J. Plasma Physics (2014), *vol.* 80, *part* 3, *pp.* 405–415. © Cambridge University Press 2013
doi:10.1017/S0022377813001360

Solitary Langmuir waves in two-electron temperature plasma

V. V. Prudkikh†

Department of Physics, Southern Federal University, Rostov-on-Don, Russia

(Received 23 July 2013; revised 17 October 2013; accepted 21 October 2013; first published online 25 November 2013)

Nonlinear interaction of Langmuir and ion-acoustic waves in two-electron temperature plasma is investigated. New integrable wave interaction regime was discovered, this regime corresponds to the Langmuir soliton with three-hump amplitude, propagating with a speed close to the ion–sound speed in the conditions of strong non-isothermality of electronic components. It was discovered that besides the known analytical solution in the form of one- and two-hump waves, there exists a range of solutions in the form of solitary waves, which in the form of envelope has multi-peak structure and differs from the standard profiles described by hyperbolic functions. In case of fixed plasma parameters, different group velocities correspond to the waves with different number of peaks. It is found that the Langmuir wave package contains both even and uneven numbers of oscillations. Low-frequency potential here has uneven number of peaks. Interrelation of obtained and known earlier results are also discussed.

1. Introduction

Solitary Langmuir waves are considered to be fundamental objects of modern plasma physics' investigation. They are interesting as both steady nonlinear structures and component of strong Langmuir turbulence (Kingsep et al. 1973).

Langmuir soliton is a dip of electron and ion density, propagating in plasma with sub-sound speed. If the pulse propagation speed u is considerably higher than the thermal ion speed $u \gg v_{ti}$, then the wave potential is negative. Alternatively, wave potential at $u \ll v_{ti}$ may be both positive and negative (Yan'kov 1979). It is important that at $u \ll c_s$ (c_s is the ion–sound speed) the condition of plasma quasi-neutrality is valid, this fact allows to find an exact dependence of charge density as a function of pondermotive force, acting from a high-frequency component of field on plasma electrons.

On the other hand, at $u \leqslant c_s$, contribution of ion dynamics and spatial separation of charges in the field of pulse localization allows existence of analytical integrable regimes of basic set equations. In the work of Rao and Shukla (1997), integrable regimes of solitary Langmuir wave propagation were found for two-electron temperature plasma. Such plasma represents a model, including two kinds of electrons with distinct temperatures, and is often used to study a large number of problems (Jones et al. 1975; Temerin et al. 1982; Isichenko et al. 1985). Solutions obtained by Rao and Shukla (1997) correspond to stationary propagating solitary waves, form of which has one- and two-hump structures. Here the wave of one-hump

† Email address for correspondence: slavadhb@mail.ru

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

form is specific for two-temperature electron plasma, and does not have any analogue in plasma with one-temperature electrons.

In the present work we investigate the propagation of solitary Langmuir pulses in two-temperature electron plasma, which leads to the system of two coupled equations of the second order for amplitude of Langmuir wave and low-frequency potential. It is discovered that at near-sonic soliton propagation, when Langmuir wave amplitude is significantly lower than the electrostatic potential, there is a solution of equation system, describing the wave of three-hump profile. Besides, new types of solutions were found in the form of solitary waves with both even and uneven numbers of peaks of high-frequency amplitude, and uneven number of electrostatic potential peak. However, these solutions are not analytical, and are obtained by means of numerical solution of initial equation system. To a certain extent such solutions are similar to the ones that appear in case of investigation of problems on intensive laser beam interaction with cold plasma (Kaw et al. 1992; Kuehl and Zhang 1993; Poornakala et al. 2002; Xie and Du 2006), or in the case of propagating near-sonic ion-acoustic waves in magnetized plasma (Prudskikh 2010).

Besides, a question on interrelation of obtained numerical solution in the form of multi-peak solitary waves with analytical solutions of the work of Rao and Shukla (1997) was studied. At a fixed level of plasma electron non-isothermality, two different values of pulse group velocity correspond to wave with one and two field peaks. It is found that within the interval between this group velocity, there is a discrete set of group velocities (Mach number), for each of which a wave is also solitary. Transfer to highly non-isothermic plasma is accompanied by the concentration of eigenvalue spectrum of the system of equations corresponding to solitary waves, which allows almost uninterrupted transformation of numerical solutions into analytical ones.

The paper is organized as follows. In Sec. 2 we derive a set of equations of the problem. In Sec. 3 there is an analysis of integrable regimes. Section 4 contains the results of numerical findings of solitary waves of coupled equation set. Discussion of the results and brief summary is given in Sec. 5.

2. Basic equations

The set of equations describing high-frequeny moving of two kinds of electrons is reduced to equation:

$$\left(\frac{\partial^2}{\partial t^2} - 3v_{th}^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2}{\partial t^2} - 3v_{tc}^2\frac{\partial^2}{\partial x^2}\right)E(x,t) + \omega_{ph}^2\left(\frac{\partial^2}{\partial t^2} - 3v_{tc}^2\frac{\partial^2}{\partial x^2}\right)E(x,t) + \omega_{pc}^2\left(\frac{\partial^2}{\partial t^2} - 3v_{th}^2\frac{\partial^2}{\partial x^2}\right)E(x,t) = 0. \quad (2.1)$$

Here $v_{t\alpha} = (T_\alpha/m)^{1/2}$ is the thermal speed of electrons, $\omega_{p\alpha} = (4\pi e^2 n_{e\alpha}/m)^{1/2}$ is the plasma frequency, m is the electron weight, $\alpha = (h, c)$, and h and c are referred to hot and cool electronic components respectively.

In linear approach, (2.1) leads to the following dispersion relation:

$$1 - \frac{\omega_{ph0}^2}{\omega^2 - 3k^2v_{th}^2} - \frac{\omega_{pc0}^2}{\omega^2 - 3k^2v_{tc}^2} = 0, \quad (2.2)$$

where $\omega_{p\alpha 0}$ is the plasma frequency corresponding to the unperturbed electron density, $n_{e\alpha 0}$.

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

Let us seek a solution of (2.1) in the form

$$E(x,t) = \mathscr{E} \exp i(kx - \omega t), \tag{2.3}$$

where $\mathscr{E}$ is the slowly varying amplitude. Then, taking into consideration $\omega \approx \omega_p \gg kv_{th}, kv_{tc}$, introducing $\omega_p^2 = \omega_{ph}^2 + \omega_{pc}^2$, slow modulation of electron density $n_{e\alpha} = n_{e\alpha 0} + \delta n_{e\alpha}$, $\delta n_{e\alpha} \ll n_{e\alpha 0}$, and maintaining the lowest order of nonlinearity after standard calculations, we obtain

$$i\varepsilon\left(\frac{\partial \mathscr{E}}{\partial t} + V_g \frac{\partial \mathscr{E}}{\partial x}\right) + \frac{3}{2}\eta \frac{\partial^2 \mathscr{E}}{\partial x^2} = \frac{1}{2} N \mathscr{E}, \tag{2.4}$$

where $\varepsilon = (m/m_i)^{1/2}$, m_i is the ion weight, $V_g = 3kv_{te}^2/(c_s\omega_{p0})$ is the normalized linear group velocity (ratio group velocity to ion–sound speed), $\eta = v_{te}^2/(\lambda_D^2\omega_{p0}^2)$, $v_{te}^2 = (n_{h0}v_{th}^2 + n_{c0}v_{tc}^2)/n_0$, $\omega_{p0} = (4\pi e^2 n_0/m)^{1/2}$, $n_0 = n_{h0} + n_{c0}$, $c_s = (T_{\text{eff}}/m_i)^{1/2}$ is the ion–sound speed, $T_{\text{eff}} = n_0 T_h T_c/(n_{h0}T_c + n_{c0}T_h)$ is the effective temperature, $\lambda_D = (T_{\text{eff}}/4\pi e^2 n_0)^{1/2}$ is the Debye length defined in terms of effective temperature. Spatial coordinate x is normalized with respect to λ_D, time t is normalized to ω_{pi}^{-1}, where $\omega_{pi} = (4\pi e^2 n_0/m_i)^{1/2}$ is the ion plasma frequency, $N = (\delta n_{eh} + \delta n_{ec})/n_0$ is the full low-frequency perturbation of electron density.

If we suppose that the thermal speed of both electron components is substantially higher than the wave group velocity $v_{th}, v_{tc} \gg V_g c_s$, then equations of slow electron motion can be written as

$$0 = e\frac{\partial \phi}{\partial x} - T_\alpha \frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial x} - \frac{e^2}{2m\omega^2}\frac{\partial}{\partial x}|\mathscr{E}|^2. \tag{2.5}$$

Here the last term is connected with the action of pondermotive force from high-frequency component of the field. For the formulation of the equation, we considered that the phase velocity of Langmuir wave is significantly higher than the group velocity so that time derivative of the pondermotive member may be neglected.

Let us turn to the coordinate $\xi = x - Mt$, where M is the Mach number, normalized to ion–sound speed, and integrated to (2.5), we find the expressions for the density of hot and cold electron components:

$$n_h = n_{h0}\exp[\alpha_h(\phi - |\mathscr{E}|^2)], \qquad n_c = n_{c0}\exp[\alpha_c(\phi - |\mathscr{E}|^2)], \tag{2.6}$$

where $\alpha_h = T_{\text{eff}}/T_h$, $\alpha_c = T_{\text{eff}}/T_c$, expression $e\mathscr{E}/\sqrt{2mT_{\text{eff}}}\omega$ is rewritten as $\mathscr{E}$.

Assume that thermal ion speed is significantly lower than the pulse group velocity. In this case the equations governing ion motion shall be written in the following form:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{2.7}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x}. \tag{2.8}$$

Here the density n_i is normalized to n_0, speed v_i is normalized to c_s, and ϕ is normalized to T_{eff}/e. In a moving frame defined by coordinate $\xi = x - Mt$, the expression for ion density, following from (2.7) and (2.8) is then written as

$$n_i = \frac{n_0}{\left(1 - \frac{2\phi}{M^2}\right)^{1/2}}. \tag{2.9}$$

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

Let us seek a solution for (2.4) in the form

$$\mathscr{E} = E \exp i(X(x) + T(t)). \tag{2.10}$$

Here $E = E(\xi)$, and variables $X(x)$ and $T(t)$ take into account slow variation of wave phase. Substituting expression (2.10) into (2.4), keeping the lowest order expansion quantities in (2.6), and separating the real and imaginary parts, we have the following equation:

$$3\eta \frac{d^2E}{d\xi^2} = \lambda E + \phi E, \tag{2.11}$$

where $\lambda = 2\varepsilon\delta + \varepsilon^2(M^2 - V_g^2)/3\eta$, and $\delta = dT/dt$. For the convenience, we keep the designations of the work of Rao and Shukla (1997).

Substituting expressions (2.6) and (2.9) into Poisson's equation, expanding them, and keeping the first and second order of smallness of nonlinear terms, in stationary frame one can get to

$$\frac{d^2\phi}{d\xi^2} = -\frac{1 - M^2}{M^2}\phi - \left(\frac{3}{M^4} - \Delta\right)\frac{\phi^2}{2} - E^2. \tag{2.12}$$

Here $\Delta = \alpha_h^2 N_h + \alpha_c^2 N_c$, $N_h = n_{h0}/n_0$, $N_c = n_{c0}/n_0$. Taking into mind the next order expansion, we write this equation in the following form:

$$\frac{d^2\phi}{d\xi^2} = -\frac{1 - M^2}{M^2}\phi - \left(\frac{3}{M^4} - \Delta\right)\frac{\phi^2}{2} - E^2 - \left(\frac{15}{M^6} - \Lambda\right)\frac{\phi^3}{6} - \Delta\phi E^2, \tag{2.13}$$

where $\Lambda = \alpha_h^3 N_h + \alpha_c^3 N_c$. Set of (2.11) and (2.12) or (2.11) and (2.13) describes interaction of Langmuir and ion–sound waves in two-temperature electron plasma.

3. Exact analytical solutions

Let's consider the case in when the relations between the parameters of solution allow to neglect cubic terms in the Poisson's equation, and for investigation we use a set of (2.11) and (2.12). Due to the fact that the next section contains numerical analysis, for this equation as well as the relationship of obtained us solutions with solutions, which were known before, we shall quote its exact solutions found by Rao and Shukla (1997).

Let us seek a solution of (2.11) in the form

$$E = E_{m1} \cosh^{-1}(\mu\xi) \tanh(\mu\xi), \qquad \phi = \phi_{m1} \cosh^{-2}(\mu\xi). \tag{3.1}$$

Substitute (3.1) to (2.11) and (2.12) and equate the coefficients of like powers of hyperbolic functions on left and right sides. So we have:

$$E_{m1} = \pm 6\lambda \left[\frac{1}{2}\left(\frac{3}{M^4} - \Delta\right) + \frac{1}{3\eta}\right]^{1/2}, \quad \phi_{m1} = -6\lambda, \quad \mu = (\lambda/3\eta)^{1/2}. \tag{3.2}$$

These solutions are realized under the following condition:

$$\frac{1 - M^2}{M^2} = \lambda\left[3\left(\frac{3}{M^4} - \Delta\right) + \frac{2}{3\eta}\right]. \tag{3.3}$$

Figure 1 shows the square Mach number and max wave amplitude E_{m1} as functions on the density of electron cool component at different temperature rates, $\tau = T_c/T_h$, for which solutions are realized in the form of solitary waves (3.1).

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

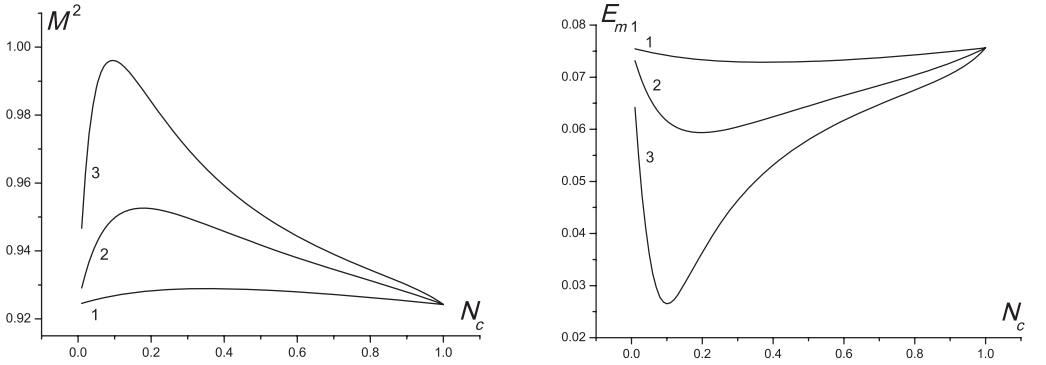


FIGURE 1. Dependence of square Mach number and max wave amplitude (3.1) on the density of cool electron component at different values of $\tau = T_c/T_h$ (λ=0.01). Curve 1 – $\tau = 0.5$, curve 2 – $\tau = 0.2$, curve 3 – $\tau = 0.1$.

Relations of (3.2) and (3.3) are valid at $\Delta = 3/M^4$, when nonlinear corrections to electron and ion density, connected with ϕ^2, into (2.12) are compensated. In this case, the wave amplitude E_{m1} and formula (3.3) turn into

$$E_{m1} = \pm\frac{6\lambda}{(3\eta)^{1/2}}, \qquad \frac{1-M^2}{M^2} = \frac{2\lambda}{3\eta}. \tag{3.4}$$

There is one more analytical solution for the set of (2.11) and (2.12) in two-temperature electron plasma. Let's find the solution in the form of

$$E = E_{m2}\cosh^{-1}(\mu\xi), \qquad \phi = \phi_{m2}\cosh^{-2}(\mu\xi). \tag{3.5}$$

In this case, by repeating the above-mentioned procedure we shall obtain the following expression:

$$E_{m2} = \pm 2\lambda\left(\frac{1-M^2}{2\lambda M^2} + \frac{2}{3\eta}\right)^{1/2}, \qquad \phi_{m2} = -2\lambda, \qquad \mu = (\lambda/3\eta)^{1/2}, \tag{3.6}$$

which takes place under the following condition:

$$\eta\left(\Delta - \frac{3}{M^4}\right) = 2. \tag{3.7}$$

In plasma with isothermal electrons, parameter Δ is equal to one and expression (3.7) is not fulfilled. However, it may be satisfied with two-temperature plasma under high non-isothermality of electron components. Figure 2 shows the dependence of $M^2(N_c)$ and $E_{m2}(N_c)$, which satisfies the requirement of positivity of radical expression (3.6) and corresponds to the condition (3.7).

Let's assume that the consideration of cubic terms is required for the description of interactions of Langmuir and ion–sound waves into Poisson's equation. Such situation is possible when the contribution of term ϕ^3 is comparable with the contribution of terms ϕ^2 and E^2. At the same time, accounting of additional nonlinear components in wave equation (2.11) is not required due to the small value of the potential amplitude ϕ compared with one. In this case the problem is described by the set of values (2.11) and (2.13). We seek the solution in the form

$$E = E_{m3}\cosh^{-1}(\mu\xi) + E_{m4}\cosh^{-3}(\mu\xi), \qquad \phi = \phi_{m3}\cosh^{-2}(\mu\xi). \tag{3.8}$$

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

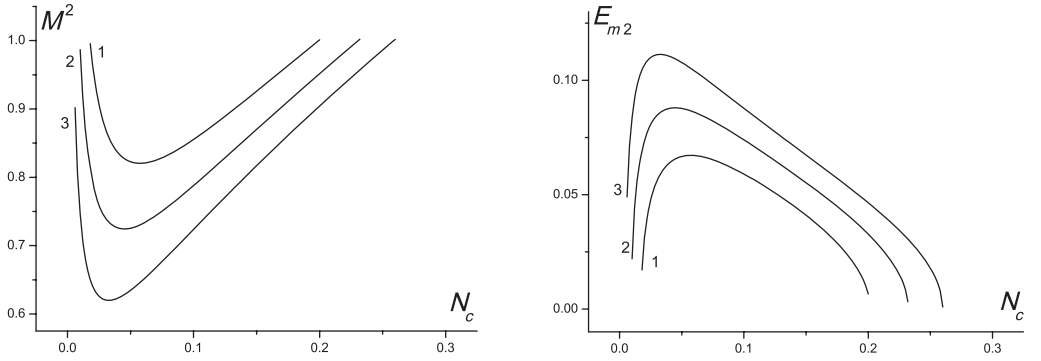


FIGURE 2. Dependence of square Mach number and max wave amplitude (3.5) on the density of cool electron component in highly non-isothermic plasma ($\tau = T_c/T_h \ll 1$, λ=0.01). Curve 1 – $\tau = 0.05$, curve 2 – $\tau = 0.04$, curve 3 – $\tau = 0.03$.

We obtain expressions for field amplitudes and parametric relations at which we have the solutions in the form (3.8). Now it is necessary to clarify the conditions of necessity of cubic term consideration. Let's require that $\max(E_{m3}, E_{m4}) \ll \phi_{m3}$. Satisfaction of this inequality allows to neglect the contribution of the last term into the right part of (2.13) as small. Substituting (3.8) into (2.11) and (2.13), we obtain

$$E_{m3} = \pm\frac{12\lambda}{\sqrt{5}}\left(\frac{3}{M^4} - \Delta + \frac{1}{3\eta}\right)^{1/2}, \qquad E_{m4} = -\frac{5}{4}E_{m3}, \qquad \phi_{m2} = -12\lambda. \tag{3.9}$$

Two corresponding parametric conditions are as follows:

$$\frac{1-M^2}{M^2} = \frac{4\lambda}{5}\left[3\left(\frac{3}{M^4} - \Delta\right) - \frac{2}{3\eta}\right], \tag{3.10}$$

$$25\left(\frac{1-M^2}{M^2} + \frac{4\lambda}{3\eta}\right) = 384\lambda^2\left(\frac{15}{M^6} - \Lambda\right). \tag{3.11}$$

According to the analysis of conditions of the link between parameters (3.10) and (3.11), three-hump solutions (3.8) are possible in plasma with significantly different temperatures for two kinds of electrons. A condition of smallness of nonlinear term $\Delta\phi E^2$ in (2.13) is fulfilled at near-sonic soliton propagation: $1 - M^2 \ll 1$ ($M^2 < 1$). Therefore, the ratio of the term $\Delta\phi E^2$ (which was excluded from consideration) to the previous term ϕ^3 is about 0.08–0.09 for $\lambda = 0.004$ and 0.05–0.06 for $\lambda = 0.003$. At the same time, contribution of the term ϕ^3 is comparable with other nonlinear terms in its right part, as coefficient $(15/M^6 - \Lambda)$ is rather high in highly non-isothermic plasma. Necessity of consideration of the third order of smallness in the expansion of electron density is connected with high sensitivity of the ratio of potential amplitudes ϕ and field E to parameter λ. This ratio is about one at $\lambda \approx 0.01 - 0.02$ and increases significantly at the decrease of λ up to the value of several thousands. So we may resume that in the case of high non-isothermality of electrons, there exist two integrable regimes of Langmuir and ion–sound waves interaction, which lead to the formation of solitary waves. The first regime, describing the wave with profile (3.5), at determined level of non-isothermality $\tau = T_c/T_h$ is realized for a certain range of cool (or hot) electron component density (see Fig. 2). Due to the existence of additional second condition of the link between parameters, second regime (wave of the form (3.8)) is possible only for some constant density ratio of two kinds of electrons. In

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

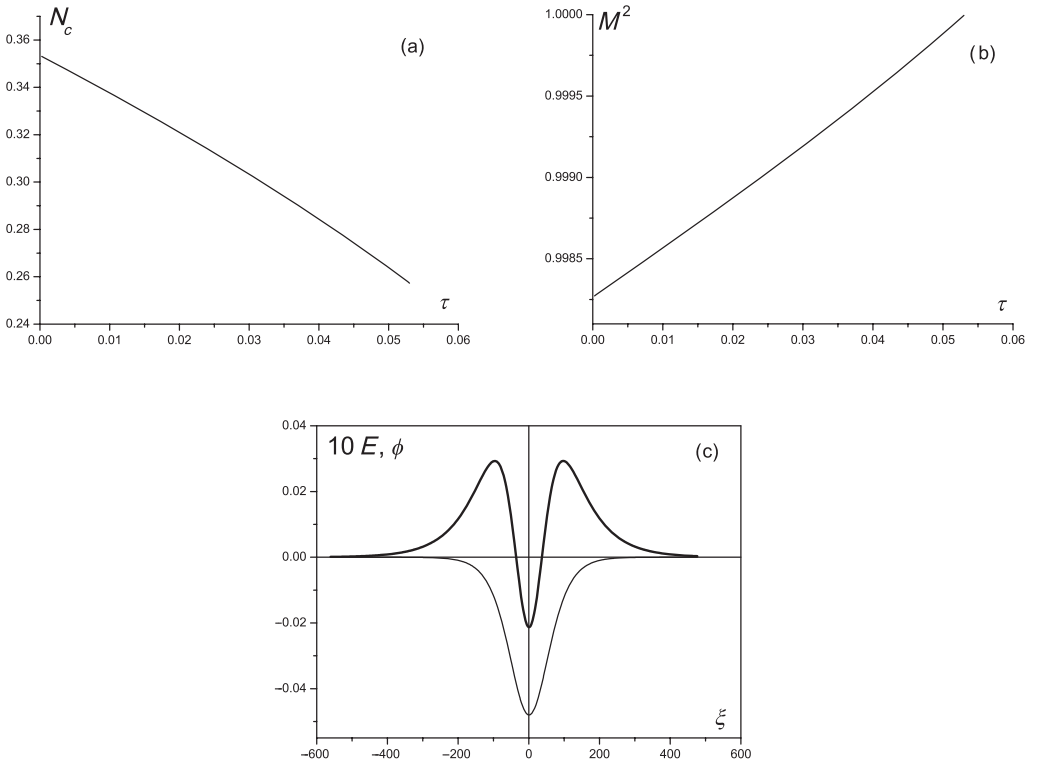


FIGURE 3. Dependence of density of cool electron component N_c (a), and square Mach number M^2 (b) on τ, as well as the form of E envelope amplitude (ten-folded in the figure) and potential ϕ (c). $\lambda = 0.004$, $\tau = 0.03$.

Fig. 3 we may see dependencies $N_c(\tau)$ and $M^2(\tau)$, for which conditions (3.10) and (3.11) are fulfilled, as well as the form of the solitary wave (3.8).

It should be noticed that the formal solution of (3.10) and (3.11) allows the existence of waves of the form (3.8) at $M^2 > 1$ ($|M^2 - 1| \ll 1$). But numerical solution of the initial set of equations does not confirm this fact. The reason of the emerged contradictions is connected with incorrectness of consideration of condition $M^2 - 1 > 0$, as potential vibrations are self-excited even in the absence of field E (see equation 2.13)). Such solutions shall be excluded from consideration because the source of low-frequency modulation of plasma density is an action at plasma electrons from ponderomotive force and, as a consequence, excitation of Coulomb field, which is connected to the charge separation.

4. Numerical analysis results

Let's consider the propagation of solitary waves in the case in which the Mach number M is not close to one. In this case, interaction between Langmuir and ion–sound waves is described by the set of (2.11) and (2.12). According to numerical analysis of this set, it allows the existence of solutions in the form of solitary waves, which differ from (3.1), (3.5), and (3.8). Appearance of such solutions may be qualitatively explained as follows. Hamiltonian of our problem has the following

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

N_E	N_ϕ	M^2	N_E	N_ϕ	M^2
2	1	0.9288242	3	3	0.9287925
	3	0.9120320		5	0.9127692
	5	0.8964000		7	0.9115109
	7	0.8835824		9	0.8973396
	9	0.8722347		11	0.8955591
	11	0.8616976		13	0.8846353

TABLE 1. Table of Mach number values for soliton solutions with N_E peaks of field E and N_ϕ peaks of potential ϕ. Calculation parameters: $\lambda = 0.01$, $N_c = 0.3$, $\tau = T_c/T_h = 0.5$.

form:

$$H = 3\eta E'^2 - \frac{1}{2}\phi'^2 + U(E, \phi), \tag{4.1}$$

where

$$U(E, \phi) = -\lambda E^2 - \frac{1 - M^2}{2M^2}\phi^2 - \left(\frac{3}{M^4} - \Delta\right)\frac{\phi^3}{6} - \phi E^2, \tag{4.2}$$

stroke denotes the derivative for ξ. Hamiltonian H is symmetric with respect to replacement $\xi \to -\xi$, $E \to -E$, $E' \to -E'$, and $\phi' \to -\phi'$, but is not symmetric with respect to replacement $\phi \to -\phi$. That is why, coupled oscillations of E and ϕ, initiating at $\xi \to -\infty$ from the value $E(\xi \to -\infty) \to 0$ and $\phi(\xi \to -\infty) \to 0$, and going through the point $\xi = 0$, shall satisfy the following conditions at this point, necessary for the return of amplitudes E and ϕ to its zero values at $\xi \to +\infty$. First, potential in the center of the wave $\phi(\xi = 0)$ shall have an extreme value, and its derivative shall become zero. It means that the number of nonlinear oscillations ϕ within the pulse may be only uneven. Second, field $E(\xi = 0)$ may have both extreme and zero values. In the latter case extremeness of the derivative $E'(\xi = 0)$ is required. According to the above-mentioned case, solutions for E may have both even and uneven numbers of oscillations.

Numerical analysis of the (2.11) and (2.12) showed that there is a set of solitary waves in which E amplitude has a two-hump form, similar to (3.1), but the potential profile ϕ is different from the function ch^{-2} and demonstrates some of the maximum and minimum values. Thus, a number of potential extremums ϕ are always uneven. Potential ϕ with different number of peaks corresponded to different Mach numbers. Results of numerical calculations are specified in Table 1. As seen in the table, all multi-peak waves are possible at Mach numbers if these are less than M_{cr}, where M_{cr} is determined in (3.3). At the decrease of Mach number, the number of potential peaks ϕ increases, and their amplitude of oscillations around some mean value decreases. In the case of low M, amplitude of potential oscillations around the mean value tends to be zero, and the distance between the peaks of field E increases. At $M \to 0$, left part of (2.12) is insignificant in comparison with the right part, so it may be neglected. In this case we have Langmuir soliton with quasi-static density perturbation of plasma components. The form of wave and distribution of spatial charge for one of the typical pulses are represented at the Fig. 4.

Analysis of (2.11) and (2.12) shows that besides the two-peak solutions for field E, there are also three- and multi-peak solutions. You may see one of such three-peak solutions in Fig. 5. Numeric values of parameters, for which three-peak solutions are discovered, are specified into the table.

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

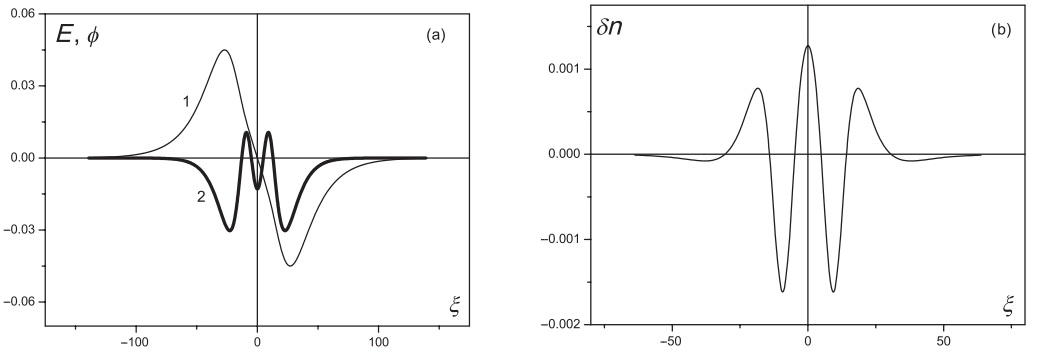


FIGURE 4. Soliton corresponding to the value $N_E/N_\phi = 2/5$ from the table. (a) Dependence $E(\xi)$ (curve 1) and $\phi(\xi)$ (curve 2); (b) Dependence of full perturbation of plasma density $\delta n = \delta N_i - \delta N_h - \delta N_c$ on ξ.

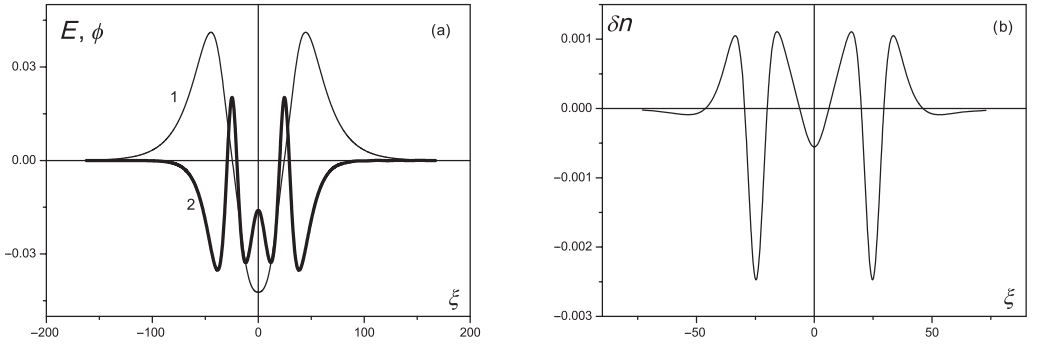


FIGURE 5. Soliton corresponding to the value $N_E/N_\phi = 3/7$ from the table. (a) Dependence $E(\xi)$ (curve 1) and $\phi(\xi)$ (curve 2); (b) Dependence of full perturbation of plasma density $\delta n = \delta N_i - \delta N_h - \delta N_c$ on ξ.

Let's consider the issue of relation between the waves of the form (3.1) and (3.5) in highly non-isothermic plasma. Solution (3.1) exists at Mach numbers M_{cr1}, which correspond to the condition (3.3); solution (3.5) is possible for the values M_{cr2}, which are found from (3.7). At $M_{cr1} < M < M_{cr2}$ there is a discrete set of Mach numbers corresponding to eigenvalues of the equation set (2.11) and (2.12), for which its solutions are solitary waves. At the ends of the range the wave has the form of (3.1) and (3.5) (correspondingly). If the plasma electron component increases non-isothermally, spectrum of Mach number, corresponding to solitary waves, remains discrete, but the interval between M successive values, for which the number of potential peaks differs by 2, is being compressed. The closer the value of M to the value determined by (3.7), the greater is the compression of Mach numbers. Besides, here the distance between the successive peaks of field E increases and the potential ϕ has very small oscillation amplitude. Middle curve, around which potential vibrations are performed, is almost described by the function $\cosh^{-2}$. At M, which tends to the value given by the ratio in (3.7), the distance between the peaks of field E tends to infinity, and the oscillation amplitude E with respect to the curve $\cosh^{-2}$ becomes zero, which corresponds to the analytical solution (3.5). Above-mentioned facts are illustrated in Fig. 6, where the transformation of three-hump soliton to one-hump soliton of the form (3.5) is depicted as an example.

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

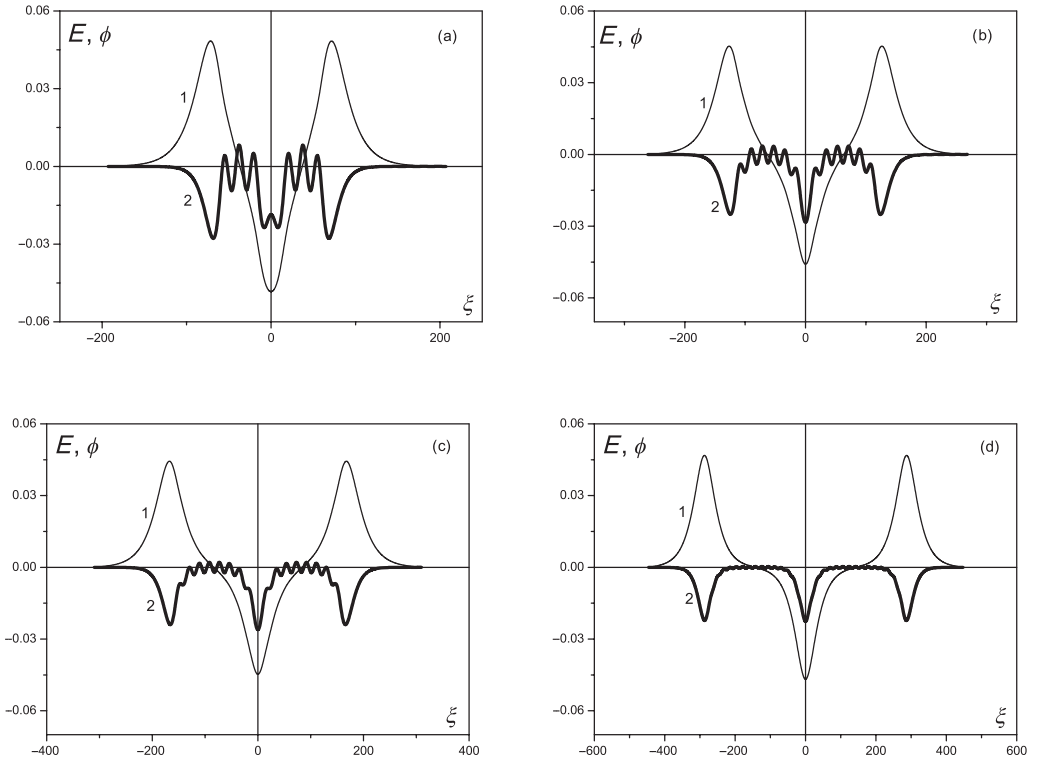


FIGURE 6. Three-peak soliton deformation at the increase of plasma non-isothermality level (N_{ξ}=0.3). Index 1 indicates field amplitudes E, index 2 indicates potential ϕ. (a) – $\tau = 0.5$, $M^2 = 0.8825285$; (b) – $\tau = 0.2$, $M^2 = 0.8965284$; (c) – $\tau = 0.15$, $M^2 = 0.9009753$; (d) – $\tau = 0.1$, $M^2 = 0.8931238$.

5. Main conclusions

In this paper we analyzed nonlinear interaction of Langmuir and ion–sound waves in two-temperature electron plasma. It was found that at near-sound propagation ($1 - M^2 \ll 1$), there exists some new integrable regime of equation set, which describes solitary Langmuir pulses of three-peak form. Such solutions take place in case of fulfillment of two conditions, (3.10) and (3.11), and are possible at a certain ratio between the density values of two electron components. Using numerical analysis, we discovered solitary waves in which field amplitude E has one or several peaks. The form of ion–sound perturbation has both one peak, corresponding the analytical results of Rao and Shukla (1997), and several peaks, the number of which is always uneven. Such ion–sound oscillations are frozen in the wave and propagate steadily together with it. Into a motionless frame, it corresponds to the full switch-off of plasma oscillations after wave transmission. Waves with different number of potential oscillations propagate at different group velocities. In plasma with higher level of electron non-isothermality, the spectrum of Mach number eigenvalues is highly concentrated at $M^4 \to 3\eta/(\Delta\eta - 2)$, and the transfer of the obtained numerical solutions to the analytical solution of the kind (3.5) is performed almost continuously.

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press

REFERENCES

Isichenko, M. B., Chukbar, K. V. and Yan'kov, V. V. 1985 Theory of the fast evolution of the magnetic field in a two-component electron plasma. *J. Exp. Theor. Phys. Lett.* **41**, 101.

Jones, W. D., Lee, A., Gleeman, S. M. and Doucet, H. J. 1975 Propagating of ion-acoustic waves in a two-electron temperature plasma. *Phys. Rev. Lett.* **35**, 1349–1352.

Kaw, P. K., Sen, A. and Katsouleas, T. 1992 Nonlinear 1D laser pulse solitons in a plasma. *Phys. Rev. Lett.* **68**, 3172–3175.

Kingsep, A. S., Rudakov, L. I. and Sudan, R. N. 1973 Spectra of strong Langmuir turbulence. *Phys. Rev. Lett.* **31**, 1482–1484.

Kuehl, H. H. and Zhang, C. Y. 1993 One-dimensional, weakly nonlinear electromagnetic solitary waves in a plasma. *Phys. Rev. E* **48**, 1316–1323.

Poornakala, S., Das, A., Sen, A. and Kaw, P. K. 2002 Laser envelope solitons in cold overdense plasmas. *Phys. Plasmas* **9**, 1820–1823.

Prudskikh, V. V. 2010 Supersonic and near-sonic solitary ion-acoustic waves in magnetized plasma. *Plasma Phys. Rep.* **36**, 993–999.

Rao, N. N. and Shukla, P. K. 1997 Coupled Langmuir and ion-acoustic waves in two-electron temperature plasmas. *Phys. Plasmas* **4**, 636–645.

Temerin, M., Cherny, K., Lotko, W. and Mozer, F. S. 1982 Observations of double layers and solitary waves in the auroral plasma. *Phys. Rev. Lett.* **48**, 1175–1179.

Xie, B.-S. and Du, S.-C. 2006 Multipeak solitons and nonlinear modulated solutions in plasma under relativistic electromagnetic field. *Phys. Plasmas* **13**, 074504.

Yan'kov, V. V. 1979 Two types of Langmuir solitons. *J. Exp. Theor. Phys. Lett.* **29**, 160.

https://doi.org/10.1017/S0022377813001360 Published online by Cambridge University Press