J. Plasma Physics (2006), *vol.* 72, *part* 2, *pp.* 205–212. © 2005 Cambridge University Press
doi:10.1017/S0022377805003892 First published online 25 November 2005 Printed in the United Kingdom

Screening effects on elastic electron–atom collisions in partially ionized hydrogen plasmas

MYOUNG-JAE LEE[1] and YOUNG-DAE JUNG[2]

[1]Department of Physics, Hanyang University, Seoul 133-791, South Korea
[2]Department of Applied Physics, Hanyang University, Ansan Kyunggi-Do 426-791, South Korea
(ydjung@hanyang.ac.kr; yjung@ihanyang.ac.kr)

(Received 10 January 2005 and accepted 30 January 2005)

Abstract. The non-ideal collective and plasma screening effects on elastic electron–atom collisions are investigated in partially ionized hydrogen plasmas using the first-order eikonal method. The effective pseudopotential model, taking into account the screening and collective effects, is applied to describe the interaction potential in non-ideal plasmas. Semiclassical trajectory analysis is applied to the path of the projectile electron in order to investigate the variation of the eikonal phase and cross section as functions of the impact parameter, the non-ideality plasma parameter, the Debye length, and the collision energy. The results show that the collective and screening effects enhance the differential eikonal cross section for small impact parameters, but suppress the cross section for large impact parameters. It is also found that the non-ideal collective effects enhance the total eikonal cross section.

1. Introduction

Recently, elastic electron collisions (Janev et al. 1985; Shevelko and Vainshtein 1993; Salzmann 1995; Jung 1996, 1997, 2003; Shevelko and Tawara 1998; Beyer and Shevelko 2003) in plasmas have been of great interest since these processes can be used for plasma diagnostics. However, most of the literature has dealt with electron–ion elastic collision radiation rather than the electron–atom elastic collision. Recently, the electron–ion elastic collision in weakly coupled plasmas has been investigated using the Yukawa-type Debye–Hückel potential (Jung 1997). The classical Debye–Hückel model has been used to describe the properties of a low-density plasma and corresponds to a pair-correlation approximation. It has been known that the plasmas described by the Debye–Hückel model are ideal plasmas since the average interaction energy between charged particles is smaller than the average kinetic energy of a particle (Zubarev et al. 1996). However, the multiparticle correlation effects caused by the simultaneous interaction of many charged particles should be taken into account with increasing plasma density. Hence, it is necessary to take into account not only short-range collective effects but also long-range effects in the case of a plasma with a moderate density and temperature. In these circumstances, the interaction potential would not be described by the ordinary

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

Debye–Hückel model because of the strong collective effects of non-ideal particle interactions (Baimbetov et al. 1995; Arkhipov and Davletov 1998: Arkhipov et al. 2000, 2002). Then, the electron–atom elastic collision process in non-ideal plasmas would be different from those in ideal plasmas. It has long been known that the eikonal method (Burke and Joachain 1995; Kopaleishvili 1996; Jung 1996) has a great advantage since we can obtain the generalized continuum states of the projectile electron in terms of the eikonal phase, including various physical properties of the surrounding plasma. Then, using the eikonal phase and wave function, we can investigate scattering cross sections in plasmas since the phase includes all physical information. Thus, in this paper we investigate the non-ideal collective and plasma screening effects on the electron–atom elastic collisions in partially ionized hydrogen plasmas using the pseudopotential model with the eikonal analysis since the theoretical plasma spectroscopy is essential to the study of various plasma parameters. The semiclassical impact-parameter analysis is applied to the path of the projectile electron in order to investigate the variation of the eikonal phase and cross section as functions of the impact parameter, i.e. the separation between the projectile electron and the target ion.

In Sec. 2, we discuss the eikonal method and derive a closed form for the eikonal phase in electron–atom elastic collision using the semiclassical trajectory method. In Sec. 3, we obtain the eikonal cross section for electron–atom elastic collisions in non-ideal plasmas. We also investigate the non-ideal collective and plasma screening effects on the eikonal cross section for several values of the non-ideality plasma parameter. Finally, in Sec. 4, conclusions are given.

2. Eikonal analysis

Using the first-order theory arising from the Lippmann–Schwinger equation (Sakurai 1994),

$$\Psi(\mathbf{r}) = \phi(\mathbf{r}) + \int d^3\mathbf{r}\, G_0(\mathbf{r},\mathbf{r}')\, U(\mathbf{r})\, \Psi(\mathbf{r}), \tag{1}$$

where $U(\mathbf{r})$ $[= 2mV(\mathbf{r})/\hbar^2]$ is the reduced interaction potential, m is the electron rest mass, $V(\mathbf{r})$ is the interaction potential, $\Psi(\mathbf{r})$, $\phi(\mathbf{r})$ and $G_0(\mathbf{r},\mathbf{r}')$ are, respectively, the solution of the Schrödinger equation, the solution of the homogeneous equation and the Green's function such that

$$(\nabla^2 + k^2)\Psi(\mathbf{r}) = U(\mathbf{r})\, \Psi(\mathbf{r}), \tag{2}$$

$$(\nabla^2 + k^2)\phi(\mathbf{r}) = 0, \tag{3}$$

$$(\nabla^2 + k^2)G_0(\mathbf{r},\mathbf{r}') = \delta^3(\mathbf{r}-\mathbf{r}'), \tag{4}$$

where k $(= \sqrt{2mE/\hbar^2})$ is the wave number of the projectile electron, E is the collision energy and $\delta^3(\mathbf{r}-\mathbf{r}')$ is the Dirac delta function. If we have chosen the z-axis in the direction of the incident wave number k_i and using the expansion method on the outgoing free-particle Green's function $G_0^{(+)}(\mathbf{r},\mathbf{r}')$ (Joachain 1983), we then obtain the eikonal wave function $\Psi_\mathrm{E}(\mathbf{r})$, expressed as

$$\Psi_\mathrm{E}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \exp\left[i\mathbf{k}_\mathrm{i}\cdot\mathbf{r} - \frac{i}{2k_\mathrm{i}} \int_{-\infty}^{z} dz'\, U(z',\mathbf{b}) \right], \tag{5}$$

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

where **b** is the impact parameter and the integration is along a straight line parallel to the incident wave vector $k_{\rm i}$. The condition of validity of the eikonal method is known to be $|V|/E \ll 1$, where $|V|$ is a typical strength of the interaction potential. Using the cylindrical coordinates such that $\mathbf{r} = z\hat{\mathbf{n}} + \mathbf{b}$, where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the momentum transfer Δ ($\equiv \mathbf{k}_{\rm i} - \mathbf{k}_{\rm f}$), $\mathbf{k}_{\rm f}$ is the final wave vector, and the integration over the variable z, the eikonal scattering amplitude $f_{\rm E}$ can be expressed as follows:

$$f_{\rm E} = -\frac{1}{4\pi}\int d^3\mathbf{r}\, e^{i\Delta\cdot\mathbf{r}}\, U(\mathbf{r}) \exp\left[-\frac{i}{2k_{\rm i}}\int_{-\infty}^{z} dz'\, U(z', \mathbf{b})\right]$$
$$= -\frac{ik_{\rm i}}{2\pi}\int d^2\mathbf{b} \exp[i(\Delta\cdot\mathbf{b} + \chi(k_{\rm i}, \mathbf{b}))], \tag{6}$$

where $\chi(k_{\rm i}, b)$ $[= -(m/k_{\rm i}\hbar^2)\int_{-\infty}^{\infty} dz\, V(r)]$ is the eikonal phase with $r = (z^2 + b^2)^{1/2}$. Hence, the main contribution comes from the values of the impact parameter b in the vicinity of the extremum of the total eikonal phase, i.e. $\Delta + \partial\chi(k_{\rm i}, b)/\partial b = 0$. After some algebra, the differential eikonal scattering cross section for the interaction potential $V(r)$ can be represented by

$$\frac{d\sigma_{\rm E}}{db} = 2\pi b\, |\exp[i\chi(k_{\rm i}, b)] - 1|^2. \tag{7}$$

The higher-order correction to the first-order eikonal phase has been proposed within the impact parameter formalism (Burke and Joachain 1995). A recent investigation (Jung 1996) for elastic electron–ion collisions shows that the higher-order corrections to the first-order eikonal phase are found to be less than 1% for high projectile energies E ($\equiv E/Z^2Ry$) $\geqslant 10$, where Ry ($= me^4/2\hbar^2 \approx 13.6$ eV) is the Rydberg constant. Hence, in this paper, the higher-order correction to the first-order eikonal phase is neglected since the investigation of the screening effects on the high-energy electron–atom elastic collision is the main purpose of this paper.

3. Eikonal cross section

In a recent paper (Baimbetov et al. 1995), an integro-differential equation for the effective interaction potential of particle interactions, taking into account the simultaneous correlations of many particles, has been obtained on the basis of a sequential solution of the chain of Bogolyubov equations for the equilibrium distribution function of charged particles of a non-ideal plasma. In addition, the analytic expression for the pseudopotential of the particle interaction in non-ideal plasmas has been obtained by application of the spline approximation. Using the effective pseudopotential model (Baimbetov et al. 1995), taking into account the plasma screening and non-ideal collective effects, the interaction potential $V(\mathbf{r}, \mathbf{r}')$ between the projectile electron (e^-) and the hydrogen atom (H) in non-ideal plasmas is represented by

$$V(\mathbf{r}, \mathbf{r}') = -\frac{e^2}{r}e^{-r/\Lambda}\frac{1 + \gamma f(r)/2}{1 + c(\gamma)} + \frac{e^2}{|\mathbf{r} - \mathbf{r}'|}e^{-|\mathbf{r}-\mathbf{r}'|/\Lambda}\frac{1 + \gamma f(|\mathbf{r} - \mathbf{r}'|)/2}{1 + c(\gamma)}, \tag{8}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are, respectively, the positions of the projectile electron and the bound electron, Λ is the Debye length, γ ($\equiv e^2/\Lambda k_{\rm B}T$) is the non-ideality plasma parameter, $f(x) = (e^{-\sqrt{\gamma}x/\Lambda} - 1)(1 - e^{-2x/\Lambda})/5$ and $c(\gamma) \cong 0.456\gamma - 0.108\gamma^2$ is

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

the correlation coefficient, k_B is the Boltzmann constant and T is the plasma temperature. When $\gamma \to 0$, i.e. in ideal plasmas, the effective pseudopotential (8) goes over into the Debye–Hückel potential $V(\mathbf{r},\mathbf{r}') \to -(e^2/r)\,e^{-r/\Lambda} + (e^2/|\mathbf{r}-\mathbf{r}'|)\,e^{-|\mathbf{r}-\mathbf{r}'|/\Lambda}$. Recently, the eigenfunctions Ψ_{nlm} and energy eigenvalues E_{nl} for the atomic states nlm of the Schrödinger equation for the hydrogenic ion in non-ideal plasmas have been obtained by using the variation and perturbation methods (Song and Jung 2001) with the effective pseudopotential. Hence, the wave function for the ground state of the hydrogen atom in non-ideal plasmas would be represented by $\Psi_{1s}(\mathbf{r}') = (\pi\alpha_{1s}^3)^{-1/2}\,e^{-r'/\alpha_{1s}}$, where α_{1s} is the 1s variation parameter obtained by the minimization of the energy expectation value, i.e. $\partial\langle E_{1s}(\alpha_{1s})\rangle/\partial\alpha_{1s} = 0$:

$$\alpha_{1s} \cong a_0\left(A(\gamma)\left\{1-\frac{3}{4}\left[\frac{a_\Lambda}{A(\gamma)}\right]\left(1-\frac{2}{5}\gamma^{3/2}\right)+\left[\frac{a_\Lambda}{A(\gamma)}\right]^3\left(1-\frac{12}{5}\gamma^{3/2}-\frac{3}{5}\gamma^2\right)\right\}\right)^{-1}, \tag{9}$$

where a_0 $(=\hbar^2/me^2)$ is the Bohr radius, $A(\gamma) \equiv [1+c(\gamma)]^{-1}$ and a_Λ $(\equiv a_0/\Lambda)$ is the scaled reciprocal Debye length. Using the expression for the modified Helmholtz operator Green's function (Arfken and Weber 2001):

$$\frac{e^{-\xi|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = 4\pi\xi\sum_{l=0}^{\infty}\sum_{m=-l}^{l} i_l(\xi r_<)\,k_l(\xi r_>)\,Y_{lm}(\hat{\mathbf{r}})\,Y_{lm}^*(\hat{\mathbf{r}}'), \tag{10}$$

where i_l and k_l are the spherical modified Bessel functions, $r_<$ $(r_>)$ is the smaller (larger) of r and r', and Y_{lm} are the spherical harmonics, the $e^- + \mathrm{H}$ atomic interaction potential in non-ideal plasmas is found to be

$$\begin{aligned} V_{1s}(r) &= \int d^3\mathbf{r}'\,|\Psi_{1s}(\mathbf{r}')|^2\,V(\mathbf{r},\mathbf{r}'), \\ &= -\frac{e^2}{1+c(\gamma)}\left\{I\left(r,\frac{1}{\Lambda},\alpha_{1s}\right)\right. \\ &\quad \left. +\frac{\gamma}{10}\sum_{n=1}^{2}(-1)^{n-1}\left[I\left(r,\frac{\sqrt{\gamma}+2n-1}{\Lambda},\alpha_{1s}\right)-I\left(r,\frac{2n-1}{\Lambda},\alpha_{1s}\right)\right]\right\}, \end{aligned} \tag{11}$$

where the function $I(r,\zeta,\alpha_{1s})$ is represented by

$$\begin{aligned} I(r,\zeta,\alpha_{1s}) &= \frac{e^{-\zeta r}}{r} - \frac{4}{\alpha_{1s}^3[(2/\alpha_{1s})^2-\zeta^2]^2} \\ &\quad \times\left\{2\left(\frac{2}{\alpha_{1s}}\right)\frac{e^{-\zeta r}}{r} - \left[2\left(\frac{2}{\alpha_{1s}}\right)+\left[\left(\frac{2}{\alpha_{1s}}\right)^2-\zeta^2\right]r\right]\frac{e^{-(2/\alpha_{1s})r}}{r}\right\}. \end{aligned} \tag{12}$$

Using the integral representation of the second kind of modified Bessel function of order n (Abramowitz and Stegun 1965):

$$K_n(\mu) = \frac{\pi^{1/2}(\mu/2)^n}{\Gamma(n+\frac{1}{2})}\int_1^\infty dt\,e^{-\mu t}\,(t^2-1)^{n-1/2}, \tag{13}$$

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

where $\Gamma(n+\frac{1}{2})$ is the gamma function, the eikonal phase for the $e^- + \mathrm{H}$ elastic collision is obtained by

$$\chi(\bar{E},\bar{b},a_\Lambda,\gamma) = -\frac{e^2}{1+c(\gamma)}\frac{1}{\sqrt{\bar{E}}}\Bigg\{N(\bar{b},a_\Lambda,\bar{\alpha}_{1s}) + \frac{\gamma}{10}\sum_{n=1}^{2}(-1)^{n-1} \times [N(\bar{b},(\sqrt{\gamma}+2n-1)a_\Lambda,\bar{\alpha}_{1s}) - N(\bar{b},(2n-1)a_\Lambda,\bar{\alpha}_{1s})]\Bigg\}, \quad (14)$$

where $\bar{b}$ ($\equiv b/a_0$) is the scaled impact parameter and the function $N(\bar{b},\bar{\beta},\bar{\alpha}_{1s})$ is

$$N(\bar{b},\bar{\beta},\bar{\alpha}_{1s}) = 2K_0(\bar{\beta}\bar{b}) - \frac{8}{\bar{\alpha}_{1s}^3[(2/\bar{\alpha}_{1s})^2-\bar{\beta}^2]^2}\Bigg\{2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0(\bar{\beta}\bar{b}) - \left[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right) + \left[\left(\frac{2}{\bar{\alpha}_{1s}}\right)^2-\bar{\beta}^2\right]\bar{b}K_0\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right)\right]\Bigg\}, \quad (15)$$

with $\bar{\alpha}_{1s} \equiv \alpha_{1s}/a_0$ and $\bar{\beta} \equiv \beta a_0$. Hence, the scaled differential eikonal cross section in units of πa_0^2 for the $e^- + \mathrm{H}$ elastic collision in non-ideal plasmas is

$$\begin{aligned}
\frac{d\sigma_E(\bar{E},\bar{b},a_\Lambda,\gamma)}{d\bar{b}\,\pi a_0^2} &= 2\bar{b}\,|\exp[i\chi(\bar{E},\bar{b},a_\Lambda,\gamma)]-1|^2 \\
&= 2\bar{b}\Bigg|\exp\Bigg[-\frac{ie^2}{1+c(\gamma)}\frac{1}{\sqrt{\bar{E}}}\Bigg\{\Bigg[2K_0(a_\Lambda\bar{b}) - \frac{8}{\bar{\alpha}_{1s}^3[(2/\bar{\alpha}_{1s})^2-a_\Lambda^2]^2} \\
&\quad\times\Bigg[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0(a_\Lambda\bar{b}) \\
&\quad-\Bigg[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right) + \left[\left(\frac{2}{\bar{\alpha}_{1s}}\right)^2-a_\Lambda^2\right]\bar{b}K_1\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right)\Bigg]\Bigg]\Bigg] \\
&\quad+\frac{\gamma}{10}\sum_{n=1}^{2}(-1)^{n-1}\Bigg[\Bigg[2K_0((\sqrt{\gamma}+2n-1)a_\Lambda\bar{b}) \\
&\quad-\frac{8}{\bar{\alpha}_{1s}^3[(2/\bar{\alpha}_{1s})^2-((\sqrt{\gamma}+2n-1)a_\Lambda)^2]^2} \\
&\quad\times\Bigg[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0((\sqrt{\gamma}+2n-1)a_\Lambda\bar{b}) - \Bigg[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right) \\
&\quad+\left[\left(\frac{2}{\bar{\alpha}_{1s}}\right)^2-((\sqrt{\gamma}+2n-1)a_\Lambda)^2\right]\bar{b}K_1\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right)\Bigg]\Bigg]\Bigg] \\
&\quad-\Bigg[2K_0((2n-1)a_\Lambda\bar{b}) - \frac{8}{\bar{\alpha}_{1s}^3[(2/\bar{\alpha}_{1s})^2-((2n-1)a_\Lambda)^2]^2} \\
&\quad\times\Bigg[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0((2n-1)a_\Lambda\bar{b}) - \Bigg[2\left(\frac{2}{\bar{\alpha}_{1s}}\right)K_0\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right) \\
&\quad+\left[\left(\frac{2}{\bar{\alpha}_{1s}}\right)^2-((2n-1)a_\Lambda)^2\right]\bar{b}K_1\left(\frac{2}{\bar{\alpha}_{1s}}\bar{b}\right)\Bigg]\Bigg]\Bigg]\Bigg\}\Bigg] - 1\Bigg|^2. \quad (16)
\end{aligned}$$

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

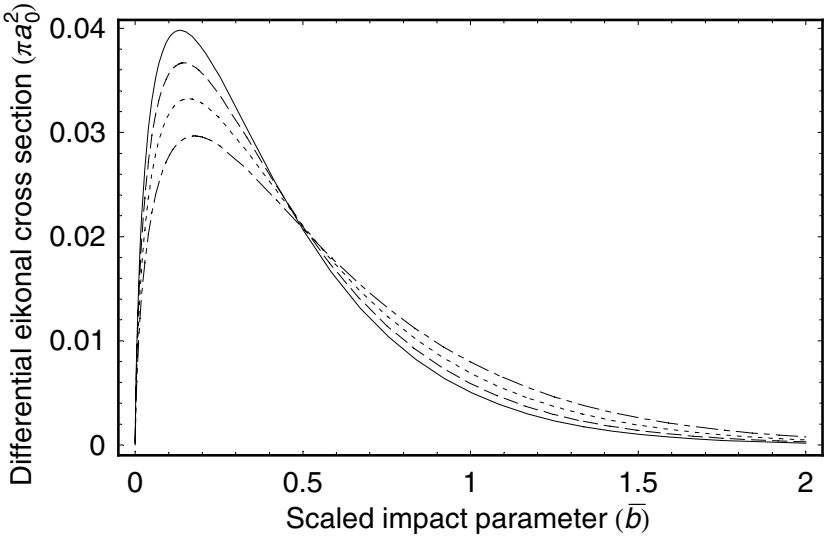


Figure 1. The differential eikonal cross section for the $e^- + \mathrm{H}$ elastic collision in non-ideal plasmas as a function of the scaled impact parameter ($\bar{b}$) when $a_\Lambda = 0.05$ and $\bar{E} = 100$. The full curve represents the case of $\gamma = 0$. The dashed curve represents the case of $\gamma = 0.2$. The dotted line represents the case of $\gamma = 0.5$. The dot-dashed curve represents the case of $\gamma = 1$.

Figure 1 shows the differential eikonal cross section for the $e^- + \mathrm{H}$ elastic collision in non-ideal plasmas as a function of the scaled impact parameter ($\bar{b}$) for various values of the non-ideality plasma parameter (γ). In addition, Fig. 2 represents the three-dimensional plot of the $e^- + \mathrm{H}$ differential eikonal cross section as a function of the scaled impact parameter and the non-ideality plasma parameter. From these figures, we found that the non-ideal collective and plasma screening effects enhance the differential eikonal cross section for small impact parameters. However, these collective and screening effects suppress the eikonal cross section for large impact parameters. Figure 3 shows the total eikonal cross section for the $e^- + \mathrm{H}$ elastic collision in non-ideal plasmas as a function of the scaled collision energy ($\bar{E}$). As we can see in this figure, it is found that the non-ideal collective effects enhance the total eikonal cross section due to weakening of the electron correlations and decrease with increasing collision energy.

4. Conclusions

The non-ideal collective and plasma screening effects on elastic electron–atom collision process are investigated in partially ionized hydrogen plasmas using the first-order eikonal method. The interaction potential in non-ideal plasmas is obtained by the effective pseudopotential model, taking into account the screening and collective effects. The semiclassical impact-parameter analysis is applied to represent the path of the projectile electron in order to investigate the variation of the eikonal cross section as a function of the impact parameter, the non-ideality plasma parameter, the Debye length and the collision energy. It is interesting to note that the non-ideal collective and plasma screening effects enhance the differential

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

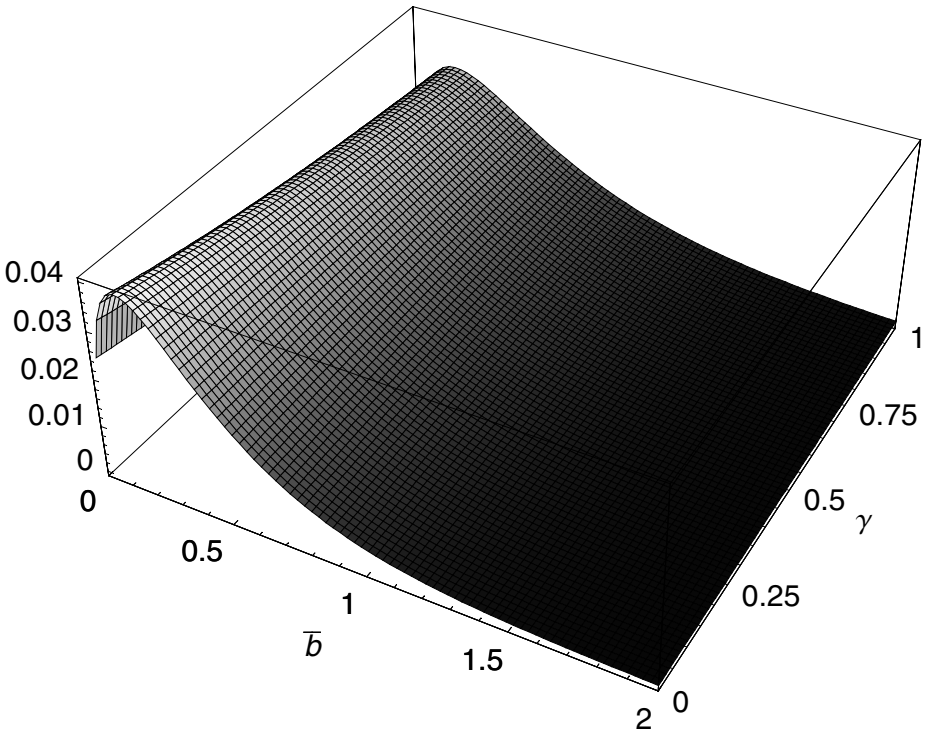


Figure 2. The three-dimensional plot of the $e^- + \mathrm{H}$ differential eikonal cross section as a function of the scaled impact parameter ($\bar{b}$) and the non-ideality plasma parameter (γ) when $a_\Lambda = 0.05$ and $\bar{E} = 100$.

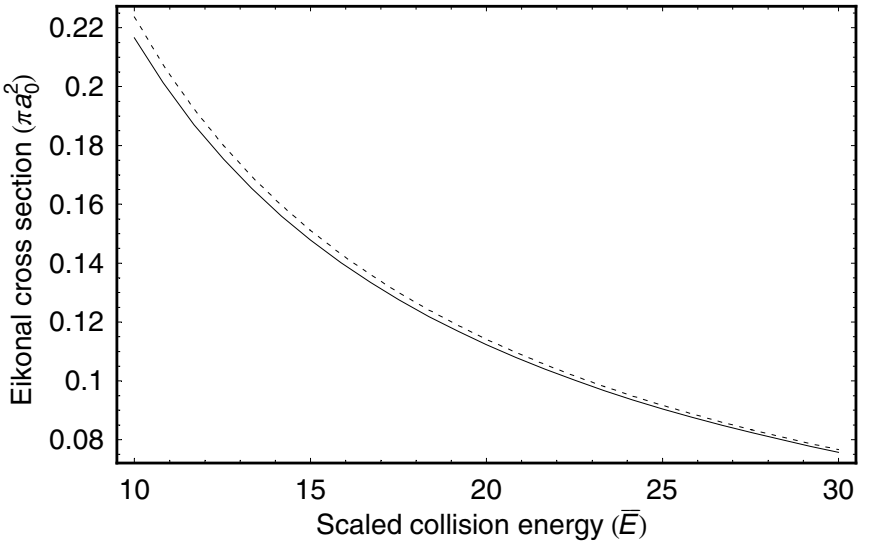


Figure 3. The total eikonal cross section for the $e^- + \mathrm{H}$ elastic collision in non-ideal plasmas as a function of the scaled collision energy ($\bar{E}$) when $a_\Lambda = 0.01$. The full curve represents the case of $\gamma = 0$. The dotted curve represents the case of $\gamma = 1$.

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press

eikonal cross section for the small impact parameter region, but suppress the cross section for the large impact parameter region. In addition, it is also found that the non-ideal collective effects enhance the total eikonal cross section due to weakening of the electron correlations. Hence, we can understand that the screening atomic effect plays an important role in atomic collisions in non-ideal plasmas. These results provide useful information on the screening and atomic effects on the elastic electron–atom collision processes in plasmas.

Acknowledgements

Y.-D. Jung gratefully acknowledges Professor T. Kato, Professor R. M. More, and Dr D. Kato for stimulating discussions and useful comments while visiting the National Institute for Fusion Science in Japan. This work was supported by the research fund of Hanyang University (HY-2004).

References

Abramowitz, M. and Stegun, A. 1965 *Handbook of Mathematical Functions*. New York: Dover.

Arfken, G. B. and Weber, H. J. 2001 *Mathematical Methods for Physicists*, 5th edn. New York: Academic Press.

Arkhipov, Yu. V. and Davletov, A. E. 1998 *Phys. Lett.* A **247**, 339.

Arkhipov, Yu. V., Baimbetov, F. B. and Davletov, A. E. 2000 *Eur. Phys. J.* D **8**, 299.

Arkhipov, Yu. V., Baimbetov, F. B., Davletov, A. E. and Starikov, K. V. 2002 *J. Plasma Phys.* **68**, 81.

Baimbetov, F. B., Nurekenov, Kh. T. and Ramazanov, T. S. 1995 *Phys. Lett.* A **202**, 211.

Beyer, H. F. and Shevelco, V. P. 2003 *Introduction to the Physics of Highly Charged Ions*. Bristol: Institute of Physics.

Burke, P. G. and Joachain, C. J. 1995 *Theory of Electron–Atom Collisions, Part 1: Potential Scattering*. New York: Plenum.

Janev, R. K., Presnyakov, L. P. and Shevelko, V. P. 1985 *Physics of Highly Charged Ions*. Berlin: Springer.

Joachain, C. J. 1983 *Quantum Collision Theory*, 3rd edn. Amsterdam: North-Holland.

Jung, Y.-D. 1996 *Phys. Plasmas* **3**, 4376.

Jung, Y.-D. 1997 *Phys. Plasmas* **4**, 4254.

Jung, Y.-D. 2003 *Appl. Phys. Lett.* **82**, 2395.

Kopaleishvili, T. 1995 *Collision Theory*. Singapore: World Scientific.

Sakurai, J. J. 1994 *Modern Quantum Mechanics*, revised edn. Reading, MA: Addison-Wesley.

Salzmann, D. 1995 *Atomic Physics in Hot Plasmas*. Oxford: Oxford University Press.

Shevelco, V. P. and Vainshtein, L. A. 1993 *Atomic Physics for Hot Plasmas*. Bristol: Institute of Physics.

Shevelco, V. P. and Tawara, H. 1998 *Atomic Multielectron Processes*. Berlin: Springer.

Song, M.-Y. and Jung, Y.-D. 2001 *Phys. Plasmas* **8**, 2983.

Zubarev, D., Morozov, V. and Röpke, G. 1996 *Statistical Mechanics of Nonequilibrium Processes, Vol. 1: Basic Concepts, Kinetic Theory*. Berlin: Akademie.

https://doi.org/10.1017/S0022377805003892 Published online by Cambridge University Press