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Robust Stable Limit Cycle Generation in Multi-Input Mechanical Systems

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SUMMARY
This paper proposes a robust controller for the generation of stable limit cycles in multi-input mechanical systems subjected to model uncertainties. The proposed idea is based on Port-Controlled Hamiltonian (PCH) model and energy-based control by considering the Hamiltonian function as the Lyapunov function. For this purpose, first, a nominal controller is designed by shaping the energy function of the system according to the structure of the desired limit cycle. Then, an additional robustifying control term is designed based on the integral sliding mode method with the selection of an appropriate sliding surface. Finally, computer simulations for two practical case studies are provided to confirm the effectiveness of the proposed controller in the generation of stable limit cycles in the presence of uncertainties.

KEYWORDS: Stable oscillation; Robust limit cycle; Multi-input mechanical systems; Uncertainties; Energy shaping, PCH model; Integral sliding mode.

1. Introduction

Repetitive patterns are demanded behaviors in many practical systems.[1–4] For example, in a walking robot, the foot motion follows a repetitive pattern. This behavior can be described by an attractive limit cycle in the phase trajectories of the dynamical system[5–8] The limit cycle is an isolated periodic orbit in the phase plane, which is a very rich dynamical behavior in the nonlinear dynamical systems. The authors of[9,10] have shown that walking and running can also be addressed as periodic motions, which are equal to specified limit cycles in the phase trajectories of the robotic system.

Generating stable oscillations have two main approaches. The first one is to solve the output tracking problem of a periodic reference signal. In this case, by the construction of the tracking error and its time derivations, the tracking problem transforms into the asymptotic stabilization of the error vector. Several solutions have been proposed according to this approach.[11–14] One of these approach drawbacks is the need for generation of the reference signal and its derivatives up to the relative degree of the system. Therefore, another approach was proposed. This is based on the extension of Lyapunov stability theorem from the stability analysis of the equilibrium point to the stability analysis of the positive invariant set.[15] In this approach, an appropriate limit cycle is selected according to the amplitude and frequency of the desired output oscillations.[8] Then, the Lyapunov function is shaped according to the desired limit cycle equation and limit cycle controller is designed for shaping this stable limit cycle in the phase trajectories of the controlled system. Backstepping control,[16] synchronization approach,[17] feedback linearization,[18] and state feedback control[5,19] are some of the designing methods employed by the second approach to generate stable oscillations in dynamical systems.

There are few achievements in the field of robust control design employing the latter approach. Whereas, it is essential to design a robust controller for mechanical systems exposed to model uncertainties due to external disturbances, model simplifications, and paramedic errors. The authors of[20] have proposed a robust controller for autonomous single-input single-output (SISO) systems with

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separable nonlinear terms. The design method consists of linearization of nonlinear elements based on describing function method, and then shaping the control loop to obtain the desired limit cycle in the closed-loop system. In the most recent studies, the robust limit cycles have been generated based on state-space representations and by different approaches like the Lyapunov redesign method,[21,22] sliding mode technique,[23] and performance recovery.[24] The authors of[25] have considered this problem for discrete-time nonlinear systems. Considering nonlinear systems with single input is a common feature for all the mentioned papers.

Based on the above, the contribution of this paper is to design a robust controller to generate robust and stable limit cycles in multi-input mechanical systems (from any order) despite uncertainties. This idea is done based on the PCH model of mechanical systems and via the energy-based control approach.

One of the significant methods in controller design for mechanical systems is an energy-based control technique.[26,27] Energy-based controllers are designed based on the stored energy in the mechanical system. The stored energy acts as the Lyapunov function and the nonlinear control methods, which are based on the Lyapunov function, may be applied in the energy-based control.

The essential step in exploiting the energy-based Hamiltonian approach is to transform the system into a PCH model. This issue first was introduced in.[28] Generally, this technique uses properties of the internal structure of the actual system in designing controllers and gives a relatively simpler controller with better performance.[29]

To the best knowledge of the authors, this is the first paper that robustly studies this issue based on the PCH model of multi-input mechanical systems. For this purpose, designing control law is done in two steps. First, the nominal controller is designed and then the robustifying term is added to it. The nominal controller is designed based on the idea of shaping the energy function of the system. The additional control law is proposed through the integral sliding mode concept to guarantee the asymptotic convergence to the desired limit cycle in the presence of uncertainties.

The sliding mode is a well-known technique since it is robust against external disturbances and model uncertainties.[30,31] Reducing the order of the system in the sliding phase is one of its advantages, which simplifies the design procedure. In general, the sliding mode technique has two key steps. The first one is designing an appropriate sliding surface, which satisfies the desired control goal of the problem, while system states are moving on the surface. The second one is the design of control law, such that state variables reach the sliding surface in a finite-time interval and remain on it. Moreover, this technique has two essential phases: reaching phase and sliding phase. Although the sliding mode technique is robust against matched uncertainties, it has some defects such as the existence of the reaching phase. Since in this phase, state variables have not reached the surface and are sensitive to unmatched uncertainties, the integral sliding mode strategy is developed to eliminate the reaching phase.[32,33] The designed surface by this strategy contains an integral term that assures trajectories of the system to be on the surface from the first time instant. The key contributions of this paper are summarized below:

This paper studies the robust limit cycle shaping in multi-input mechanical systems.

Limit cycle generation is done based on the PCH model of mechanical systems and via the energy-based control approach.

To handle the uncertainties effects, the integral sliding mode technique is used for the design of the robustifying control term.

Furthermore, the validity of the proposed approach is verified by using the simulation of two practical case studies.

The paper is organized as follows. The preliminaries are given in section 2. Section 3 considers the problem statement. Section 4 represents the main contribution of the paper, which studies the robust, stable limit cycle generation for multi-input systems. Simulation results are given in section 5 to validate the performance of the proposed controller. Lastly, conclusions are found in section 6.

2. Preliminaries

Consider the following nonlinear system:

$$\dot{x} = f(x), \tag{1}$$

where $x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T \in D \subset \mathbb{R}^n (0 \in D)$ is the state vector and $f : D \to \mathbb{R}^n$ is a continuous function on D that satisfies $f(0) = 0$. Solutions of the system (1) with the initial condition $x(0) = 0$ is denoted as $s(t, x_0)$.

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Definition 1[15]**.** A set $E \subset D \subseteq \mathbb{R}^n$ is a positively invariant set for the system (1), if $s(t, E) \subseteq E$ for all $t \geq 0$, where $s(t, E) = \{s(t, x) | \, x \in E\}$.

Definition 2[34]**.** The closed invariant set E is asymptotically stable if for any $e>0$, there exist $d > 0$ such that

$$x(0) \in U_\delta \Rightarrow x(t) \in U_\varepsilon \; \forall t \geq 0\,, \tag{2}$$

where

$$U_\delta = \left\{x \in \mathbb{R}^n | dist(x, E) < \delta\right\};\; U_\varepsilon = \left\{x \in \mathbb{R}^n | dist(x, E) < \varepsilon\right\}$$
$$dist\,(x, E) = \inf_{y \in E} \|x - y\| \tag{3}$$

and $\lim_{t\to\infty} dist\;(x, E) = 0$.

Theorem 1 (the Lyapunov set stability theorem)[34]. Consider the nonlinear system (1). Let E be a compact positively invariant set for the system (1). Assume that there exists a continuously differentiable function $V(x)$ such that:

$$V(x) = 0;\; \forall x \in E,$$
$$V(x) > \; 0;\; \forall x \in D \backslash E,$$
$$\dot{V}(x) < 0;\; \forall x \in D \backslash E,$$

then E is an asymptotically stable set.

Based on the Lyapunov set stability theorem, if $\dot{V}(x)$ be negative semi-definite, asymptotic stability could not be concluded. In this case, the following theorem is proposed:

Theorem 2 [15]**.** *Consider the nonlinear dynamic system* (1)*. Assume that $D_c \subset D$ is a compact positively invariant set for* (1) *and assume there exist a continuously differentiable function $V : D_c \to \mathbb{R}$ such that $\dot{V}(x) \leq 0; \forall x \in D_c$. let $\mathcal{R} \triangleq \left\{x \in D_c | \dot{V}(x) = 0\right\}$ and E is the largest invariant set contained in $\mathcal{R}$. If $x(0) \in D_c$, then $x(t) \to E$ as $t \to \infty$.*

3. Problem Statement

Consider the following dynamical equations, which represent the motion equations of n-degree-of-freedom mechanical systems:[29,35,36]

$$M(q)\ddot{q} + C(q, \dot{q})\,\dot{q} + B(q) = G(q)u, \tag{4}$$

where $q(t) = [q_1, q_2, \ldots, q_n]^T \in \mathbb{R}^n$ is the position vector, $\dot{q} \in \mathbb{R}^n$ is the velocity vector, $M = 0 < M^T(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis matrix, $B(q) \in \mathbb{R}^n$ is the potential forces vector, $G(q) \in \mathbb{R}^{n\times m}$ is the input coupling matrix, and $u \in \mathbb{R}^m$ is the applied torque vector. The system (4) can be written in the following PCH model:

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \nabla H(q, p) + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u, \tag{5}$$

where $p = M(q)\dot{q} \in \mathbb{R}^n$ and $H(q, p)$ is the Hamiltonian function which is the sum of the kinetic (i.e., $K(q, p)$) and potential (i.e., $V_0(q)$) energies of the system.

$$H(q, p) = K(q, p) + V_0(q).$$
$$K(q, p) = \tfrac{1}{2} p^T M^{-1}(q) P \tag{6}$$
$$V_0(q) = \int B(q) dq$$

In the presence of an uncertain term $\delta(x, t)$, which stems from parametric uncertainties, unmodeled dynamics and/or external disturbances, the system (4) can be written as follows:

$$M(q)\ddot{q} + C\,(q, \dot{q})\,\dot{q} + B(q) = G(q)\,(u + \delta(x, t)) \tag{7}$$

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with the following PCH model:

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \nabla H(q, p) + \begin{bmatrix} 0 \\ G \end{bmatrix} (u + \delta(x, t)) \\ = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} (u + \delta(x, t)) \quad . \tag{8}$$

Although $\delta(x, t)$ is unknown, it is norm-bounded and $\|\delta(x, t)\| \leq \lambda(x)$ where $\lambda(x)$ is a known positive function.

In what follows, the aim is to design the control law $u \in R^m$ for the multi-input system (8) such that it leads to generating the robust limit cycle E in the phase trajectories of the closed-loop system in the presence of uncertain term $\delta(x, t)$:

$$E = \left\{ x(t) \in \mathbb{R}^{2n} | \ \gamma(x(t)) = \mu, \ \mu > 0 \right\}. \tag{9}$$

In this regard, $\gamma(x)$ is assumed to be a positive function with a local minimum in the origin. In other words, the following conditions should hold:

$$\begin{cases} \nabla \gamma(x(t)) \mid_{x=0} = 0 \\ \nabla^2 \gamma(x(t)) \mid_{x=0} > 0 \end{cases} . \tag{10}$$

4. Robust Stable Limit Cycle Control

In this section, the desired stable limit cycle E (given in (9)) is created via the proposed limit cycle controller in the nonlinear uncertain system (8) in two steps. First, the nominal controller is designed for the nominal system (5). Then, by applying the integral sliding mode technique, an additional control term is designed to generate a robust stable limit cycle in the phase trajectories of the uncertain system (8).

5. Nominal Control Design

To design a control law for the nominal system (5), first, the energy function of the system is shaped according to the structure of the desired limit cycle. Then, the asymptotic stability of the limit cycle is proved using the system's energy function. The nominal control law is designed such that the system equations transform to the desired system ones, which guarantees convergence to the limit cycle E.

Dynamical equations of the desired system are considered as follows:

$$\dot{x} = (J(x) - R(x)) \nabla H_d(x), \tag{11}$$

where $J(x) = -J^T(x) \in \mathbb{R}^{2n \times 2n}$ and $R(x) = R^T(x) \in \mathbb{R}^{2n \times 2n} \geq 0$ is chosen as

$$R(x) = \varphi(x) G' G'^T \tag{12}$$

in which, $\varphi(x)$ is an arbitrary positive function and $G' = \begin{bmatrix} 0 & G \end{bmatrix}^T \in \mathbb{R}^{2n \times m}$. Moreover, $H_d(x)$ is the desired Hamiltonian function and is designed according to Theorem 1 such that it is positive definite with respect to set E. In other words, $H_d(x)$ must be zero for any point of set E (defined in (9)) and positive definite for some neighborhood of E (excluding E itself). Considering $H_d(x)$ as follows, satisfies the required conditions:

$$H_d(x) = \frac{1}{2} (\gamma(x) - \mu)^2, \tag{13}$$

where $\mu > 0$ and $\gamma(x)$ satisfies (10). In the next theorem, the nominal control law (u_{nom}) is designed such that state variables of the controlled nominal system (5) track one of the systems (11).

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Theorem 3. *Consider the dynamical equations (5). If the output* $y = G'^T \nabla\gamma(x)$ *be zero state observable, then the following control law guarantees the asymptotic convergence of the controlled system (5) to the limit cycle* $E = \{x(t) \in \mathbb{R}^{2n} |\ \gamma(x(t)) = \mu\}$*:*

$$u_{nom} = \left(G'^T G'\right)^{-1} G'^T \left((J(x) - R(x))\, \nabla H_d(x) - \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \nabla H(x) \right). \tag{14}$$

Furthermore, $D_c = \left\{ x \in \mathbb{R}^{2n} | H_d(x) < \frac{\mu^2}{2} \right\}$ is the region of attraction for the considered limit cycle.

Proof. Substituting the control law (14) in the equations of the nominal system (5), and considering (12), and (13), results in:

$$\dot{x} = \left(J(x) - \varphi(x) G' G'^T\right) \nabla H_d(x). \tag{15}$$

Considering (13), then $\nabla H_d(x) = (\gamma(x) - \mu)\, \nabla\gamma(x)$and therefore

$$\dot{x} = (\gamma(x) - \mu) \left(J(x) - \varphi(x) G' G'^T\right) \nabla\gamma(x). \tag{16}$$

Choosing $\gamma(x) = \mu$ leads to $\dot{x} = 0$, which means:

$$x(0) \in E \Rightarrow x(t) \in E \qquad \forall t \geq 0\,. \tag{17}$$

Therefore, E is an invariant set of the system (16). Beside, $H_d(x) = \frac{1}{2}(\gamma(x) - \mu)^2$, which represents a measure of the distance to the limit cycle, is chosen as the Lyapunov function for Theorem 1. Note that $H_d(0) = \frac{\mu^2}{2}$. Let D_c is defined as follows:

$$D_c = \left\{ x \in \mathbb{R}^{2n} | H_d(x) < \frac{\mu^2}{2} \right\} = \left\{ x \in \mathbb{R}^{2n} | \frac{1}{2}(\gamma(x) - \mu)^2 < \frac{\mu^2}{2} \right\}. \tag{18}$$

Since $H_d(0) = \frac{\mu^2}{2}$, it is concluded that $x = 0 \notin D_c$. Considering the output of the system as $y = G'^T \nabla\gamma(x)$ and taking the time derivative of $H_d(x)$ along the trajectories of system (16) results in:

$$\begin{aligned} \dot{H}_d(x) &= \nabla^T H_d(x) \dot{x} \\ &= (\gamma(x) - \mu) \nabla^T \gamma(x) \dot{x} \\ &= -(\gamma(x) - \mu)^2 \varphi(x) \nabla^T \gamma(x) G' G'^T \nabla\gamma(x). \\ &\quad + (\gamma(x) - \mu)^2 \nabla^T \gamma(x) J(x) \nabla\gamma(x) \\ &= -(\gamma(x) - \mu)^2 \varphi(x) y^T y \end{aligned} \tag{19}$$

Since $J(x) = -J^T(x)$, thus $\nabla^T \gamma(x) J(x) \nabla\gamma(x) = 0$. By defining $\mathcal{R} \triangleq \{x \in D_c | \dot{H}_d(x) = 0\}$ and taking account that the output is zero state observable (i.e., $y \equiv 0 \Rightarrow x \equiv 0$) and $x = 0 \notin D_c$, thus according to Theorem 2, if $x(0) \in D_c$, then $x(t) \to E$ (or equivalently $(\gamma(x) - \mu) \to 0$as $t \to \infty$). Consequently, the attractive limit cycle $E = \{x(t) \in \mathbb{R}^{2n} |\ \gamma(x(t)) = \mu,\ \mu > 0\ \}$ is created in the controlled system (5) via the controller (14). Moreover, the origin is surrounded by the limit cycle E and the stability of the limit cycle in the given domain of attraction D_c guarantees that all state trajectories starting in a neighborhood of the origin converges to the limit cycle and the origin is unstable.

6. Robust Control Design

In this section, the aim is to design a robustifying control law in the presence of uncertainty based on the integral sliding mode approach. In this technique, a new control law (u_{ISM}) is designed and added to the nominal control law (u_{nom}) such that the following total control law:

$$u(t) = u_{nom}(t) + u_{ISM}(t) \tag{20}$$

is robust against the norm-bounded uncertainties.

In the previous section, the nominal control law (14) was designed to assure asymptotic convergence of the state vectors of the controlled nominal system to the desired limit cycle E. In order to design the additional robustifying control term (u_{ISM}), the sliding surface ($s(x)$) is chosen as

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$$\begin{aligned} s(x) &= P\left\{x(t) - x(t_0) - \int_{t_0}^{t} \dot{x}_{nom} dt\right\}, \\ x(t_0) &= x_{nom}(t_0) \end{aligned} \tag{21}$$

where $P \in \mathbb{R}^{m\times 2n}$ is an arbitrary matrix and is chosen such that PG' be invertible. Moreover, $x(t)$ is the state vector of the uncertain system (8) and $x_{nom}(t)$ is the state vector of the controlled nominal system (5) with the nominal controller (14). According to the sliding surface (21), it is inferred that at $t = t_0$, the state vector of the system is located on the sliding surface. Therefore, in this technique, the reaching phase is eliminated. Moreover, the term

$$\int_{t_0}^{t} \dot{x}_{nom} dt = x_{nom}(t) - x_{nom}(t_0) \tag{22}$$

represents the state of the closed-loop nominal system (5) via the nominal control law (u_{nom}). Thus, the sliding surface contains a term, which is the difference between the actual value of the state vector (in the presence of uncertainties) and the nominal value of it (in the absence of uncertainties). Therefore, if the difference becomes zero (by designing an appropriate robustifying term u_{ISM}), the state vector of the perturbed system will track the trajectories of the state vector of the controlled nominal system, and therefore converges to the desired limit cycle E in the presence of model uncertainties. According to the equations of the controlled nominal system (5), the sliding surface becomes:

$$s(x) = P\left\{x(t) - x\,(t_0) - \int_{t_0}^{t} \left(\begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \nabla H + G' u_{nom}\right) dt\right\} \tag{23}$$

in what follows, a theorem is given which provides the appropriate additional robustifying controller (u_{ISM}).

Theorem 4. *Consider the system* (8)*. The controller* $u = u_{nom} + u_{ISM}$*, where* u_{nom} *is as given in equation* (14) *and* u_{ISM} *is given in* (24)*, guarantees robust asymptotic convergence to the limit cycle* E *despite the uncertain term* $\delta(x, t)$*:*

$$u_{ISM}(t) = -\rho(x) \frac{\left(PG'\right)^T s(x)}{\left\|\left(PG'\right)^T s(x)\right\|}, \tag{24}$$

where $\rho(x)$ is chosen such that $\rho(x) \geq \lambda(x) + \beta_0$ (β_0 is a small positive constant).

Proof. Consider $V(s) = \frac{1}{2} s(x)^T s(x) = \frac{1}{2} \|s(x)\|^2$ as the Lyapunov function candidate. By taking the time derivative of $V(s)$ along the trajectories of the system (8), one has:

$$\dot{V}(s) = \dot{s}(x)^T s(x). \tag{25}$$

According to (21), (24), (5), and (8), one may write:

$$\begin{aligned} \dot{s}(x) &= P(\dot{x}(t) - \dot{x}_{nom}(t)) \\ &= PG'(u_{ISM}(t) + \delta(x, t)) \\ &= -PG'\rho(x) \frac{\left(PG'\right)^T s(x)}{\left\|\left(PG'\right)^T s(x)\right\|} + PG'\delta(x, t) \end{aligned}. \tag{26}$$

Thus

$$\begin{aligned} \dot{V}(s) &= -\frac{\rho(x)}{\left\|\left(PG'\right)^T s(x)\right\|} \left(PG'\left(PG'\right)^T s(x)\right)^T s(x) + \left(PG'\delta(x, t)\right)^T s(x) \\ &\leq -\frac{\rho(x)}{\left\|\left(PG'\right)^T s(x)\right\|} \left\|\left(PG'\right)^T s(x)\right\|^2 + \delta^T(x, t)\left(PG'\right)^T s(x) \\ &= -\rho(x) \left\|\left(PG'\right)^T s(x)\right\| + \delta^T(x, t)\left(PG'\right)^T s(x) \end{aligned}. \tag{27}$$

Considering $\|\delta(x, t)\| \leq \lambda(x)$, (27) leads to the following inequality

$$\begin{aligned} \dot{V}(s) &\leq -\rho(x) \left\|PG'\right\| \|s(x)\| + \|\delta(x, t)\| \left\|PG'\right\| \|s(x)\| \\ &\leq -\rho(x) \left\|PG'\right\| \|s(x)\| + \left\|PG'\right\| \|s(x)\| \lambda(x) \end{aligned}. \tag{28}$$

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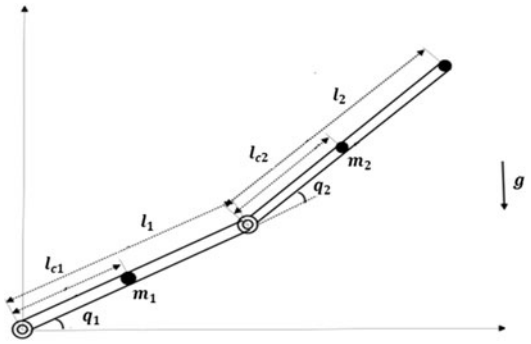


Fig. 1. The two-link robot manipulator system.

Assuming $\rho(x) \geq \lambda(x) + \beta_0$, where $\beta_0 > 0$, then the relation (28) yields to:

$$\begin{aligned}\dot{V}(s) &\leq -(\lambda(x) + \beta_0) \left\| PG' \right\| \|s(x)\| + \left\| PG' \right\| \|s(x)\| \lambda(x) \\ &= -\beta_0 \left\| PG' \right\| \|s(x)\| \\ &= -\beta_0 \left\| PG' \right\| \sqrt{2V(s)}. \end{aligned} \tag{29}$$

The above relation assures that all trajectories of the system starting from the sliding surface cannot leave it.[37] Therefore, $s(x) = 0\ \forall t \geq 0$ which means that the perturbed controlled system (8) follows the controlled nominal system, and therefore the phase trajectories asymptotically converge to the desired limit cycle E. □

Remark 1. The proposed controller may lead to a chattering problem due to discontinuity in the relation (24). To avoid the phenomena, the following relation may be used for a small positive value of ε:[38,39]

$$u_{ISM}(t) = -\rho(x) \frac{\left(PG'\right)^T s(x)}{\left\| (PG')^T s(x) \right\| + \varepsilon}.$$

7. Simulation Results

In this section, two case studies are considered to evaluate the performance of the proposed control.

First case study: Consider a two-link robot manipulator which is shown in Fig. 1.

Considering $q = [q_1, q_2]^T \in \mathbb{R}^2$ as the angular position vector and $u = [u_1, u_2]^T \in \mathbb{R}^2$ as the control torques of motors; the equations (7) are satisfied for the following parameters:[40]

$$\begin{aligned} M(q) &= \begin{bmatrix} \bar{m}_1 + \bar{m}_2 + 2\bar{m}_3 cosq_2 & \bar{m}_2 + \bar{m}_3 cosq_2 \\ \bar{m}_2 + \bar{m}_3 cosq_2 & \bar{m}_2 \end{bmatrix} \\ C(q, \dot{q}) &= \begin{bmatrix} -\bar{m}_3 \dot{q}_2 sinq_2 & -\bar{m}_3 (\dot{q}_1 + \dot{q}_2) sinq_2 \\ \bar{m}_3 \dot{q}_1 sinq_2 & 0 \end{bmatrix} \\ B(q) &= \begin{bmatrix} \bar{m}_4 g\, cosq_1 + \bar{m}_5 g \cos(q_1 + q_2) \\ \bar{m}_5 g \cos(q_1 + q_2) \end{bmatrix} \\ G &= I_{2\times 2}, \end{aligned} \tag{30}$$

where

$$\begin{aligned} \bar{m}_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + m_P l_1^2 \\ \bar{m}_2 &= m_2 l_{c2}^2 + I_2 + m_P l_2^2 \\ \bar{m}_3 &= m_2 l_1 l_{c2} + m_P l_1 l_2 \end{aligned}$$

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Table I. Parameters of the two-link robot manipulator system.

Parameter	**Definition**
m_i	mass of ith link
l_i	length of ith link
l_{ci}	distance from ith node to the ith link center of mass
I_i	moment of inertia of ith link
m_p	mass of load

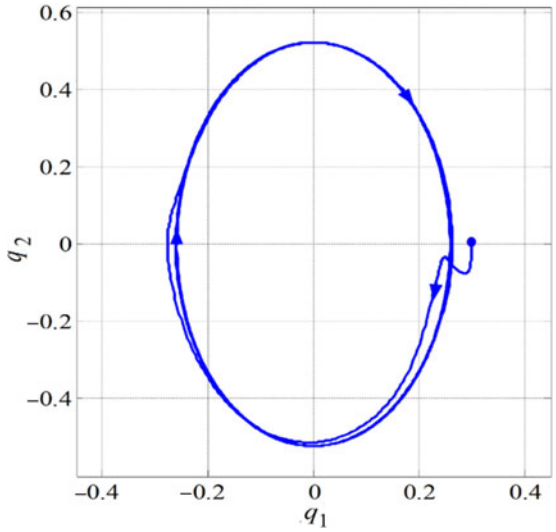


Fig. 2. Convergence of the phase trajectory of the closed-loop system to the desired limit cycle.

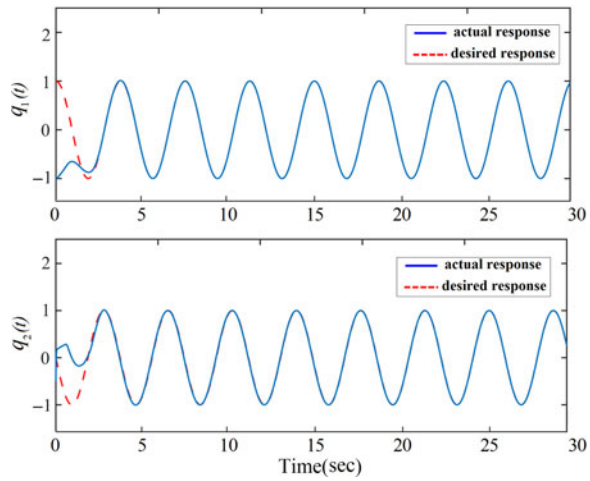


Fig. 3. The time responses of real and desired angular positions.

$$\bar{m}_4 = m_1 l_{c2} + m_2 l_1 + m_P l_1$$

$$\bar{m}_5 = m_2 l_{c2} + m_P l_2$$

and the parameters are introduced in Table I.

Furthermore, the limit cycle $E = \{x(t) \in \mathbb{R}^{2n} |\, \gamma(x(t)) = \mu, \}$ is considered as $E = \{x(t) \in \mathbb{R}^{2n} |\, 4q_1^2 + q_2^2 = 0.25 \}$ and the uncertain term is selected as $\delta(x, t) = 0.5\, sin\,(2t)$. Figure 2 depicts that phase trajectories of the controlled system, by the proposed control law (20), could appropriately converge to the desired limit cycle. Furthermore, the effectiveness of the

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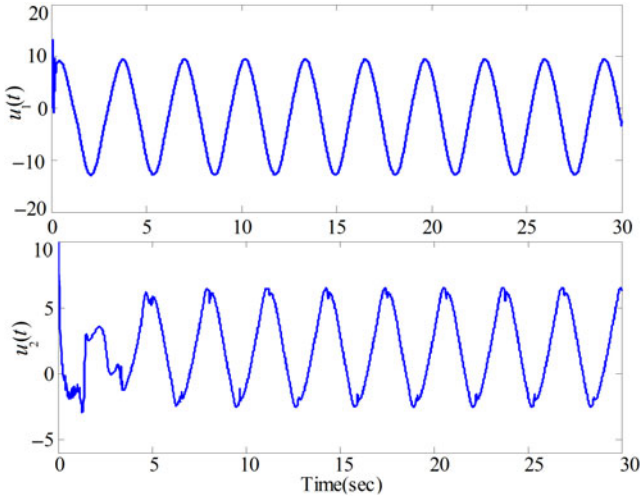


Fig. 4. The time responses of robust control inputs.

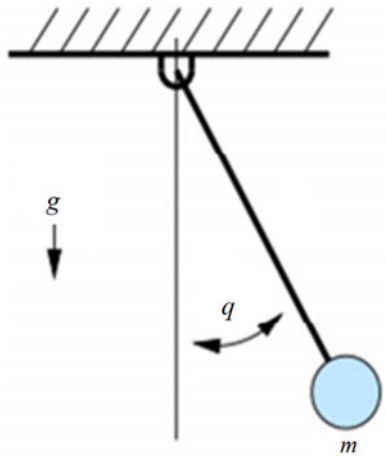


Fig. 5. Inverted pendulum system.

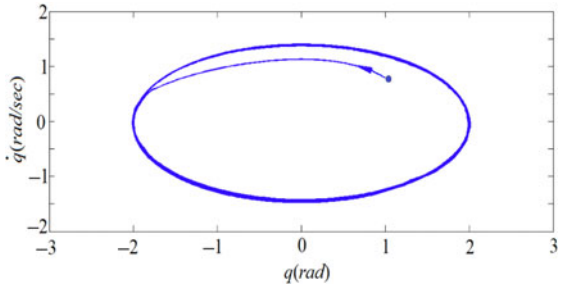


Fig. 6. Convergence of closed-loop system (33) to the desired limit cycle in phase plan.

designed controller to generate the desired oscillations is shown in Fig. 3. The time responses of the proposed robust controllers are illustrated in Fig. 4.

Second case study: Consider the inverted pendulum that is shown in Fig. 5. The motion equation of the inverted pendulum is as follows:[36]

$$\ddot{q} + \frac{k_0}{m}\dot{q} + \frac{g}{l}\sin(q) = \frac{1}{ml^2}(u + \delta)\,, \tag{31}$$

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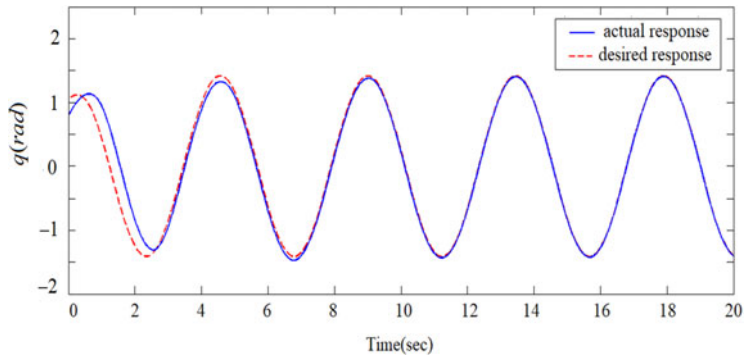


Fig. 7. The time response of the angular position of the closed-loop system.

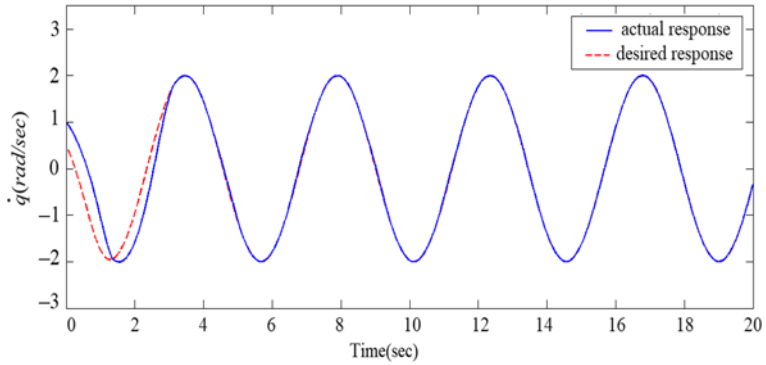


Fig. 8. The time response of angular velocity of the closed-loop system.

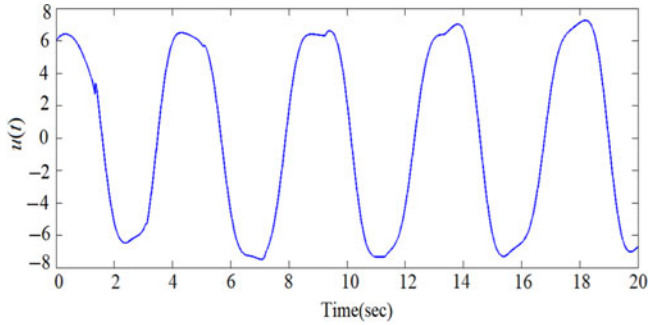


Fig. 9. The time response of robust control input.

where m is mass of the pendulum, l is the length of the arm, g is the gravity constant, k_0 is the friction coefficient, and q is the angular position of the pendulum. Moreover, u is the applied torque, δ is the uncertainty and $x = \left[q \;\; p \right]^T$, where $p = \dot{q}$.

Simulations are done by choosing system and controller parameters as $m = 1\,kg$, $l = 1\;\; m$, $g = 9.8\;\; m/s^2$, $k_0 = 0.002$, $\xi = 1$, $\varphi(x) = 5$, $\gamma(x) = 0.5q^2 + 0.25p^2$, and $\rho(x) = 0.6$. Moreover, uncertainty is considered as $\delta(x, t) = 0.5\, sin\,(3t)$. Figure 6 depicts that phase trajectories of the closed-loop system could appropriately converge to the desired limit cycle E. Furthermore, the effectiveness of the designed controller to generate the desired oscillations in the state variables are shown in Figs. 7 and 8. The time history of the proposed robust controller is illustrated in Fig. 9.

8. Conclusion

This paper studied the problem of robust limit cycle generation for multi-input mechanical systems subjected to uncertainties. The design procedure contained two steps. First, a nominal control law was designed based on the idea of shaping the energy function of the PCH model of the system. In

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the second step, by making use of an integral sliding mode method and designing a suitable sliding surface, an additional robustifying control term was proposed. Finally, simulations were done for two practical case studies and the results verified the effectiveness of the proposed control approach in the presence of uncertainties.

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