Aircraft gust load estimation due to atmospheric turbulence under different flight conditions

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ABSTRACT

Based on power spectral technique and Lyapunov approach, methodology to determine the vertical gust load on aircraft encountering atmospheric turbulence under different flight conditions is presented in this paper. Modified longitudinal short period aircraft equations of motion to reflect gust inputs are solved. Family of five linear dynamics models of increasing gust excitation complexity are developed to describe the normal load factor throughout an aircraft due to vertical gust. These models (except Model 2) give a rapid estimation of normal load factor in case complete data are not readily available. Numerical model constructed for a Boeing 747 jet transport is solved to illustrate the results. These results show that Model 5 exhibits higher frequency contents when compared with other models under different flight conditions. The normal load factor of aircraft is estimated at different probabilities of not exceeding the corresponding load factor value based on statistical technique. The Models 1, 3, 4 and 5 predict the load factor with maximum 5% error when compared with Model 2 which considered all gust penetration effects. Finally, the results show a good agreement with the published work in load factor determination, at different probabilities of not exceeding this value when encountering a turbulent vertical gust.

NOMENCLATURE

a_n	normal acceleration
[**A**]	plant matrix
{**B**}	control vector matrix
d	gradient distance (cf. FAR 25)
$E(x^2)$	the expectation for the mean square of x
FDF	frequency-dependent function
g	gravity
H	height
G_{nw_g}	aircraft transfer function
G_{wg}	Dryden vertical gust transfer function
L_2	the scale of vertical turbulence gust
M	Mach number
M_q	pitch damping
M_α	longitudinal stability derivative
$M_{\dot{\alpha}}$	pitching moment due to angle-of-attack rate
M_δ	pitching moment due to elevator
n	the white noise input
PSD	power spectral density
q	pitch rate
T	time
V	aircraft velocity

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$x(t)$ original time-varying random signal or function
w_g vertical gust velocity
x state variables
Z_q normal force due to pitch rate
$Z_{\dot{\alpha}}$ normal force due to angle-of-attack rate
Z_α normal force due to angle-of-attack
Z normal force due to elevator
α_g angle-of-attack reflect gust input
δ control input
ϕ_{w_g} power spectral density
ϕ_{nw_g} aircraft normal load factor with σ_w = 0·328m/s^{-1} (1ft /sec)
σ_w root mean square (RMS) of stochastic gust.
σ_n normal load factor

$$\omega = \frac{\pi V}{d}$$

1.0 INTRODUCTION

Disturbances due to natural or artificial nonuniformity of the wind and pressure fields are a frequent occurrence during regular operation of aircraft and missiles[1]. The effects of atmospheric turbulence on many of the modern sophisticated technological systems have become an important design parameter from both structural and performance aspects. Aircraft encounters with turbulence represent a serious safety threat for airlines. Often in aircraft model simulation development, the gust effects of the atmosphere are neglected for various reasons and removed in the final form of the equations. Here, gust effects are the key excitation of interest. Turbulence refers to an irregular or disturbed flow in atmosphere that produces gusts and eddies. It may be caused by many factors such as storms and thunderstorms that generate powerful and unexpected gusts, and high winds blowing over or through mountains, and other obstructions.

The studies of gust loading and gust response were conducted. Etkin[2] described how the models of the wind have to accommodate two sorts of events that are perceived as discrete gust, as well as the phenomenon described as continuous turbulence.

Schiess[3] discussed the application of three statistical methods in combination to model wind gusts for use in aircraft flight simulation. The approach combined principle components analysis, time series analysis and probability distribution methods to analyze and simulate the wind gust components.

Kim, Kabe and Lee[4] presented a new Monte Carlo atmospheric flight gust loads analysis approach. The procedure used forcing functions that were derived by extracting the turbulent, short-duration, non-persistent components of measured wind profiles. Buck, Bowles and Newman[5] predicted aircraft transient motion resulting from flight within a turbulent atmospheric environment, coupled with validation using flight test data. Linear dynamic models were developed for describing the normal acceleration throughout an aircraft cabin due to vertical gust excitation.

The main statistical features that need to be considered were: stationary, homogeneity, probability distribution, and correlations and spectra. The power spectral approach offered a more realistic representation of the nature of atmospheric turbulence, and it allowed more rational consideration of design and operational variations such as configuration changes, mission changes and aircraft degrees of freedom.

The object of this paper is to determine the normal load factor of aircraft under the excitation of turbulence atmosphere, based on spectral technique and Lyapunov approach.

Family of five linear dynamics models is developed to determine the gust load and describe the aircraft response due to vertical gust effect under different flight conditions. To the best of author's knowledge, no attempts have been made to determine the gust load and investigate the effect of atmospheric turbulence on aircraft response with these various models for the base line Boeing 747 jet transport aircraft model. Both the application and the models are novel.

2.0 AERODYNAMICS MODEL AND STABILITY DERIVATIVES OF SELECTED AIRCRAFT

The aircraft selected as a model in this research work is the Boeing 747 jet transport. The Boeing 747 is very large four-fanjet intercontinental transport designed to operate from existing international airports. The selected aircraft operates with M = 0·5, 0·65 and 0·8 at altitudes of 6,096m (20,000ft) and with M = 0·7, 0·8 and 0·9 at altitudes of 12,192m (40,000ft). The airframe fixed-co-ordinate systems, geometrical configuration, inertia properties and aerodynamic model are a hybrid from Refs 6 and 7. The values of aerodynamics model and the stability derivatives are shown in Table 1(a). The aerodynamic model is based on representative experimental data[6].

In an attempt to better understand the nature of atmospheric turbulence, to provide data through which mathematical modeling of turbulence may be made, and to improve means for treating the response of aircraft in turbulent air, many experimental studies have been made to predict and measure the vertical gust velocity in various circumstances, using aircraft probing (such as NASA probe used in flight measurement of turbulence[11] and NASA B757-200 research aircraft[5]). In current work, the aircraft is excited under different values of turbulent vertical gust (σ_w = 6·7, 7·3 and 7·9m/s^{-1}) that are detected experimentally from Refs 2 and 8 under different flight condition.

3.0 MODIFIED AIRCRAFT EQUATIONS OF MOTION TO REFLECT GUST INPUT

The use of the short period dynamic model will provide an insight as to import of increasing the airframe degrees of freedom when representing the airframe dynamics. The simplified set of short period equations of motion can be expressed as[9]

$$\begin{aligned}(V - Z_{\dot{\alpha}})\dot{\alpha} &= Z_\alpha \alpha + (V + Z_q)q + Z_\delta \delta \\ -M_{\dot{\alpha}}\dot{\alpha} + \dot{q} &= M_\alpha \alpha + M_q q + M_\delta \delta\end{aligned} \quad \ldots (1)$$

The two coupled linear Equations (1) will be restated as functions solely of and by the use of algebraic substitutions, i.e.,

$$\dot{\alpha} = \frac{Z_\alpha}{(V - Z_{\dot{\alpha}})}\alpha + \frac{V + Z_q}{(V - Z_{\dot{\alpha}})}q + \frac{Z_\delta}{(V - Z_{\dot{\alpha}})}\delta \quad \ldots (2)$$

$$\dot{q} = \left[M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{(V - Z_{\dot{\alpha}})}\right]\alpha + \left[M_q + \frac{(V + Z_q)M_{\dot{\alpha}}}{(V - Z_{\dot{\alpha}})}\right]q + \left[M_\delta + \frac{M_{\dot{\alpha}} Z_\delta}{(V - Z_{\dot{\alpha}})}\right]\delta$$

A further simplification can be made by recognising that both $Z_{\dot{\alpha}}$ and Z_q are nearly zero in magnitude. Equation (2) becomes

$$\begin{aligned}(\dot{\alpha} &= \frac{Z_\alpha}{V}\alpha + q + \frac{Z_\delta}{V}\delta \\ \dot{q} &= \left[M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{V}\right]\alpha + \left[M_q + M_{\dot{\alpha}}\right]q + \left[M_\delta + \frac{M_{\dot{\alpha}} Z_\delta}{V}\right]\delta\end{aligned} \quad \ldots (3)$$

The longitudinal model (short period response) modified to reflect gust inputs of $\alpha_g(t)$ and $q_g(t)$ in place of control inputs δ. Equation (3) becomes

$$\dot{\alpha} = \frac{Z_\alpha}{V}\alpha + q + \frac{Z_\alpha}{V}\alpha_g + 0\, q_g \quad \ldots (4)$$

$$\dot{q} = \left[M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{V}\right]\alpha + \left[M_q + M_{\dot{\alpha}}\right]q + \left[M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{V}\right]\alpha_g + M_q\, q_g$$

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Table 1(a)
Aerodynamics properties and stability derivatives of Boeing 747 aircraft with W = 288,757kg, S = 510·966m^2 and c = 8·324m

Height [m]	Mach	$C_{L\alpha}$ [rad^{-1}]	$C_{m\alpha}$ [rad^{-1}]	$C_{m\dot{\alpha}}$ rad^{-1}]	M_α [s^{-2}]	$M_{\dot{\alpha}}$ [s^{-1}]	Z_α/V [s^{-1}]	M_q [s^{-1}]
6,096	0.50	4·68	–1·1555	–3·33	–0·891	–0·067	–0·426	–0·421
6,096	0·65	4·63	–0·9555	–4·00	–1·248	–0·106	–0·550	–0·535
6,096	0·80	4·39	–0·6000	–5·33	–1·187	–0·173	–0·641	–0·669
12,192	0·70	5·07	–1·1550	–4·88	–0·708	–0·060	–0·280	–0·284
12,192	0·80	4·97	–1·0200	–6·44	–0·819	–0·091	–0·316	–0·339
12,192	0·90	5·65	–1·6000	–8·33	–1·626	–0·132	–0·404	–0·401

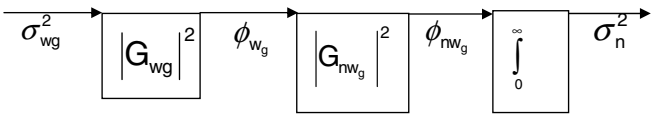


Figure 1. Load factor response to a vertical gust input.

$$M'_\alpha = M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{V} \qquad M'_q = M_q + M_{\dot{\alpha}}$$

Substituting the values of M'_α and M'_q in Equation (4)

$$\dot{\alpha} = \frac{Z_\alpha}{V}\alpha + q + \frac{Z_\alpha}{V}\alpha_g + 0\,q_g$$
$$\dot{q} = M'_\alpha\,\alpha + M'_q\,q + M'_\alpha\alpha_g + M_q\,q_g \qquad \ldots (5)$$

The longitudinal equations of motion can be written in state space form as follow

$$\begin{Bmatrix}\dot{\alpha}\\ \dot{q}\end{Bmatrix} = \begin{bmatrix}Z_\alpha/V & 1\\ M'_\alpha & M'_q\end{bmatrix}\begin{Bmatrix}\alpha\\ q\end{Bmatrix} + \begin{Bmatrix}Z_\alpha/V\\ M'_\alpha\end{Bmatrix}\alpha_g + \begin{Bmatrix}0\\ M_q\end{Bmatrix}q_g \qquad \ldots (6)$$

The normal acceleration output is given by

$$\Delta n_Z = -Z_\alpha(\alpha + \alpha_g)/g \qquad \ldots (7)$$

4.0 AIRCRAFT RESPONSE MODELS TO RANDOM GUST

The power spectrum represents a frequency viewpoint for describing the square of random variable that is originally considered in time domain. The power spectral will be denoted by $\phi(\omega)$.Thus, it is indicated to be a function of the radian frequency ω. If $\phi(\omega)$ is the PSD of the quantity $(y)t$, then $\phi(\omega)d\omega$ is the contribution to $\bar{y}^2$ or, σ_y^2 of frequencies between ω and $d\omega$.

The aircraft normal load factor, in response to a turbulent vertical gust may be found by the series application by Dryden vertical gust model's transfer function $|G_{wg}|^2$ to the aircraft transfer function of normal load factor to vertical gust input $|G_{nwg}|^2$. This statement can be shown in Fig. 1. The expectation of the normal load factor response is obtained by integrating the power spectral density $(\int_0^\infty \phi_{nwg}(\omega)\,d\omega)$. The Dryden vertical gust model may be expressed in a transfer function format as Refs (9, 12)

$$\phi_{w_g}(\omega) = \left|G_{w_g}(\omega)\right|^2 \sigma_w^2 \text{ and } G_{w_g}(\omega) = G_{w_g}(s)\big|_{s=i\omega} \qquad \ldots (8)$$

The transfer function $G_{wg}(s)$ can be expressed in terms of the Laplace transform variable as follows,

$$G_{w_g}(s) = K\frac{(s+\theta)}{(s+\lambda)^2} \qquad \ldots (9)$$

Where, $K = (3V/\pi L_2)^{1/2}$, $\theta = V/\sqrt{3}\,L_2$ and $\lambda = V/L_2$

The aircraft longitudinal response is based on short period approximation where it is noted that $\alpha = w/V$ and $\alpha_g = w_g/V$. The state variable form,

$$\dot{x} = \mathbf{A}x + \mathbf{B}w_g \text{ and}$$
$$a_n = \mathbf{C}x + \mathbf{D}w_g \qquad \ldots (10)$$

Where $\mathbf{A} = \begin{bmatrix}\frac{Z_\alpha}{V} & V\\ M'_\alpha & M'_q\end{bmatrix}$, $\mathbf{B} = \begin{bmatrix}\frac{Z_\alpha}{V} & \frac{M'_\alpha}{V}\end{bmatrix}^T$, $\mathbf{C} = \begin{bmatrix}\frac{Z_\alpha}{V} & 0{\cdot}0\end{bmatrix}$,

$\mathbf{D} = \begin{bmatrix}\frac{Z_\alpha}{V}\end{bmatrix}$ whereas $\{x\} = [w\ q]^T$.

The transfer function of $G_{nwg}(s)$ becomes

$$G_{nw_g}(s) = \mathbf{C}[\mathbf{sI} - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D} \qquad \ldots (11)$$

which leads to PSD as:

$$\phi_{nw_g}(\omega) = \left|G_{nw_g}(s)\right|^2_{s=i\omega} \qquad \ldots (12)$$

and the output PSD as:

$$\phi_n(\omega) = \phi_{w_g}(\omega)\,\phi_{nw_g}(\omega) \qquad \ldots (13)$$

Finally, the normal load factor is obtained by the integration of the output power spectral density

$$E(n_z^2) = \int_0^\infty \phi_n(\omega)\,d\omega \qquad \ldots (14)$$

5.0 LYAPUNOV EQUATION USAGE FOR AIRCRAFT LOAD FACTOR DETERMINATION

The frequency variation of the power spectral density function provided information concerning the spectral content of the gust input and the load factor output. In additional to providing a perspective of the frequency content, clues were also given as to whether the rigid-airframe assumption should be modified to include the influence of the structural dynamic responses. Lyapunov equation is an alternate approach to determine the respective variances without numerical integration. Also the state space

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Table 1(b)
Model assumption

Short period PSD concept Model 1	Short period PSD concept Model 2	Short period PSD concept Model 3	Lyapunov's approach Model 4	Lyapunov's approach Model 5
$M_{\dot{\alpha}} = 0{\cdot}0$	$M_{\dot{\alpha}} \neq 0{\cdot}0$	$M_{\dot{\alpha}} \neq 0{\cdot}0$	$M_{\dot{\alpha}} \neq 0{\cdot}0$	$M_{\dot{\alpha}} \neq 0{\cdot}0$
$q_g = 0{\cdot}0$	$q_g \neq 0{\cdot}0$	$q_g = 0{\cdot}0$	$q_g = 0{\cdot}0$	$q_g = 0{\cdot}0$

technique was used in present work to obtain the covariance matrix directly. The Lyapunov equation can be expressed as[9]

$$\mathbf{A}\,\mathbf{E}[x\,x^T] + \mathbf{E}[x\,x^T]\,\mathbf{A}^{\mathbf{T}} + \mathbf{B}\,\sigma^2\,\mathbf{B}^{\mathbf{T}} = [0] \qquad \ldots (15)$$

The product of two vectors will result in square matrix:

$$x\,x^T = \underbrace{\{x\}}_{(n*1)}\ \underbrace{[x]}_{(1*n)} = \underbrace{[x\,x^T]}_{(n*n)}$$

The Dryden turbulence model for vertical gust has been put into transfer function format and used as an input when making an estimate of the aircraft load factor spectral response. The transfer function, when considered in the time domain, involves a second-order differential equation, i.e.,

$$\ddot{w}_g + 2\lambda\,\dot{w}_g(t) = K[\dot{n} + \theta\,n(t)] \qquad \ldots (16)$$

Equation (16) can be converted to state-space format by several methods. The following approach is in accord with Ref. 10 and requires the inclusion of an auxiliary state variable due to the presence of the $\dot{n}$ term in the input. Assume

$$y_1 = w_g - \beta_o\,n$$

$$y_2 = \dot{w}_g - \beta_o\,\dot{n} - \beta_1\,n = \dot{y}_1 - \beta_1\,n$$

Taking the time derivative of y_2 and including the statement of Equation (16) yields

$$\dot{y}_2 = \ddot{w}_g - \beta_o\,\ddot{n} - \beta_1\,\dot{n} = (-\lambda^2\,w_g - 2\lambda\,\dot{w}_g + K\,\dot{n} + K\,\theta\,n) - \beta_o\,\ddot{n} - \beta_1\,\dot{n}$$

Consideration of these three relations leads to

$$\beta_o = 0, \qquad \beta_1 = K, \qquad w_g = y_1, \qquad \dot{w}_g = y_2 + K\,n$$

This provides the two state equations

$$\begin{aligned} \dot{y}_1 &= y_2 + K\,n \\ \dot{y}_2 &= -\lambda^2\,y_1 - 2\lambda\,y_2 + K(\theta - 2\lambda)\,n \end{aligned} \qquad \ldots (17)$$

Or in matrix notation,

$$\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & -2\lambda \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \begin{Bmatrix} K \\ K(\theta - 2\lambda \end{Bmatrix} n \qquad \ldots (18)$$

The use of Lyapunov's equation removes the integration limit difficulty, but requires a conversion of the transfer function into state-variable format.

Lyapunov's equation may be extended to the variance estimation of aircraft load factor by appropriate partitioning of the state-space relations.

The y_1 and y_2 components of Equation (18) for Dryden vertical gust model are recognised as

$$\mathbf{Y} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} w_g \\ y_2 \end{Bmatrix}$$

and Equation (18) may be expressed as

$$\dot{\mathbf{Y}} = \mathbf{F}\,\mathbf{Y} + \mathbf{G}\,n(t) \qquad \ldots (19)$$

where **F** and **G** corresponding matrices of Equation (18).

A suitable form of the short-period of an aircraft can be obtained from a variation of Equation (3) with α replaced by V/W. In terms of $w_g\,(= y_1)$ and y_2, the short-period equation becomes

$$\begin{Bmatrix} \dot{w} \\ \dot{q} \end{Bmatrix} = \begin{bmatrix} Z_\alpha / V & V \\ M'_\alpha / V & M'_q \end{bmatrix} \begin{Bmatrix} w \\ q \end{Bmatrix} + \begin{bmatrix} Z_\alpha / V & 0 \\ M'_\alpha / V & 0 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

And may be expressed symbolically as

$$\dot{\mathbf{X}} = \mathbf{A}\,\mathbf{X} + \mathbf{B}\,\mathbf{Y} \qquad \ldots (20)$$

Equations (19) and (20) are coupled by virtue of the **Y** state vector and combined by partitioning as follows:

$$\dot{\mathbf{Z}} = \begin{Bmatrix} \dot{\mathbf{X}} \\ \dot{\mathbf{Y}} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \begin{Bmatrix} \mathbf{X} \\ \mathbf{Y} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{G} \end{Bmatrix} n(t) \text{ or in symbolic format}$$

$$\dot{\mathbf{Z}} = \mathbf{H}\,\mathbf{Z} + \mathbf{M}\,n(t) \qquad \ldots (21)$$

The normal acceleration response for aircraft is

$a_n = (Z_\alpha / V)\,w + (Z_\alpha / V)\,w_g$ which result in

$$a_n = \mathbf{C}\,\mathbf{Z} + \mathbf{D}\,n \qquad \ldots (22)$$

where $\mathbf{C} = \begin{bmatrix} (Z_\alpha / V & 0 & Z_\alpha / V & 0 \end{bmatrix}$ and $\mathbf{D} = 0$

When the output from Lyapunov's equation is the covariance matrix **Q**, i.e.,

$$\mathbf{Q} = \mathbf{E}[\mathbf{Z}\,\mathbf{Z}^{\mathbf{T}}]$$

Then the variance of the normal acceleration is given by

$$\sigma_n^2 = \mathbf{C}\,\mathbf{Q}\,\mathbf{C}^{\mathbf{T}} \qquad \ldots (23)$$

Five models with different gust excitation complexities are used in present work. The assumptions used in each models are presented in Table 1(b). Models 1 and 5 are the simplest models for short period, and Lyapunov approach. As shown, the value of $M_{\dot{\alpha}}$ (part of aerodynamic damping), and the pitch rate gust signal q_g are zero. Models 3 and 4 are considered the effects of $M_{\dot{\alpha}}$ and ignore the pitch rate gust signal in two approaches while Model 2, accounts all gust penetration effects of the aircraft in short period response.

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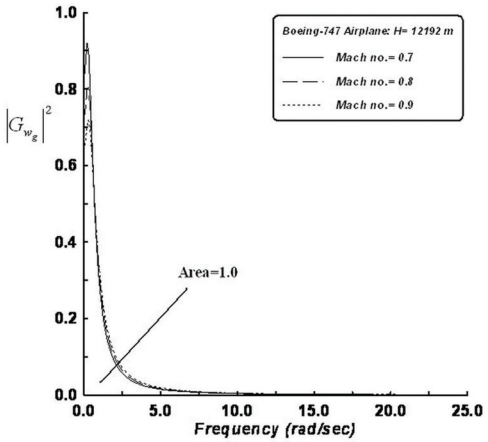


Figure 2(a). Dryden vertical gust input at M = 0·5, 0·65 and 0·8, H = 6,096m.

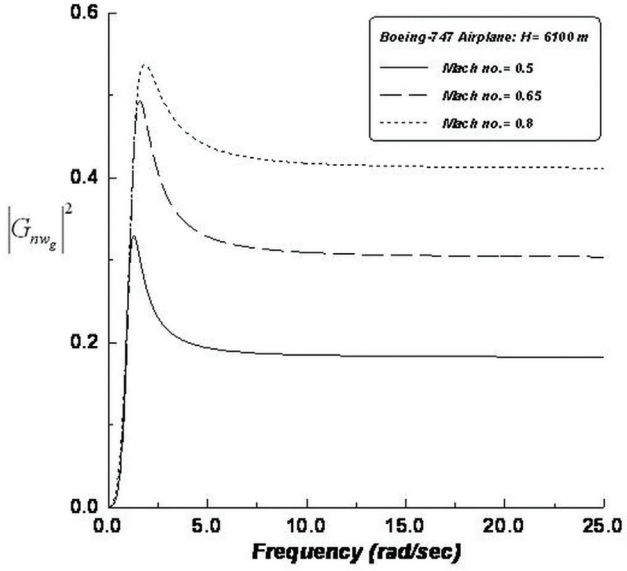


Figure 2(b). Aircraft transfer function at M = 0·5, 0·65 and 0·8, H = 6,096m.

6.0 NUMERICAL SIMULATION AND DISCUSSION

Given the aerodynamics model and stability derivatives in Table 1(a) and Ref. 6, methodology to determine the vertical gust load on Boeing 747 aircraft encountering continuous atmospheric turbulence under different flight conditions is presented in current work. The random process can be modeled by an appropriate power spectral density. The transfer function approach will be applied here to determine the aircraft gust response based on short period approximation. The area under the power spectrum curve represents the mean square of load factor. The numerical estimation for σ_w (normalised input) and σ_n (output) were determined by using trapezoidal integral approximation for finite frequency range $0 < \omega < 25$. The normalised gust transfer function Equation (8) would yield to = 0·382m/s^{-1}(1ft/sec) if ω_{max} were infinite; however, the frequency truncation results in estimation, which corresponds to 0·6% error.

Figure 2(a) shows the spectral representation of Dryden vertical gust model when normalised to unit area at mentioned Mach with

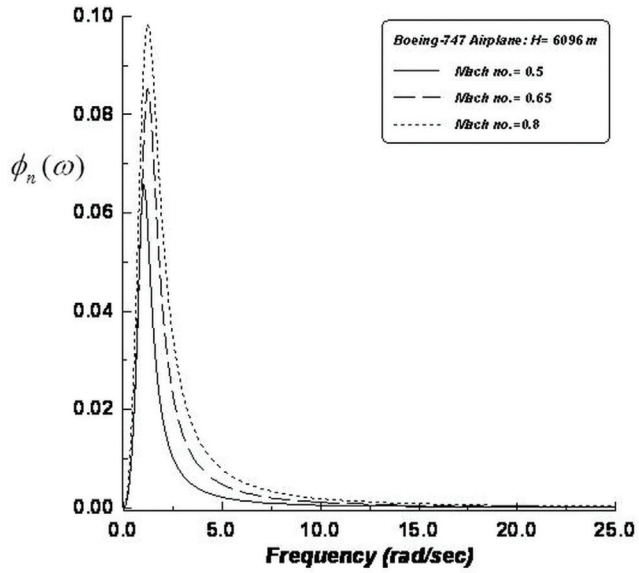


Figure 2(c). Aircraft normal load factor: H = 6,096m $\phi(\omega)$ for σ = 0·328m/s^{-1} (1ft/sec).

altitude H = 6,069m. The aircraft normal load factor transfer function due to vertical gust input is shown in Fig. 2(b). Figure 2(c) represents the frequency distribution of aircraft normal acceleration (product of $|G_{wg}(\omega)|^2$ $|G_{nw}(\omega)|^2$).

The three-sigma values for aircraft normal load factor estimation are 0·622g, 0·8036g and 0·9468g at Mach = 0·5, 0.65 and 0·8 respectively (H = 6,096m) with the probability of 99·7% not exceeding this value when encountering a turbulent vertical gust at variance σ_w = 6·7m/s^{-1} (Fig. 2). In addition the spectral of Dryden vertical gust model when normalised to unit area, aircraft normal load factor transfer function due to vertical gust input and the frequency distribution of aircraft normal acceleration at Mach = 0·7, 0·8 and 0·9 with altitude H = 12,192m are shown in Figs 3(a), (b) and (c), respectively. The values of three-sigma load factor are 0·5219g, 0·5781g and 0·7151g with σ_w = 6·7m/s^{-1} (depending on Mach and altitude of Fig. 3).

The five models presented in this paper to estimate the load factor of Boeing 747 which is excited by vertical gust under different flight conditions have been considered. To show the reliability of suggested models, a comparison between the Models 1, 3, 4 and 5 with Mode 2 (which takes into accounts all gust penetration effects of the aircraft in short period response) has been made. The results of estimated load factor of related models are shown in Figs 4, 5, 6 and 7 with probability 99·2% and 99·7% of not exceeding the corresponding load factor. For more details, the difference in load factor values for suggested models when compared with Model 2 at H = 6,096m and H = 12,192m are shown in Tables 2(a), (b). The numerical results show that Model 5 exhibits higher frequency contents and gives a rapid estimation of normal load factor in case the complete data are not readily available.

To validate the numerical results, a comparison between the present work and data Ref. 9 was made to determine the load factor for Lockheed aircraft when operating at M = 0·75 and altitude 6,096m (20,000ft).The results show a good agreement with 1·6% error. These verification results can be shown in Fig. 8.

To emphasise the importance of gust load intensity and its effect, the aircraft was excited by different values of gust input which are mentioned in Refs 2 and 8.

The results show that the load factor and σ_w are directly related under different flight conditions. This effect is shown in Fig. 9 and Fig. 10 with probability 99·7% of not exceeding load factor. Based

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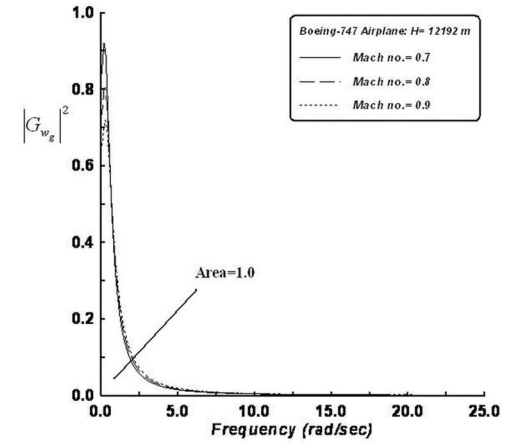


Figure 3(a). Dryden vertical gust input at M = 0·7, 0·8 and 0·9, H = 12,192m.

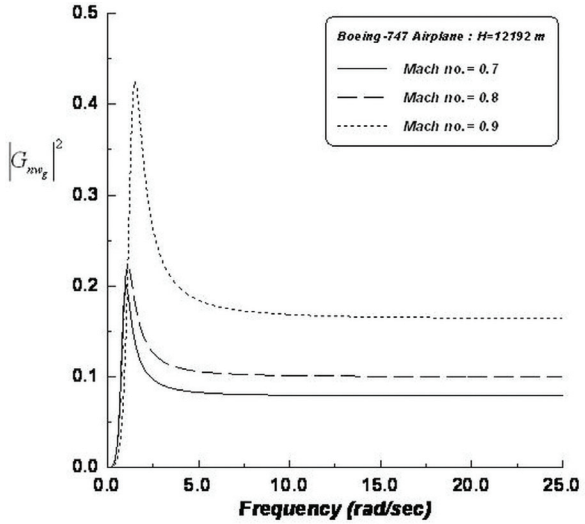


Figure 3(b). Aircraft transfer function at M = 0·7, 0·8 and 0·9, H = 12,192m.

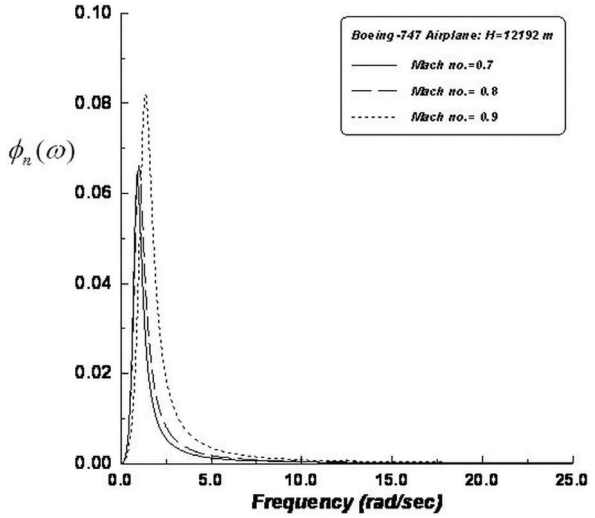


Figure 3(c). Aircraft normal load factor: H = 12,192m $\phi(\omega)$ for σ = 0·328m/s^{-1} (1ft/sec).

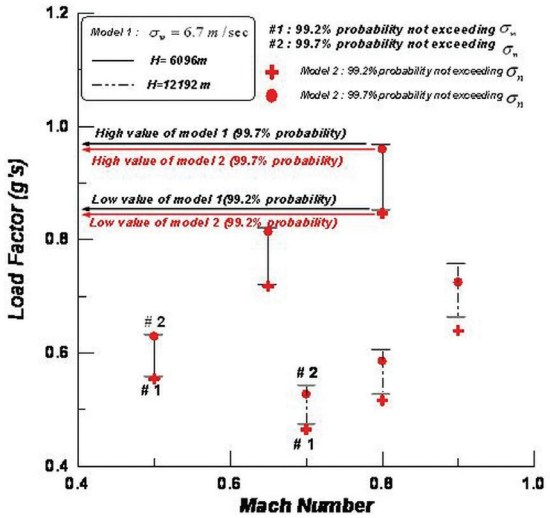


Figure 4. Comparison results of load factor values for Models 1 and 2 at H = 6,096m and H = 12,192m.

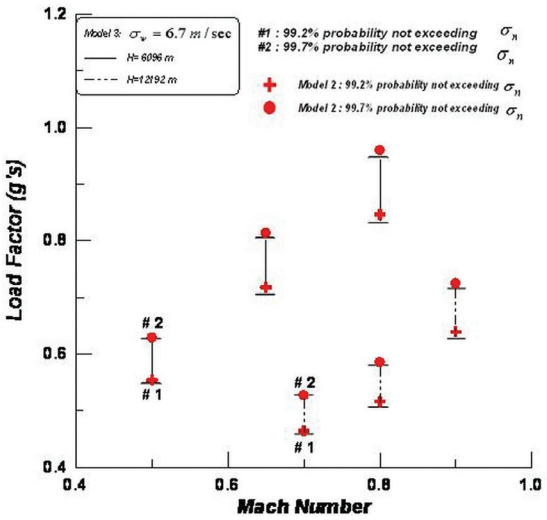


Figure 5. Comparison results of load factor values for Models 3 and 2 at H = 6,096m and H = 12,192m.

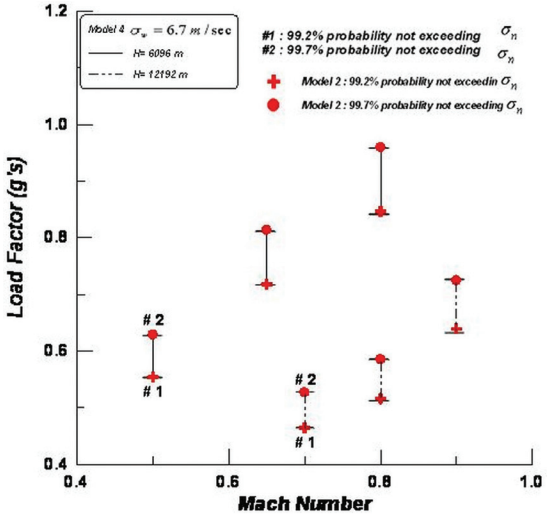


Figure 6. Comparison results of load factor values for Models 4 and 2 at H = 6,096m and H = 12,192m.

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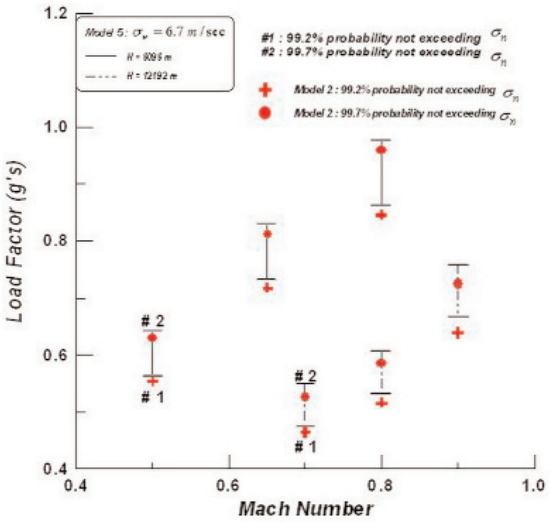


Figure 7. Comparison results of load factor values for Models 5 and 2 at H = 6,096m and H = 12,192m.

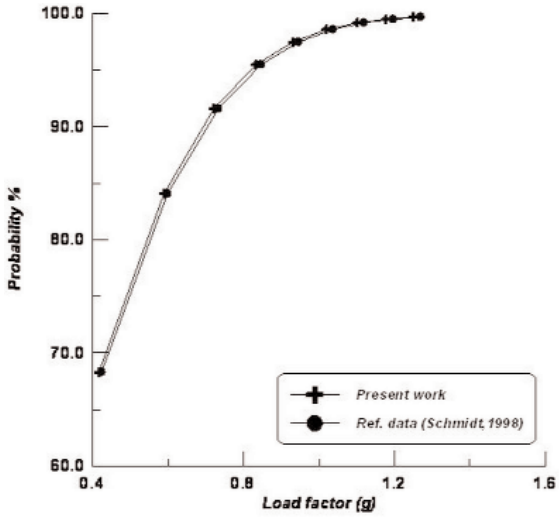


Figure 8. Lockheed jet transport comparison results of peak normal load factor (Model 3): H = 6,096m, M = 0·75.

on the statistical, the estimation of load factor for Model 5 which exhibits higher frequency contents at different values of probability are presented in Fig. 11 and Fig. 12. According to previous information, this paper provides increased motivation to improve airplane response in gust turbulence atmosphere by using modern optimal control methods.

Table 2(a)
Difference in load factor for suggested models % at H= 6,096m

Comparison Models	Mach no	Difference in load factor value %: H = 6,096m 99·7% probability of not exceeding load factor	99·2% probability of not exceeding load factor	Increasing and decreasing wrt Model 2
Model 1 & 2	0·50	0·788	0·894	Higher
	0·65	1·0	0·951	Higher
	0·80	0·75	0·808	Higher
Model 3 & 2	0·50	0·974	1·11	Lower
	0·65	1·28	1·26	Lower
	0·80	1·37	1·37	Lower
Model 4 & 2	0·50	0·413	0·288	Lower
	0·65	0·356	0·376	Lower
	0·80	0·281	0·295	Lower
Model 5 & 2	0·50	1·565	1·598	Higher
	0·65	1·951	1·792	Higher
	0·80	1·860	1·786	Higher

Table 2(b)
Difference in load factor for suggested models % at H = 12,192m

Comparison Models	Mach no	Difference in load factor value %: H = 6,096m 99·7% probability of not exceeding load factor	99·2% probability of not exceeding load factor	Increasing and decreasing wrt Model 2
Model 1 & 2	0·70	2·4	2·36	Higher
	0·85	2·74	2·60	Higher
	0·90	3·79	3·68	Higher
Model 3 & 2	0·70	0·970	1·02	Lower
	0·85	1·29	1·29	Lower
	0·90	1·25	1·39	Lower
Model 4 & 2	0·70	0·400	0·450	Lower
	0·85	0·618	0·623	Lower
	0·90	0·485	0·590	Lower
Model 5 & 2	0·70	3·980	2·790	Higher
	0·85	3·330	3·408	Higher
	0·90	4·986	4·612	Higher

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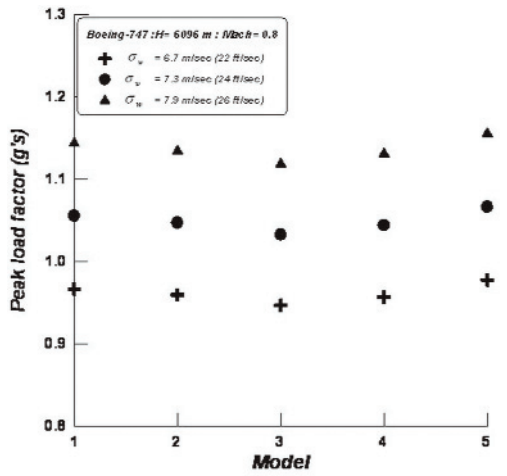


Figure 9. Effects of σ on peak normal load factor on all models at H = 6,096m, M = 0·8.

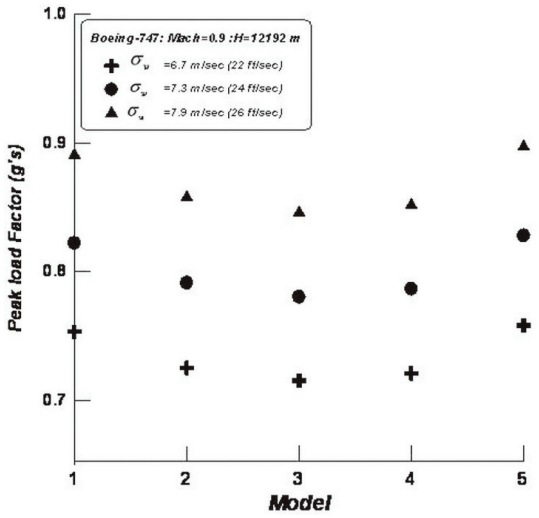


Figure 10. Effects of σ on peak normal load factor on all models at H = 6,096m, M = 0·8.

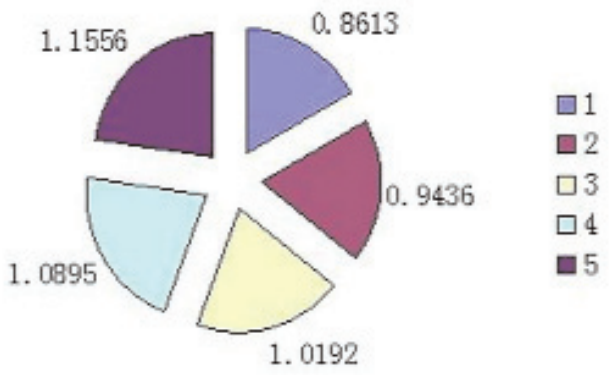


Figure 11. Boeing 747 load factor (Model 5): H = 6,096m, M = 0·8 at different probability not exceeding this value: σ = 7·9ms^{-1}.

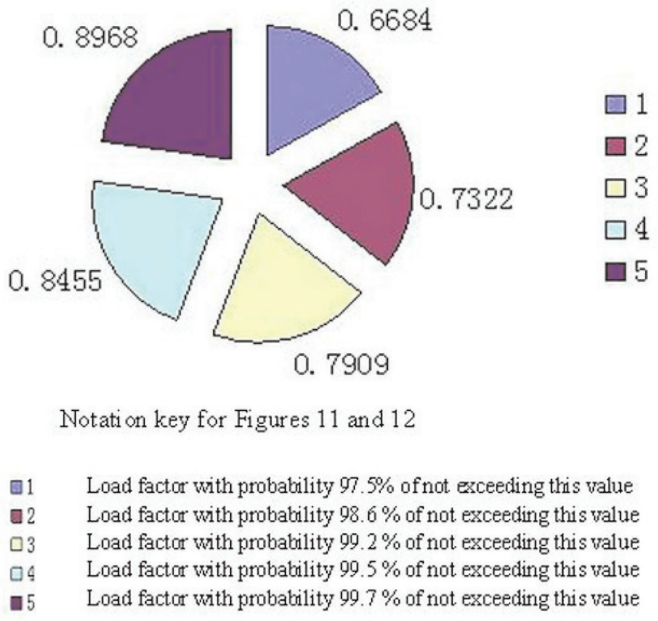


Figure 12. Boeing 747 load factor (Model 5): H = 12,192m, M = 0·9 at different probability not exceeding this value: σ = 7·9ms^{-1}.

7.0 CONCLUSIONS

Based on power spectral technique and Lyapunov approach, methodology to determine the vertical gust load of aircraft in continuous atmospheric turbulence under different flight conditions is presented. Modified longitudinal equations of motion including the effects of atmospheric gust are solved. Five linear dynamics models are developed to describe the normal load throughout an aircraft when it encounters a vertical gust. The numerical simulation model was built by using the MATLAB software. The following conclusions have been obtained in the present paper:

(a) The numerical results depend on the intensity of gust excitation intensity and evaluation criteria which are considered.
(b) Models 5 exhibits higher frequency contents when compared with all other models under different flight conditions. The models give a rapid estimation of normal load factor in case complete data are not readily available. These models predict the load factor with maximum 5% error when compared with Model 2 which considered all gust penetration effects.
(c) It can be concluded that the agreement between the finite frequency limit on integration of the spectral distribution and Lyapunov's results is well within the accuracy.

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