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A new variable structure controller for robot manipulators with a nonlinear PID sliding surface

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SUMMARY

In this paper a new sliding mode controller for set-point control of robot manipulators is proposed. The controller does not use any part of the robot dynamics in the control law. Thus, it is structurally simpler than other sliding mode controllers where the control law uses a nominal model of the robot dynamics. The controller uses a new nonlinear Proportional-Integral-Derivative (PID) sliding surface. The stability of the controlled robot dynamics is proved. On applying the boundary-layer approach to remove chattering, a nonlinear PID controller exists inside the boundary layer. This PID controller ensures that the error tend to zero asymptotically if there is no disturbances applied to the robot dynamics.

KEYWORDS: Sliding mode control; PID control; Set-point control; Robot control; Asymptotic stability.

1. Introduction

High precision motion control has become an essential requirement in today's advanced mechanical systems such as robot manipulators. As performance requirements become more stringent, classical controllers can no longer provide satisfactory results. Although various approaches to the design of better controllers have been proposed in the literature, control problems associated with system uncertainties, presence of high-order dynamics, and system's inherent nonlinearities remain big challenges for control engineers. Sliding mode control (SMC) as a sub-class of variable structure control (VSC) is the most acceptable approach to controller design in these cases. The technique has been receiving much attention recently. When the system is in the sliding mode, it emulates a prescribed reduced order system and becomes insensitive to parameter variations and disturbances. The reduced order system is represented by what is known as the switching surface. This surface may be linear, nonlinear, or time-varying.[1,2] In the control of robot manipulators, the widely used sliding surface is a linear hyperplane that results in a PD-type surface.[3,4] Nonlinear PD sliding surface is also possible.[5] Linear Proportional-Integral-Derivative (PID) sliding surfaces were also used.[6,7] It was shown[6] that linear PID sliding surface results in high overshoots when compared with linear PD sliding surface but the tracking accuracy is better when smoothing the control law.

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The common problem in the application of sliding mode control is the chattering phenomena where the trajectories of the controlled system do not slide along the sliding surface but actually cross it at high frequency. Chattering is undesirable in both itself and because it may excite unmodelled high-frequency dynamics. However, any attempt to attenuate the control chatter degrades the tracking performance of the sliding mode control and is a source of a non-zero sliding variable. This requires the search for additional control system, which compensates for this drawback.

In this paper, we introduce a new sliding mode controller for set point control of robot manipulators. The controller uses a novel nonlinear PID sliding surface. Unlike the standard sliding mode control design approach, the controller does not use any part of the robot dynamics in the control law. Thus, it is structurally simpler than other sliding mode controllers. The controller does not use any additional control system to rectify the problem associated with attenuating the control chatter. This makes the controller simple both in structure and design itself.

We should refer here to the fact that although some global nonlinear PID regulators have been reported in the literature,[8–15] which are based on Lyapunov and passivity theory, they do not take into account the effects of external disturbances or unmodelled higher dynamics. Besides, the use of fixed gains in these controllers results in the need of complicated performance tuning procedures. Since sliding mode control takes into account the various types of disturbances, the existing controller should be better than global PID regulators.

Through this paper we use notations $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ to indicate the smallest and the largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $\mathbf{A}(\mathbf{x})$ for any $\mathbf{x} \in \mathsf{R}^n$. The norm of a vector $\mathbf{x}$ is defined as $\|\mathbf{x}\| = \sqrt{\mathbf{x}^{\mathrm{T}}\mathbf{x}}$, and that of matrix $\mathbf{A}(\mathbf{x})$ it is defined as the corresponding induced norm $\|\mathbf{A}\| = \sqrt{\lambda_M\{\mathbf{A}^{\mathrm{T}}\mathbf{A}\}}$.

2. Rigid Robots Dynamics

In the presence of disturbances, the model of an n-link rigid manipulator with revolute joints is given by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_d = \boldsymbol{\tau}, \tag{1}$$

where $\mathbf{q} \in \mathsf{R}^n$ denotes the joint angle, $\mathbf{M}(\mathbf{q}) \in \mathsf{R}^{n\times n}$ denotes the inertia matrix, which is symmetric and positive definite,

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$\mathbf{g}(\mathbf{q}) \in \mathsf{R}^n$ denotes the gravitational force, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \in \mathsf{R}^n$ is the centrifugal and Coriolis forces, and $\boldsymbol{\tau} \in \mathsf{R}^n$ is the control input. The disturbance input (including system uncertainties and unmodelled dynamics) is $\boldsymbol{\tau}_d \in \mathsf{R}^n$ and will be defined later.

The above robot dynamics are characterized by the following properties[15–17]:

P1: The inertia matrix is bounded and satisfies

$$\lambda_m\{\mathbf{M}\}\,\|\mathbf{x}\|^2 \le \mathbf{x}^{\mathrm{T}}\mathbf{M}(\mathbf{q})\,\mathbf{x} \le \lambda_M\{\mathbf{M}\}\,\|\mathbf{x}\|^2 \quad \forall \mathbf{x} \in \mathsf{R}^n. \tag{2}$$

P2: The centrifugal and Coriolis term satisfies

$$\mathbf{C}(\mathbf{x}, \mathbf{y})\,\mathbf{z} = \mathbf{C}(\mathbf{x}, \mathbf{z})\,\mathbf{y}, \qquad \forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathsf{R}^n, \tag{3}$$

and there exists a bounded constant $k_c > 0$ such that

$$\|\mathbf{C}(\mathbf{x}, \mathbf{y})\,\mathbf{z}\| \le k_c\,\|\mathbf{y}\|\|\mathbf{z}\| \qquad \forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathsf{R}^n. \tag{4}$$

P3: The matrix $[\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}]$ is skew-symmetric. This implies

$$\mathbf{y}^{\mathrm{T}}\left\{\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}}\right\}\mathbf{y} = 0 \quad \forall \mathbf{y} \in \mathsf{R}^n \tag{5}$$

and

$$\dot{\mathbf{M}}(\mathbf{q}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \{\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\}^{\mathrm{T}}. \tag{6}$$

P4: The gravity vector $\mathbf{g}(\mathbf{q})$ is obtained from the robot potential energy $U(\mathbf{q})$ by the relation

$$\mathbf{g}(\mathbf{q}) = \frac{\partial U(\mathbf{q})}{\partial \mathbf{q}} \tag{7}$$

with

$$\|\mathbf{g}(\mathbf{q})\| \le k_v \qquad \forall \mathbf{q} \in \mathsf{R}^n \tag{8}$$

and

$$\|\mathbf{g}(\mathbf{x}) - \mathbf{g}(\mathbf{y})\| \le k_g\,\|\mathbf{x} - \mathbf{y}\| \qquad \forall \mathbf{x}, \mathbf{y} \in \mathsf{R}^n, \tag{9}$$

where k_v and k_g are bounded positive constants.

P5: It is possible to choose $\mathbf{K} \in \mathsf{R}^{n\times n}$, a positive definite diagonal matrix, large enough such that the following two inequalities are satisfied simultaneously for a sufficiently small positive scalar[15] ε:

$$\mathbf{s}(\mathbf{e})^{\mathrm{T}}\,\mathbf{K}\mathbf{s}(\mathbf{e}) + \mathbf{s}(\mathbf{e})^{\mathrm{T}}\,[\mathbf{g}(\mathbf{q}) - \mathbf{g}(\mathbf{q}_d)] \ge \varepsilon\,\|\mathbf{s}(\mathbf{e})\|^2, \tag{10}$$

$$\sum_{i=1}^{n} K_i S(e_i) + U(\mathbf{q}) - U(\mathbf{q}_d) + \mathbf{e}^{\mathrm{T}}\mathbf{g}(\mathbf{q}_d) \ge \varepsilon\,\|\mathbf{s}(\mathbf{e})\|^2, \tag{11}$$

where K_i is an element of $\mathbf{K}$, $\mathbf{e} = \mathbf{q}_d - \mathbf{q}$ is the joint position error, $\mathbf{s}(\mathbf{x})$ is the saturated vector function, $\mathbf{s}(\mathbf{x}) = [s(x_1)\,s(x_2)\cdots s(x_n)]^{\mathrm{T}}$, and $S(x_i)$ is a scalar potential function whose derivative is the ith element of $\mathbf{s}(\mathbf{x})$, i.e. $s(x_i) = \frac{\mathrm{d}S(x_i)}{\mathrm{d}x_i}$. These two functions have the following properties[15,18]:

(a) $S(x_i)$ is unbounded, twice continuously differentiable and

$$S(x_i) > 0\,\forall x_i \ne 0 \quad \text{and} \quad S(0) = 0. \tag{12}$$

(b) The saturated vector function $\mathbf{s}(\mathbf{x})$ whose elements are denoted by $s(x_i)$ belongs to the set of all continuous differentiable increasing functions defined by Kelly.[10] With $b > 0$ and $1 \ge a > 0$, they satisfy

$$\begin{cases} |x_i| \ge |s(x_i)| \ge a\,|x_i| & \forall x_i \in \mathsf{R} : |x_i| < b \\ b \ge |s(x_i)| \ge ab & \forall x_i \in \mathsf{R} : |x_i| \ge b \\ 0 \le \dfrac{\mathrm{d}s(x_i)}{\mathrm{d}x_i} \le 1 & \forall x_i \in \mathsf{R} \\ x_i s(x_i) > 0 & \forall x_i \ne 0, \\ s(0) = 0 \end{cases} \tag{13}$$

$$\begin{cases} \|\mathbf{x}\| \ge \|\mathbf{s}(\mathbf{x})\| \ge a\,\|\mathbf{x}\| & \text{if } \|\mathbf{x}\| < b \\ b\sqrt{n} \ge \|\mathbf{s}(\mathbf{x})\| \ge ab & \text{if } \|\mathbf{x}\| \ge b. \end{cases} \tag{14}$$

3. Discontinuous Sliding Mode Controller Design

The proposed controller is defined by

$$\boldsymbol{\tau} = \mathbf{K}_1\boldsymbol{\sigma} + \mathbf{K}_2\mathbf{Sgn}(\boldsymbol{\sigma}), \tag{15}$$

where $\boldsymbol{\sigma} \in \mathsf{R}^n$ is the switching function, $\mathbf{K}_1$ and $\mathbf{K}_2 \in \mathsf{R}^{n\times n}$ are diagonal positive definite matrices, and $\mathbf{Sgn}(\boldsymbol{\sigma})$ is the vector function with $\mathbf{Sgn}(\boldsymbol{\sigma}) = [\mathrm{Sgn}(\sigma_1)\ \mathrm{Sgn}(\sigma_2)\ \cdots\ \mathrm{Sgn}(\sigma_n)]^{\mathrm{T}}$. Before designing the sliding surface $\boldsymbol{\sigma}$, we define the novel functions $s(x_i)$ and $S(x_i)$ as follows:

$$\forall i = 1, 2, \ldots, n,$$

$$s(x_i) = \begin{cases} 1 - \frac{1}{Q}exp(P - Qx_i) & \forall x_i \ge \frac{P}{Q} \\ x_i & \forall |x_i| < \frac{P}{Q} \\ -1 + \frac{1}{Q}exp(P + Qx_i) & \forall x_i \le -\frac{P}{Q} \end{cases}, \tag{16}$$

$$S(x_i) = \begin{cases} x_i + \frac{1}{Q^2}exp(P - Qx_i) - \frac{2PQ+2-P^2}{2Q^2} & \forall x_i \ge \frac{P}{Q} \\ \frac{1}{2}x_i^2 & \forall |x_i| < \frac{P}{Q}, \\ -x_i + \frac{1}{Q^2}exp(P + Qx_i) - \frac{2PQ+2-P^2}{2Q^2} & \forall x_i \le -\frac{P}{Q} \end{cases} \tag{17}$$

where $P,\ Q \in \mathsf{Z}_+$ (a set of positive integers) and $Q = P + 1$.

We also define the following $n \times n$ diagonal matrix $\mathbf{B}(\cdot)$,

$$0 \le \mathbf{B}(\mathbf{x}) = \frac{\partial \mathbf{s}(\mathbf{x})}{\partial \mathbf{x}} \le \mathbf{I} \quad \forall \mathbf{x} \in \mathsf{R}^n, \tag{18}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix.

The scalars P and Q must be chosen carefully such that with $b = 1$, $a = 1 - \frac{1}{Q}e^{(P-Q)}$, the function $s(x_i)$ defined by Eq. (16) satisfies Eq. (13) and its saturated vector function $\mathbf{s}(\mathbf{x})$ satisfies Eq. (14). The most important fact here is that

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the function $s(x_i)$ has been designed specifically so that the vector function $\mathbf{s}(\mathbf{x})$ satisfies the following condition:

$$\mathbf{s}(\mathbf{x}) = \mathbf{x} \quad \text{if} \quad |x_i| \ \{= |s(x_i)|\} < \frac{P}{Q} \qquad \forall i = 1:n. \quad (19)$$

In terms of the vector's norm, we can write this condition as follows:

$$\mathbf{s}(\mathbf{x}) = \mathbf{x} \qquad \text{if} \quad \|\mathbf{x}\| \ \{= \|\mathbf{s}(\mathbf{x})\|\} < \frac{P}{Q}. \quad (20)$$

For constant $\mathbf{q}_d$ (set-point control), the proposed sliding surface $\boldsymbol{\sigma}$ is a nonlinear PID function defined by

$$\boldsymbol{\sigma} = -\dot{\mathbf{q}} + \alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon}), \quad (21)$$

$$\dot{\boldsymbol{\upsilon}} = (\alpha\gamma + \beta)\mathbf{s}(\mathbf{e}) - \gamma \dot{\mathbf{q}}, \quad (22)$$

where α, β, and γ are positive constants.

On the sliding surface $\{\boldsymbol{\sigma} = \mathbf{0}\}$ the robot dynamics are defined by

$$\dot{\mathbf{q}} = \alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon}), \quad (23)$$

$$\dot{\boldsymbol{\upsilon}} = (\alpha\gamma + \beta)\mathbf{s}(\mathbf{e}) - \gamma \dot{\mathbf{q}}. \quad (24)$$

The equilibrium of system (23) and (24) requires $\dot{\mathbf{q}} = \dot{\boldsymbol{\upsilon}} = \mathbf{0} \Rightarrow \mathbf{s}(\mathbf{e}) = \mathbf{0} \Rightarrow \mathbf{e} = \mathbf{0} \Rightarrow \mathbf{q} = \mathbf{q}_d \Rightarrow \mathbf{s}(\boldsymbol{\upsilon}) = \mathbf{0} \Rightarrow \boldsymbol{\upsilon} = \mathbf{0}$. Hence, the origin is the equilibrium point of the system. If the system states remain on the chosen sliding surface, the error will tend to zero asymptotically $\forall t \geq 0$. This can be achieved if we obtain global asymptotic stability of dynamics (23) and (24). This can be proved by considering the following Lyapunov function:

$$V(\mathbf{e}, \boldsymbol{\upsilon}) = \sum_{i=1}^{n} S(e_i) + \sum_{i=1}^{n} S(\upsilon_i).$$

This function is positive definite and radially unbounded because of equations (12) and (17). The time derivative of this function along with trajectories of system dynamics (23) and (24) is given by

$$\begin{aligned} \dot{V} &= -\mathbf{s}(\mathbf{e})^{\mathrm{T}} \dot{\mathbf{q}} + \mathbf{s}(\boldsymbol{\upsilon})^{\mathrm{T}} \dot{\boldsymbol{\upsilon}} \\ &= -\mathbf{s}(\mathbf{e})^{\mathrm{T}} [\alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon})] + \mathbf{s}(\boldsymbol{\upsilon})^{\mathrm{T}} [\beta \mathbf{s}(\mathbf{e}) - \beta\gamma \mathbf{s}(\boldsymbol{\upsilon})] \\ &= -\alpha \|\mathbf{s}(\mathbf{e})\|^2 - \beta\gamma \|\mathbf{s}(\boldsymbol{\upsilon})\|^2. \end{aligned}$$

The function $\dot{V}$ is negative definite. Hence, global asymptotic stability is guaranteed. The speed of convergence to the origin is proportional to the values of α, β, and γ.

After choosing the switching surface, the next step in the sliding mode control design technique is to prove that the control law (15) will drive the state of the system towards the sliding surface in a finite time and then maintains the state on that surface once it has been reached. To verify this, we will use the following Lyapunov function:

$$V(\boldsymbol{\sigma}) = \tfrac{1}{2} \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) \boldsymbol{\sigma}. \quad (25)$$

This function has the following properties: $V(\boldsymbol{\sigma}) > 0 \, \forall \boldsymbol{\sigma} \neq \mathbf{0}, V(\mathbf{0}) = 0$. The time derivative of this Lyapunov function along trajectories of the robot dynamics (1) under the control law (15) is given by

$$\begin{aligned} \dot{V} &= \tfrac{1}{2} \boldsymbol{\sigma}^{\mathrm{T}} \dot{\mathbf{M}}(\mathbf{q}) \boldsymbol{\sigma} + \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) [-\ddot{\mathbf{q}} - \alpha \mathbf{B}(\mathbf{e}) \dot{\mathbf{q}} + \beta \mathbf{B}(\boldsymbol{\upsilon}) \dot{\boldsymbol{\upsilon}}] \\ &= \tfrac{1}{2} \boldsymbol{\sigma}^{\mathrm{T}} \dot{\mathbf{M}}(\mathbf{q}) \boldsymbol{\sigma} - \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) [\alpha \mathbf{B}(\mathbf{e}) + \beta\gamma \mathbf{B}(\boldsymbol{\upsilon})] \dot{\mathbf{q}} \\ &\quad + \boldsymbol{\sigma}^{\mathrm{T}} [\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_d - \mathbf{K}_1 \boldsymbol{\sigma} - \mathbf{K}_2 \mathbf{Sgn}(\boldsymbol{\sigma})] \\ &\quad + \beta(\alpha\gamma + \beta) \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) \mathbf{B}(\boldsymbol{\upsilon}) \mathbf{s}(\mathbf{e}). \end{aligned}$$

Using equations (3), (5), and (21) yields

$$\begin{aligned} \dot{V} &= \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) [\alpha \mathbf{B}(\mathbf{e}) + \beta\gamma \mathbf{B}(\boldsymbol{\upsilon})] [\boldsymbol{\sigma} - \alpha \mathbf{s}(\mathbf{e}) - \beta \mathbf{s}(\boldsymbol{\upsilon})] \\ &\quad + \beta(\alpha\gamma + \beta) \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) \mathbf{B}(\boldsymbol{\upsilon}) \mathbf{s}(\mathbf{e}) \\ &\quad + \boldsymbol{\sigma}^{\mathrm{T}} [\mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_d - \mathbf{K}_1 \boldsymbol{\sigma} - \mathbf{K}_2 \mathbf{Sgn}(\boldsymbol{\sigma})] \\ &\quad + \alpha \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{C}(\mathbf{q}, \mathbf{s}(\mathbf{e})) [-\boldsymbol{\sigma} + \alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon})] \\ &\quad + \beta \boldsymbol{\sigma}^{\mathrm{T}} \mathbf{C}(\mathbf{q}, \mathbf{s}(\boldsymbol{\upsilon})) [-\boldsymbol{\sigma} + \alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon})]. \end{aligned}$$

At this position we have to define the disturbance $\boldsymbol{\tau}_d$.

Assumption 1. The disturbance input is assumed to be bounded and defined by the relation[19]

$$\|\mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_d\| \leq \rho, \quad (26)$$

where ρ is a bounded positive constant.

By applying Eqs. (2), (4), (8), (14), (18), and (26) we obtain

$$\begin{aligned} \dot{V} &\leq -\mu_1 \|\boldsymbol{\sigma}\|^2 - \mu_2 \|\boldsymbol{\sigma}\| \\ &\leq -\mu_2 \|\boldsymbol{\sigma}\|, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \mu_1 &= \lambda_m\{\mathbf{K}_1\} - (\alpha + \beta\gamma) \lambda_M\{\mathbf{M}\} - (\alpha + \beta) k_c \sqrt{n}, \\ \mu_2 &= \lambda_m\{\mathbf{K}_2\} - (\alpha^2 + \beta^2 + \alpha\beta + \beta^2\gamma) \lambda_M\{\mathbf{M}\} \sqrt{n} \\ &\quad - (\alpha + \beta)^2 n k_c - \rho. \end{aligned}$$

If the constant scalars μ_1 and μ_2 are positive, then the function $\dot{V}$ (Eq. (27)) is negative definite $\forall \mathbf{e}, \dot{\mathbf{q}}, \boldsymbol{\upsilon} \in R^n - \Omega$, where Ω is the set defined as $\Omega = \{\mathbf{e}, \dot{\mathbf{q}}, \boldsymbol{\upsilon} : \boldsymbol{\sigma}(\mathbf{e}, \dot{\mathbf{q}}, \boldsymbol{\upsilon}) = \mathbf{0}\}$. From Eqs. (25) and (27), it follows that all initial conditions lying off Ω will be attracted to Ω (global sliding condition[2]). The convergence here is exponential so that the sliding mode occurs in a finite time.

The control law (15) is discontinuous across the sliding surface $\{\boldsymbol{\sigma} = \mathbf{0}\}$, which leads to chattering. The chatter is normally undesirable in practice, since it may excite unmodelled high-frequency dynamics, which could result in instability. This problem can be alleviated by introducing the so-called boundary layer around the switching surface. The stability analysis of the system in this case is performed in the following section.

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4. Continuous Approximation of the Sliding Mode Control Law

In the boundary layer approach, the signum function is replaced by the following saturated vector function,

$$\mathbf{Sat}(\boldsymbol{\sigma}) = \begin{cases} \phi^{-1}\boldsymbol{\sigma} & \text{if} \quad \|\boldsymbol{\sigma}\| \le \phi \\ \mathbf{Sgn}(\boldsymbol{\sigma}) & \text{otherwise,} \end{cases} \tag{28}$$

where $\phi > 0$ is the boundary layer thickness, which is made fixed in this work.

The new control law is now defined by

$$\boldsymbol{\tau} = \mathbf{K}_1\boldsymbol{\sigma} + \mathbf{K}_2\mathbf{Sat}(\boldsymbol{\sigma}). \tag{29}$$

The stability of the robot system (1) with the sliding surface (21) under the continuous control law (29) is established as follows:

(i) $\|\boldsymbol{\sigma}\| > \phi$: In this case the control law (15) is a discontinuous one and we can choose the Lyapunov function (25) to obtain the conclusion (Eq. (27)). With $\mu_1 > 0$ and $\mu_2 > 0$, it follows that $\boldsymbol{\sigma}$ approaches the boundary layer in a finite time.

(ii) $\|\boldsymbol{\sigma}\| \le \phi$: In this case the control law can be written as follows:

$$\boldsymbol{\tau} = \mathbf{K}\boldsymbol{\sigma} = \alpha\mathbf{K}\mathbf{s}(\mathbf{e}) - \mathbf{K}\dot{\mathbf{q}} + \beta\mathbf{K}\mathbf{s}(\boldsymbol{\upsilon}), \tag{30}$$

$$\dot{\boldsymbol{\upsilon}} = (\alpha\gamma + \beta)\,\mathbf{s}(\mathbf{e}) - \gamma\dot{\mathbf{q}}, \tag{31}$$

where

$$\mathbf{K} = \mathbf{K}_1 + \phi^{-1}\mathbf{K}_2. \tag{32}$$

Using the same Lyapunov function (25), its time derivative along the trajectories of Eqs. (1) and (30), after applying Eqs. (2)–(5), (8), (14), (18), (21), and (26), is given by

$$\begin{aligned} \dot{V} &\le -\mu_3\|\boldsymbol{\sigma}\|^2 + \mu_4\|\boldsymbol{\sigma}\| \\ &\le -\kappa\|\boldsymbol{\sigma}\|^2 + 0.5\mu_4, \end{aligned} \tag{33}$$

where

$$\begin{aligned} \mu_3 &= \lambda_m\{\mathbf{K}\} - (\alpha + \beta\gamma)\,\lambda_M\{\mathbf{M}\} - (\alpha+\beta)\,k_c\sqrt{n}, \\ \mu_4 &= \eta\lambda_M\{\mathbf{M}\}\sqrt{n} + (\alpha+\beta)^2\,nk_c + \rho, \\ \eta &= (\alpha+\beta)^2 - \alpha\beta + \beta^2\gamma, \\ \kappa &= \mu_3 - 0.5\mu_4, \end{aligned}$$

and we should have $\mu_3 > 0$ and $\kappa > 0$.

The structure of Eq. (33) shows that the uniform ultimate boundedness is guaranteed[20] and the tracking error can be reduced arbitrarily by adjusting the value of κ.

The control laws (30) and (31) can also be viewed as a nonlinear PID controller. In the absence of disturbance ($\boldsymbol{\tau}_d = \mathbf{0}$), global asymptotic stability of the robot dynamics is guaranteed by this controller. This can be proved as follows:

The equilibrium of the robot dynamics (1) in a closed loop with the existed PID control law requires $\dot{\mathbf{q}} = \dot{\boldsymbol{\upsilon}} = \mathbf{0} \rightarrow \mathbf{s}(\mathbf{e}) = \mathbf{0} \rightarrow \mathbf{e} = \mathbf{0} \rightarrow \mathbf{q} = \mathbf{q}_d \rightarrow \mathbf{s}(\boldsymbol{\upsilon}^*) = \beta^{-1}\mathbf{K}^{-1}\mathbf{g}(\mathbf{q}_d)$. If $\mathbf{K}$ is chosen such that

$$\lambda_m\{\mathbf{K}\} > \frac{Q\rho}{\beta P}, \tag{34}$$

then the unique equilibrium point is $(\mathbf{e}^*, \dot{\mathbf{q}}^*, \boldsymbol{\upsilon}^*) = (\mathbf{0}, \mathbf{0}, \beta^{-1}\mathbf{K}^{-1}\mathbf{g}(\mathbf{q}_d))$. The proof of this statement is as follows:

At $(\mathbf{e} = \mathbf{0}, \dot{\mathbf{q}} = \mathbf{0})$ we have

$$\begin{aligned} \|\mathbf{s}(\boldsymbol{\upsilon}^*)\| &= \|\beta^{-1}\mathbf{K}^{-1}\mathbf{g}(\mathbf{q}_d)\| \\ &\le \beta^{-1}\lambda_M\{\mathbf{K}^{-1}\}\rho = \frac{\beta^{-1}\rho}{\lambda_m\{\mathbf{K}\}}. \end{aligned}$$

If the condition (34) is true, then

$$\|\mathbf{s}(\boldsymbol{\upsilon}^*)\| < \frac{P}{Q}.$$

Using Eq. (20), it follows that

$$\mathbf{s}(\boldsymbol{\upsilon}^*) = \boldsymbol{\upsilon}^* = \beta^{-1}\mathbf{K}^{-1}\mathbf{g}(\mathbf{q}_d).$$

The asymptotic stability of the existing nonlinear PID controller is now established by choosing the following radially unbounded Lyapunov function:

$$\begin{aligned} V = {} & \frac{1}{2}[\gamma\dot{\mathbf{q}} - (\alpha\gamma+\beta)\,\mathbf{s}(\mathbf{e})]^{\mathrm{T}}\,\mathbf{M}(\mathbf{q})\,[\gamma\dot{\mathbf{q}} - (\alpha\gamma+\beta)\,\mathbf{s}(\mathbf{e})] \\ & - \frac{1}{2}(\alpha\gamma+\beta)^2\,\mathbf{s}(\mathbf{e})^{\mathrm{T}}\,\mathbf{M}(\mathbf{q})\,\mathbf{s}(\mathbf{e}) + \gamma(\alpha\gamma+\beta)\sum_{i=1}^{n} K_i S(e_i) \\ & + \gamma^2\left[\sum_{i=1}^{n}(\alpha K_i)\,S(e_i) + U(\mathbf{q}) - U(\mathbf{q}_d) + \mathbf{e}^{\mathrm{T}}\mathbf{g}(\mathbf{q}_d)\right] \\ & + \beta\gamma\sum_{i=1}^{n} K_i[S(\upsilon_i) - S(\upsilon_i^*) - s(\upsilon_i^*)(\upsilon_i - \upsilon_i^*)]. \end{aligned} \tag{35}$$

Choosing $(\alpha\mathbf{K})$ large enough so that Eq. (11) is satisfied, makes the fourth term positive definite. Using the properties of functions $S(\cdot)$, $s(\cdot)$, and the fact that $\mathbf{s}(\boldsymbol{\upsilon}^*) = \boldsymbol{\upsilon}^*$ or $s(\upsilon_i^*) = \upsilon_i^*$, it is easy to show that the last term is also positive definite. If V_1 designates the sum of the second and third terms, then

$$\begin{aligned} V_1 = {} & \gamma(\alpha\gamma+\beta)\sum_{i=1}^{n} K_i S(e_i) - \frac{1}{2}(\alpha\gamma+\beta)^2\,\mathbf{s}(\mathbf{e})^{\mathrm{T}}\,\mathbf{M}(\mathbf{q})\,\mathbf{s}(\mathbf{e}) \\ \ge {} & \sum_{i=1}^{n}\gamma(\alpha\gamma+\beta)\,\lambda_m\{\mathbf{K}\}S(e_i) \\ & - \frac{1}{2}(\alpha\gamma+\beta)^2\,\lambda_M\{\mathbf{M}\}\,\|\mathbf{s}(\mathbf{e})\|^2 \\ = {} & (\alpha\gamma+\beta)\sum_{i=1}^{n}\psi_i, \end{aligned}$$

where

$$\psi_i = \gamma\lambda_m\{\mathbf{K}\}S(e_i) - \tfrac{1}{2}(\alpha\gamma+\beta)\,\lambda_M\{\mathbf{M}\}s(e_i)^2.$$

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The Lyapunov function (35) is positive definite if V_1 is positive definite. The latter is true provided that

$$e_i \left(\frac{\mathrm{d}\psi_i}{\mathrm{d}e_i} \right) > 0 \qquad \forall e_i \neq 0.$$

This requires

$$\lambda_m\{\mathbf{K}\} > \gamma^{-1} (\alpha\gamma + \beta) \lambda_M\{\mathbf{M}\}, \tag{36}$$

where we have used the facts

$$0 \leq \frac{\mathrm{d}s(e_i)}{\mathrm{d}e_i} \leq 1 \quad \text{and} \quad s(e_i) = \frac{\mathrm{d}S(e_i)}{\mathrm{d}e_i}.$$

The time derivative of the Lyapunov function (35) along trajectories of systems (1), (30), and (31) is given by

$$\begin{aligned}\dot{V} = {} & \tfrac{1}{2}\gamma^2 \dot{\mathbf{q}}^{\mathrm{T}} \dot{\mathbf{M}}(\mathbf{q}) \dot{\mathbf{q}} + \gamma^2 \dot{\mathbf{q}}^{\mathrm{T}} \{-C(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} - \mathbf{g}(\mathbf{q}) + \alpha \mathbf{K} \mathbf{s}(\mathbf{e}) \\ & - \mathbf{K}\dot{\mathbf{q}} + \beta \mathbf{K} \mathbf{s}(\boldsymbol{\upsilon})\} + \gamma(\alpha\gamma + \beta)\, \mathbf{s}(\mathbf{e})^{\mathrm{T}} \{C(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} \\ & + \mathbf{g}(\mathbf{q}) - \alpha \mathbf{K} \mathbf{s}(\mathbf{e}) + \mathbf{K}\dot{\mathbf{q}} - \beta \mathbf{K} \mathbf{s}(\boldsymbol{\upsilon})\} - \gamma(\alpha\gamma + \beta) \\ & \times \mathbf{s}(\mathbf{e})^{\mathrm{T}} \dot{\mathbf{M}}(\mathbf{q}) \dot{\mathbf{q}} + \gamma [\beta \mathbf{K} \mathbf{s}(\boldsymbol{\upsilon}) - \mathbf{g}(\mathbf{q}_d)]^{\mathrm{T}} [(\alpha\gamma + \beta) \\ & \times \mathbf{s}(\mathbf{e}) - \gamma \dot{\mathbf{q}}] + \gamma(\alpha\gamma + \beta) \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{B}(\mathbf{e}) \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}} \\ & - (2\alpha\gamma^2 + \beta\gamma) \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{K} \mathbf{s}(\mathbf{e}) + \gamma^2 \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{g}(\mathbf{q}) - \gamma^2 \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{g}(\mathbf{q}_d).\end{aligned}$$

Using Eqs. (5) and (6), $\dot{V}$ is reduced to

$$\begin{aligned}\dot{V} = {} & -\gamma^2 \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{K} \dot{\mathbf{q}} - \gamma(\alpha\gamma + \beta) \mathbf{s}(\mathbf{e})^{\mathrm{T}} [C(\mathbf{q}, \dot{\mathbf{q}})]^{\mathrm{T}} \dot{\mathbf{q}} \\ & + \gamma(\alpha\gamma + \beta) \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{B}(\mathbf{e}) \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}} - \gamma(\alpha\gamma + \beta) \mathbf{s}(\mathbf{e})^{\mathrm{T}} \\ & \times [\alpha \mathbf{K}] \mathbf{s}(\mathbf{e}) - \gamma(\alpha\gamma + \beta) \mathbf{s}(\mathbf{e})^{\mathrm{T}} [\mathbf{g}(\mathbf{q}_d) - \mathbf{g}(\mathbf{q})].\end{aligned}$$

Applying Eqs. (2), (4), (10), and (18), we obtain

$$\begin{aligned}\dot{V} \leq {} & -\{\gamma^2 \lambda_m\{\mathbf{K}\} - \gamma(\alpha\gamma + \beta)(\lambda_M\{\mathbf{M}\} + k_c\sqrt{n})\} \|\dot{\mathbf{q}}\|^2 \\ & - \varepsilon\gamma(\alpha\gamma + \beta) \|\mathbf{s}(\mathbf{e})\|^2.\end{aligned}$$

The time derivative of the Lyapunov function is negative semi-definite if

$$\lambda_m\{\mathbf{K}\} > \gamma^{-1} (\alpha\gamma + \beta)(\lambda_M\{\mathbf{M}\} + k_c\sqrt{n}). \tag{37}$$

Although $\dot{V}$ is negative semi-definite, we are still able to conclude global asymptotic stability. This follows directly from applying the LaSalle's invariance principle. Hence, the error converges to zero asymptotically.

From Eqs. (32), (34), (36), and (37), the gain $\mathbf{K}$ must be chosen such that

$$\begin{aligned}\lambda_m\{\mathbf{K}\} &= \lambda_m\{\mathbf{K}_1 + \phi^{-1}\mathbf{K}_2\} = \lambda_m\{\mathbf{K}_1\} + \phi^{-1}\lambda_m\{\mathbf{K}_2\} \\ &> \max\left\{ \frac{Q\rho}{\beta P}, \gamma^{-1}(\alpha\gamma + \beta)\left(\lambda_M\{\mathbf{M}\} + k_c\sqrt{n}\right) \right\}.\end{aligned} \tag{38}$$

Note that the product $(\alpha\mathbf{K})$ must satisfy Eqs. (10) and (11).

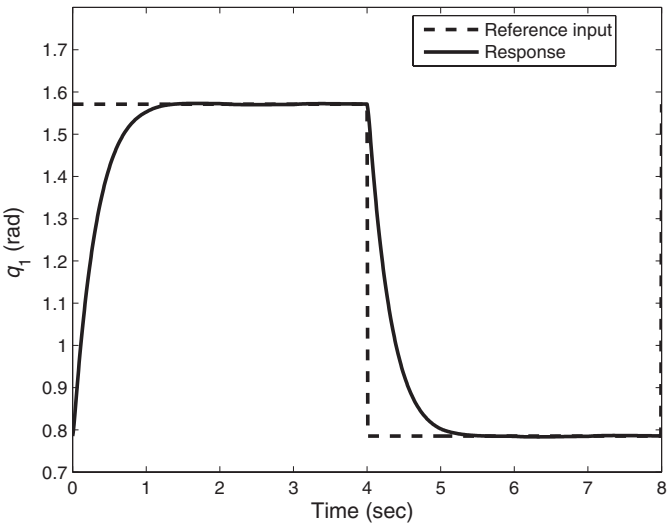


Fig. 1. The response of the first link to a step input.

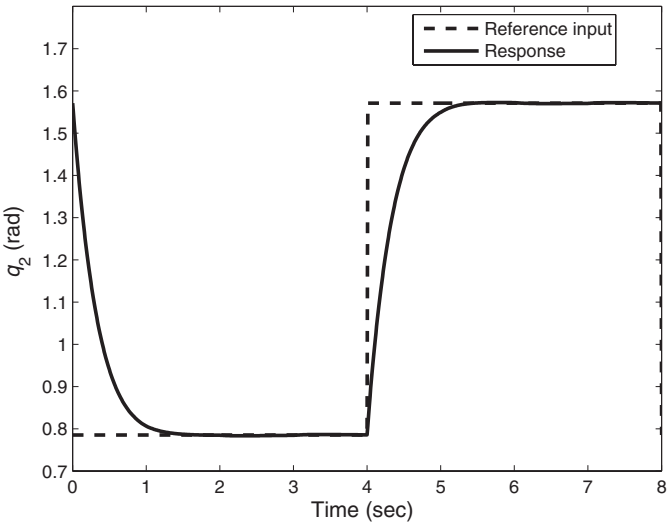


Fig. 2. The response of the second link to a step input.

5. Performance Analysis

Linear PID sliding surfaces may cause integrator windup and overshooting problems. These are critical in robot applications. The proposed nonlinear sliding surface overcomes the first problem by using the saturated integral function. Overshoots are also minimized by limiting the speed and the acceleration around the sliding surface (inside the boundary layer). In order to prove these statements, we introduce the following propositions.

Proposition 1. Inside the boundary layer ($\|\boldsymbol{\sigma}\| \leq \phi$), the norm of the speed satisfies

$$\|\dot{\mathbf{q}}\| \leq \phi + (\alpha + \beta)\sqrt{n}.$$

Proof.

$$\begin{aligned}\|\dot{\mathbf{q}}\| &= \|\dot{\mathbf{q}} - \{\alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon})\} + \{\alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon})\}\| \\ &\leq \|\dot{\mathbf{q}} - \alpha \mathbf{s}(\mathbf{e}) - \beta \mathbf{s}(\boldsymbol{\upsilon})\| + \|\alpha \mathbf{s}(\mathbf{e}) + \beta \mathbf{s}(\boldsymbol{\upsilon})\| \\ &\leq \|-\boldsymbol{\sigma}\| + \alpha\|\mathbf{s}(\mathbf{e})\| + \beta \|\mathbf{s}(\boldsymbol{\upsilon})\| \\ &\leq \phi + (\alpha + \beta)\sqrt{n},\end{aligned}$$

where Eq. (14) has been used.

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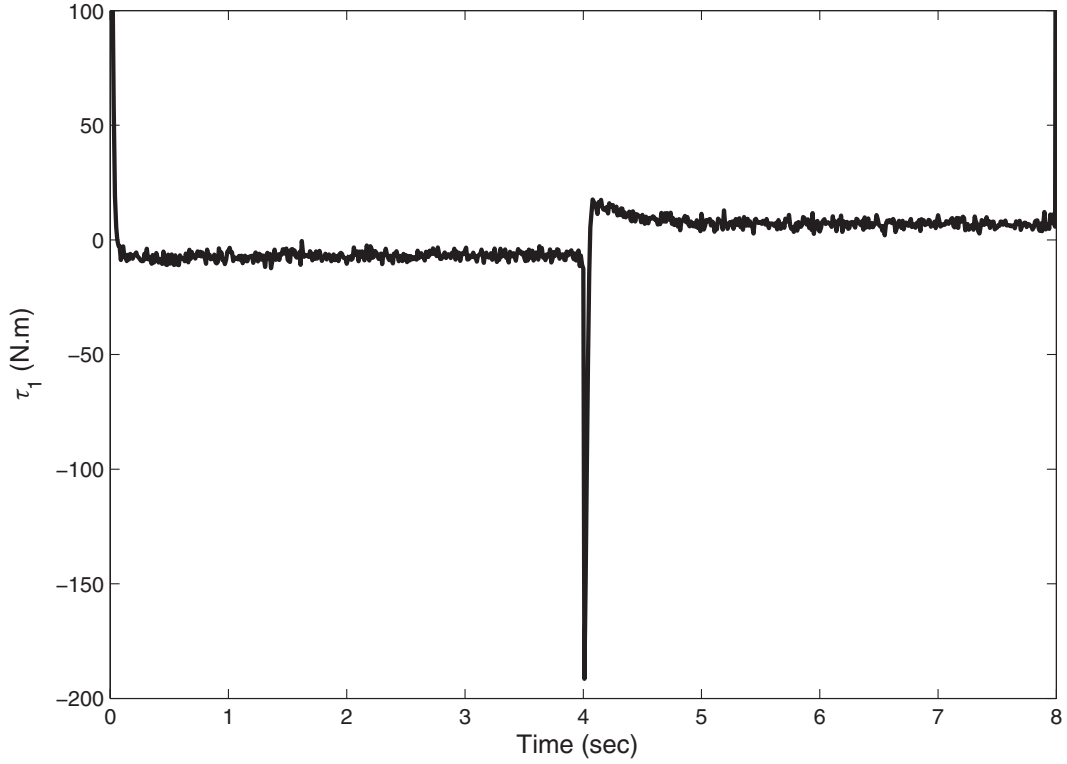


Fig. 3. The control signal of the first joint.

Proposition 2. Inside the boundary layer the acceleration is bounded.

Proof.

$$\begin{aligned}\|\ddot{\mathbf{q}}\| &= \left\|\mathbf{M}(\mathbf{q})^{-1}\left[\boldsymbol{\tau} - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q}) - \boldsymbol{\tau}_d\right]\right\| \\ &\leq \left\|\mathbf{M}(\mathbf{q})^{-1}\right\| \left\|\boldsymbol{\tau} - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q}) - \boldsymbol{\tau}_d\right\| \\ &\leq \left\|\mathbf{M}(\mathbf{q})^{-1}\right\| \left\{\|\boldsymbol{\tau}\| + \|\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\| + \|\mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_d\|\right\} \\ &\leq \left\|\mathbf{M}(\mathbf{q})^{-1}\right\| \left\{\|\mathbf{K}\|\,\|\boldsymbol{\sigma}\| + k_c\,\|\dot{\mathbf{q}}\|^2 + \rho\right\} \\ &\leq \left\|\mathbf{M}(\mathbf{q})^{-1}\right\| \left\{\lambda_M\{\mathbf{K}\}\phi + k_c(\phi + (\alpha+\beta)\sqrt{n})^2 + \rho\right\}.\end{aligned}$$

Since the mass matrix is bounded, the acceleration is also bounded.

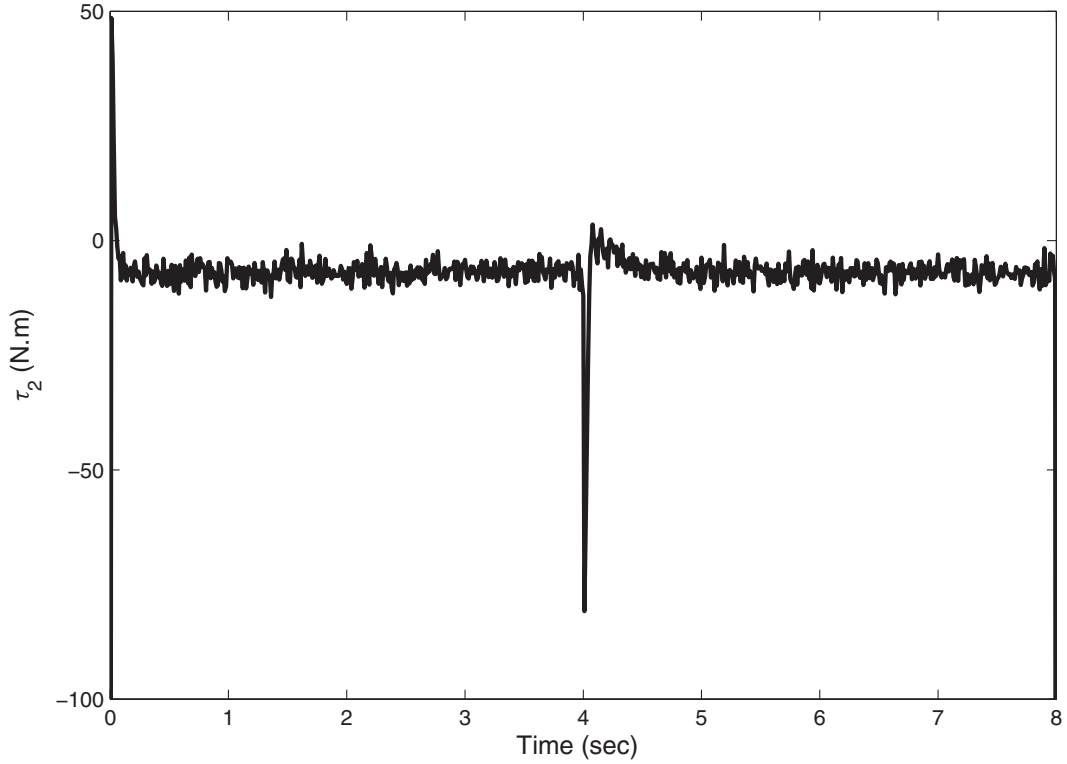


Fig. 4. The control signal of the second joint.

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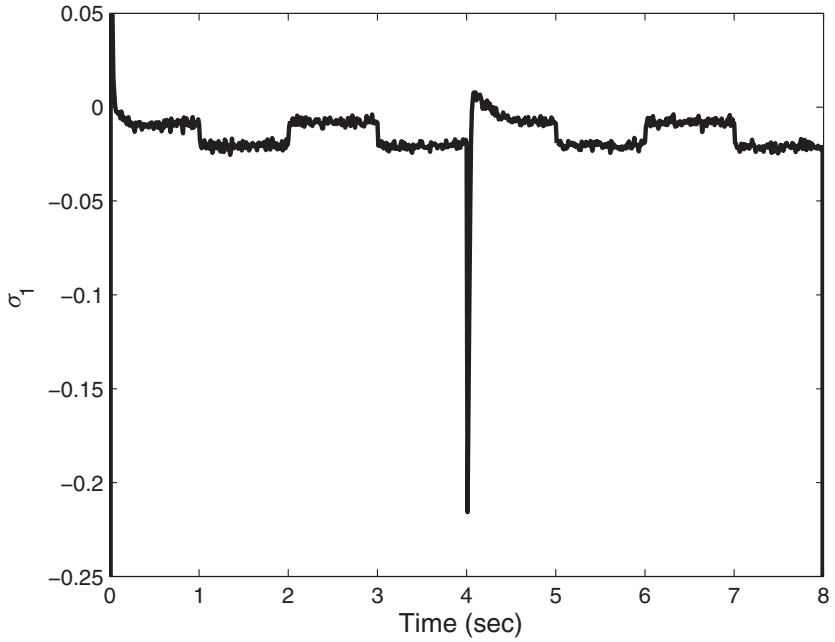


Fig. 5. The sliding variable of the first joint.

6. Numerical Example

In this section, an example of two degrees of freedom planar elbow robot with revolute joints is considered[21] and simulation results are presented. Assuming that $m_1 = m_2 = 1$ (kg) and $l_1 = l_2 = 1$ (m), the inertia matrix is defined by

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} 3 + 2Cos(q_1) & 1 + Cos(q_2) \\ 1 + Cos(q_2) & 1 \end{bmatrix}.$$

The Coriolis matrix has the following form

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -\dot{q}_2 Sin(q_2) & -(\dot{q}_1 + \dot{q}_2) Sin(q_2) \\ \dot{q}_1 Sin(q_2) & 0 \end{bmatrix}.$$

The gravity vector is defined by

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} 19.62Cos(q_1) + 9.81Cos(q_1 + q_2) \\ 9.81Cos(q_1 + q_2) \end{bmatrix}.$$

The above robot dynamics results in the following constants:

$$\lambda_M\{\mathbf{M}\} \approx 5.83,\ k_v \approx 31,\ k_g \approx 33.5,\ k_c = 0.71.$$

During simulations each link has been subjected to a set of step input signals. The disturbance applied to each joint is a random signal (mean is zero, variance is 3 (N^2 m^2))

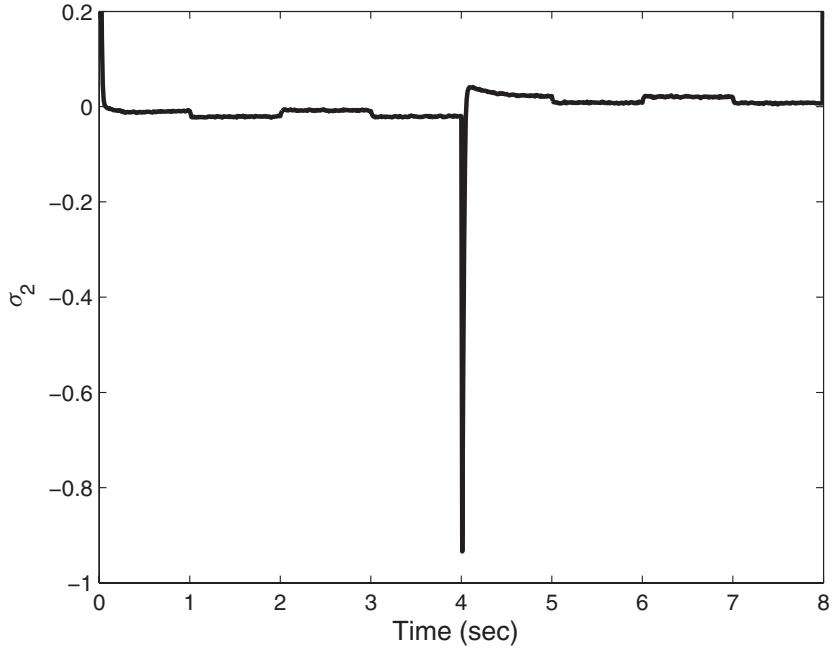


Fig. 6. The sliding variable of the second joint.

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plus a square wave signal of magnitude 3 N.m and frequency 0.5 Hz. The design parameters of the controller are

$$\lambda_m\{\mathbf{K}_1\} = \lambda_M\{\mathbf{K}_1\} = 42,\ \lambda_m\{\mathbf{K}_2\} = \lambda_M\{\mathbf{K}_2\} = 230,$$
$$\phi = 0.5,\ \alpha = \gamma\ =\ 3,\ \beta = 1,\ P = 3.$$

On applying the control law (29), the simulation results in the form of system responses and control signals are shown in Figs. 1–4. Figures 5 and 6 show how the sliding variables σ_1 and σ_2 change along the time. The results show that there is no chattering in the responses or the control signals. It is also clear that the performance is uniform and overshoots are negligible. It is found that the parameter β should be small to minimize overshoots.

7. Conclusions

In this paper, a new set-point controller for robot manipulators based on sliding mode control has been proposed. The control law has used a novel nonlinear PID sliding surface. In preventing the chattering phenomena by the boundary layer approach, a nonlinear PID controller has been found inside the boundary layer. The stability analysis of the robot dynamics under this PID controller has been performed. Analysis of the robot motion under the proposed controller has been performed by simulation.

References

1. Y. Stepanenko and C. Su, "Variable structure control of robot manipulators with nonlinear sliding manifolds," *Int. J. Control* **58**(2), 285–300 (1993).
2. J. Slotine and S. Sastry, "Tracking control of non-linear systems using sliding surfaces, with applications to robot manipulators," *Int. J. Control* **38**(2), 465–492 (1983).
3. P. Muraca and P. Pugliese, "A variable-structure regulator for robotic systems," *Automatica* **33**(7), 1423–1426 (1997).
4. N. Kim, C. Lee and P. Chang, "Sliding mode control with perturbation estimation: Application to motion control of parallel manipulator," *Control Eng. Practice* **6**(11), 1321–1330 (1998).
5. Y. Li, B. Eriksson and J. Wikander, "Sliding Mode Control of Two-Mass Positioning Systems," *Proceedings of the 14th IFAC Triennial World Congress on Automatic Control*, Beijing, China (1999).
6. Y. Stepanenko, Y. Cao and C. Su, "Variable structure control of robotic manipulator with PID sliding surfaces," *Int. J. Robust Nonlinear Control* **8**, 79–90 (1998).
7. I. Eker, "Sliding mode control with PID sliding surface and experimental application to an electromechanical plant," *ISA Trans.* **45**(1), 109–118 (2006).
8. J. Meza, V. Santibaez and V. Hernandez, "Saturated Nonlinear PID Global Regulator For Robot Manipulators: Passivity Based Analysis," *Proceedings of 16th IFAC World Congress on Automatic Control*, Czech Republic (2005).
9. J. Kasac, B. Novakovic, D. Majetic and D. Brezak, "Performance Optimization of Saturated PID Controller for Robot Manipulators," **In**: *Proceedings of the 10th IEEE International Conference on Methods and Models in Automation and Robotics* (Miedzyzdroje, Poland, 2004) pp. 843–848.
10. R. Kelly, "Global positioning of robot manipulators via PD control plus a class of nonlinear integral actions," *IEEE Trans. Autom. Control* **43**(7), 934–938 (1998).
11. J. Kasac, B. Novakovic, D. Majetic and D. Brezak, "Performance Tuning for a New Class of Globally Stable Controllers for Robot Manipulators," *Proceedings of 16th IFAC World Congress on Automatic Control* (Czech Republic, 2005).
12. J. Alvarez-Ramirez, R. Kelly and I. Cervantes, "Semiglobal stability of saturated linear PID control for robot manipulators," *Automatica* **39**(4), 989–995 (2003).
13. I. Cervantes and J. Alvarez-Ramirez, "On the PID tracking control of robot manipulators," *Syst. Control Lett.* **42**, 37–46 (2001).
14. A. Loria, E. Lefeber and H. Nijmeijer, "Global asymptotic stability of robot manipulators with linear PID and PI^2 D control," *SACTA* **3**(2), 138–149 (2000).
15. S. Arimoto, *Control Theory of Nonlinear Mechanical Systems: A Passivity-Based and Circuit-Theoretic Approach* (Oxford University Press, Oxford, UK, 1996).
16. R. Ortega, A. Loria, P. Niclasson and H. Sira-Ramirez, *Passivity-Based Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications* (Springer-Verlag, London, 1989).
17. M. Spong and M. Vidysagar, *Robot Dynamics and Control* (Wiley, New York, 1998).
18. H. Yazarel, C. Cheah and H. Liaw, "Adaptive SP-D control of robotic manipulator in the presence of modeling error in a gravity regressor matrix: theory and experiment," *IEEE Trans. Robot. Autom.* **18**(3), 373–379 (2002).
19. S. Kirchoff and W. Melek, "A saturation-type robust controller for modular manipulators arms," *Mechatronics* **17**, 175–190 (2007).
20. M. Zeinali and L. Notash, " Adaptive sliding mode control with uncertainty estimator for robot manipulators," *Mech. Mach. Theory* **45**, 80–90 (2010).
21. F. Lewis, C. Abdallah and D. Dawson, *Control of Robot Manipulators* (Maxwell Macmillan, New York, 1993).

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