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LINKS BETWEEN THE SOLVABILITY OF MATRIX AND SCALAR DIOPHANTINE EQUATIONS

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Abstract

We investigate solutions to the matrix Diophantine equation $X^n + Y^n = n!\,Z^n$ within specific classes of 2×2 integer matrices. Firstly, we introduce a class of 2×2 matrices $G_2(d)$ and show that $X^n + Y^n = n!\,Z^n$ admits a solution in $G_2(d)$ if and only if the classical Diophantine equation $x^n + y^n = n!\,z^n$ has a solution in $\mathbb{Q}(\sqrt{d})$. We further present a constructive theorem for generating matrix solutions over a general number field K. Finally, we introduce a determinant-fixed class of matrices $G_2(d, l)$, and prove that the Fermat-type equation $X^n + Y^n = 2Z^n$ has only the trivial solution in $G_2(d, l)$ whenever $n > 2$.

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1. Introduction

Diophantine analysis is a significant area within number theory. Diophantine equations appear naturally when we look for integer or rational solutions of algebraic or geometric problems. Such equations arise in many areas, such as the study of integer points on curves, approximation of real numbers by rational numbers, dynamical systems and cryptography. In cryptography, elliptic curves defined by such equations are used in modern public-key encryption systems. As another example, the geometric problem of finding right-angled triangles whose side lengths are integers leads, by the Pythagorean theorem, to the equation $x^2 + y^2 = z^2$.

In 1637, Fermat proposed that for any integer $n \geq 3$, the equation $x^n + y^n = z^n$ admits no positive integer solutions. In 1990, Ribet [10] demonstrated that the equation $x^n + y^n = 2z^n$ lacks nontrivial solutions for $n > 2$ (the trivial solutions include $x = y = z$). In 1995, Elkies [4] examined the equation $x^3 + y^3 = nz^3$. In contrast to these scalar equations, Mallick and Mishra [8] showed that the Diophantine equation $X^n + Y^n = 2Z^n$ admits infinitely many solutions within various structured matrix classes when $n = 3$. A question that remains unanswered is whether there is a relation between the set of solutions of a matrix Diophantine equation and the solutions of the

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corresponding scalar equation over certain number fields. Answering this will provide a key motivation for studying matrix Diophantine equations.

In 1937, Erdős and Obláh [5] established that the equation $x^p \pm y^p = n!$ possesses no solutions for prime exponents $p > 2$. In 2025, Bhoi *et al.* [9] showed that Diophantine equations with binary recurrence sequences possess only finitely many solutions. Guy [6, page 219] asked for a proof that the equation

$$x^n + y^n = n!\, z^n \tag{1.1}$$

has no integer solutions when n is a positive integer greater than 2. Contrary to this conjecture, Jena [7] demonstrated in 2018 that the Diophantine equation $x^n + y^n = n!\, z^n$ admits infinitely many primitive integer solutions in the specific case $n = 3$.

Matrix equations such as $AX = B$, $AX + XB = C$ (Sylvester equation), $A^TX + XA - XBR^{-1}B^TX + Q = 0$ (Riccati equation), $\dot{X}(t) = AX(t) + X(t)B$ (matrix differential equation), $AX = \lambda BX$ (generalised eigenvalue problem), $AX - XA = C$ (commutator equation), $X^2 + AX + B = 0$ (matrix polynomial equation) and $X^n + Y^n = Z^n$ (matrix Fermat equation) arise naturally in a wide range of mathematical and engineering contexts, including control theory, signal processing, systems theory, stability analysis, partial differential equations, data fitting, machine learning, quantum mechanics and operator theory. These equations often contain very important information about a system, such as its structure and whether it is stable. Strong theories and fast, reliable methods to solve matrix equations are major topics of research.

Let $d \neq 0$ be a fixed integer. Białek and Grytczuk [1] introduced the matrix class

$$G_2(d) = \left\{ \begin{pmatrix} a & b \\ db & a \end{pmatrix} \in M_2(\mathbb{Z}) : a, b \in \mathbb{Z} \setminus \{0\} \right\}$$

and studied the solutions of the Fermat equation $X^n + Y^n = Z^n$ in $G_2(d)$.

Clearly, $G_2(d)$ is closed under the usual multiplication. Basic results dealing with $G_2(d)$, which are required to prove our three main results stated below, are collected in Section 2. In 1998, Cao and Grytczuk [2] obtained partial results on Fermat-type equations in the set $G_2(d)$ under certain conditions.

We establish relationships between solutions of the matrix Diophantine equation

$$X^n + Y^n = n!\, Z^n$$

for 2×2 integer matrices in $G_2(d)$ and solutions of the corresponding scalar equation (1.1) in the number field $\mathbb{Q}(\sqrt{d})$ in the following two theorems.

THEOREM 1.1. *Let $n > 1$ be an integer. The matrix Diophantine equation*

$$X^n + Y^n = n!\, Z^n$$

admits a solution in $G_2(d)$ if and only if there exist numbers $\theta, \eta, \omega \in \mathbb{Q}(\sqrt{d})$ satisfying

$$\theta^n + \eta^n = n!\, \omega^n.$$

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THEOREM 1.2. *Let K be a number field and let $n \in \mathbb{N}$. For any $p, q, r \in K$ satisfying the equation*

$$p^{2n} + q^{2n} = n!\, r^{2n},$$

the matrices

$$P = \begin{pmatrix} 0 & 1 \\ p & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 1 \\ q & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 1 \\ r & 0 \end{pmatrix}$$

satisfy the related matrix Diophantine equation

$$P^{4n} + Q^{4n} = n!\, R^{4n}.$$

We next introduce a determinant-fixed family of matrices associated with $G_2(d)$. For a fixed $l \in \mathbb{Z}$ and a nonzero square-free integer d, define

$$G_2(d, l) = \left\{ \begin{pmatrix} a & b \\ db & a \end{pmatrix} : a, b \in \mathbb{Z},\ \det \begin{pmatrix} a & b \\ db & a \end{pmatrix} = l \right\} \subseteq M_2(\mathbb{Z}).$$

The final result of this article, stated below, concerns a solution of a matrix Diophantine equation in $G_2(d, l)$.

THEOREM 1.3. *Let $X, Y, Z \in G_2(d, l)$. Then the Diophantine equation*

$$X^n + Y^n = 2Z^n \tag{1.2}$$

admits only trivial solutions for $n > 2$.

The equations $x^n + y^n = n!\, z^n$ and $x^n + y^n = 2z^n$ arise in supercongruences, norm equations, elliptic curves and symmetry problems. The connections developed in Theorems 1.1 and 1.2 linking matrix Diophantine equations and scalar Diophantine equations may serve as a useful foundation for further research on the arithmetic and structural properties of such equations.

2. Background results

In proving the theorems, we make use of the following lemmas.

LEMMA 2.1 [1, Lemma 1]. *If $A^n = \left(\begin{smallmatrix} \alpha & \beta \\ d\beta & \alpha \end{smallmatrix}\right)$ with $A = \left(\begin{smallmatrix} a_1 & a_2 \\ da_2 & a_1 \end{smallmatrix}\right) \in G_2(d)$ and $n \geq 2$, then*

$$\alpha = \frac{1}{2}[(a_1 + a_2\sqrt{d})^n + (a_1 - a_2\sqrt{d})^n] \tag{2.1}$$

and

$$\beta = \frac{1}{2\sqrt{d}}[(a_1 + a_2\sqrt{d})^n - (a_1 - a_2\sqrt{d})^n]. \tag{2.2}$$

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LEMMA 2.2 [1, Lemma 3]. *Let $d \neq 0$ be a fixed integer and let $X = \left(\begin{smallmatrix} x_1 & x_2 \\ x_3 & x_4 \end{smallmatrix}\right) \in M_2(\mathbb{Z})$. If, for some integer $n \geq 2$, the matrix X satisfies $X^n = \left(\begin{smallmatrix} \alpha & \beta \\ d\beta & \alpha \end{smallmatrix}\right)$, where $\alpha, \beta \in \mathbb{Z} \setminus \{0\}$, then $X \in G_2(d)$.*

3. Proof of the main results

The proof techniques for Theorems 1.1 and 1.2 are adapted from those employed in Theorems 1 and 2 of Białek and Grytczuk [1]. For completeness, we give a concise proof of these results below.

PROOF OF THEOREM 1.1. Let $d \neq 0$ be a fixed integer, and suppose that $X, Y, Z \in G_2(d)$ are given by

$$X = \begin{pmatrix} x_1 & y_1 \\ dy_1 & x_1 \end{pmatrix}, \quad Y = \begin{pmatrix} x_2 & y_2 \\ dy_2 & x_2 \end{pmatrix}, \quad Z = \begin{pmatrix} x_3 & y_3 \\ dy_3 & x_3 \end{pmatrix}.$$

Assume that the matrices X, Y, Z satisfy the matrix Diophantine equation

$$X^n + Y^n = n!\, Z^n, \tag{3.1}$$

for some integer $n \geq 2$. By Lemma 2.1,

$$X^n = \begin{pmatrix} \alpha_1 & \beta_1 \\ d\beta_1 & \alpha_1 \end{pmatrix}, \quad Y^n = \begin{pmatrix} \alpha_2 & \beta_2 \\ d\beta_2 & \alpha_2 \end{pmatrix}, \quad Z^n = \begin{pmatrix} \alpha_3 & \beta_3 \\ d\beta_3 & \alpha_3 \end{pmatrix},$$

where

$$\alpha_i = \frac{1}{2}[(x_i + y_i\sqrt{d})^n + (x_i - y_i\sqrt{d})^n] \quad \text{and} \quad \beta_i = \frac{1}{2\sqrt{d}}[(x_i + y_i\sqrt{d})^n - (x_i - y_i\sqrt{d})^n]$$

for $i = 1, 2, 3$. Thus, by (3.1),

$$n!\,\alpha_3 = \alpha_1 + \alpha_2 \quad \text{and} \quad n!\,\beta_3 = \beta_1 + \beta_2. \tag{3.2}$$

Now, substituting α_i, β_i for $i = 1, 2, 3$ in (3.2) and adding the resulting equations, we obtain

$$(x_1 + y_1\sqrt{d})^n + (x_2 + y_2\sqrt{d})^n = n!\,(x_3 + y_3\sqrt{d})^n.$$

Setting $x_1 + y_1\sqrt{d} = \theta$, $x_2 + y_2\sqrt{d} = \eta$ and $x_3 + y_3\sqrt{d} = \omega$, we have

$$\theta^n + \eta^n = n!\,\omega^n, \quad \text{where } \theta, \eta, \omega \in \mathbb{Q}(\sqrt{d}).$$

Conversely, suppose that $\theta, \eta, \omega \in \mathbb{Q}(\sqrt{d})$ satisfy

$$\theta^n + \eta^n = n!\,\omega^n. \tag{3.3}$$

Then we may write

$$\theta = x_1 + y_1\sqrt{d}, \quad \eta = x_2 + y_2\sqrt{d}, \quad \omega = x_3 + y_3\sqrt{d}$$

and their conjugates

$$\overline{\theta} = x_1 - y_1\sqrt{d}, \quad \overline{\eta} = x_2 - y_2\sqrt{d}, \quad \overline{\omega} = x_3 - y_3\sqrt{d},$$

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for some integers x_i, y_i, $1 \le i \le 3$. Taking conjugates on both sides of (3.3) yields

$$(\overline{\theta})^n + (\overline{\eta})^n = n!\,(\overline{\omega})^n. \tag{3.4}$$

From equations (3.3) and (3.4),

$$\frac{1}{2}[\theta^n + \overline{\theta}^n] + \frac{1}{2}[\eta^n + \overline{\eta}^n] = \frac{n!}{2}[\omega^n + \overline{\omega}^n], \tag{3.5}$$

$$\frac{1}{2\sqrt{d}}[\theta^n - \overline{\theta}^n] + \frac{1}{2\sqrt{d}}[\eta^n - \overline{\eta}^n] = \frac{n!}{2\sqrt{d}}[\omega^n - \overline{\omega}^n]. \tag{3.6}$$

By defining

$$\frac{1}{2}[\theta^n + \overline{\theta}^n] = \alpha_1, \quad \frac{1}{2}[\eta^n + \overline{\eta}^n] = \alpha_2, \quad \frac{1}{2}[\omega^n + \overline{\omega}^n] = \alpha_3,$$

$$\frac{1}{2\sqrt{d}}[\theta^n - \overline{\theta}^n] = \beta_1, \quad \frac{1}{2\sqrt{d}}[\eta^n - \overline{\eta}^n] = \beta_2, \quad \frac{1}{2\sqrt{d}}[\omega^n - \overline{\omega}^n] = \beta_3,$$

equations (3.5) and (3.6) become

$$n!\,\alpha_3 = \alpha_1 + \alpha_2 \quad \text{and} \quad n!\,\beta_3 = \beta_1 + \beta_2. \tag{3.7}$$

Now consider the matrices

$$X_1 = \begin{pmatrix} \alpha_1 & \beta_1 \\ d\beta_1 & \alpha_1 \end{pmatrix}, \quad Y_1 = \begin{pmatrix} \alpha_2 & \beta_2 \\ d\beta_2 & \alpha_2 \end{pmatrix}, \quad Z_1 = \begin{pmatrix} \alpha_3 & \beta_3 \\ d\beta_3 & \alpha_3 \end{pmatrix},$$

where $\beta_i \neq 0$ for $1 \le i \le 3$. By (3.7),

$$X_1 + Y_1 = n!\,Z_1. \tag{3.8}$$

From Lemmas 2.1 and 2.2, it follows that there exist matrices X, Y, Z satisfying $X_1 = X^n$, $Y_1 = Y^n$ and $Z_1 = Z^n$. Substituting these into (3.8) gives $X^n + Y^n = n!\,Z^n$ and the matrices

$$X = \begin{pmatrix} x_1 & y_1 \\ dy_1 & x_1 \end{pmatrix}, \quad Y = \begin{pmatrix} x_2 & y_2 \\ dy_2 & x_2 \end{pmatrix} \quad \text{and} \quad Z = \begin{pmatrix} x_3 & y_3 \\ dy_3 & x_3 \end{pmatrix} \quad \text{are in } G_2(d). \qquad \square$$

REMARK 3.1. Ribet [10] showed that the Diophantine equation $x^n + y^n = 2z^n$ lacks nontrivial solutions for $n > 2$, except for the case $x = y = z$. Similar to Theorem 1.1, we can establish a relationship between the solutions of the matrix analogue of this equation in $G_2(d)$ and its solutions in the number field $\mathbb{Q}(\sqrt{d})$.

More generally, let $n > 1$ and a_n, b_n, c_n be integers. Then, the matrix Diophantine equation

$$a_n X^n + b_n Y^n = c_n Z^n$$

admits a solution in $G_2(d)$ if and only if there exist numbers $\theta, \eta, \omega \in \mathbb{Q}(\sqrt{d})$ satisfying

$$a_n \theta^n + b_n \eta^n = c_n \omega^n.$$

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PROOF OF THEOREM 1.2. For any matrix of the form $P = \left(\begin{smallmatrix} 0 & 1 \\ p & 0 \end{smallmatrix}\right)$, we have $P^2 = pI$. Consequently, with P, Q, R as in the statement of the theorem,

$$P^{4n} + Q^{4n} = p^{2n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + q^{2n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = n!\, r^{2n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = n!\, R^{4n}.$$

Thus, $P^{4n} + Q^{4n} = n!\, R^{4n}$, as required. □

As in Remark 3.1, there is an extension of Theorem 1.2 to the matrix Diophantine equation $a_n P^{4n} + b_n Q^{4n} = c_n R^{4n}$ for any integers a_n, b_n, c_n.

If we set $n = 1$ and $K = \mathbb{Q}$ in Theorem 1.2, we obtain the matrix form of Fermat's equation of exponent 4 with rational entries, which admits infinitely many solutions. This gives the following corollary. (For more details see [3].)

COROLLARY 3.2. *Let $K = \mathbb{Q}$, $p, q, r \in \mathbb{Z} \setminus \{0\}$ and $n = 1$. Then the matrix Diophantine equation $P^4 + Q^4 = R^4$ admits infinitely many solutions of the form*

$$P = \begin{pmatrix} 0 & 1 \\ p & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 1 \\ q & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 1 \\ r & 0 \end{pmatrix},$$

where $p = (u^2 - v^2)w$, $q = 2uvw$ and $r = (u^2 + v^2)w$, for $u > v, w \geq 1$.

REMARK 3.3. Let K be a number field and let $n \in \mathbb{N}$. For any $p, q, r \in K$ satisfying the equation

$$p^{2n} + q^{2n} = 2r^{2n},$$

the matrices

$$P = \begin{pmatrix} 0 & 1 \\ p & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 1 \\ q & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 1 \\ r & 0 \end{pmatrix}$$

satisfy the related matrix Diophantine equation $P^{4n} + Q^{4n} = 2R^{4n}$.

PROOF OF THEOREM 1.3. Suppose that $X = \left(\begin{smallmatrix} x_1 & y_1 \\ dy_1 & x_1 \end{smallmatrix}\right)$, $Y = \left(\begin{smallmatrix} x_2 & y_2 \\ dy_2 & x_2 \end{smallmatrix}\right)$, $Z = \left(\begin{smallmatrix} x_3 & y_3 \\ dy_3 & x_3 \end{smallmatrix}\right)$ is a solution of (1.2) with $\det X = \det Y = \det Z = l$. By Lemma 2.1 and in a similar manner to the proof of Theorem 1.1, we obtain

$$X^n = \begin{pmatrix} \alpha_1 & \beta_1 \\ d\beta_1 & \alpha_1 \end{pmatrix}, \quad Y^n = \begin{pmatrix} \alpha_2 & \beta_2 \\ d\beta_2 & \alpha_2 \end{pmatrix}, \quad Z^n = \begin{pmatrix} \alpha_3 & \beta_3 \\ d\beta_3 & \alpha_3 \end{pmatrix}$$

and

$$\alpha_i = \frac{1}{2}({\sigma_i}^n + {\rho_i}^n), \quad \beta_i = \frac{1}{2\sqrt{d}}({\sigma_i}^n - {\rho_i}^n),$$

where $\sigma_i = x_i + y_i\sqrt{d}$ and $\rho_i = x_i - y_i\sqrt{d}$ for $i = 1, 2, 3$. The assumption $X^n + Y^n = 2Z^n$ yields

$$\alpha_1 + \alpha_2 = 2\alpha_3 \quad \text{and} \quad \beta_1 + \beta_2 = 2\beta_3 \tag{3.9}$$

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and consequently,

$$\sigma_1{}^n + \sigma_2{}^n = 2\sigma_3{}^n \quad \text{and} \quad \rho_1{}^n + \rho_2{}^n = 2\rho_3{}^n. \tag{3.10}$$

Furthermore, $\det X = \det Y = \det Z = l = x_i^2 - dy_i^2 = \sigma_i\rho_i$ for $i = 1, 2, 3$. Multiplying equations (3.10) now gives

$$(\sigma_1{}^n + \sigma_2{}^n)(\rho_1{}^n + \rho_2{}^n) = (\sigma_1\rho_1)^n + (\sigma_1\rho_2)^n + (\sigma_2\rho_1)^n + (\sigma_2\rho_2)^n = 4\sigma_3{}^n\rho_3{}^n$$

and, since $\sigma_i\rho_i = l$ for each i, this gives

$$-2l^n + (\sigma_1\rho_2)^n + (\sigma_2\rho_1)^n = 0. \tag{3.11}$$

If $l = \sigma_i\rho_i = 0$, then $\sigma_i = 0$ or $\rho_i = 0$. In this case, $\alpha_i = \frac{1}{2}(\sigma_i{}^n + \rho_i{}^n) = 2^{n-1}x_i^n$ for $i = 1, 2, 3$. Thus, (3.9) implies that

$$x_1^n + x_2^n = 2x_3^n. \tag{3.12}$$

But, from the result given by Ribet [10], (3.12) has only a trivial solution $x_1 = x_2 = x_3$ for $n > 2$. Similarly, we can obtain $y_1 = y_2 = y_3$ so that $X = Y = Z$.

We can now assume that $l \neq 0$. Since $l = \sigma_1\rho_1 = \sigma_2\rho_2$, dividing both sides of (3.11) by $\sigma_1\rho_1$ gives

$$-2 + \left(\frac{\rho_2}{\rho_1}\right)^n + \left(\frac{\rho_1}{\rho_2}\right)^n = 0. \tag{3.13}$$

Taking $(\rho_2/\rho_1)^n = t$, we can rewrite (3.13) as $-2 + t + 1/t = 0$ so that $t = 1$ and consequently $(\rho_2/\rho_1)^n = 1$. Here we have two cases.

Case I. If n is odd, then $\rho_2/\rho_1 = 1$ so that $\rho_2 = \rho_1$. From (3.10), $\rho_1{}^n + \rho_2{}^n = 2\rho_3{}^n$ so $\rho_1{}^n = \rho_3{}^n$. Therefore, we have $\rho_1 = \rho_2 = \rho_3$, which implies that $x_1 = x_2 = x_3$ and $y_1 = y_2 = y_3$ and so $X = Y = Z$. Hence, (1.2) has only a trivial solution.

Case II. If n is even, then $\rho_2/\rho_1 = \pm 1$, that is, $\rho_2 = \pm\rho_1$. From (3.10), $\rho_1{}^n + \rho_2{}^n = 2\rho_3{}^n$ so that $\rho_3 = \pm\rho_1$. Therefore, we have $X = Y = Z$, $X = Y = -Z$, $X = -Y = Z$ or $-X = Y = Z$. Hence, (1.2) has only trivial solutions. □

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