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# THE LIKELIHOOD OF EFFECTIVE LOWER BOUND EVENTS

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This paper provides estimates of the probability of an economy hitting its effective lower bound (ELB) on the nominal interest rate and of the expected duration of such an event for eight advanced economies. To that end, a mean-adjusted panel vector autoregression with static interdependencies and the possibility of regime change is estimated. The simulation procedure produces ELB risk estimates for both the short term, where the current phase of the business cycle plays an important role, and the medium term, where the occurrence of an ELB situation is determined mainly by the equilibrium values of macroeconomic variables. The paper also discusses the ELB event probability estimates with respect to previous approaches used in the literature.

**Keywords:** Effective Lower Bound, ELB Risk, Panel VAR, Regime Change

## 1. INTRODUCTION

The recent experience of a prolonged period of extraordinarily low monetary policy rates in advanced countries has rekindled interest in macroeconomic issues relating to the effective lower bound (ELB) on the nominal interest rate. Before the Great Financial Crisis (GFC), the only relevant example of an ELB event was Japan, which, however, was viewed as a peculiar case. It was not generally believed that advanced countries could experience long and recurrent periods of being stuck at the ELB. This opinion has now changed, and novel research relating to the ELB has been published.

There are two important policy questions underlying this research work: what is the probability of an ELB event at a given point in time and how often can an ELB event be expected to occur in general? The usual approach to answering the latter involves conducting model-based stochastic dynamic simulations and estimating the stationary distribution of the short-term interest rate, with the area below a given lower bound defining the ELB risk. ELB risk estimates of this type are usually based on calibrated linear models and are highly sensitive to the

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equilibrium values of macroeconomic variables assumed (see, e.g., Reifschneider and Williams (2000), Coenen (2003), and Kiley and Roberts (2017)).[1]

When the examination of the likelihood of an ELB event is related to a specific point in time, more data-driven approaches are employed. Statistical models are used to estimate a distribution forecast for the interest rate. In contrast to the other approach, this allows uncertainty in model parameters, latent variables, and measurement errors to be taken into account and enables some simple form of nonlinearity (Chung et al. (2012)). Nakata (2017a) employs survey data on macroeconomic projections to bring the standard stochastic simulations to actual data. However, data-driven approaches are not suitable for estimating ELB risk in the medium and long term, because they often do not possess well-defined unconditional moments.[2]

The aim of this paper is to draw on the above-mentioned approaches and to provide estimates of the probability and expected duration of ELB events. To that end, the paper combines stochastic simulations around equilibrium values and distribution forecasting reflecting current observed data. The results are therefore relevant to both the short term, where the ELB likelihood is driven mainly by the current phase of the business cycle, and the medium term, where the ELB risk is determined mainly by equilibrium values. Dealing with both time scales in one modeling framework makes the estimates for the short and medium term consistent with each other. Furthermore, the approach addresses some of the contentious issues of the previous approaches and should therefore deliver more accurate ELB likelihood estimates.

I adopt an empirical strategy and base ELB likelihood estimates on recent data from advanced countries, exploiting the fact that many of them have recently experienced, or are still experiencing, nominal interest rates at their ELB. As a consequence, the ELB risk estimates have to rely less on calibrated parameters and can be based more on estimated quantities. The framework thus accounts for parameter uncertainty, which has been found to be important for realistically assessing ELB likelihood (Chung et al. (2012)). On the other hand, the time span of the macro data capturing ELB events is still short, suggesting that it is appropriate to exploit the panel nature of that data.

Furthermore, the data-driven approach depends less heavily on assumptions about the equilibrium values of macroeconomic variables than do studies based on stochastic simulations. The assumptions about the equilibrium values enter the estimation procedure in the form of priors and are confronted with the observed data during the Bayesian estimation process. Finally, as an ELB situation implies a possibility of regime change, the modeling framework allows for change in the shock transmission mechanism and shock volatilities.

To quantitatively assess the occurrence of ELB events, I employ the mean-adjusted panel vector autoregression (VAR) technique, which allows for static interdependencies and threshold behavior. The model is estimated on data for eight advanced economies over the period 1999Q1–2016Q4 and provides estimates of ELB risk in both the short and the medium term. In addition, the

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modeling framework makes it possible to analyze the impact of various assumptions employed by the techniques used to estimate ELB risk in previous studies.

It turns out that, in the short term, the ELB risk is currently the highest for Japan and Sweden and the lowest for Canada, Norway, and the USA. In the medium term, the probability of an ELB situation ranges from 0.01 for Canada to 0.16 for Japan. A supplementary analysis suggests that the calibration of the steady-state values of macroeconomic variables can lead to ELB risk underestimation due to the fact that the calibrated value of the equilibrium interest rate is high and is assumed to be known with certainty. It is also shown how the empirical approach gains from the panel nature of data by partially pooling country-specific information and by improving the efficiency of estimates.

From the policy maker's perspective, the estimates should be taken into account in the conduct of monetary policy. Higher ELB risk during a recession calls for more aggressive easing of monetary policy (ceteris paribus), whereas higher ELB risk during a boom justifies slower normalization of rates (see, e.g., Williams (2014), and references therein). Similarly, if there is agreement on the optimal size of the central bank balance sheet, the speed of balance sheet normalization from current levels should reflect the medium-term probability of return to the ELB and to unconventional measures that would increase the size of the sheet again.

The structure of the paper is as follows. Section 2 discusses recent monetary policy rates in advanced countries. Section 3 presents the model. Section 4 explains the estimation approach and describes the data. Section 5 presents the results and considers the role of calibration, regime change, and panel structure in ELB risk estimation. It also includes several robustness checks. Section 6 concludes. Supplementary materials can be found in Online Appendixes A, B, and C.

## 2. THE ELB IN ADVANCED COUNTRIES

Figure 1 shows monetary policy rates in eight advanced economies since 1999. In general, two cycles can be observed, one peaking around the year 2000 and the second peaking in 2007 just before the GFC hit the world economy. The observed monetary policy rate profiles also suggest a downward trend in the equilibrium interest rate. The short time span of the observations, however, makes any conclusion about the trend uncertain.

In the aftermath of the 2008 crisis, advanced economies approached or even crossed the zero bound with their interest rates. Once a country was not able or willing to ease monetary policy further using the policy rate, unconventional measures were employed. The USA, the UK, and the euro area used large-scale asset purchases aimed at lowering long-term interest rates. Japan initially adopted the same approach to quantitative easing and then followed up with additional quantitative and qualitative monetary easing in 2013. It also introduced a negative monetary policy rate in mid-2016. Negative policy rates have been seen in

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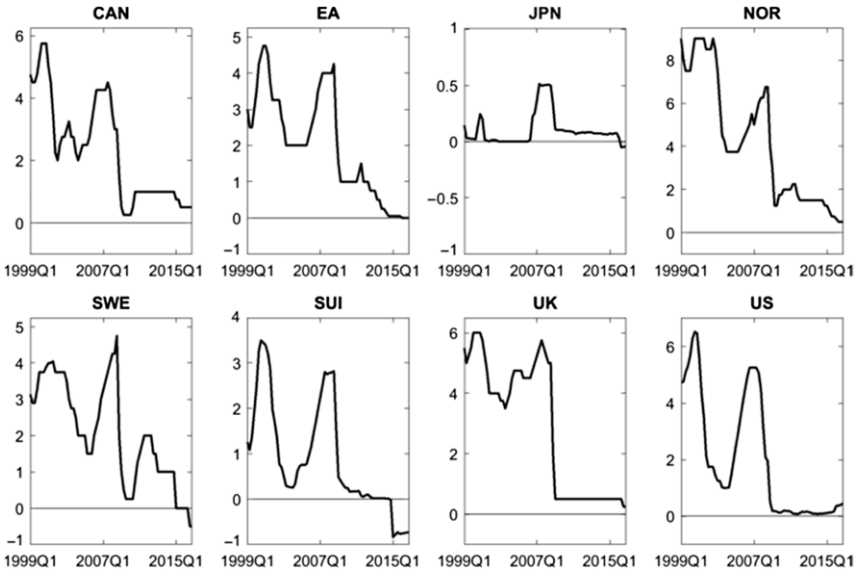



**FIGURE 1.** Monetary policy rates in advanced countries.

Switzerland and Sweden recently as well. Furthermore, Switzerland imposed an exchange rate floor to prevent currency appreciation between 2011 and 2015.

Figure 1 suggests some degree of homogeneity and close interconnectedness in monetary policy stance across countries. On the other hand, significant heterogeneity is present in the data, especially after the start of the GFC. Monetary policy rules presumably changed when economies hit the ELB, and the changes differ across countries. The shock propagation mechanism and shock volatilities might also be different across countries after the GFC. Therefore, the modeling framework should be able to exploit similarities, but at the same time to be general enough to allow for differences between countries.

## 3. MODEL

The modeling framework is built on the panel VAR structure, which allows for cross-sectional heterogeneity and static interdependencies. Cross-sectional heterogeneity allows for differences in the dynamics of macroeconomic variables across countries. This option is especially important for the period of unconventional monetary policy, because countries employed different strategies for easing monetary policy further. Static interdependencies allow for correlation of reduced-form shocks across countries. The GFC spreads across the advanced economies primarily through the financial markets within one quarter, so accounting for static interdependencies is crucial, probably more so than accounting for dynamic ones.[3] Ignoring static interdependencies, if present, would lead to less efficient estimates and consequently to less accurate ELB risk estimates. Note

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also that static interdependence is the only interconnection between the countries in the model setup. Without that, the model would be effectively a set of single-country models. Another connection between the countries is implemented by the estimation procedure discussed in Section 4.

The model is formulated in mean-adjusted form to deal directly with the steady state of macroeconomic variables. Mean adjustment for VAR models is introduced in Villani (2009) and allows the steady state of a macroeconomic variable to be treated as a single parameter. As such, a prior on the steady state can be formulated and the parameter directly estimated. The posterior of the steady state then defines the long-run dynamics of the model. Explicit treatment of the steady state leads to well-defined ELB risk in the medium and long term.

For $N$ countries and for $t = p+1, \ldots, T$, the model is formulated as follows:

$$\begin{aligned}\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{N,t} \end{pmatrix} &= \begin{pmatrix} A_1^1 & 0 & \cdots & 0 \\ 0 & A_2^1 & \cdots & 0 \\ \vdots & \vdots & 0 & \vdots \\ 0 & 0 & \cdots & A_N^1 \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{N,t-1} \end{pmatrix} + \cdots \\ &+ \begin{pmatrix} A_1^p & 0 & \cdots & 0 \\ 0 & A_2^p & \cdots & 0 \\ \vdots & \vdots & 0 & \vdots \\ 0 & 0 & \cdots & A_N^p \end{pmatrix} \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{N,t-p} \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix} x_t \qquad (1) \\ &- \begin{pmatrix} A_1^1 F_1 \\ A_2^1 F_2 \\ \vdots \\ A_N^1 F_N \end{pmatrix} x_{t-1} - \cdots - \begin{pmatrix} A_1^p F_1 \\ A_2^p F_2 \\ \vdots \\ A_N^p F_N \end{pmatrix} x_{t-p} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{N,t} \end{pmatrix}. \end{aligned}$$

The $n \times 1$ vector $y_{c,t} = \left(y_{c1,t}, \ldots, y_{cn,t}\right)'$ denotes the vector of $n$ endogenous variables for country $c$ and time $t$. The coefficient matrices $A_c^l$ relating to the $l$th lag of the vector of endogenous variables for country $c$ are of dimension $n \times n$.

The vector of endogenous variables contains real GDP (gross domestic product) growth, CPI (consumer price index) inflation, the short-term interest rate, and the spread between the 10-year government bond yield and the policy rate.[4] The set of endogenous variables is usually chosen according to the shocks that are to be identified. However, structural shock identification is not performed. The main aim of the choice of variables is to capture both conventional and unconventional monetary policy rules, that is, the interest rate rule and the quantitative reaction function reflected by the spread.

The question is whether this minimal set can also represent the other unconventional monetary policy measures contained in our data set. Obviously, the vector of endogenous variables can deal with negative interest rates. There could be problems with the Swiss exchange rate floor, as the exchange rate is not

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included in the vector of endogenous variables, and with the multipolicy approach employed in Japan, which is presumably difficult to represent with such a small set of endogenous variables. The effect of unconventional monetary policy measures is captured to the extent to which they are reflected by short-term interest rates, long-term government bond yields, and monetary policy rates.

The $m \times 1$ vector $x_{c,t} = \left(x_{c1,t}, \ldots, x_{cm,t}\right)'$ in (1) contains exogenous variables, which are the same across countries. It comprises constant terms only. The coefficient matrices relating to exogenous variable $F_c$ are of dimension $n \times m$. The country-specific matrices $F_c$ include the steady states of the endogenous variables, because it follows from (1) that if the process for $y_{c,t}$ is stationary then

$$E\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{N,t} \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix}. \tag{2}$$

Finally, the vector of error terms $\left(\varepsilon_{1,t}, \ldots, \varepsilon_{N,t}\right)'$ in (1) is distributed independently and normally, with zero mean and covariance matrix $\Sigma$ of dimension $Nn \times Nn$. The error covariance matrix is generally nondiagonal, allowing for residual correlation between countries. Two lags are assumed in the benchmark specification. A different number of lags is discussed in Section 5.2.

Let $\bar{y}$ denote a $NnT \times 1$ data vector for all countries:

$$\bar{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix},$$

where the $nT \times 1$ vector $y_c$ is defined as $y_c = vec\,(Y_c)$, $c = 1, \ldots, N$. $Y_c$ is a $T \times n$ matrix of observations for country $c$, and the operator $vec\,(\cdot)$ denotes the column-wise transformation of the matrix into a vector. Let $\bar{X}$ denote an $(NnT) \times \left(Nn^2p\right)$ data matrix

$$\bar{X} = \begin{pmatrix} \bar{X}_1 & 0 & \cdots & 0 \\ 0 & \bar{X}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \bar{X}_N \end{pmatrix},$$

where $\bar{X}_i = I_n \otimes X_i$, with $X_i$ being a $T \times (np)$ country-specific data matrix comprising up to $p$ lags of $Y_i$, and $\bar{Z} = I_{Nn} \otimes Z$, where

$$Z = \begin{pmatrix} x_1' & -x_0' & \cdots & -x_{1-p}' \\ x_2' & -x_1' & \cdots & -x_{2-p}' \\ \vdots & \vdots & \ddots & \vdots \\ x_T' & -x_{T-1}' & \cdots & -x_{T-p}' \end{pmatrix}.$$

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Finally, define

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}, \delta = \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_N \end{pmatrix}, \text{ and } \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix},$$

where $\beta_c = vec \begin{pmatrix} \left(A_c^1\right)' \\ \vdots \\ \left(A_c^p\right)' \end{pmatrix}$, and $\delta_c = vec \begin{pmatrix} F_c' \\ F_c' \left(A_c^1\right)' \\ \vdots \\ F_c' \left(A_c^p\right)' \end{pmatrix}$ for $c = 1, ..., N$.

The model (1) can then be written as follows:

$$\bar{y} = \bar{X}\beta + \bar{Z}\delta + \varepsilon, \quad \varepsilon \sim N\left(0_{NnT\times 1}, \bar{\Sigma}\right). \tag{3}$$

As discussed in Section 2, the data set presumably includes changes in policy rules when the ELB was hit and unconventional monetary policies were introduced. Moreover, as suggested by Clark (2011), allowing for changes in shock volatilities results in more accurate density forecasts, even though the time perspective of Clark (2011) is longer than in our case. Translating this into the context of this paper, a more accurate density forecast means a more accurate estimate of ELB risk. Model (3) is therefore extended in such a way that it allows for two regimes in the form of possible threshold behavior driven by the endogenous variables. Due to the short time series available for the estimation, only one regime change is considered. Whether the change is driven by a change in dynamic coefficients (e.g., in policy rules), by a change in shock volatility, or by both is decided on the basis of model likelihood comparison within the estimation procedure.

Combining regime change with the mean-adjustment procedure enforces to take a stand on the issue of whether the mean-adjustment procedure should be applied to both regimes or to one regime only. Strong prior information exists for the "normal times" regime with standard monetary policy conduct, so mean adjustment is imposed in that regime only. For the other regime, a simple constant term is included in the data matrices.[5]

Allowing for two regimes with mean adjustment applied in one regime only, the model takes the form:

$$\begin{aligned} \bar{y}^{(1)} &= \bar{X}^{(1)}\beta^{(1)} + \varepsilon^{(1)} && y_t^{TR} < r \\ \bar{y}^{(2)} &= \bar{X}^{(2)}\beta^{(2)} + \bar{Z}^{(2)}\delta + \varepsilon^{(2)} && y_t^{TR} \geq r, \end{aligned} \tag{4}$$

where $\varepsilon^{(1)} \sim N\left(0, \bar{\Sigma}^{(1)}\right)$, $\varepsilon^{(2)} \sim N\left(0, \bar{\Sigma}^{(2)}\right)$, and the matrices $\bar{\Sigma}^{(1)}$ and $\bar{\Sigma}^{(2)}$ are in general nondiagonal. Data matrices and vectors with superscripts denoting the regime refer to the subsamples of $\bar{X}$ and $\bar{y}$ relating to the relevant regime. Note that the data matrix $\bar{X}^{(1)}$ includes the constant term.

The switch between the two regimes is driven according to the threshold variable, $y_t^{TR}$, and the threshold $r$, and the switch date is the same for all countries in the panel. The threshold variable is a function of the endogenous variables

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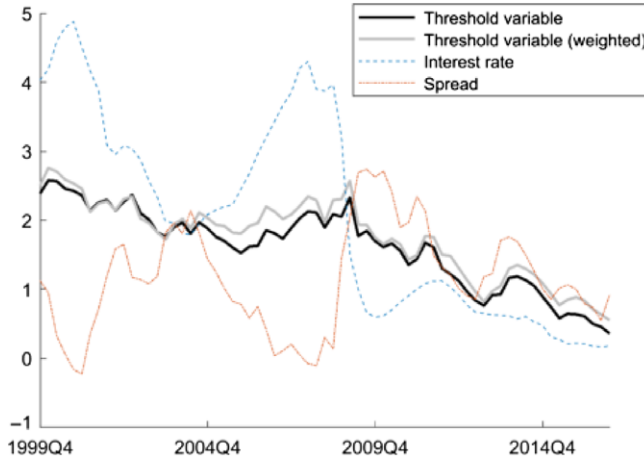



**FIGURE 2.** The threshold variables, the average short-term interest rate, and the average spread across countries.

and is defined as the combination of the short-term interest rate and the spread (Figure 2). More precisely, the threshold variable is the average of the average short-term interest rate across countries and the average spread across countries, both lagged by one quarter. So, it combines conventional and unconventional monetary policy measures. The definition of the threshold variable reflects the belief that a regime change is expected when the world economy is around the ELB and unconventional policies aimed at lowering the spread are employed. The threshold parameter $r$ is estimated.

The threshold variable is based on simple means across countries. Weighting each country in the sample according to its size yields a threshold variable that possesses a more profound economic interpretation: the larger the country, the stronger its global influence. Figure 2 shows an alternative threshold variable that takes into account the country's weight defined as the share of nominal GDP in US dollars. It follows that the difference between the two threshold variables is small, reflecting the synchronization of financial variables in advanced economies. A sensitivity check is conducted with the weighted threshold variable in Subsection 5.2.[6]

## 4. ESTIMATION

The estimation approach is Bayesian and combines estimation of a mean-adjusted VAR (Villani (2009)), a hierarchical linear model for VAR (Jarocinski (2010)), and Bayesian estimation of a threshold VAR (Chen and Lee (1995), and Koop and Potter (2003)). To estimate the model, the country-specific estimates of some parameters exploit information from all countries by means of an exchangeable prior. Such information pooling represents a second way in which countries are

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connected on top of the assumed static interdependencies. It is assumed that the coefficients at the lagged values of the endogenous variables stacked in vector $\beta_c^{(r)}$ are distributed normally around a regime-specific common mean $b^{(r)}$:

$$\beta_c^{(r)} = b^{(r)} + b_c^{(r)}, c = 1, \ldots, N,\ r = 1, 2 \quad (5)$$

where $b_c^{(r)} \sim N\left(0, \Sigma_c^{b,(r)}\right)$. The spread of the country-specific vectors of the dynamic parameters around the common mean is driven by the overall tightness parameter $\lambda^{(r)}$ defining the variance $\Sigma_c^{b,(r)}$ (see formula B3).

The exchangeable prior is not used for the parameters capturing the steady state, $F_c$. Strong prior information on the steady states of the endogenous variables is available, so there is no need to pool information across countries through the common mean for those parameters.

The estimation procedure simulates the posterior distributions based on likelihood, priors, and conditional priors, respectively. The vector of model parameters contains dynamic parameters $\beta^{(r)}$, hyperparameters $b^{(r)}$ and $\Sigma_c^{b,(r)}$, error variances $\bar{\Sigma}^{(r)}$, the steady states included in the vector $\delta$, and the threshold $r$ that determines the regimes. Using conditionally conjugate priors yields conditional posterior densities that are easy to draw from within the Gibbs sampler. The conditional posterior of the threshold cannot be expressed by a standard density function, and a Metropolis step is used to take a draw of the parameter. The specification of the prior distributions and a description of the sampler can be found in Online Appendix B.

The ELB risk is estimated during the estimation of the model. For a given draw of model parameters, iterated forecasts for up to 1048 quarters are computed using random draws of shocks from a given distribution. The entire posterior distribution of the model parameters is thus used for the ELB risk simulation to account for parameter uncertainty. Note that regime switching is allowed for during the simulation, because the threshold variable is a function of the endogenous variables. Then, for a given period, the proportion of forecasts that are below or at the ELB is computed to estimate the probability of an ELB event in that period. The medium-term ELB risk is computed as the average ELB risk for last 1000 quarters, that is, the 49th to 1048th quarters, for which the initial conditions are already filtered out. Various quantiles of the medium-term ELB risk distribution constituted of the ELB probabilities in the 1000 forecasted periods are computed to obtain credible intervals. In addition to the estimate of the ELB risk, the expected duration of the ELB event is computed as the average number of quarters for which the simulated forecast remains at the ELB. The number of forecasts simulated equals the number of draws used for the computation of the posterior distribution moments, that is, 10,000 simulations are employed.

During the forecasting, the ELB constraint is imposed in a straightforward manner, because the model is backward looking. The value of the ELB is imposed whenever the one-period-ahead value of the interest rate falls below the ELB. This approach can be interpreted as passive conventional monetary policy. The monetary authority does not lower the interest rate when the estimated interest

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**TABLE 1.** Imposed effective lower bounds

| | ELB |
|---|---|
| Canada | −0.5 |
| Euro area | −0.3125 |
| Japan | −0.0376 |
| Norway | 0 |
| Sweden | −0.78 |
| Switzerland | −0.84 |
| UK | 0 |
| USA | 0 |

*Note*: The ELB imposed on the short-term interest rate.

rate rule suggests doing so. Even with the interest rate rule switched off, unconventional monetary policy still affects the economy according to the estimated spread equation.

The numerical value of the ELB is set differently for each country. First of all, it is important to stress that the ELB is imposed on the short-term nominal interest rate, because the policy rate is not directly included in the vector of endogenous variables. For those countries which have not experienced negative policy rates, the ELB is set to zero (Norway, the UK, and the USA). In addition, the choice for the USA is driven by comparability with other studies. For countries with negative policy rates, the ELB is set to the lowest value of the short-term interest rate in the sample. Finally, following Witmer and Yang (2016), the ELB for Canada is set to –0.5.[7] The ELB values are reported in Table 1.

The ELB values do not enter the estimation procedure and thus do not affect the estimation of the model parameters. They enter the simulation of the ELB risk and the expected duration of the ELB spell only. As a consequence, a different ELB for a country only affects the country-specific results (the exception is regime change timing, which relates to all countries). Different reasoning for ELB values across countries therefore does not represent an obstacle.

### 4.1. Data

Real GDP and CPI are seasonally adjusted and enter the vector of endogenous variables as the first difference of their logs. The two series are downloaded from the Bank for International Settlements database. The spread variable is made up of the 10-year bond yield and the monetary policy rate. The bond yields are downloaded from the OECD MEI (Organization for Economic Co-operation and Development Main Economic Indicators) database. Euro area long-term bond yields are downloaded from the ECB FM (European Central Bank Financial Market) database. They are constructed from AAA-rated bond yields.[8] Yields for the UK are downloaded from Bank of England Statistics and yields for the USA from the Federal Reserve Economic Data database. Monetary policy rates are

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obtained from the IMF IFS (International Monetary Fund International Financial Statistics). The Swiss National Bank targets the 3-month Swiss franc Libor, so the Libor is used as the policy rate. The policy rate for the euro area is the interest rate on main refinancing operations. Finally, short-term interest rates are 3-month interbank rates obtained from the OECD MEI database.

The data are quarterly and the data set covers the period 1999Q1–2016Q4. The choice of quarterly data, as opposed to monthly data, is driven by the fact that the panel VAR specification with static and without dynamic interdependencies is more reasonable in a setting with quarterly data.

The sources of the data employed in the robustness checks are as follows. The nominal GDP values and national currency to US dollar exchange rates used for the computation of the alternative threshold variable are downloaded from the OECD MEI database. The data set of the time series longer than the benchmark data set covers the period 1985Q1–2016Q4. For Canada, the overnight money market rate (IMF IFS) is taken because the time series of the monetary policy rate starts in 1992. For Japan, the benchmark 10-year bond yield as computed by the ECB is used. The longer time series of real GDP for Sweden is taken from the OECD MEI database. The long-term bond yields for Sweden are taken from the Sveriges Riksbank website.

## 5. RESULTS

The results are based on 100,000 iterations, with 5000 iterations as a burn-in period. Every tenth draw is used for inference to deal with autocorrelation of draws. Convergence diagnostics of the sampler and additional estimation results are presented in Online Appendix C.

The posterior mean of the threshold $r$ is 1.27, implying that regime 1 covers the period 2012Q1–2016Q4 and regime 2 the period 1999Q2–2011Q4.[9] Regime 1 thus covers the period when advanced economies hit or approached their ELBs, launched their unconventional monetary policy measures, and gradually reduced the spread between long- and short-term interest rates (Figure 3).

To provide some reassurance about the usefulness of the model for the question at hand, out-of-sample forecasting ability of the model is examined. Figure 4 relates the estimated ELB risk for the euro area and the expectations of 3M Euribor for quarters covering the period 2011Q1–2016Q4. So, regarding the first data point, the ELB risk is based on the model estimated on the period 1999Q1–2010Q4 and expectations are taken as of the end of 2010Q4, the second data point is based on data ending by 2011Q1 etc. Expectations of 3M Euribor are considered for the horizons of 1 quarter and 4 quarters. The horizon of 1 quarter assesses mainly the out-of-sample fit of VAR dynamics; the horizon of 4 quarters also takes into account the mean adjustment.

It turns out that at the horizon of 1 quarter the profile of interest rate expectations is inverted to the profile of ELB risk estimates. The expectations approaching the ELB correspond to the increase in the ELB risk. The increase of the risk is higher; the closer is the expected interest rate to the ELB. At the horizon

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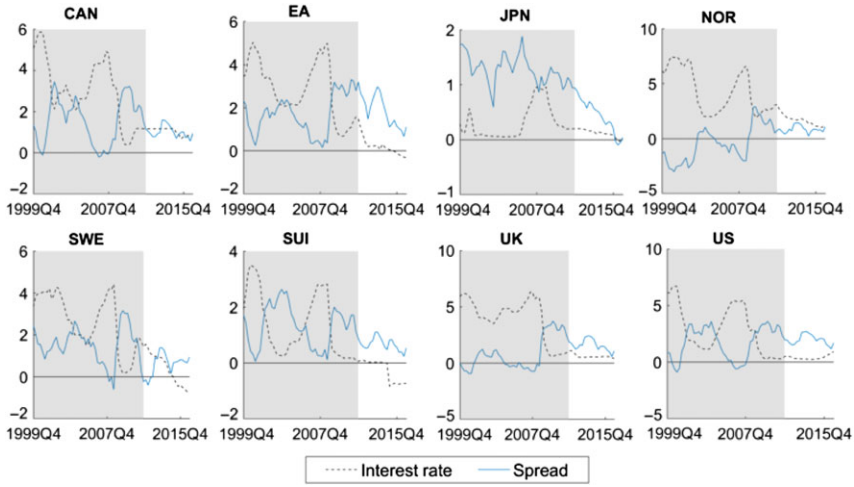
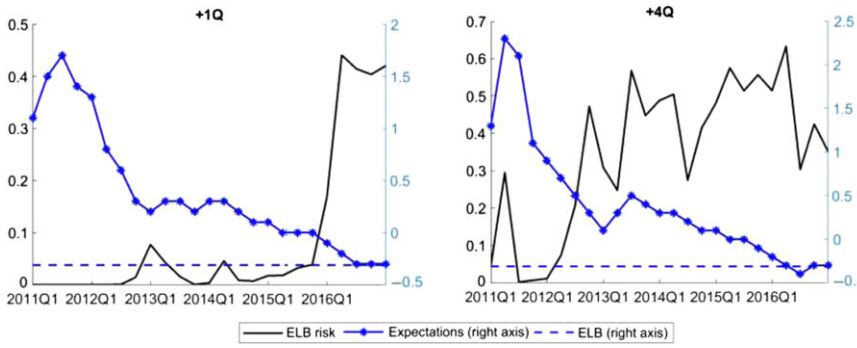



**FIGURE 3.** The interest rate and spread in regime 1 (white) and regime 2 (grey).




*Note*: Expectations of 3M Euribor are constructed from Euribor, Euribor FRA and Euribor swap rates smoothed by a cubic spline.

**FIGURE 4.** ELB risk and expectations of 3M Euribor for the euro area for horizons 1 and 4 quarters.

of 4 quarters, the relationship is noisier. However, the general picture remains suggesting the model is an appropriate tool for the ELB risk estimation.[10]

The estimated ELB risk based on the full data set and for all countries is presented in Figure 5. The probability of the short-term interest rate being at the ELB in 2017Q1 is driven to a great extent by whether the relevant economy was stuck at the ELB in the last quarter used for the estimation, that is, in 2016Q4. So, for the euro area, Japan, and Sweden, which are at their assumed ELBs, the ELB risk is in the range of 0.41–0.59. For the other countries, the ELB risk is lower. For Canada, Norway, and the USA, it is virtually zero for 2017Q1.

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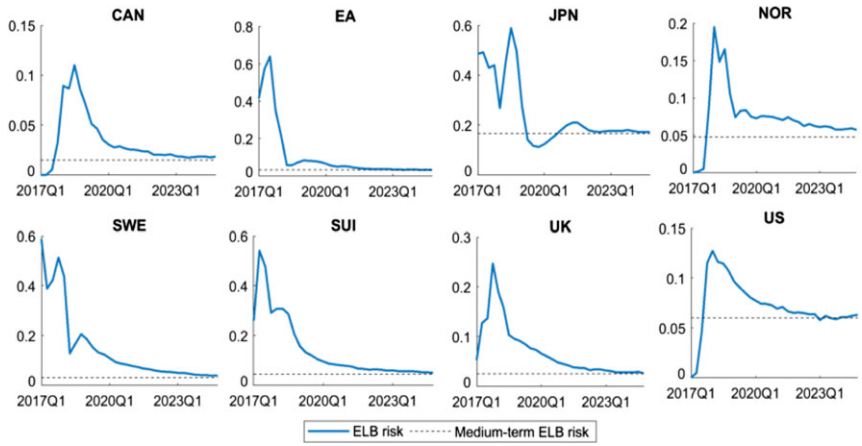



**FIGURE 5.** ELB risk and medium-term ELB risk.

The ELB risk profile during the first eight forthcoming quarters (2017Q1–2018Q4) reflects the current phase of, and the outlook for, the business cycle. For the euro area, the ELB risk increases in the first three quarters and then declines due to a delay in its economic recovery with respect to other advanced economies.[11] For Canada, Norway, and the USA, the ELB risk is very low for the first three quarters and rises after that, because these countries experience an expansionary phase of the business cycle in the first quarters of 2017. The comparison across countries is also influenced by the different ELBs. The lower the ELB, the lower the probability of the economy being at the ELB, all other things being equal. The same set of interest rate forecasts hit the exogenously given ELB less frequently if the value of the ELB is lower. Such fact should be taken into account when comparing, for example, the zero ELB risk in 2017Q1 for Canada and the USA. If the ELB equals zero for Canada and for the USA, the Canadian ELB risk in 2017Q1 would probably be nonzero, implying a higher tendency of the Canadian economy to reach a low interest rate environment than that estimated for the US economy.

The ELB risk always converges to a value which is by definition close to the medium-term ELB risk. The medium-term ELB risk is determined mainly by the posterior of the interest rate steady state, the assumed ELB, and the estimated average size of the shocks to macroeconomic variables. It ranges from 0.01 (Canada) to 0.16 (Japan). The numbers can be interpreted as meaning the percentage of the time (quarters) the ELB is binding. For Canada, the economy is at the ELB 1% of the time, that is, for one quarter in 25 years. For Japan, on the other hand, the ELB is estimated to be binding 16% of the time, that is, for 4 years every 25 years.

By combining the short-run outlook based on the business cycle and medium-run simulations drawing on steady-state values, nonmonotonicity of ELB risk measures can arise. For example, the ELB risk in the euro area initially increases

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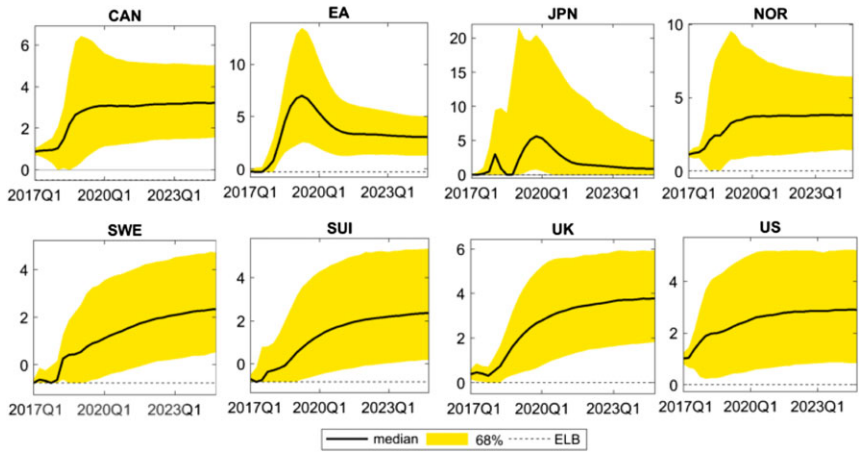



**FIGURE 6.** Density forecasts of the short-term interest rate (median and centered 68% of the posterior distribution).

and then drops from 64% in 2017Q3 to 35% in 2017Q4. The explanation of this phenomenon is illustrated by Figure 6, which shows the density forecasts and the 68% credible set of short-term interest rates. The ELB risk relates to the mass of the density forecast that is at the ELB line. Usually, the density forecasts widen smoothly over the forecast horizon. However, the mean adjustment disciplines the density forecast; hence, we observe a widening in the short term and a constant width in the medium term. As a consequence, nonmonotonicity of the median and quantiles of the density forecast arises, especially if the short-term economic conditions imply the interest rate moving away from its steady-state value. Nonmonotonicity of the quantiles of the interest rate density forecast implies nonmonotonicity of the ELB risk estimates.

The presented ELB risk estimates can be compared with the results provided by the recent literature. Assuming a steady-state nominal interest rate of 3% and imposing an ELB of zero, Kiley and Roberts (2017) estimate the frequency of ELB events for the USA to be 17.4% based on the dynamic stochastic general equilibrium (DSGE) model of Lindé et al. (2016) and 31.7% based on the FRB (Federal Reserve Board)/US model.[12] The estimate based on the DSGE model is close to the estimates in Hills et al. (2016) and Nakata (2017b), which draw on calibrated DSGE models and provide an estimated probability of around 14%. Here the same ELB as in Kiley and Roberts (2017) is assumed, and the steady-state nominal interest rate is estimated at 2.79%. The ELB risk is then 5.79%, which is much lower than the figures obtained by Kiley and Roberts (2017). Lower ELB risk estimates can also be found in the literature based on the estimated nonlinear DSGE models. Gust et al. (2017) found for the USA that the average probability of hitting the ZLB (zero lower bound) is 4%, while Richter and Throckmorton (2016) estimated the probability at around 5%.

Part of the difference can be explained by the fact that our ELB risk estimates include the effects of unconventional monetary policy, which lowers the

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**TABLE 2.** Probability of an ELB event in next 3 years

| CAN | EA | JPN | NOR | SWE | SUI | UK | USA | Nakata (2017) |
|---|---|---|---|---|---|---|---|---|
| 0.25 | 0.96 | 0.97 | 0.39 | 0.93 | 0.79 | 0.51 | 0.33 | 0.40–0.50 |

probability of hitting/staying at the ELB. Whenever the US economy is simulated to be in regime 1, the estimated impact of unconventional monetary policy is present in the simulation. Unconventional measures are not included in Kiley and Roberts (2017). Another reason suggested by the plain comparison is that nonlinear models could deliver lower ELB risk—the effect of nonlinearity on ELB risk estimates is discussed in Subsection 5.1.

Nakata (2017a) combines survey-based macroeconomic projections and stochastic simulations of the FRB/US model and defines the ELB risk as the probability that the federal funds rate will be constrained by the ELB for at least one quarter in the next 3 years. For 2016Q4, the ELB risk is estimated to exceed 50% for all three survey-based projections considered (Survey of Primary Dealers, Survey of Professional Forecasters, and Summary of Economic Projections). Excluding the period of elevated macroeconomic volatility in the 1970s and 1980s, which reduces the estimated size of shocks, the ELB risk lies between 40 and 50%. Assuming the ELB consistent with Nakata (2017a), Nakata's ELB risk for the USA estimated within the presented framework is 33%. Nakata's ELB risk measures for other countries are reported in Table 2.

Regarding the ELB risk estimates, a few notes are worth making for Japan. The ELB risk in both the short term and the medium term does not seem too realistic. The ELB risk estimates in the short and medium terms do not exceed 0.6, even though the Japanese economy was at, or close to, its ELB for the majority of the quarters covered by the data set. The medium-term ELB risk is 0.16, which is at odds with the expectations of the majority of observers of the Japanese economy. In the following text, a few reasons for taking the ELB risk and duration estimates for Japan with caution are put forward. They involve the heterogeneity of the regime break date with respect to the other countries in the sample and the estimated shock volatilities.

The estimated ELB risk can be complemented with estimates of the expected duration of the ELB event, that is, the average number of quarters the economy is stuck at the ELB given that it hits the ELB in a given quarter or remains at the ELB from the previous quarter (Figure 7). Note that the expected duration cannot be computed if no simulated forecast hits the ELB (as is the case for Canada, Norway, and the USA in 2017Q1).

The highest medium-term expected duration (2.7 quarters) is obtained for the euro area, while the lowest expected duration (2.1 quarters) is observed for Sweden. For the USA, the medium-term expected duration is estimated to be 2.43 quarters. Hills et al. (2016) and Nakata (2017b) estimated the expected duration at 9 quarters. A much shorter duration of the ELB spell is estimated by

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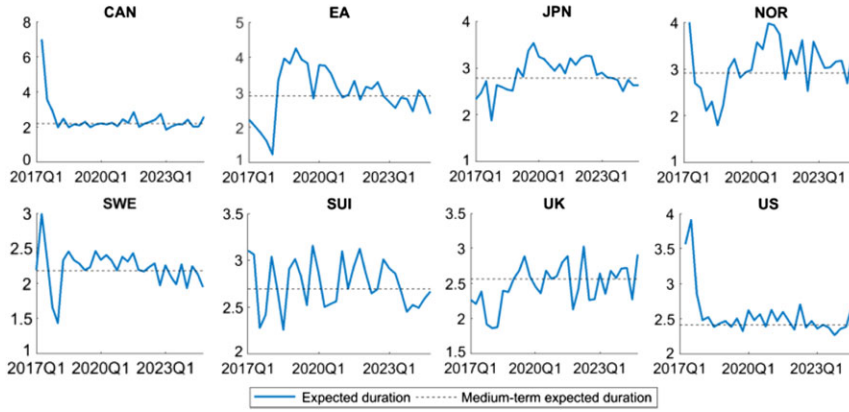



**FIGURE 7.** Expected duration and medium-term expected duration.

Gust et al. (2017) and Richter and Throckmorton (2016) who give an average duration of around 3 quarters. Similarly to the estimates of the ELB risk, the estimate of 2.43 quarters for the USA is closer to the literature dealing with estimated nonlinear DSGE models than calibrated linearized DSGE models.[13]

The ELB risk and duration estimates involve unconventional monetary policy, as it is reflected in the data used for the estimation. So, the estimate for the USA assumes a relatively strong unconventional easing of monetary policy at the ELB, while the estimate for Canada draws on the estimated conventional monetary policy only, because the Bank of Canada has never employed unconventional measures in the post-2008 period. The two countries' ELB risk should be compared with this consideration in mind. If no unconventional monetary policy had been conducted in the USA after 2008, the ELB risk would be higher and the difference with respect to the ELB risk in Canada would be more profound.

### 5.1. The Roles of Various Model Properties

The versatility of the model allows us to examine the various approaches to ELB risk estimation employed in the literature so far. First, mean adjustment sheds some light on the role of the usual calibration of the interest rate steady state in stochastic simulations based on structural macroeconomic models. Next, comparing specifications with and without regime change can help us understand the role of nonlinear models for ELB risk estimation. Finally, looking at the role of panel structure suggests how important pooling information is in the estimation. All these aspects are discussed in detail in Online Appendix A. This subsection provides a short summary of the results.

The calibration of the interest rate steady state realized through the tight prior on the interest rate steady-state parameter affects the medium-term ELB risk estimates significantly; the error implied by an inappropriate calibration of the parameter can be large. On the other hand, retaining the possibility to discipline

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the first moment of the interest rate distribution (through mean adjustment in the case of this paper) leads to inaccuracy of the ELB risk estimates as well. So, it turns out that an approach to ELB risk estimation that takes into account the uncertainty of the calibrated value of the interest rate steady state can help substantially. If the calibrated value is not consistent with the data, the uncertainty imposed on the prior allows the calibrated value to be changed based on the observed data. Furthermore, if the calibrated value is close to the value favored by the data, the uncertainty related to the prior itself does not affect the results considerably.

Comparing the estimated model coefficients between the two regimes reveals significant differences in the dynamics and shock volatilities, with the shock volatilities dominating the regime change. Moreover, the difference in the ELB risk estimates between the specifications with and without regime change is minor for all countries. This observation contrasts with the recent literature for the USA, which has found that theoretical nonlinear models (Richter and Throckmorton (2016), Gust et al. (2017)) provide much lower ELB risk estimates than linear theoretical models. Importantly, theoretical nonlinear models and the empirical model in this paper produce comparable medium-term ELB risk estimates. The empirical model takes into account unconventional monetary policy measures reflected by the data. Accounting for unconventional measures rather than for nonlinearity seems to be an important factor for realistic assessment of ELB risk estimates. It follows that understanding the effect of nonlinearity and unconventional monetary policy on ELB risk estimation is an important step in understanding the likelihood of ELB situations.

Finally, pooling information in the form of a common mean for dynamic coefficients and allowing for shock correlation across countries affects the ELB risk estimates substantially. From the econometric point of view, it makes more sense to work with a panel structure and ignore the possibility of different optimal regime change dates across countries. Note that the same regime change date is imposed for all countries. Put differently, the benefits in terms of the accuracy of the ELB risk stemming from higher efficiency of the estimates overcome the costs represented by the inaccuracy stemming from different regime change dates. As a consequence, the estimates for countries with a regime change date estimated by country-specific VAR that is close to the regime change date estimated by panel VAR are presumably more accurate than the ELB risk estimates for countries that diverge from the rest of the countries in terms of optimal regime change date. Online Appendix A.3 suggests that this is indeed the case for Japan and to some extent also for Norway.

### 5.2. Robustness Checks

In this subsection, two robustness checks are presented. The first check deals with the alternative definition of the threshold variable introduced in Section 3. Next, the benchmark model is estimated on a longer time series and a narrower set of

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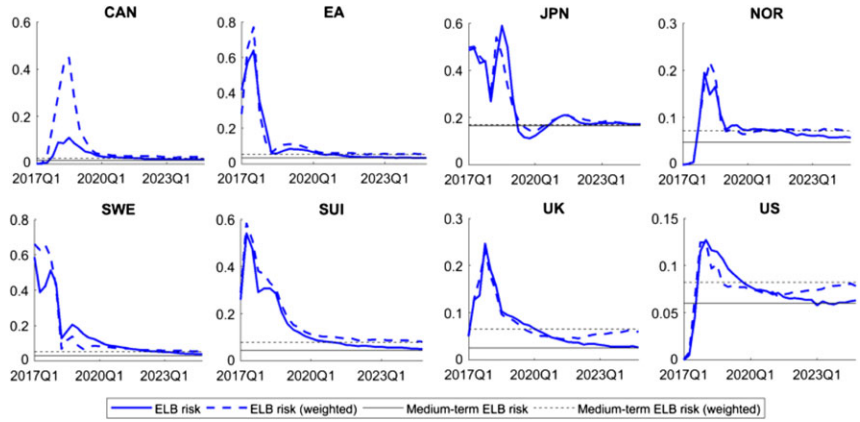



**FIGURE 8.** ELB risk for the benchmark threshold variable (solid lines) and for the threshold variable computed as the weighted average of the countries' short-term interest rate and spread (dashed lines).

countries to discuss the robustness of the results and a different number of lags in the model.

The estimation results for the model with the threshold variable based on the weighted average of the country's short-term interest rate and spread are almost identical, as the two alternative threshold variables are very similar (see Figure 2)—the posterior mean of the threshold equals 1.46, while regime 1 covers the period 2012Q1–2016Q4 and regime 2 the period 1999Q2–2011Q4. The posterior means of $\lambda^{(1)}$ and $\lambda^{(2)}$ are almost the same as in the benchmark case (0.15 and 0.05). However, the ELB risk and expected duration differ.

Figure 8 compares the ELB risk estimates for the benchmark threshold variable and the threshold variable based on weighted averages. Note that for the weighted threshold variable it is assumed during the simulation of the ELB risk and expected duration that the weights equal the last observed weights. The exchange rate is not an endogenous variable in the model, so projections of nominal GDP in dollars are not computed during the ELB risk simulations. Figure 8 shows that the short-term ELB risk differs mainly in the case of Canada. The medium-term ELB risks are in general higher by 3p. p. at most.[14]

As noted in Section 3, the choice of the threshold variable is a compromise between proper economic interpretation of the variable and the expected forecasting accuracy of the exercise. As the focus is on the forecasting part, the benchmark threshold variable is the unweighted average, which can be properly simulated because all the elements of the threshold variable are simulated in the forecasting exercise.

The second robustness check involves using a longer time series for the estimation. The benchmark data set starts in 1999Q1 due to the data availability of the euro area macroeconomic series. As a consequence, two lags are imposed in

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**TABLE 3.** Marginal likelihood

| Number of lags | |
|---|---|
| 1 | −25598 |
| 2 | −22199 |
| 3 | −20072 |
| 4 | −18832 |
| 5 | −15497 |

**TABLE 4.** ELB risk in the medium term for the panel VAR model estimated on samples starting in 1999Q1 and 1985Q1

| | Benchmark | Longer sample |
|---|---|---|
| Canada | 0.01 | 0.15 |
| | (0.007, 0.014) | (0.122, 0.182) |
| Japan | 0.20 | 0.17 |
| | (0.183, 0.209) | (0.117, 0.214) |
| Norway | 0.03 | 0.14 |
| | (0.025, 0.033) | (0.118, 0.167) |
| Sweden | 0.02 | 0.10 |
| | (0.013, 0.028) | (0.071, 0.124) |
| Switzerland | 0.02 | 0.24 |
| | (0.020, 0.038) | (0.195, 0.292) |
| UK | 0.03 | 0.15 |
| | (0.025, 0.036) | (0.114, 0.185) |
| USA | 0.04 | 0.13 |
| | (0.039, 0.048) | (0.089, 0.163) |

*Note*: 95% credible intervals reported in parenthesis.

the model, so as not to shorten the data set even more. As a robustness exercise, the data set starting in 1985Q1 excluding the euro area is used for the model estimation. Furthermore, the longer time series allows us to choose the number of lags in the usual way rather than impose it. The marginal likelihood of the model given the data is therefore computed to decide on the optimal number of lags (see Table 3).

The marginal likelihood suggests 5 lags in the model.[15] The implied medium-term ELB risk is significantly higher than in the benchmark case (see Table 4). The higher ELB risk estimates based on the longer data sample are a consequence of lower efficiency of the estimates of the heavily parameterized model. The lower efficiency is evidenced by much wider credible intervals. Part of the difference can also be explained by the higher posterior means of the interest rate steady-state values estimated using the longer data sample. For example, for Canada, the posterior mean is 4.09 instead of the 3.38 estimated in the benchmark case.

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## 6. CONCLUSIONS

This paper provides new estimates of the probability and expected duration of an ELB event for eight advanced economies. Such estimates are very important for policy makers, especially those in central banks, because the likelihood of the economy being stuck at the ELB is related to monetary policy conduct and central bank balance sheet size management.

On the one hand, the task is simple, because the employed methodology always results in a number, and the available data, which cover only a few ELB events, preclude any rigorous ex-post assessment of the accuracy of the ELB risk estimates. On the other hand, without such a forecasting accuracy exercise, the reasoning for the approach underlying the estimates has to be very clear and sound. In this paper, I motivate all the features of the model with the aim of obtaining accurate ELB risk estimates. In addition, the model features are related to the contentious aspects of previous approaches to estimating ELB risk.

So, mean adjustment is present to obtain well-behaved long-run dynamics and to incorporate out-of-data information on equilibrium values less strictly than during the calibration of the model. As a consequence, uncertainty relating to the equilibrium values is present in the ELB risk estimates. Next, allowing for regime change seems to be necessary, as I work with a statistical model, not a structural one, although the results are not much affected by assuming regime change. Finally, the panel nature of the data is a particular advantage in the situation where only a short time span of data is available and accounting for static interdependencies is necessary (and sufficient) if financial markets play an important role in shock transmission between countries. The multicountry perspective may enrich the country-specific estimates available so far in the literature.

Some methodological issues are still open. The ELB situation can be viewed as a tail event. As such, the focus of the modeling procedure should be on accurate modeling of distribution tails, especially for the interest rate distribution. While the regime change can be viewed as an attempt to go in this direction, the assumption of normally distributed errors may not be too realistic. Extending the methodology to include shocks exhibiting fat tails represents a natural next step for future research.

## SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100519000968.

### *NOTES*

1. A recent example of an estimated DSGE model used to estimate the ELB risk is Gust et al. (2017).

2. Probit-type models could, in principle, be employed to estimate the ELB risk. However, different models have to be estimated for different horizons of interest. Moreover, duration of the ELB spell could not be examined within these types of model.

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3. Another reason for not implementing dynamic interdependencies in our model is that allowing for them greatly increases the number of estimated parameters.

4. The same vector of endogenous variables is employed in Baumeister and Benati (2013) who examine the effect of unconventional monetary policy based on large-scale asset purchases for UK and US economies.

5. To simplify the notation, we assume that mean adjustment is applied to regime 2. The estimation procedure shows that this is indeed the case—the implied steady state of the threshold variable lies above the estimated threshold.

6. As suggested by one referee, another alternative choice of threshold variable would be to estimate the optimal weight between the demeaned simple average of the short-term interest rate and the spread and construct the threshold variable such that the threshold is zero. The estimation procedure would thus deal with the weight and not with the threshold.

7. This value is derived from the cost of storing cash. The estimate is originally for the policy rate; we consider it as the ELB for the short-term interest rate.

8. An alternative is to obtain yields from overnight index swap rates—for details see ECB (2014).

9. The 95% credible interval for the threshold implies two different regime structures. On top of the regime decomposition implied by the posterior mean, the regime decomposition with regime 1 covering 2010Q4 and 2011Q4–2016Q4 belongs to the credible interval. As a robustness check, the regime break date is also detected by drawing on the marginal likelihood of the data for models with all possible regime break dates (with at least 20 observations in a regime). The results confirm the benchmark estimate of the break date with regime 1 covering the period 2012Q1–2016Q4. The estimation of the marginal likelihood is described in Online Appendix B.5.

10. The fall of the expected 3M Euribor below the imposed lower bound in 2016 may seem implausible. Note, however, that the value of the ELB for the euro area is defined as the lowest short-term interest rate observed in the sample, that is, over the period 1999–2016. Expectations were considerably affected by the ECB's announcement of a comprehensive package to further ease monetary policy in March 2016. As a consequence, expectations fell below the previously observed interest rate values.

11. Going back to the motivation mentioned in the Introduction, the ELB risk is estimated to decrease substantially by the beginning of 2018 in the euro area. Such timing justifies the reduction of the monthly pace of asset purchases planned for 2018.

12. Assuming the equilibrium nominal interest rate to be equal to 4.5%, Reifschneider and Williams (2000) estimate the ZLB risk to be around 5%. The difference of the estimate between Kiley and Roberts (2017) and Reifschneider and Williams (2000) demonstrates the importance of the assumption on the equilibrium interest rate.

13. When comparing predictions of ELB spell duration, estimates from calibrated linearized DSGE models seem to be closer to surveys than estimates based on estimated nonlinear models. For example, Survey of Professional Forecasters and Primary Dealers Surveys suggest for the USA an expected duration starting at 5 quarters at the beginning of 2011, increasing close to 10 quarters in 2012 and 2013, and then steadily decreasing toward zero at the end of 2015. The expected duration based on panel VAR does not exceed 2 quarters throughout the ZLB period in the USA. In addition to the simplistic dynamics in the panel VAR model, forward guidance not captured by endogenous variables could explain the difference between the estimated expected duration and survey data.

14. All the medium-term ELB risk estimates are significantly different in the sense of nonoverlapping 95% credible intervals.

15. The model is estimated using a Gibbs chain of length 1000 with a burn-in of 5000 to keep the estimation time feasible.

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