J. Plasma Physics (2000), *vol.* 64, *part* 2, *pp.* 125–130. Printed in the United Kingdom
© 2000 Cambridge University Press

Generalized dispersive Alfvén waves

P. K. SHUKLA[1] and L. STENFLO[2]

[1] Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(ps@tp4.ruhr-uni-bochum.de)

[2] Department of Plasma Physics, UmeåUniversity, S-90187 Umeå, Sweden
(lennart.stenflo@physics.umu.se)

(Received 10 December 1999)

Abstract. Generalized dispersion relations for Alfvén waves are derived, accounting for arbitrary values of $\omega/\omega_{ci}, k_\perp\rho_i$, and $k_\perp\lambda_e$, where $\omega\,(\omega_{ci})$ is the wave (ion gyro) frequency, $k_\perp = 2\pi/\lambda_\perp$ the perpendicular (to the external magnetic field line) wavenumber, and $\rho_i(\lambda_e)$ the ion gyroradius (electron skin depth). Our dispersion relations are appropriate for deducing the relationships of the dispersive (inertial and kinetic) Alfvén waves with other plasma modes. The present results can be considered as a prerequisite for understanding some salient features of the broadband electromagnetic/electrostatic waves that are frequently observed by the Freja and FAST spacecraft in the auroral zone of the Earth's ionosphere.

1. Introduction

Observations from various satellites (namely Viking, Freja, and FAST) reveal clear signatures of broadband low-frequency (in comparison with the ion gyrofrequency ω_{ci}) electromagnetic and electrostatic waves (Berthelier et al. 1991; Louran et al. 1994; Wahlund et al. 1998; Chaston et al. 1999) in the Earth's auroral zone as well as in the polar magnetosphere. The observed fluctuation spectra are typically in the frequency domain 1–500 Hz, and cover a wide range of perpendicular spatial scales $\lambda_\perp$ ($\approx$ 30–7000 m). The ratio between the wave magnetic and electric field components determines whether the waves are electrostatic or electromagnetic. The short-scale (in comparison with $\rho_i = v_{ti}/\omega_{ci}$ and $\lambda_e = c/\omega_{pe}$, where v_{ti} is the ion thermal velocity, c the speed of light, and ω_{pe} the electron plasma frequency) waves are necessarily electrostatic, since the associated $\delta E/\delta B$ fluctuations are quite large for those short-wavelength modes. Recently, however, Stasiewicz et al. (2000) reanalysed the Freja data and confirmed the presence of broadband fluctuation spectra comprising inertial Alfvén (Hasegawa and Uberoi 1982; Goertz 1985; Shukla and Stenflo 1995; Lysak and Lotko, 1996) and electron-acoustic (Mikhailovskii 1974) waves. It was noted that at shorter wavelengths (in comparison with ρ_i and λ_e), the waves are primarily electrostatic. Accordingly, it is of practical interest to examine the wave spectra that span from long to short wavelengths.

In this brief report, we present the spectra of dispersive Alfvén waves (DAWs), including arbitrary values of ω/ω_{ci}, $\rho_i/\lambda_\perp$, and $\lambda_e/\lambda_\perp$, and discuss the mathematical details and the physics of other plasma modes that are related to the DAWs. Thus, the present investigation can contribute to a better understanding of long-

https://doi.org/10.1017/S0022377800008412 Published online by Cambridge University Press

and short-scale dispersive Alfvén waves in magnetoplasmas, such as those in the Earth's auroral zone as well as in the Large Alfvén Plasma Device (LAPD) (see e.g. Gekelman 1999) at the University of California at Los Angeles.

2. The wave spectra

Let us consider a uniform ionospheric plasma containing a homogeneous magnetic field $\hat{\mathbf{z}}B_0$, where B_0 is the geomagnetic field strength and $\hat{\mathbf{z}}$ the unit vector along the z direction. In the presence of low-frequency (in comparison with the electron gyrofrequency $\omega_{ce} = eB_0/m_e c$, where e is the magnitude of the electron charge and m_e the electron mass), long-wavelength (in comparison with the electron gyroradius $\rho_e = v_{te}/\omega_{ce}$, where v_{te} is the electron thermal velocity) electric $\mathbf{E} = -\boldsymbol{\nabla}\phi - (1/c)\hat{\mathbf{z}}\partial_t A_z$ (where ϕ is the electrostatic potential and A_z the parallel (to $\hat{\mathbf{z}}$) component of the vector potential) and magnetic $\mathbf{B}_\perp = \boldsymbol{\nabla}A_z \times \hat{\mathbf{z}}$ fields, the perpendicular (to $\hat{\mathbf{z}}$) component of the electron fluid velocity is

$$\mathbf{v}_{e\perp} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi - \frac{cT_e}{eB_0 n_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp n_{e1} + \frac{c}{B_0\omega_{ce}}\partial_t \boldsymbol{\nabla}_\perp \phi, \tag{1}$$

where T_e (T_i) is the electron (ion) temperature, and n_{e1} ($= n_e - n_0 \ll n_0$) is a small electron number density perturbation of the equilibrium plasma number density n_0.

The parallel component of the electron fluid velocity v_{ez} is determined from Ampère's law

$$v_{ez} \approx \frac{c}{4\pi n_e e}\nabla_\perp^2 A_z, \tag{2}$$

and the parallel component of the electron momentum equation

$$\partial_t v_{ez} = \frac{e}{m_e}\partial_z \phi + \frac{e}{m_e c}\partial_t A_z - \frac{T_e}{m_e n_0}\partial_z n_{e1}, \tag{3}$$

where the parallel ion motion has been neglected, because we are isolating the ion-sound branch.

Substituting for the electron fluid velocities from (1) and (2) into the electron continuity equation and (3), we obtain respectively

$$\partial_t n_{e1} + \frac{cn_0}{B_0\omega_{ce}}\partial_t \nabla_\perp^2 \phi + \frac{c}{4\pi e}\partial_z \nabla_\perp^2 A_z = 0 \tag{4}$$

and

$$\left(1 - \lambda_e^2 \nabla_\perp^2\right)\partial_t A_z + c\partial_z\left(\phi - \frac{T_e}{en_0}n_{e1}\right) = 0, \tag{5}$$

where $\lambda_e = c/\omega_{pe}$ and $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency. We recall that for $|\partial_t| \ll v_{te}|\partial_z|$, the inertial term $\lambda_e^2\nabla_\perp^2$ on the left-hand side of (5) is negligible (Stéfant 1971; Hasegawa and Chen 1976), whereas for $|\partial_t| \gg v_{te}|\partial_z|$, the pressure-gradient term $-T_e\partial_z n_{e1}/en_0$ on the right-hand side of (5) can be dropped (Goertz 1985; Petviashvili and Pokhotelov 1992; Lysak and Lotko 1996; Leneman et al. 1999; Gekelman 1999).

Combining (4) and (5) and Fourier-transforming the resulting equation by supposing that A_z, ϕ, and n_{e1} are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$, where $\mathbf{k} = \mathbf{k}_\perp + \hat{\mathbf{z}}k_z$ is the wave vector and ω the wave frequency, we obtain for the electron number density perturbation associated with the inertial Alfvén wave (IAW), for which

https://doi.org/10.1017/S0022377800008412 Published online by Cambridge University Press

$|\partial_t| \gg v_{te}|\partial_z|$,

$$n_{e1} = -\frac{k_z^2 c^2 k_\perp^2}{4\pi e(1+b_e)\omega^2}\phi, \tag{6}$$

where $b_e = k_\perp^2 \lambda_e^2$. For simplicity, the contribution of the electron polarization drift has been ignored here. On the other hand, for the kinetic Alfvén wave (KAW) with $|\partial_t| \ll v_{te}|\partial_z|$, we have

$$n_{e1} = -\frac{k_z^2 c^2 k_\perp^2}{4\pi e(\omega^2 - k_z^2 v_A^2 b_s)}\phi, \tag{7}$$

where $b_s = k_\perp^2 \rho_s^2$, $\rho_s = c_s/\omega_{ci}$, and $c_s = (T_e/m_i)^{1/2}$ is the ion-sound velocity.

The parallel electric fields associated with the IAW and the KAW are respectively

$$E_z = i\left[\frac{m_e\omega^2(1+b_e)}{b_e e k_z n_0}\right] n_{e1}, \tag{8}$$

and, if also $\omega/k_z \ll b_e v_{te}$,

$$E_z = -i\frac{T_e k_z}{e n_0} n_{e1}. \tag{9}$$

On the other hand, the wave magnetic field is determined from

$$\mathbf{B}_\perp = \frac{c}{\omega}\,\mathbf{k} \times \mathbf{E}. \tag{10}$$

Next, we present the expression for the ion number density perturbation n_{i1}, which is deduced from the linearized Vlasov equation (Rao and Kaup 1992). We have

$$n_{i1} = -\frac{k^2}{4\pi e}\chi_i \phi, \tag{11}$$

where the ion susceptibility is (Brambilla 1998)

$$\chi_i \approx \frac{1}{k^2\lambda_{Di}^2}\left[1 - \Gamma_0(b_i) - \frac{2\omega^2\Gamma_1(b_i)}{\omega^2 - \omega_{ci}^2}\right] \equiv \frac{\alpha}{k^2\lambda_{Di}^2}, \tag{12a}$$

where $\lambda_{Di} = (T_i/4\pi n_0 e^2)^{1/2}$ is the ion Debye radius, T_i is the ion temperature, $\Gamma_{0,1} = I_{0,1}\exp(-b_i)$, I_0 (I_1) is the zero- (first-) order modified Bessel function, $b_i = k_\perp^2\rho_i^2$, $\rho_i = v_{ti}/\omega_{ci}$, $v_{ti} = (T_i/m_i)^{1/2}$, $\omega_{ci} = eB_0/m_i c$, and m_i is the ion mass. For $b_i \ll 1$, we have from (12)

$$\chi_i \approx -\frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \tag{12b}$$

whereas for $b_i \gg 1$, we obtain

$$\chi_i \approx \frac{1}{k^2\lambda_{Di}^2}. \tag{12c}$$

Equations (6), (7), and (11) can be combined in the quasineutrality approximation, namely $n_{e1} \approx n_{i1}$, yielding

$$\omega^2\left[1 - \Gamma_0(b_i) - \frac{2\omega^2\Gamma_1(b_i)}{\omega^2 - \omega_{ci}^2}\right] = \frac{\omega_A^2 b_i}{1+b_e}, \tag{13}$$

for the coupled IA–ion-cyclotron (IC) waves, and

$$(\omega^2 - \omega_A^2 b_s)\left[1 - \Gamma_0(b_i) - \frac{2\omega^2\Gamma_1(b_i)}{\omega^2 - \omega_{ci}^2}\right] = \omega_A^2 b_i, \tag{14}$$

https://doi.org/10.1017/S0022377800008412 Published online by Cambridge University Press

for the coupled KA–IC waves. Here we have used the notation $v_A = B_0/(4\pi n_0 m_i)^{1/2}$ and $\omega_A = k_z v_A$.

Equations (13) and (14) are our new dispersion relations that do not impose any restriction on the values of ω/ω_{ci}, b_i, and b_e.

The parallel electric fields associated with the IAWs and KAWs are respectively

$$E_z = \frac{k_z \omega^2}{k_\perp k_z^2 c_a^2} \alpha |\mathbf{E}_\perp| \tag{15}$$

and

$$E_z = \frac{c^2 k_z^3 \lambda_{De}^2 |\mathbf{k}_\perp \cdot \mathbf{E}_\perp|}{\omega^2 - \omega_A^2 b_s}, \tag{16}$$

where $c_a = (T_i/m_e)^{1/2}$. We note that for $b_i \gg 1$, $\alpha \approx 1$, whereas for $b_i \ll 1$, we have $\alpha \approx -b_i \omega_{ci}^2/(\omega^2 - \omega_{ci}^2)$, which reduces to $\alpha \approx b_i$ for $\omega \ll \omega_{ci}$.

Several comments are in order. First, for $b_i \ll 1$, we have $\Gamma_0 \approx 1 - b_i$ and $\Gamma_1 = \frac{1}{2} b_i$, so that (13) and (14) reduce to

$$\omega^2 \approx \frac{\omega_A^2}{1 + b_e} \left(1 - \frac{\omega^2}{\omega_{ci}^2} \right) \tag{17}$$

and

$$(\omega^2 - \omega_A^2 b_s) \approx \omega_A^2 \left(1 - \frac{\omega^2}{\omega_{ci}^2} \right). \tag{18}$$

For $\omega \ll \omega_{ci}$, we obtain from (17) and (18)

$$\omega \approx \frac{\omega_A}{(1 + b_e)^{1/2}} \tag{19}$$

and

$$\omega \approx \omega_A (1 + b_s)^{1/2}, \tag{20}$$

which are the frequencies of the IA and KA waves (Hasegawa and Uberoi 1982) in low-β ($\beta = 8\pi n_0 T_e/B_0^2 \ll m_e/m_i$) and intermediate-$\beta$ ($m_e/m_i \ll \beta \ll 1$) magnetoplasmas, respectively. For $b_e \gg 1$, we have from (19)

$$\omega = \frac{k_z}{k_\perp} \omega_{gm}, \tag{21}$$

which is the frequency of the modified convective cell (MCC) (Okuda and Dawson 1973). Here $\omega_{gm} = (\omega_{ce}\omega_{ci})^{1/2}$ is the geometric mean (or the lower-hybrid resonance) frequency. Equation (21) is also referred to as the inertial Alfvén resonance cone (Stasiewicz et al. 1997), characterized by $k_z/k_\perp = \omega/\omega_{gm}$. Since the frequency of the MCC is much smaller than ω_{ci}, we must have $k_\perp/k_z \gg (m_i/m_e)^{1/2}$; the latter dictates that the MCC propagates almost perpendicular to $\hat{\mathbf{z}}$.

Secondly, for $\omega \ll \omega_{ci}$, we obtain from (13) and (14)

$$\omega^2 = \frac{\omega_A^2 b_i}{[1 - \Gamma_0(b_i)](1 + b_e)}, \tag{22}$$

and (Hasegawa and Chen 1976)

$$\omega^2 = \omega_A^2 b_s + \frac{\omega_A^2 b_i}{[1 - \Gamma_0(b_i)]}, \tag{23}$$

https://doi.org/10.1017/S0022377800008412 Published online by Cambridge University Press

which, for $b_i \gg 1$, yield

$$\omega = \frac{k_z v_A k_\perp \rho_i}{(1 + k_\perp^2 \lambda_e^2)^{1/2}}, \tag{24}$$

and (Hasegawa and Chen 1976)

$$\omega \approx k_z v_A k_\perp \rho_s \left(1 + \frac{T_i}{T_e}\right). \tag{25}$$

It is interesting to note that for $k_\perp \lambda_e \gg 1$, (24) gives the frequency of the electrostatic electron-acoustic wave (EEAW) (Mikhailovskii 1974)

$$\omega \approx k_z \left(\frac{T_i}{m_e}\right)^{1/2} \equiv k_z c_a, \tag{26}$$

in which the restoring force comes from the pressure of the unmagnetized ions and the electron mass provides the inertia. We note that (26) is also deduced from (13) in the limit of $b_i \gg 1$ and $b_e \gg 1$, without assuming that $\omega \ll \omega_{ci}$. The dispersion to the EEAW comes from electron polarization drift, and the dispersive EEAW is governed by $\omega = k_z c_a/(1 + k_\perp^2 \rho_a^2)^{1/2}$, where $\rho_a = c_a/\omega_{ce}$.

Thirdly, we retain the finite-ω/ω_{ci} effect, and obtain from (17) and (18)

$$\omega = \frac{\omega_A}{(1 + b_e + b_f)^{1/2}} \tag{27}$$

and

$$\omega = \frac{\omega_A (1 + b_s)^{1/2}}{(1 + b_f)^{1/2}}, \tag{28}$$

which are valid for $b_i \ll 1$. Here $b_f = k_z^2 \lambda_i^2$, $\lambda_i = c/\omega_{pi}$ is the ion skin depth, and ω_{pi} is the ion plasma frequency. For $b_e \ll b_f$, (27) is the dispersion relation for magnetic field-aligned dispersive Alfvén waves (see e.g. Verheest 1990) in a cold plasma, whereas for $b_f \gg 1$, (28) yields $\omega \approx \omega_{ci}(1 + k_\perp^2 \rho_s^2)^{1/2}$, which is the frequency of the dispersive ion-cyclotron wave.

3. Summary and conclusions

To summarize, we have investigated the dispersive properties of Alfvén waves with arbitrary values of ω/ω_{ci}, b_i, and b_e. It appears that the newly found dispersive Alfvén-wave spectra are broad in that they are capable of reproducing the frequencies of the inertial and kinetic waves, which are long-wavelength (in comparison with the ion gyroradius) electromagnetic waves, as well as those electrostatic MCCs that have short wavelengths (in comparison with the electron skin depth). On the other hand, the general dispersion relations also correctly yield the frequency of the short-wavelength (in comparison with the ion gyroradius and the electron skin depth) electron-acoustic waves. Furthermore, the spectra of magnetic-field-aligned dispersive Alfvén and electromagnetic ion-cyclotron waves are also retrieved from our general dispersion relations. In conclusion, we stress that the results of the present investigation have clarified the underlying physics of long- and short-wavelength IA and KA waves, as well as their relationships with other plasma modes in a uniform magnetoplasma. In fact, our results are useful for describing the salient features of the broadband fluctuations that account for a transition from long- to short-wavelength modes. For the latter, the ion-fluid response ceases to hold and one has to resort to a hybrid approach involving fluid electrons and kinetic ions.

https://doi.org/10.1017/S0022377800008412 Published online by Cambridge University Press

Finally, we emphasize that (13) accounts for the broadband spectra of IAWs, which are analysed by Stasiewicz et al. (2000) in the data from the Freja satellite.

Acknowledgements

This research was partially supported by the Deutsche Forschungsmeinschaft through the Sonderforschungsbereich 191 as well as by the Swedish Natural Science Research Council and the International Space Science Institute at Bern (Switzerland) through its international team 'Dust Plasma Interaction in Space'.

References

Berthelier, A., Cerisier, J. C., Berthelier, J. J. and Rezeau, L. 1991 Low frequency magnetic turbulence in the high-latitude topside ionosphere: low-frequency waves or field-aligned currents? *J. Atmos. Terr. Phys.* **53**, 333.

Brambilla, M. 1998 *Kinetic Theory of Plasma Waves: Homogeneous Plasmas*, p. 333. Clarendon Press, Oxford.

Chaston, C., Carlson, C. W., Paria, W. J., Ergun, R. E. and McFadden, J. P. 1999 FAST observations of inertial Alfvén waves in the dayside aurora. *Geophys. Res. Lett.* **26**, 647.

Gekelman, W. 1999 Review of laboratory experiments on Alfvén waves and their relationship to space observations. *J. Geophys. Res. Lett.* **104**, 14 417.

Goertz, C. K. 1985 Auroral arc formation – kinetic and magnetohydrodynamic effects. *Space Sci. Rev.* **42**, 499.

Hasegawa, A. and Chen, L. 1976 Kinetic processes in plasma heating by resonant mode conversion of Alfvén wave. *Phys. Fluids.* **19**, 1924.

Hasegawa, A. and Uberoi, C. *The Alfvén Wave*. Report DOE/TIC No. 11197, Technical Center, US Department of Energy, Washington, DC.

Leneman, D., Gekelman, W. and Maggs, J. 1999 Laboratory observations of shear Alfvén waves launched from a small source. *Phys. Rev. Lett.* **82**, 2673.

Louran, P., Wahlund, J.-E., Chust, T., de Feraudy, H., Roux, A., Holback, B., Dovner, P. O., Erikson, A. I. and Holmgren, G. 1994 Observation of kinetic Alfvén waves by the FREJA spacecraft. *Geophys. Res. Lett.* **21**, 1847.

Lysak, R. L. and Lotko, W. 1996 On the kinetic dispersion relation for shear Alfvén waves. *J. Geophys. Res.* **101**, 5085.

Mikhailovskii, A. B. 1974 *Theory of Plasma Instabilities: Instabilities of a Homogeneous Plasma*, Vol. 1, pp. 149–154. Consultants Bureau, New York.

Okuda, H. and Dawson, J. M. 1973 Theory and numerical simulation on plasma diffusion across a magnetic field. *Phys. Fluids*, **16**, 408.

Petviashvili, V. I. and Pokhotelov, O. A. 1992 *Solitary Waves in Plasmas and in the Ionosphere*. Gordon and Breach Science Publishers, New York.

Rao, N. N. and Kaup, D. J. 1992 Excitation of electron cyclotron harmonic waves in ionospheric modification experiments. *J. Geophys. Res.* **97**, 6323.

Shukla, P. K. and Stenflo, L. 1995 Nonlinear Alfvén waves. *Physica Scripta* **T60**, 32.

Stasiewicz, K., Gustafsson, G., Marklund, G., Lundqvist, P.-A., Clemmons, J. H. and Zanetti, L. 1997 Cavity resonators and Alfvén resonance cones observed on Freja, *J. Geophys. Res.* **102**, 2565.

Stasiewicz, K., Khotyaintsev, Y., Berthomier, M. and Wahlund, J.-E. 2000 Identification of widespread turbulence of dispersive Alfvén waves. *Geophys. Res. Lett.* **27**, 173.

Stéfant, R. J. 1971 Alfvén wave damping from finite gyroradius coupling to the ion acoustic mode. *Phys. Fluids* **13**, 440.

Verheest, F. 1990 Nonlinear parallel Alfvén waves in cometary plasmas. *Icarus*, **86**, 273.

Wahlund, J. E., Eriksson, A. I., Holback, B., Boehm, M., Bonel, J., Kintner, P. M., Seyler, C. E., Clemmons, J. H., Eliasson, L., Knudsen, D., Norqvist, P. and Zanetti, L. 1998 Broadband ELF plasma emission during auroral energization: slow ion-acoustic waves. *J. Geophys. Res.* **103**, 4343.

https://doi.org/10.1017/S0022377800008412 Published online by Cambridge University Press