The Relation between Refractive Index and Density. By D. BURNETT, B.A., Clare College.

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Varying ideas of the mechanism of the propagation of light and of the constitution of matter have led at different times to different formulae for the dependence of the refractive index n of a medium on its density ρ, notably those of Laplace, $\frac{n^2-1}{\rho} = \text{constant}$, and of Lorenz* and Lorentz†, $\frac{n^2-1}{n^2+2}\frac{1}{\rho} = \text{constant}$. Recently Macdonald‡ has given a theory which seems more in accord with the modern concept of atomic structure and finds a formula $\frac{n^2-1}{n\rho} = \text{constant}$ for a non-magnetic medium. It has therefore seemed worth while to examine a number of experimental results to see whether this new formula gives as satisfactory agreement as that of Lorenz and Lorentz; it is found that it does in most cases, and is occasionally the better.

The assumptions made by the different authors about the structure of matter may be noted. Lorenz postulates a distribution of homogeneous, transparent molecules in free space, whose volumes and refractive indices do not change when the density changes; his result is that, though n and ρ are not the same for a molecule as for the matter in bulk, yet in the limiting case of infinitely long waves $\frac{n^2-1}{n^2+2}\frac{1}{\rho}$ is the same, and this relation holds approximately for the wave lengths of light. Lorentz applies the theory of electrons, with the assumption that each electron in the substance is acted on by a force, of the nature of a resistance, which is proportional to its velocity. Macdonald supposes that the difference between matter and free space can be attributed entirely to the presence of electric charges, the amplitudes of whose motion depend on the density of the matter; various simple assumptions as to the nature of this dependence lead to the formulae already quoted.

His results may be given briefly as follows: let $ie^{i(pt+\epsilon)}$ be the

* Lorenz, *Oeuvres Scientifiques*, Copenhagen, 1898, vol. I, p. 211.
† Lorentz, *Theory of Electrons*, Ch. IV.
‡ Macdonald, *Proc. Roy. Soc.* A, vol. 113, p. 237 (1926).

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part of the electric current, in the direction of the electric force in the incident wave, associated with one of the charges, i_0 the value of i at the centre of a sphere, large enough to permit the assumption that it is homogeneous and has the same electric constants as the matter in bulk, yet with a radius small compared with the wave length of light; then in a non-magnetic medium $\frac{n^2-1}{n^2+2}$ is proportional to Σi_0, so that the Lorenz-Lorentz formula involves the assumption that Σi_0 is proportional to the density. It is also found that $\Sigma |i|$ is equal to (n^2-1) multiplied by a factor depending on n and on the angle of incidence of the light on the surface of the medium. In these cases *the amplitude of the incident waves is supposed unaltered* when the density varies. Macdonald then puts forward the hypothesis that the amplitude of the motion of a charge is the same in two media of the same kind but of different densities provided that *the density of electric energy is the same in both*, and this of course is not consistent with the amplitude of the incident waves being the same in the two cases. This assumption leads to $\frac{n^2-1}{n\rho} =$ constant for a non-magnetic medium and

$$\frac{n^2-1}{n\rho} = k\sqrt{1+k'^2\rho^2} + k'\sqrt{1+k^2\rho^2}$$

for a magnetic medium, k and k' being constants.

Experimental results are available for a large number of substances. In the case of gases the refractive index is so near unity that it is not always possible to differentiate between the formulae. Magri's* results for air up to pressures of 200 atmospheres are quoted by Lorentz in support of his formula, and the agreement is good. No other results have been found for such a wide range of density, but it is to be noted that recent authors† give $\frac{n-1}{\rho} =$ constant within the limit of experimental error for pressures from one to ten atmospheres; this has been found for air, oxygen, hydrogen and carbon dioxide.

In view of this uncertainty, it has seemed advisable to consider liquids rather than gases. Variation of the density of a liquid is obtained by varying the temperature, and this introduces the possibility of a shift of the absorption bands. Such an effect has been observed in specimens of glass‡ and some other crystalline

* Magri, *Phys. Zeit.* vol. 6, p. 629 (1905).

† Cf. Posejpal, *Journ. de Phys.* vol. 2, p. 87 (1921) and vol. 4, p. 451 (1923); Opláden, *Zeit. für Phys.* vol. 42, p. 160 (1927).

‡ Gibson, *Phys. Rev.* vol. 7, p. 194 (1916); Peters, *Sc. Papers of the Bureau of Standards*, 1926.

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media*, and in solutions of certain colouring matters†, but information about other substances is scanty. Henri‡ has studied the absorption of most of the substances considered in this paper and he does not seem to have noted any temperature effect, so that it is probably justifiable to assume that in these cases the shift of the bands is inappreciable in the range of temperature involved.

The following table indicates the kind of results obtained: in it A and B represent the formulae $\frac{n^2-1}{n^2+2}\frac{1}{\rho}$ = constant and $\frac{n^2-1}{n\rho}$ = constant respectively.

Substance	Best Formula	Author
Water, Ice, and Steam	*A*	Lorenz§
Water	*B*	Hall and Payne‖
Alcohol (liq. and vap.)	*A*	Lorenz§
Alcohol (liq.)	*B*	Hall and Payne‖; Ketteler¶
Benzene	*A* or *B*	Landolt-Bornstein**
Ether (liq. and vap.)	*A*	Lorenz§
Ether (liq.)	*A* or *B*	Landolt-Bornstein**
Ethyl Bromide	*A* or *B*	Weegmann††
Aniline	*B*	Landolt-Bornstein**

In the cases in which an extended series of observations is available there is a steady variation in the same sense in $\frac{n^2-1}{n^2+2}\frac{1}{\rho}$ and $\frac{n^2-1}{n\rho}$. Some of the figures obtained from the measurements of Hall and Payne on pure water and alcohol may be quoted:

* Cf. Martin, *Proc. Roy. Soc.* A, vol. 96, p. 185 (1919).
† Cf. Houstoun, *Ann. der Phys.* vol. 21, p. 535 (1906).
‡ Cf. Henri, *Études de Photochimie*, Paris, 1919.
§ *Loc. cit.* p. 332.
‖ Hall and Payne, *Phys. Rev.* vol. 20, p. 249. The densities are from the *Smithsonian Tables*, 7th ed.
¶ Ketteler, *Wied. Ann.* vol. 33, p. 508 (1888).
** Landolt-Bornstein, *Tabellen.* (The values given are from different authors and only those which are consistent have been used.)
†† Weegmann, *Zeit. Ph. Chem.* vol. 2, p. 234 (1888). (A large number of similar compounds are also investigated.)

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Water.

$t°$ C.	20	30	40	50	60	70
n	1·33335	1·33228	1·33087	1·32930	1·32754	1·32547
ρ	·99823	·99568	·99225	·98807	·98324	·97781
$\frac{n^2-1}{n\rho}$	·58439	·58422	·58401	·58399	·58406	·58398
$\frac{n^2-1}{n^2+2}\frac{1}{\rho}$	·20626	·20618	·20610	·20607	·20608	·20603

Alcohol.

$t°$ C.	16	20	24	28	32	36	40
$\frac{n^2-1}{n\rho}$	·7915	·7918	·7920	·7922	·7925	·7927	·7929
$\frac{n^2-1}{n^2+2}\frac{1}{\rho}$	·22402	·22412	·22420	·22427	·22434	·22442	·22450

Lorenz gives values of n and ρ for ice, water and steam; the values of the constants calculated from them are:

	Ice	Water at 10° C.	Water at 20° C.	Steam
$\frac{n^2-1}{n\rho}$	·5903	·5841	·5839	·6203
$\frac{n^2-1}{n^2+2}\frac{1}{\rho}$	·2080	·20615	·2061	·2068

It is evidently only in the case of steam that the Lorenz-Lorentz formula is the better.

The following tables were calculated from the values of n and ρ given in Landolt-Bornstein:

Aniline.

ρ	1·016	1·0216	1·0220	1·02478	1·02792
n	1·5828	1·58629	1·58632	1·58818	1·59073
$\frac{n^2-1}{n\rho}$	·9360	·9357	·9354	·9354	·9360
$\frac{n^2-1}{n^2+2}\frac{1}{\rho}$	·3288	·3286	·3285	·3285	·3286

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Benzene.

ρ	·8801	·88341	·8867	·89137
n	1·49646	1·4988	1·50119	1·50381
$\frac{n^2-1}{n\rho}$	·9411	·9414	·9418	·9411
$\frac{n^2-1}{n^2+2}\frac{1}{\rho}$	·33218	·33226	·33237	·33209

Ethyl Ether.

ρ	·7141	·7157	·7166	·7183
n	1·3519	1·35032	1·35112	1·35246
$\frac{n^2-1}{n\rho}$	·8573	·8520	·8526	·8535
$\frac{n^2-1}{n^2+2}\frac{1}{\rho}$	·3028	·3008	·3011	·3015

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