Robotica (2002) volume 20, pp. 553–561. © 2002 Cambridge University Press
DOI: 10.1017/S0263574702004381 Printed in the United Kingdom

Residual vibration reduction for a flexible structure using a modified input shaping technique

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(Received in Final Form: June 13, 2002)

SUMMARY

This paper presents a modified input shaping method to reduce the motion-induced vibration of a linear time-varying system after a rest-to-rest motion. Shaping parameters were obtained using the concept of modal-filtered impulse response. The *conventional* shaping method can be said just a special case of the proposed shaping method. The effectiveness of this proposing method was checked using some examples of both moderate and considerably fast time-varying systems. With a rest-to-rest motion control of a two-link flexible manipulator, this study also demonstrates that this method can be expanded to nonlinear cases.

KEYWORDS: Vibration reduction; Input shaping technique; Time-varying impulse sequence; Two-link manipulator

1. INTRODUCTION

There has been a great interest in finding control methods that can reduce vibration in a wide variety of mechanical and structural systems. In particular, vibration control of flexible machines, such as robot manipulators, becomes important as the demand for faster and lighter systems increases. When a light and flexible machine moves fast, its inertia can produce a deformation, and results in a long lasting vibration in case of light damping. Generally, this unwanted vibration limits the operation of high performance positioning systems such as robots, optical telescopes and space structures, and some effective techniques to reduce this vibration are needed.

Among the feedforward control techniques, input shaping is one particular group of control strategies that has received recent attention. This is largely based on the idea introduced by Smith[1] with his posicast control scheme. Unfortunately, posicast control is sensitive to modeling errors. Input shaping has been used successfully after a method for incorporating robustness was presented[2] and implemented on a variety of system. Residual vibrations of coordinate measuring machines,[3–5] cranes,[6–7] long-reach manipulators,[8] and silicon-handling robots[9] have been effectively improved with this input shaping technique. Although input shaping was developed to reduce residual vibration, it has been also proven to be beneficial for trajectory following applications.[10, 11, 12] Since its simplicity allows it readily generate feedforward commands for real-time actuators, input shaping has been shown to be effective in reducing vibration in both open and closed loop systems.[10]

Robust input shaping techniques for improving its insensitivity to modeling errors and parameter variations have been also studied[2, 13–16] Furthermore, input shaping has been investigated in both discrete time domain applications[17, 18] and frequency domain application.[19] Many researchers have extended their work to generalize the input shaper for a multimode vibration system[20, 21] and for a time-varying system.[22] Recently, an adaptive input shaping algorithm[23] was proposed using adaptive filtering technique in time domain. Variable amplitude command shaping techniques to curtail its residual vibration were also studied.[24, 25]

Although numerous studies have been carried out for input shaping to reduce motion-induced vibration, the majority of the applications are restricted to a linear time-invariant system or systems that have slowly time varying parameters. Only several papers[22, 24, 25] handled the issue of flexible time-varying systems. When a designer attempts to apply the previous input shaping technique to these moderate or considerably fast time-varying systems, a penalty of a long time maneuver is unavoidable and sometimes it might be impossible to reduce its vibration for a system having rapidly changing parameters. For that reason, an efficient technique to reduce vibration of a time-varying dynamic system is required. In this study, a modified technique is proposed to shape the nominal input for a considerably fast time-varying system in order to reduce its residual vibration after the rest-to-rest motion.

In the *primitive* robust input shaping method,[2] the derived impulse sequence consists of the primary and the second impulse with constant delay and magnitude, since it is based on the linear time-invariant system theory. To cope with the time-varying characteristics of vibrational modes, this work extends the previous shaping concept to time-varying shaping. In the meantime, the effectiveness of this method is demonstrated and its control performance checked and explained on the modal characteristics of both moderate and considerably fast time-varying systems.

In this work, a brief review of input shaping and a modified input shaper is proposed in the next section. In Section 3, the performance of input shapers is evaluated in various time-varying situations. In order to demonstrate the effectiveness of the proposed method not only in time varying but also in nonlinear systems, a flexible two-link manipulator is tested as a numerical example in Section 4,

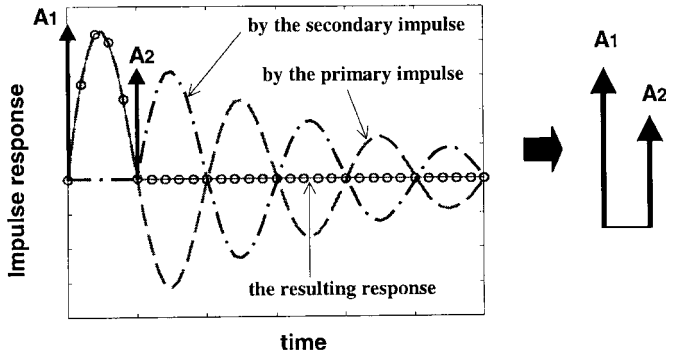
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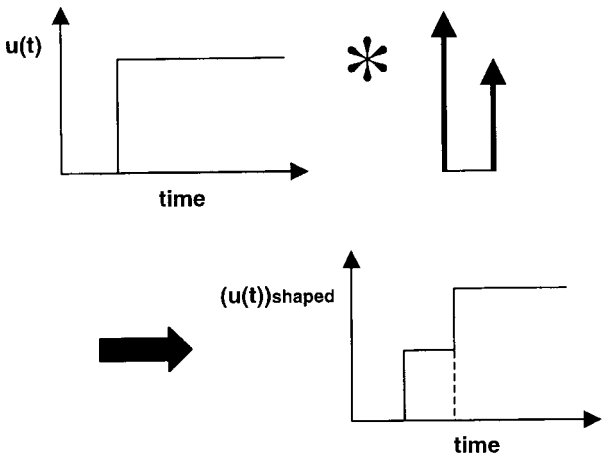
and finally, concluding remarks are given in the last section.

2. INPUT SHAPER OF LINEAR TIME-VARYING SYSTEMS

The original input shaping idea can be explained with the impulse response of a decoupled single degree-of-freedom linear vibratory system. According to the linear system theory, a linear vibratory system can be decoupled into a cascaded set of second-order single degree-of-freedom systems, and each decoupled system has the impulse response of a decaying sinusoidal function, as demonstrated in Figure 1-a. To eliminate the residual vibration after loading the second impulse, the time delay and magnitude of the second impulse can be properly obtained. The impulse sequence is called as an input shaper. The vibration reduction can be achieved by convoluting any arbitrary input with the impulse sequence for obtaining the shortest actual system input, which renders the same desired vibration-free motion. The sequence, therefore, can be treated as the prefilter for any given input to the system. The input time lengthening penalty, which comes from prefiltering the input force, equals to the length of the impulse sequence (i.e. the order of one cycle of oscillation for the sequence). Figure 1-b shows the process of convoluting an input with two impulses.



(a) Input shaping idea



(b) Input shaping process

Fig. 1. Input shaping idea and process.

The impulse sequence derived by the previous shaping method can be obtained from the linear time-invariant (LTI) system theory, and consists of the impulses having constant time delays and magnitudes. (In this work, the input shaping method using the constant two-impulse sequence is called a *conventional* method.) Many researchers have also shown that the robustness of the conventional shaping method can be improved by using multiple impulse sequences.[2, 13–16] The robust impulse shaping method can be applicable to a robot manipulator control in the case that the mass matrix is slowly varying. However, in the case of rapidly changing parameters, the conventional shaping technique is not good enough to obtain a successful residual vibration reduction, and it sometimes results in a worse performance. To overcome this problem, a modified input shaper is needed.

The state-space equation of a n-th order linear time-varying (LTV) system can be expressed as:

$$\dot{x}(t) = \underset{n\times n}{A}(t)\,x(t) + \underset{n\times m}{B}(t)\,u(t) \tag{1}$$

where $x(t) \in R^{n\times 1}$ is the state vector, $u(t) \in R^{m\times 1}$ is the control input, $A(t) \in R^{n\times n}$ is the system matrix and $B(t) \in R^{n\times m}$ is the input coefficient matrix. It is assumed that the equation of motion is a correct model for the plant and the actuator is suitably located for the system to be controllable. The system response can be expressed using the state transition matrix, $\Phi_x(t, t_0)$, as follows:

$$x(t) = \Phi_x(t, t_0)x(t_0) + \int_{t_0}^{t} (\Phi_x(t, \tau)B(\tau)u(\tau))d\tau \tag{2}$$

For a rest-to-rest motion, where the initial conditions, $x(t_0)$, of the states are zero, the necessary and sufficient condition of vibration-free motion at an arbitrary time t after the maneuver period T becomes:

$$\int_{t_0}^{t} \Phi_x(t, \tau)B(\tau)u(\tau)d\tau = 0, \quad t \geq T. \tag{3}$$

When the system input is an impulse sequence, $u(t) = \delta(t-t_1) + \alpha(t_2)\cdot\delta(t-t_2)$, $t_1 < t_2$ with a positive real coefficient $\alpha(t_2)$, the vibration-free condition can be expressed as Equation (4).

$$\int_{t_0}^{t} \Phi_x(t, \tau)B(\tau)u(\tau)d\tau = \Phi_x(t, t_1)B(t_1) + \Phi_x(t, t_2)B(t_2)\alpha(t_2)$$

$$= \Phi_x(t,T)\cdot[\Phi_x(T, t_1)B(t_1) + \Phi_x(T, t_2)B(t_2)\alpha(t_2)], \quad t \geq T \geq t_2 \tag{4}$$

Since the state transition matrix $\Phi_x(t,T)$ always has full ranks, the vibration-free condition becomes:

$$\Phi_x(T, t_1)B(t_1) + \Phi_x(T, t_2)B(t_2)\alpha(t_2) = 0. \tag{5}$$

For a simple notation, a vector, $\beta(T, \tau)$, is defined as Equation (6):

$$\beta(T, \tau) \underline{\Delta} \Phi_x(T, \tau)\cdot B(\tau), \; T \geq \tau \tag{6}$$

where the vector, $\beta(T, \tau)$ represents an influence on the system at a specific time T for a given impulse loading at time τ. Taking the vector, $\beta(T, \tau)$, the necessary and

https://doi.org/10.1017/S0263574702004381 Published online by Cambridge University Press

sufficient condition to obtain a vibration-free motion of the system exposed to an impulse sequence can be expressed as:

$$[\beta(T, t_1)+\beta(T, t_2)\alpha(t_2)]=0 \tag{7}$$

To minimize the residual vibration after time T, we should obtain the proper values of t_2 ($t_2=t_1+\Delta(t_1)$) and $\alpha(t_2)$ such that

$$\min_{t_2\geq t_1}\| \beta(T, t_1)+\beta(T, t_2)\alpha(t_2) \| \tag{8}$$

The vector, $\beta(T, \tau)$ is a set of state space responses at time T over the impulse loading $\delta(\tau)$ at time τ and has both displacement and velocity states. For fast moving systems, the magnitudes of velocities are considerably larger than those of displacements and an attempt to just minimize the vector norm can introduce an erroneous switching time for the second impulse. For example, direct searching input parameters to minimize the vector norm of the Equation (8) may result in a sort of residual-velocityminimizing impulse sequence. After the second impulse loading, therefore, the system can restart its free oscillation due to the leftover displacement (deflection) term.

In order to effectively review the dynamic influence of an impulse loading on the system, a modal-filtered vector $\beta_M(T, t)$ is proposed in this work.

$$\beta_M(T, t)\underline{\Delta}U^T(T)\cdot\Phi_x(T, t)\cdot B(t) \tag{9}$$

where $U(T)$ is an instantaneous left eigen matrix of the system, $A(t)$, at time $t=T$, and obtained from Equation (10).

$$A(T)\cdot V(T)=V(T)\cdot\Lambda(T) \tag{10a}$$

$$U^T(T)\cdot V(T)=V(T)\cdot U^T(T)=I_{n\times n} \tag{10b}$$

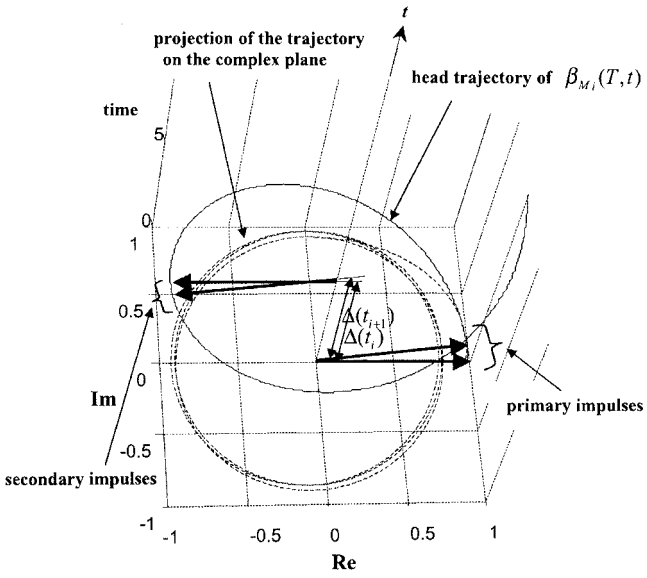
where $V(T)$ is an instantaneous right eigen matrix, and $\Lambda(T)$ is a diagonal eigenvalue matrix. $I_{n\times n}$ represents the identity matrix. The modal-filtered vector $\beta_M(T, t)$ denotes the modal response of the system (reviewed at time T) over an impulse input (loaded at time t), and it can be estimated with a specific modal matrix and state transition matrix. The i-th element, $\beta_{Mi}(T, t)$, of the vector $\beta_M(T, t)$, which is a complex value, can be considered as the i-th modal impulse response due to the impulse input.

The necessary and sufficient condition to eliminate the i-th modal response (phasor) induced by an impulse sequence can be expressed by Equation (11):

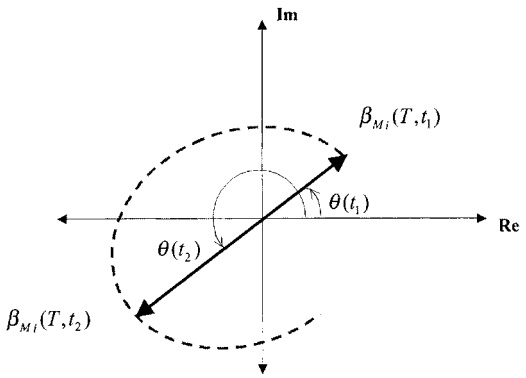
$$\beta_{Mi}(T, t_1)+\beta_{Mi}(T, t_2)\cdot\alpha(t_2)=0 \tag{11}$$

The input shaper design is to search the time t_2 and the gain $\alpha(t_2)$ numerically to satisfy the condition of Equation (11). Figure 2 shows two phasor vectors in the complex plane, each representing the i-th modal state $\beta_{Mi}(T, t_1)$ (or $\beta_{Mi}(T, t_2)$) at time T over the impulse input at time t_1 (or t_2). Each modal phasor vector in complex plane can be understood as a phasor rotating around the origin with a time-varying speed and magnitude. Although the modal decomposition of LTV system is less accurate than that of LTI system, the effects of an impulse, which is loaded at time t, upon the i-th modal response can be always estimated at a specific time T, since the state transition matrix has full rank,[26] and the instantaneous modal matrix, $U^T(T)$, at a specific time T is a set of n independent vectors. As a special case, the phasor $\beta_{Mi}(T, t)$ can have zero magnitude instantaneously, which means that the impulse loaded at that time doesn't have any influence on the i-th model response at the time T and the impulse doesn't need any counter-acting impulse. With the concept of the model phasor vector cancellation, both the secondary impulse loading time, t_2, and its proper magnitude (or gain), $\alpha(t_2)$, can be numerically obtained using the phase and magnitude of the complex values as in Figure 2-b.

In the previous paper,[25] a similar input shaping method using a time varying impulse sequence (TVIS) was proposed. But in that paper, the effects of the first impulse were calculated at the loading time of the secondary impulse using an instantaneous modal matrix and modal transition matrix. The basic concept, therefore, can be explained with



(a) trajectory of the *i*-th modal phasor, $\beta_{Mi}(T,t)$ and its cancellation concept



(b) projected plot of Fig 2-(a) on the complex plane (numerical search of $t_2=t_1+\Delta(t_1)$ so as to satisfy $|\theta(t_2)-\theta(t_1)|=\pi$)

Fig. 2. Modal impulse response cancellation in complex plane.

https://doi.org/10.1017/S0263574702004381 Published online by Cambridge University Press

the above Equation (8) by replacing the final time (T) with the instance of the secondary impulse loading (t_2) such as in Equation (12).

$$\beta_{Mi}(t_2, t_1)+\beta_{Mi}(t_2, t_2)\cdot\alpha(t_2)=0 \tag{12}$$

where the modal phasor, $\beta_{Mi}(*,*)$, is the i-th component of the vector quantity, as expressed in Equation (13).

$$\begin{aligned}\beta_M(t_2, t_1)&=\{U^T(t_2)\cdot\Phi_x(t_2, t_1)\cdot B(t_1)\}\\&=\{U^T(t_2)\cdot\Phi_x(t_2, t_1)\cdot V(t_1)\cdot U^T(t_1)\cdot B(t_1)\},\\&=[U^T(t_2)\cdot\Phi_x(t_2, t_1)\cdot V(t_1)]\cdot\{U^T(t_1)\cdot B(t_1)\}\end{aligned} \tag{13a}$$

$$\begin{aligned}\beta_M(t_2, t_2)&=\{U^T(t_2)\cdot\Phi_x(t_2, t_2)\cdot B(t_2)\}\\&=\{U^T(t_2)\cdot B(t_2)\}\end{aligned} \tag{13b}$$

In Equation (13a), generally, the modal state transition matrix, $[U^T(t_2)\cdot\Phi_x(t_2,t_1)\cdot V(t_1)]$, is non-diagonal for an LTV system, i.e. each modal state can be affected by other modes. In that work, however, assuming that the effects of other modes upon the i-th modal state transition are negligible, Equation (12) was approximately solved as follows:

$$\begin{aligned}&Diag(U^T(t_2)\cdot\Phi_x(t_2, t_1)\cdot V(t_1))_{ii}\cdot\{U^T(t_1)\cdot B(t_1)\}_i\\&\quad+\alpha(t_2)\cdot\{U^T(t_2)\cdot B(t_2)\}_i=0\end{aligned} \tag{14}$$

where $Diag(U^T(t_2)\cdot\Phi_x(t_2, t_1)\cdot V(t_1))_{ii}$ is the i-th diagonal element of the diagonal matrix, $Diag(U^T(t_2)\cdot\Phi_x(t_2, t_1)\cdot V(t_1))$, and $\{U^T(*)\cdot B(*)\}_i$ is the i-th element of the vector quantity, $\{U^T(*)\cdot B(*)\}$. Equation (14) is the design condition for the TVIS input shaping method, and the shaping parameters, t_2 and $\alpha(t_2)$, were numerically found in the complex plane so as to eliminate the residual vibration at time t_2.

Although the assumption on the negligible effects of other modes makes the concept of the TVIS input shaping method simple, the negligible amounts can be accumulated on the i-th modal response and worsen the input shaping performance, especially in fast time-varying systems. The accumulation depends not only on the degree of coupling with other modes but also on the degree of the varying rate of the modal matrix.

On the other hand, the proposed shaping method in this work eliminates the i-th modal component from the residual vibration at time T completely. For a rest-to-rest motion of an LTV system, if system parameters are fixed (time-invariant) after the move, the i-th modal component in the residual vibration can be permanently eliminated by this proposed method.

In the previous TVIS method, furthermore, approximated modal transition matrices were used to link the relationship of the second modal input with the primary one, and sometimes that approximation can worsen vibration reduction at the end position. To search the secondary impulse loading time, t_2, the eigenvalue problem was repeatedly solved at every time step. Thus the method requires all the information of the instantaneous modal space and the state transition matrix sampled at each loading time (i.e. both the first and the second loading time); needless to say, its computation is very heavy. But the method proposed in this work requires similar procedures but does not need the calculation of all the instantaneous modal space information, since it evaluates the effects of the input upon the modal response using the modal matrix at just a specific time and its state transition matrix.

The proposed shaping scheme, which is based on the modal-filtered impulse response concept, is an extended idea of the *conventional* input shaping method. Therefore, it gives exactly the same shaper as that of the conventional method for the case of an LTI system. Its consistency in regard to the conventional method for the case of LTI system can be also proven by deriving a closed form expression of the state transition matrix. As for a constant system matrix $\underset{n\times n}{A}$, the state transition matrix of Equation (9) is expressed as a closed form of exponential functions and thus the modal matrix becomes constant.[26] The real and imaginary parts of the modal-filtered impulse response, $\beta_{Mi}(T,t)$ are simplified into sinusoidal functions with a constant amplitude and frequency if the system is undamped. If the damping is included, the amplitudes are modulated by an exponentially decaying function and its frequency is replaced by a damped natural frequency. From the condition of Equation (11), we can derive the equations for the switching time t_2 and the gain $\alpha(t_2)$, which are identical with the case of the conventional input shaping method and, therefore, both are time invariant.

3. PERFORMANCE ANALYSIS OF INPUT SHAPERS

To demonstrate the weakness of the constant impulse shaping and to show the input design procedure in this proposed input shaping scheme, a flexible model having timevarying stiffness and contact elements, as shown in Figure 3-a, is numerically tested. The equations of the motion for the model can be expressed as follows:

$$m\ddot{x}+k(t)\,x=b(t)\,f(t) \tag{15}$$

where $f(t)$ is the external force exerted upon the body. The stiffness $k(t)$ denotes the time varying system property. The coefficient $b(t)$ is to investigate the effects of the time-varying contact elements upon the system control input. For simplicity of the performance analysis, an undamped single degree-of-freedom system is tested. Since there are many LTV systems being time varying for a finite duration, the time-varying elements of the example are assumed to have time-dependencies as per Equation (16):

$$\omega_n(t)=\sqrt{\frac{k(t)}{m}}=\omega_0\cdot\{1+\frac{\varepsilon_w}{2}(1-\cos(2\pi f_k t))\}\ \text{ for }\ 0\le t\le T_0 \tag{16a}$$

$$\omega_n(t)=\omega_n(T_0)\ \text{ for }\ t>T_0 \tag{16b}$$

$$b(t)=b_0\cdot\{1+\frac{\varepsilon_b}{2}(1-\cos(2\pi f_b t))\}\ \text{ for }\ 0\le t\le T_0 \tag{16c}$$

$$b(t)=b(T_0)\ \text{ for }\ t>T_0 \tag{16d}$$

where the minimum natural frequency ω_0 is set to $2\pi\times 3.4$ *rad/sec*, the minimum contact coefficient, b_0, is 1

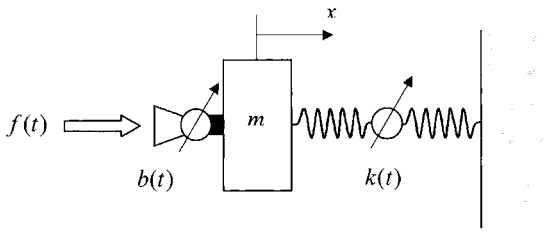
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and the time varying period, T_0, is 1 sec. The maximum magnification value of natural frequency, ε_w, and the maximum magnification value of contact coefficient, ε_b, are all set to 100%. The stiffness and the contact element change in a harmonic pattern with the frequency f_k and f_b, respectively. In this study two simulation cases are tested to discuss the performance of the input shapers: Case A demonstrates a moderate time-varying system ($f_k=1.0\,Hz$ and $f_b=2.0Hz$) and Case B is a considerably fast time-varying system ($f_k=3.0Hz$ and $f_b=5.0Hz$). The natural frequency and the contact property, therefore, fluctuate with time t, as shown in Figures 3-b and 3-c.

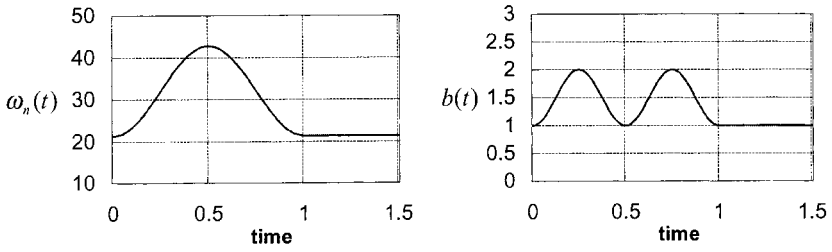
Figure 4 shows the numerically obtained shaping parameters for the secondary impulse, t_2 ($t_2=t_1+\Delta(t_1)$) and $\alpha(t_2)$ for Case A using Equation (11). The parameters of the proposed shaping scheme are very close to those of the previous TVIS shaper. The switching delay time fluctuates in the band of 0.07 sec to 0.15 sec and finally converges into a specific value at the end of time-varying period. The shaping gains for both shaping schemes behave also in very close pattern. In spite of the 100% variation of $b(t)$, the gain value $\alpha(t_2)$ does not exceed over 2 or under 0.5.

For a fast time-varying system like Case B, however, both schemes present different shaping parameters, as shown in Figure 5. Although the stiffness and contact properties vary in simple harmonic oscillation forms, it is clear that the behaviors of the shaping parameters in time are not harmonic or simple. These complexities are due to the coupling effects between the time varying stiffness and the time varying contact element. After the time-varying period, the switching delay times for the both schemes converge into that of the conventional shaping method, and the shaping gains become 1 (one), which is the gain for the conventional shaping method.

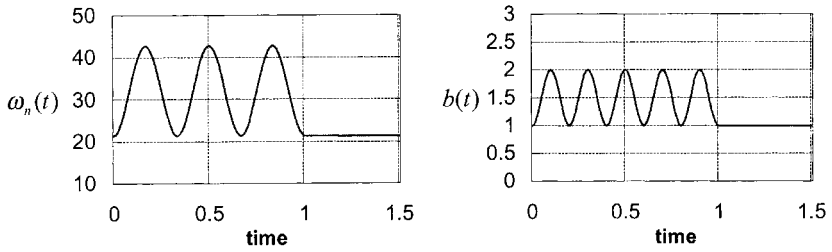
A bang-bang command, having 0.8 sec length and a single switching at 0.4 sec, is used as a nominal input for a rest-to-rest motion. To verify the performance of each input shaping method, the control inputs are obtained by shaping the nominal input in three different shaping methods, respectively: the conventional method, the previous TVIS shaping method and the proposed method in this work. Figure 6 shows the differences of residual vibration for those three shaping methods. It can be observed that the modified input shaper is very effective in reducing its residual vibration. For a slowly time varying system (Case A), the previous TVIS shaper shows almost the same performance with the method suggested in this study. But for a considerably fast time varying system (Case



(a) Test example for an LTV system



(b) Case A



(c) Case B

Fig. 3. Time-varying natural frequency and contact element for testing cases.

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B), the suggested method is superior to the previous TVIS input shaping method in reducing residual vibration. This suggested method, furthermore, has a quite efficient computing advantage over the previous TVIS method, which requires all the eigenproperties of all the time steps.

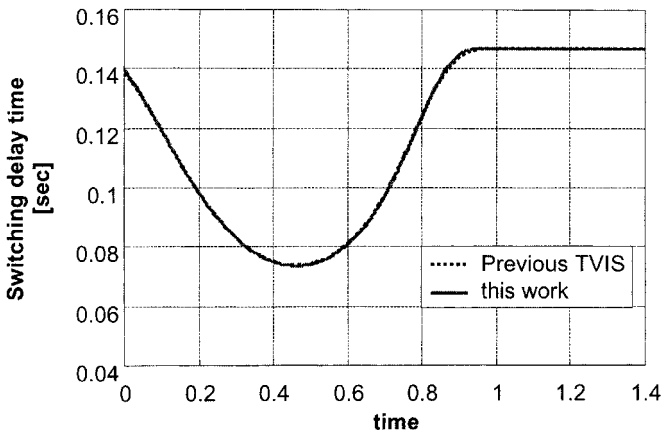
4. APPLICATION TO NONLINEAR SYSTEMS

Although the shaping technique has been developed for the feedforward control of linear time-varying system, it can be applied to a group of nonlinear systems to reduce its residual vibration.[12, 24] Flexible multibody systems undergoing a large rest-to-rest motion can be successfully controlled using this input shaping by linearizing the nonlinear dynamic equations about their nominal states. Based on the assumptions that the rigid body maneuvering motion is already known and the variations of their nominal states are small, its governing nonlinear equations can be transformed into a linear time-varying form of the state equation by the Taylor expansion. In the LTV state space equation, the system matrix and the input coefficient matrix can be simply expressed using timevarying coefficients perturbed by the known rigid body maneuvers.

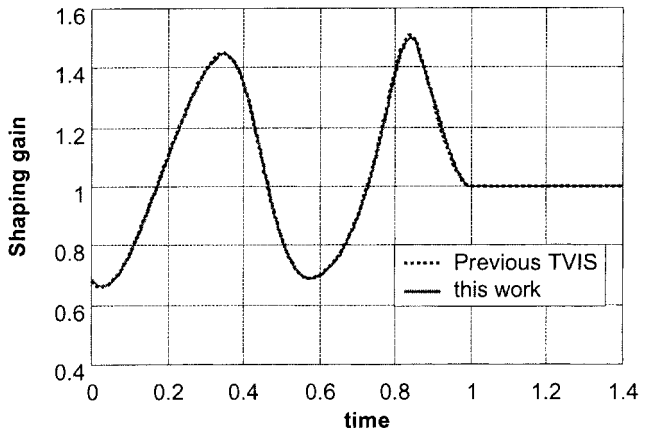
A simple two-link manipulator is taken as an example to show the applicability of this shaping technique to a nonlinear multibody system. The system shown in Figure 7-a has two uniform beams connected by a revolute 'elbow' joint. The beams are assumed to be flexible only in the direction transverse to their length in the plane of motion, so that there are no out-of-plane deflections and no axial elongation of the links as the arm moves. The assumed mode method is used to model the link deflection, $y_i(x_i, t)$, along the i-th *virtual* rigid link, fixed at the joint coordinate:

$$y_i(x_i,t) = \sum_{j=1}^{n_i} \phi_{ij}(x_i) q_{ij}(t). \qquad (17)$$

where $\phi_{ij}(x_i)$ and $q_{ij}(t)$ are clamped-free beam mode shapes and generalized coordinates, respectively, for the j-th mode of the i-th link. n_i denotes the number of modes considered

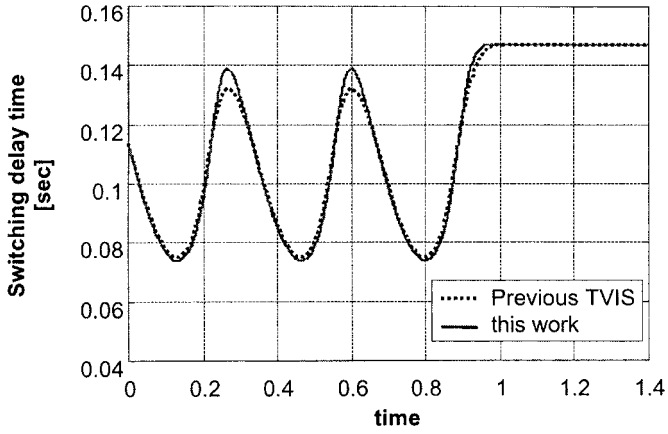


(a) Switching (Delay) Time

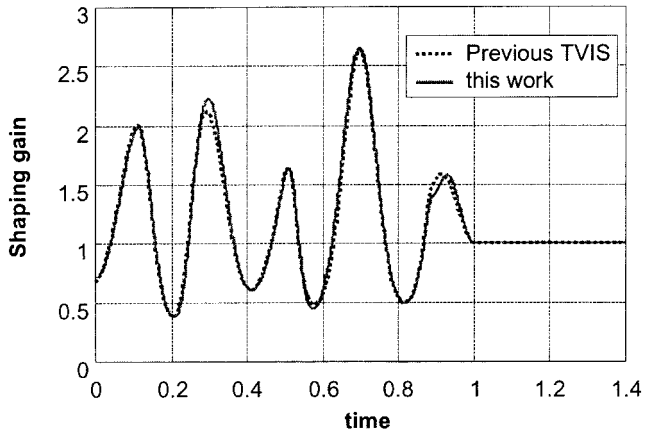


(b) Shaping Gain

Fig. 4. Numerically obtained input shaping parameters (CASE A: moderate time-varying case).



(a) Switching (Delay) Time



(b) Shaping Gain

Fig. 5. Numerically obtained input shaping parameters (CASE B: fast time-varying case).

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for the i-th link. Using Lagrange's equation, the dynamic equation of the flexible two-link manipulator can be obtained as:

$$M\ddot{z}+Kz+f(z,\dot{z})+f_g(z)=Q \tag{18}$$

$$z \triangleq [\theta_1 \;\; \theta_2 \;\; q_{11} \;\cdots\; q_{1n_1} \;\; q_{21} \;\cdots\; q_{2n_2}]^T$$

where z is the set of the generalized coordinates, and θ_1 and θ_2 are the joint angles, representing the rigid body motion of each link. The nonlinear force $f(z,\dot{z})$ is a column vector with the elements as follows:

$$f_i(z,\dot{z})=\sum_{j=1}^{n_1+n_2+2}\left(\dot{z}_j e_i^T \frac{\partial M}{\partial z_j}\dot{z}\right)-\frac{1}{2}\dot{z}^T\frac{\partial M}{\partial z_i}\dot{z} \tag{19}$$

The gravitational nonlinear force $f_g(z)$ and the generalized force Q are the column vectors having some degree of dependency on the kinematics of the system, respectively. The Equation (18) is highly nonlinear due to cross-products between the generalized velocities, and the dependency of the mass matrix upon the configuration of the system. Its present form, therefore, is unsuitable for shaping control, which requires a linear plant model.

To linearize the equations of motion, it is necessary to restrict the configuration and motion of the manipulator about a nominal set of kinematics. This requires nominal values, z_0 and $\dot{z}_0$, to be specified for the generalized coordinates and their velocities, respectively. It is then possible to perform a Taylor expansion of the equation about the given set of generalized coordinates and velocities. For small changes in the coordinates and velocities, the system model can be approximated by a first-order expression. The zeroth-order terms can be set to zero leaving the following linear differential equation:

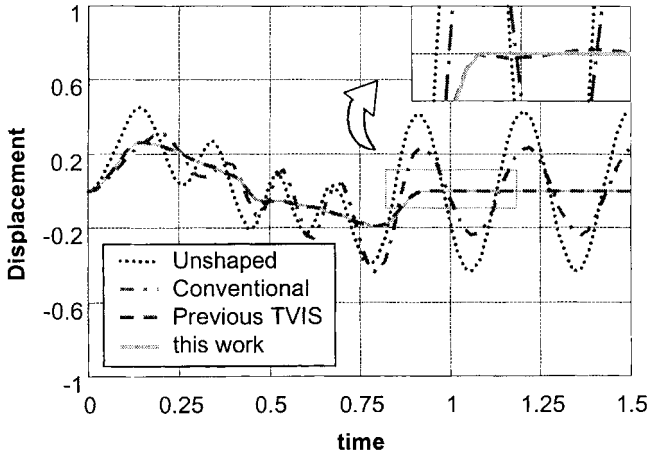
$$M(\dot{z}_0)\delta\ddot{z}+C_f\delta\dot{z}+(K+K_m+K_f+K_g)\delta z=0 \tag{20}$$

where:

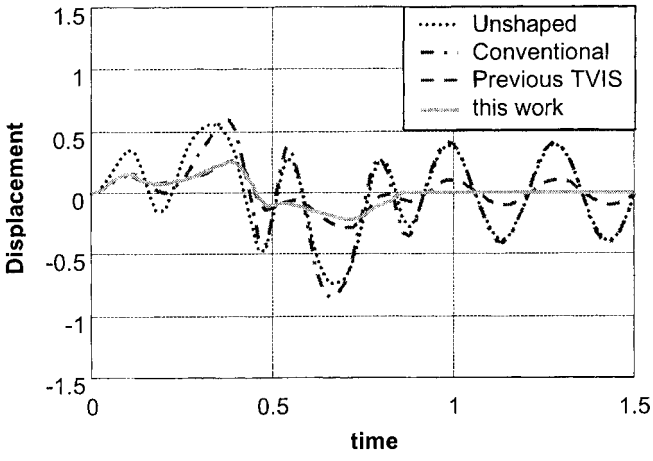
$$C_f=\left[\frac{\partial f}{\partial \dot{z}_1}\cdots\frac{\partial f}{\partial \dot{z}_{n_1+n_2+2}}\right]\Bigg|_{z_0,\,\dot{z}_0}, \tag{21a}$$

$$K_m=\left[\frac{\partial M}{\partial z_1}\ddot{z}_0\cdots\frac{\partial M}{\partial z_{n_1+n_2+2}}\ddot{z}_0\right]\Bigg|_{z_0}, \tag{21b}$$

$$K_f=\left[\frac{\partial f}{\partial z_1}\cdots\frac{\partial f}{\partial z_{n_1+n_2+2}}\right]\Bigg|_{z_0,\,\dot{z}_0}, \tag{21c}$$

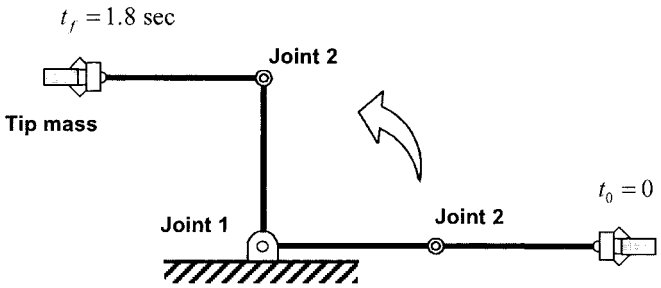


(a) Residual vibration of CASE A

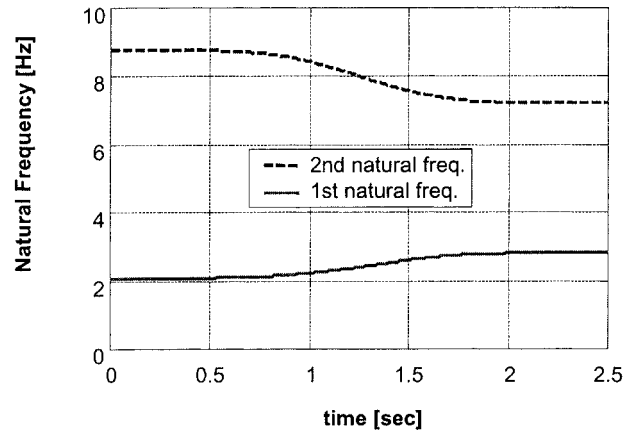


(b) Residual vibration of CASE B

Fig. 6. Residual vibration.



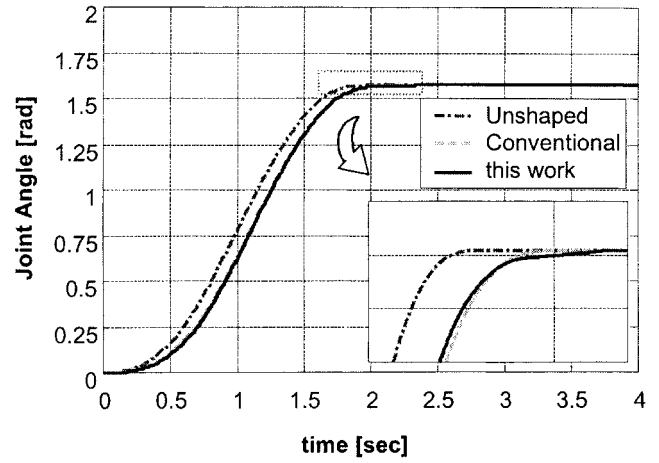
(a) Prescribed point-to-point maneuver



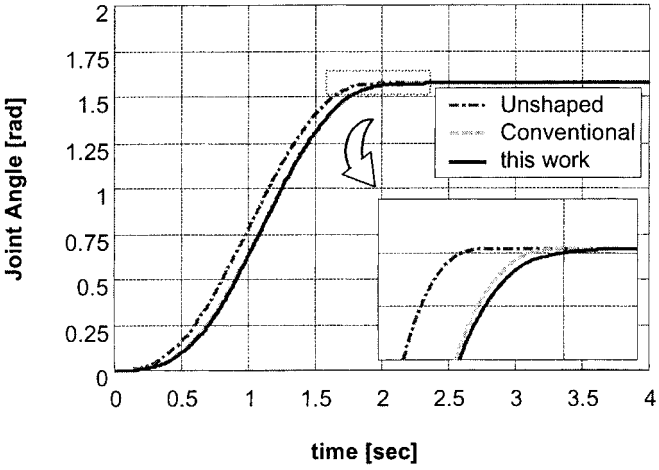
(b) Time-varying instantaneous natural frequencies

Fig. 7. Point-to-point motion and its instantaneous natural frequencies of a flexible two-link manipulator.

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(a) Commands at Joint 1



(b) Commands at Joint 2

Fig. 8. Shaped inputs at joint 1 & 2.

$$K_g = \left[\frac{\partial f_g}{\partial z_1} \cdots \frac{\partial f_g}{\partial z_{n_1+n_2+2}} \right] \Bigg|_{z_0}. \tag{21d}$$

Using the coefficient matrices of the Equation (21) and defining the state vector as Equation (22), the system matrix $A(t)$ and the input coefficient matrix $B(t)$ of the Equation (1) can be constructed and both matrices depend on the nominal states, $x_0(t)$. Since it is difficult to estimate the deflections of each link in advance, all the nominal vales of the modal deflection coordinates, $q_{ij}(t)$, during the motion are assumed to be negligible and set to zeros.

$$x(t) = \{z^T(t), \dot{z}^T(t)\}^T \tag{22}$$

A rest-to-rest motion of the manipulator will be controlled to verify the performance of the proposed shaping method. As an elementary effort to reduce residual vibration, a quintic polynomial like Equation (23) is used for the

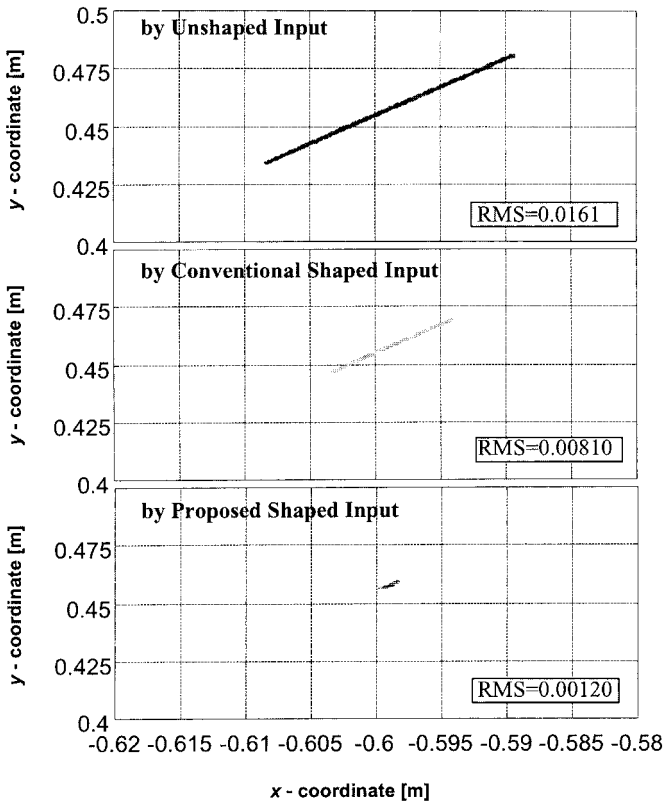


Fig. 9. Tip vibration.

nominal trajectory of each joint angle in order to generate a smooth trajectory, which is able to specify the position, velocity, and acceleration at both boundaries. From the six constraints on the position, velocity, and acceleration of the initial and final time, the coefficients a_i can be determined.

$$\theta(t) = \sum_{j=0}^{5} a_j \cdot t^j. \tag{23}$$

To demonstrate the performance of the proposed scheme, a fast and large maneuver is set with the initial and the final configurations of the Figure 7-a, where t_f is the time to perform the given rest-to-rest motion. For the nominal motion trajectory, the fundamental natural frequency, estimated by using quasi-static modal analysis at every instant, varies within the range from 2.1 Hz to 2.8 Hz, as shown in Figure 7-b.

From the known nominal states, the shaping parameters, t_2 and $\alpha(t_2)$, of Equation (11) can be determined numerically in the complex plane. Using the shaping parameters, the original path for the desired joint angles is modified into the shaped one as the solid line in Figure 8, where the dashed line represents the path shaped by the conventional input shaping method. Both shaped inputs require a slightly longer time of maneuver compared with that of the original input. When these shaped inputs are given, the residual vibration at the tip of the second link is shown in Figure 9. The conventional shaping method shows a 50% reduction in residual vibration over the unshaped input and the proposed shaping method displays a 93% reduction. It is clear that the

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proposed shaping method can reduce effectively the residual vibration of the flexible system.

5. CONCLUSIONS

The research objective of this work is to develop an efficient method to reduce motion-induced vibration by modifying a nominal input. A modified input shaping method was proposed, which can be applicable to a time-varying system. Comparing with the previous TVIS method, the proposed method in this work can be used for a fast time-varying system in order to reduce residual vibration more effectively, since the proposed method doesn't neglect the effects of other modes and the effects of the changes in the modal matrix. In addition, the proposed method is more efficient in computing process than the previous TVIS method. With a simple example of an LTV system, the performance of the modified input shaper was evaluated under several time-varying situations, comparing with other input shaping methods. The proposed method, derived for the control of an LTV system, can be applicable to a nonlinear system as well, by linearizing the state space equations about a nominal motion. It is obvious from the example simulations that the proposed method shows good performance in residual vibration reduction for flexible structures.

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