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PROBLEMS AND SOLUTIONS

PROBLEMS

00.4.1. *Simple Applications of the Cox–Tsiatis Result on Unidentifiability of Dependent Competing Risks Models with Regressors*, proposed by S.K. Sapra. Cox (1962) and Tsiatis (1975) showed that, for every dependent competing risks model without regressors, there is an equivalent independent competing risks model. Heckman and Honroe (1989) showed that the Cox–Tsiatis result can be overturned through introduction of regressors in the dependent competing risks model. The following simple examples illustrate the Cox–Tsiatis result and construct independent risks models that are equivalent to the given dependent competing risks models.

Consider the following *dependent* competing risks models in which we observe $T = \min(T_1, T_2)$, and δ, where $\delta = 1$ if $T = T_1$ and 2 if $T = T_2$. T_1 and T_2 are latent durations of risks 1 and 2, respectively. Suppose that (T_1, T_2) have the joint survivor functions

$$S(t_1, t_2) = \exp(-(\lambda_1 t_1 + \lambda_2 t_2)^{\alpha}), \qquad 0 < \alpha \leq 1, \qquad \lambda_1, \lambda_2 > 0, \tag{1}$$

$$S(t_1, t_2) = [\beta/(\lambda_1 t_1 + \lambda_2 t_2 + \beta)]^{\alpha+1}, \qquad \alpha > -1, \qquad \beta > 0, \qquad \lambda_1, \lambda_2 > 0. \tag{2}$$

Construct *independent* competing risks models that are equivalent to the *dependent* competing risks models in (1) and (2).

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Cox, D.R. (1962) *Renewal Theory.* London: Methuen.

Tsiatis, A. (1975) A nonidentifiability aspect of competing risks. *Proceedings of the National Academy of Sciences USA* 72, 20–22.

Heckman, J.J. & B. Honroe (1989) The identifiability of the competing risks model with regressors. *Biometrika* 76, 325–330.

00.4.2. *A Necessary and Sufficient Condition for the Convergence of the Sum of Weighting Random Variables to a Normal Distribution*, proposed by Hua Liang. Prove the following theorem.

THEOREM. *Let $X_1, \ldots, X_n$ be i.i.d. random variables with zero mean and finite variance σ^2. X_1 is a non-Gaussian distribution. Assume $\{a_{ni}; i = 1, \ldots, n, n \geq 1\}$ is a sequence of constants satisfying $\sum_{i=1}^{n} a_{ni}^2 = 1$. The sufficient and necessary condition that $\sum_{i=1}^{n} a_{ni} X_i$ converges to the normal distribution $N(0, \sigma^2)$ is $\max_{1 \leq i \leq n} |a_{ni}| \to 0$.*

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