level using MatLab installed on a Dell Optiplex 980 desktop incorporating one 3.2GHz Intel Core i5-650 processor and two 2GB DDR3 SDRAM memory cards. The operating system is 32-bit Windows 7 Enterprise. The average computation time is listed in Table I. This table shows that the computation time of the D-K method is not affected by noise since it is a closed-form approach, while noise increases the average computation time of the PN method since it is an iterative process.

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Table I. Average computation time of D-K and PN methods under different noise levels.

	No noise (ms)	0.5% noise (ms)	1% noise (ms)	2% noise (ms)
D-K method	629.6	720.0	668.2	698.5
PN method	409.0	5404.9	15112.2	24880.6

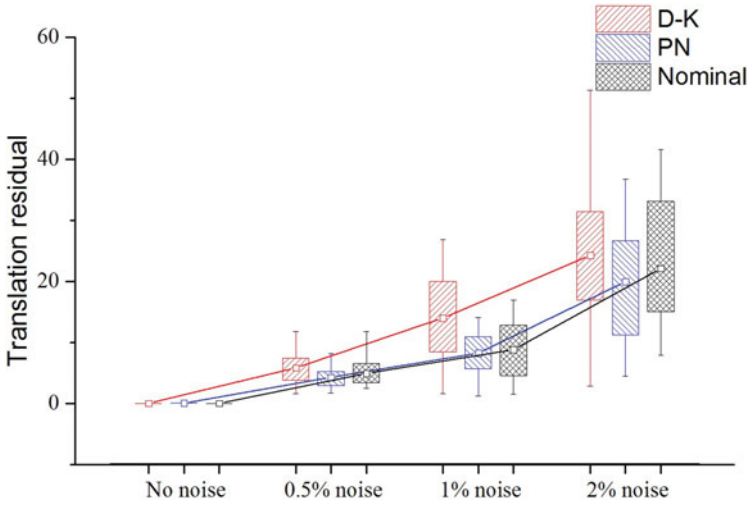


Fig. 5. Translation residuals obtained from the D-K method and the PN method.

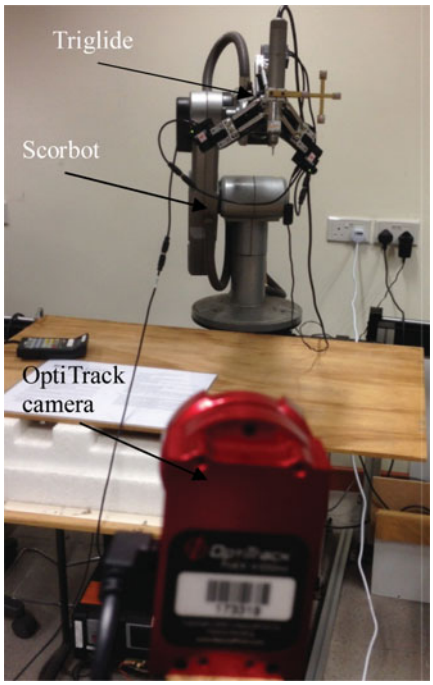


Fig. 6. Experimental setup of a hybrid robot.

It is noted that shorter computation time can be obtained with the PN method due to the one step process of the PN method if there is no noise. However, with the presence of noise, the PN method requires longer time than the D-K method.

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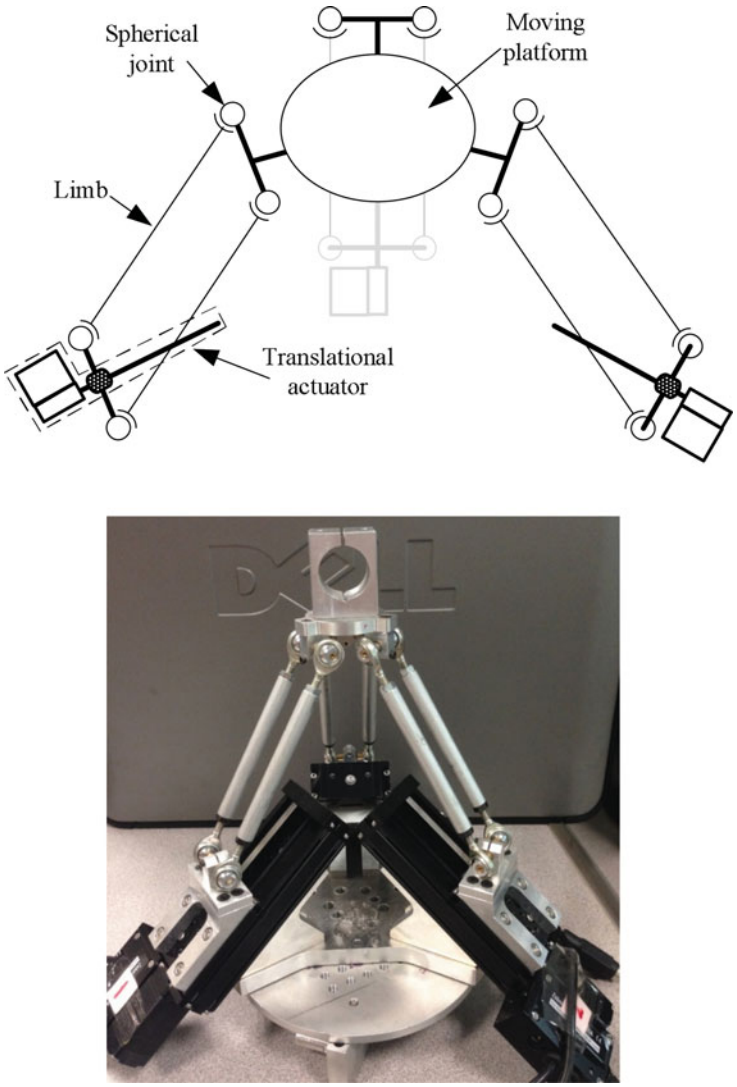


Fig. 7. The structure of the triglide. (a) The mechanism of the triglide. (b) The triglide.

6. Experiments

In this section, the D-K method and PN method are compared using real data obtained from a hybrid robot, which comprises a serial robot Scorbot-ER VII (Eshed Robotec Inc.) and a self-constructed 3-DOF purely translational parallel manipulator triglide, as shown in Fig. 6. Figure 7 depicts the structure of the triglide. The tool pose of the hybrid robot is captured using an OptiTrack system (Natural point Inc.) with three cameras. The computer and programming environment used for the implementation of the proposed methods are the same as that described in Section 5.

In this experiment, 40 measurements are obtained by adjusting the joint configurations of the serial robot and the parallel manipulator. As each step of the first two steps of the D-K method requires the number of poses measured to be $n \geq 3$, and the serial robot has a much larger workspace than the triglide, 30 poses are obtained when the parallel manipulator is locked, and the remaining 10 measurements are obtained when the serial robot is locked. Similar to the simulation, both methods are implemented several times with different initial starting points to obtain the final results. Figure 8 shows the rotation residuals of the final result, and the translation residuals are depicted in Fig. 9. Their average values are listed in Table II with the average computation times of these two methods. The average time is obtained by performing each method 100 times using the same set of real data.

As depicted in Fig. 8, the rotation residuals obtained using the PN method overlaps significantly with the rotation residuals obtained using the D-K method. Table II shows that the average rotation

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Table II. Average residuals and average computation time obtained with D-K and PN method.

	Average rotation residual	Average translation residual/mm	Average time/ms
D-K method	0.0113	4.8609	1322.9
PN method	0.0109	1.1004	1873.6

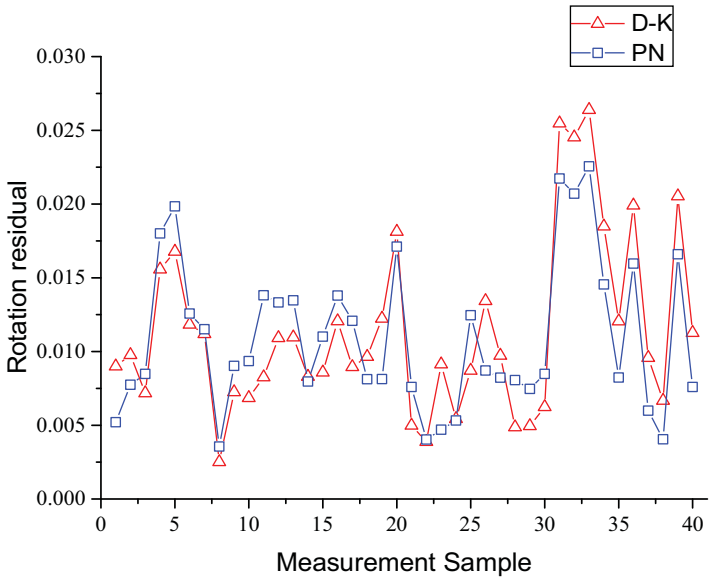


Fig. 8. Rotation residuals obtained with the D-K method and the PN method.

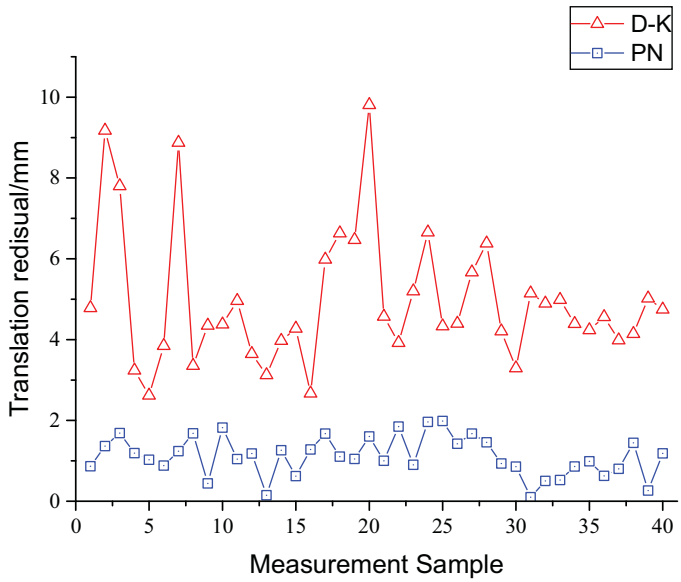


Fig. 9. Translation residuals obtained with the D-K method and the PN method.

residual of the PN method is slightly smaller than that of the D-K method, while the difference of the translation residuals between them is significant, as shown in Fig. 9. Figure 9 illustrates that all the translation residuals of the PN method are located in the interval of 0 to 2mm. The average residual is 1.1004 as listed in Table II. In contrast, the D-K method produces the translation residuals which maximum is about 10mm. Although the difference between rotation residuals is very small, the smaller translation residuals show that the PN method is more accurate than the D-K method.

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Table II shows that the computation time of the PN method is longer than that of the D-K method. This finding is consistent with the simulation result.

7. Conclusion

This paper proposes two different methods for the registration of a hybrid robot. Various solutions have been described by many researchers for HEC and RWHEC. To the best of the authors' knowledge, this paper is a first attempt to propose methods to address this issue for a hybrid robot. This issue can be solved using the D-K method, which is a closed-form solution, with three steps. Besides the D-K method, the PN method solves the problem using a nonlinear iterative algorithm.

Simulation results show that both methods are capable of obtaining globally optimal solutions. The methods are robust to noise. With the simulation results, it is found that the PN method is more accurate than the D-K method due to the degradation in the D-K method, which propagates computation errors to its final result, and the orthogonal constraint, which uses a closest orthogonal matrix to replace non-orthogonal results obtained. In practice, the D-K method requires shorter computation time and is not affected by noise, which can be contributed to its closed-form. Different from the D-K method, the better accuracy and longer computation time of the PN method have been validated with an experiment.

In conclusion, the proposed methods can obtain optimal solutions of the registration of a hybrid robot. The D-K method can be used to present an approximate solution under the requirement of shorter computation time or narrow the search area for the PN method, and the PN method is suitable for refining the solution.

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