J. Plasma Physics (2006), *vol.* 72, *part* 6, *pp.* 1197–1200. © 2006 Cambridge University Press
doi:10.1017/S0022377806005927 Printed in the United Kingdom

Diamagnetic equilibrium of spherical torus obtained in a two-fluid model with flow

T. KANKI[1] and M. NAGATA[2]

[1]Department of Maritime Science and Technology, Japan Coast Guard Academy, 5-1 Wakaba, Kure, Hiroshima 737-8512, Japan

[2]Department of Electrical Engineering, University of Hyogo, 2167 Shosha, Himeji 671-2201, Japan

(Received 22 July 2005 and accepted 29 January 2006)

Abstract. Two-fluid flowing equilibrium configurations of a helicity-driven spherical torus (HD-ST) in the realistic region including the spherical flux conserver and the coaxial helicity source are numerically determined by using the combination of the finite difference and the boundary element methods. It is found from the numerical results that the ultra low-q HD-ST with diamagnetic toroidal field and high-β (volume average β value, $\langle\beta\rangle \cong 18\%$) can be obtained by reversing the toroidal field over the whole region and by increasing the toroidal ion flow up to the Alfvén Mach number $M_A \cong 0.7$. The ion diamagnetic drift velocity is larger than the $\mathbf{E} \times \mathbf{B}$ drift velocity and has the same direction.

1. Introduction

There have been many experiments to sustain spherical torus plasmas by helicity drive alone. It is one of the most popular methods for helicity-driven spherical torus (HD-ST) operation to utilize a magnetized coaxial plasma gun for both the formation and sustainment phases, i.e. coaxial helicity injection (CHI). In these experiments, toroidal ion flow related to the toroidal mode number $n = 1$ mode is observed during CHI [1]. This observation has raised the crucial issue of the equilibrium and stability properties of flowing plasmas. Recently, the importance of a two-fluid model with flow [2, 3] has attracted considerable interest in the high-β operation regimes of spherical torus. However, the details of how such a two-fluid model with flow affects the equilibrium configurations of HD-ST are not numerically investigated. The purpose of this study is to numerically determine the flowing two-fluid equilibrium configurations of HD-ST and to explore their properties. In this paper, we focus our attention on the diamagnetic equilibrium.

2. Numerical model

In Fig. 1, we show the spherical flux conserver (FC) and the coaxial helicity source of a helicity-injected spherical torus (HIST) device [4]. Let us use the cylindrical coordinate system (r, θ, z) and take the symmetry axis as the z-axis. At first, we assume that the bias flux extends over the entire space. Next, we assume that the plasma exists only in Ω_1, and that the magnetic flux generated by the plasma current extends over all Ω_1 and Ω_2. Hereafter, dimensionless variables are used,

https://doi.org/10.1017/S0022377806005927 Published online by Cambridge University Press

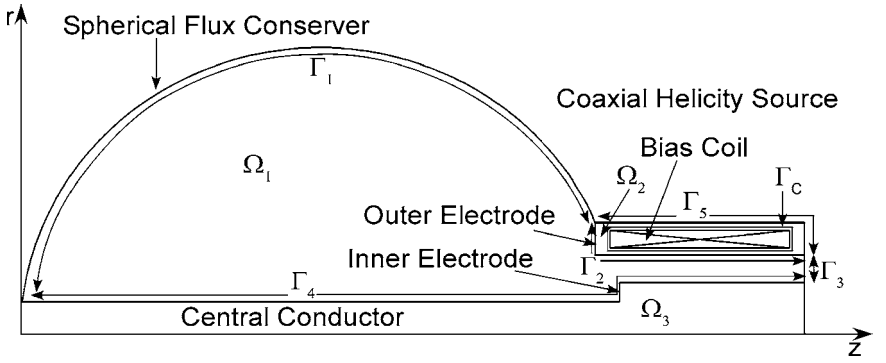


Figure 1. Model of the flux conserver and the helicity source.

and the details of the normalized variables scheme are described elsewhere [5]. The axisymmetric flowing two-fluid equilibrium [5] of HD-ST can be described by the coupled pair of differential equations for the generalized stream functions Ψ_i and Ψ_e,

$$r^2\frac{d\psi_\mathrm{i}}{d\Psi_\mathrm{i}}\nabla\cdot\left(\frac{d\psi_\mathrm{i}}{d\Psi_\mathrm{i}}\frac{\nabla\Psi_\mathrm{i}}{r^2}\right)=S_*^2(\psi_\mathrm{i}-\psi_\mathrm{e})\frac{d\psi_\mathrm{i}}{d\Psi_\mathrm{i}}+S_*^2(\Psi_\mathrm{e}-\Psi_\mathrm{i})+r^2\frac{dH_\mathrm{i}}{d\Psi_\mathrm{i}},\qquad(2.1)$$

$$\Delta^*\Psi_\mathrm{e}=S_*^2(\psi_\mathrm{i}-\psi_\mathrm{e})\frac{d\psi_\mathrm{e}}{d\Psi_\mathrm{e}}+S_*^2(\Psi_\mathrm{e}-\Psi_\mathrm{i})-r^2\frac{dH_\mathrm{e}}{d\Psi_\mathrm{e}}.\qquad(2.2)$$

Here Δ^* denotes the Grad–Shafranov operator and S_* is defined as the ratio of the system size scale to the ion skin depth. The size parameter $S_*=16.0$ is given by using the density $n=5.0\times10^{19}$ m^{-3} obtained in the HIST experiments. The flow stream functions ψ_α and the generalized stream functions Ψ_α are introduced to express the species flow velocities and the poloidal part of the generalized vorticitiy of each species, respectively. The total enthalpies H_α and ψ_α are arbitrary surface functions of their respective surface variables Ψ_α, respectively, $H_\mathrm{i}(\Psi_\mathrm{i})=p_\mathrm{i}+u_\mathrm{i}^2/2+\phi_\mathrm{E}$, $H_\mathrm{e}(\Psi_\mathrm{e})=p_\mathrm{e}-\phi_\mathrm{E}$, and $\psi_\alpha=\psi_\alpha(\Psi_\alpha)$. Here p_α, u_α, and ϕ_E are the pressure, the flow velocity, and the scalar potential regarding the steady electric field. In this study we consider the special case of purely toroidal ion flow, i.e. $\psi_\mathrm{i}(\Psi_\mathrm{i})=0$. In addition, we assume the remaining arbitrary functions: $dH_\mathrm{i}/d\Psi_\mathrm{i}=C_{H_{\mathrm{i}0}}+C_{H_{\mathrm{i}1}}\Psi_\mathrm{i}$, $dH_\mathrm{e}/d\Psi_\mathrm{e}=C_{H_{\mathrm{e}0}}+C_{H_{\mathrm{e}1}}\Psi_\mathrm{e}+C_{H_{\mathrm{e}3}}\Psi_\mathrm{e}^3$, and $\psi_\mathrm{e}(\Psi_\mathrm{e})=C_{B_\mathrm{T}}+C_{\psi_{\mathrm{e}1}}\Psi_\mathrm{e}+C_{\psi_{\mathrm{e}2}}\Psi_\mathrm{e}^2$, where C are constant parameters. Here $C_{H_{\mathrm{i}0}}$ and $C_{H_{\mathrm{i}1}}$ are related to the value of toroidal ion flow at the edge and the maximum value of toroidal ion flow in the core region, respectively. The structures of total enthalpies H_α are determined by the combination of $C_{H_{\mathrm{e}0}}$, $C_{H_{\mathrm{e}1}}$, $C_{H_{\mathrm{e}3}}$, $C_{H_{\mathrm{i}0}}$ and $C_{H_{\mathrm{i}1}}$. Also, $C_{\psi_{\mathrm{e}1}}$ and $C_{\psi_{\mathrm{e}2}}$ denote the parameters for self-toroidal field structure. The parameters $C_{H_{\mathrm{e}3}}$ and C_{B_T} affect the pressure profile and the strength of the vacuum toroidal field, respectively.

Next let us consider the boundary conditions for (2.1) and (2.2). We set $\Psi_\mathrm{e}=\Psi_\mathrm{bias}$ on Γ_1, Γ_3, Γ_4, and Γ_5 because the bias flux extends all over the space. Here Ψ_bias represents the bias flux produced by the bias coil current I_bias. Ampère's law on the coil surface Γ_C can be written as $I_\mathrm{bias}=\oint_{\Gamma_\mathrm{C}} r^{-1}\partial\Psi_\mathrm{e}/\partial n\,dl$. Here $\mathbf{n}$ denotes the unit vector whose direction is outward normal to Γ_C. The boundary condition for Ψ_e on the surface Γ_C is obtained by setting at an unknown constant. After the linearization of the equation reduced by substituting (2.1) into (2.2) under the condition of

https://doi.org/10.1017/S0022377806005927 Published online by Cambridge University Press

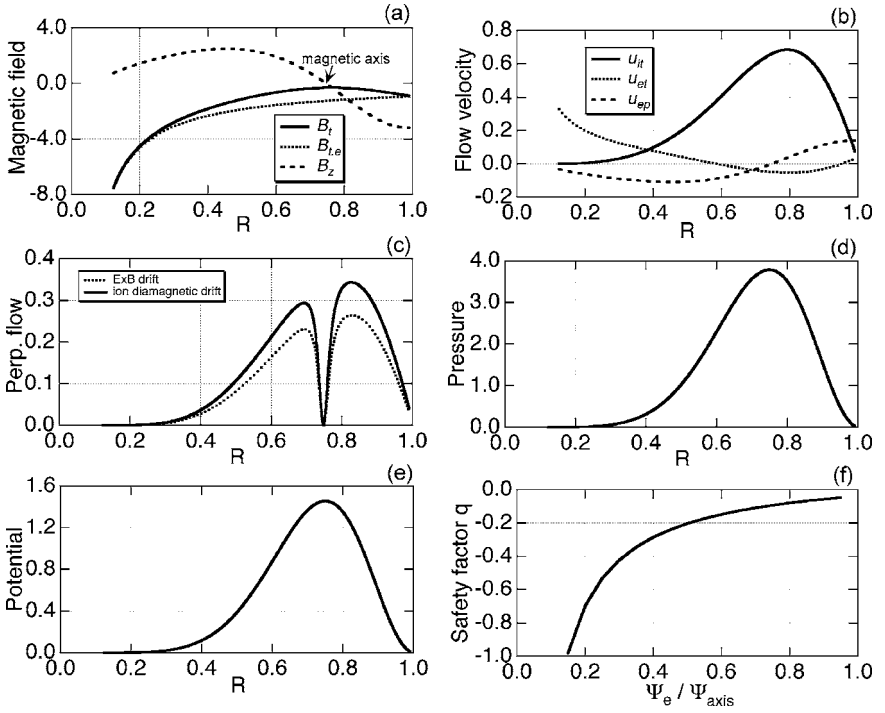


Figure 2. Ultra low-q HD-ST equilibrium with $C_{H_{i0}} = 0.0$, $C_{H_{i1}} = 28.0$, $C_{H_{e0}} = 0.0$, $C_{H_{e1}} = 4.0$, $C_{H_{e3}} = -1.0$, $S_* C_{\psi_{e1}} = -0.7$, $S_* C_{\psi_{e2}} = 0.0$, $S_* C_{B_T} = 0.926$, and $I_{\rm bias}/I_t = 0.5$. The radial profiles at the midplane of the magnetic fields (the solid, dotted, and dashed curves indicate the toroidal field, the vacuum toroidal field, and the poloidal field, respectively), flow velocities (the solid, dotted, and dashed curves indicate the toroidal ion flow, the toroidal electron flow, and the poloidal electron flow, respectively), toroidal components of the fluid drifts perpendicular to the magnetic field (the solid and dotted curves indicate the ion diamagnetic and $\mathbf{E} \times \mathbf{B}$ drifts, respectively), total pressure, and electrostatic potential are shown in (a), (b), (c), (d), and (e), respectively. Here the ion diamagnetic and $\mathbf{E} \times \mathbf{B}$ drifts are calculated from $\mathbf{B} \times \nabla p_{\rm i}/S_* B^2$ and $\mathbf{E} \times \mathbf{B}/S_* B^2$, respectively. The temperature ratio $T_{\rm i}/T_{\rm e}$ is assumed as 1.0. The safety factor q as a function of the normalized poloidal flux function $\Psi_{\rm e}/\Psi_{\rm axis}$ is shown in (f). Here $\Psi_{\rm axis}$ is Ψ_e at the magnetic axis.

purely toroidal ion flow [5], the problem can be numerically solved by using the combination of the finite difference and the boundary element methods [6]. This computation is implemented so that the total toroidal current $I_{\rm t}$ is constant.

3. Numerical results

Numerical investigations of two-fluid flowing equilibria for discharges on a HIST device in which the vacuum toroidal field $B_{\rm t.e}$ in a pre-formed CHI produced spherical torus is reversed show that the magnetic configurations change from the high-q HD-ST ($q > 1$) with low-β (volume average β value, $\langle\beta\rangle \cong 2\%$) and paramagnetic toroidal field $B_{\rm t}$ through the helicity-driven spheromak and reversed-field pinch to the ultra low-q HD-ST ($0 < q < 1$) with high-β ($\langle\beta\rangle \cong 18\%$) and diamagnetic $B_{\rm t}$ as $B_{\rm t.e}$ near the central conductor decreases and reverses sign [7].

Figure 2 shows the radial profiles at the midplane and the safety factor q in the ultra low-q HD-ST equilibrium. It is found from Fig. 2(a) that the diamagnetic

https://doi.org/10.1017/S0022377806005927 Published online by Cambridge University Press

field structure has the high-β characteristics that B_t at the major radius $R = 0.76$ is approximately zero. Figure 2(b) shows that the toroidal electron flow velocity u_{et} has the same direction as the toroidal ion flow velocity u_{it} near the central conductor. At $R = 0.79$, u_{it} referenced to the Alfvén speed [5] has a maximum of about 70% of the Alfvén speed. The ion diamagnetic velocity is larger than the $\mathbf{E} \times \mathbf{B}$ drift velocity and has the same direction, as shown in Fig. 2(c). The pressure is similar in peaked profile near the magnetic axis to the electrostatic potential, as shown in Fig. 2(d) and 2(e). Figure 2(f) shows that the ultra low-q HD-ST has negative q-values varied from -1 to 0 due to the reversed B_t profile over the whole region. This diamagnetic equilibrium of HD-ST has not been observed theoretically and experimentally, yet. It can be obtained by changing two parameters $C_{H_{il}}$ and C_{B_T} related to the strength of the toroidal ion flow and the vacuum toroidal field, respectively. The evidence of this observation can be verified by above continuous equilibrium transition from the high-q HD-ST obtained in the experiment to the the ultra low-q HD-ST. This observation is considered to be closely related to two-fluid effect with large toroidal ion flow.

4. Conclusions

By means of the combination of the finite difference and the boundary element methods, we have numerically investigated the flowing two-fluid equilibria for discharges on a HIST device in which $B_{t.e}$ in a pre-formed CHI produced spherical torus is reversed. The ultra low-q HD-ST can be obtained by reversing B_t over the whole region and by increasing u_{it} up to the Alfvén Mach number $M_A \cong 0.7$. The ion diamagnetic velocity is larger than the $\mathbf{E} \times \mathbf{B}$ drift velocity and has the same direction. The ultra low-q HD-ST shows the high-β characteristics and diamagnetism that B_t at $R = 0.76$ is approximately zero and the absolute value of $B_{t.e}$ is larger than that of B_t over the core region.

References

[1] McCollam, K. J. and Jarboe, T. R. 2002 Magnetic relaxation in coaxial helicity injection. *Plasma Phys. Contr. Fusion* **44**, 493–517.

[2] Steinhauer, L. C. and Ishida, A. 1997 Relaxation of a two-species magnetofluid. *Phys. Rev. Lett.* **79**, 3423–3426.

[3] Yoshida, Z. and Mahajan, S. M. 2002 Variational principles and self-organization in two-fluid plasmas. *Phys. Rev. Lett.* **88**, 095001.

[4] Nagata, M., Kanki, T., Fukumoto, N. and Uyama T. 2003 The internal magnetic field structures and current density profiles in the helicity injected spherical torus plasma driven by coaxial helicity injection. *Phys. Plasmas* **10**, 2932–2939.

[5] Yamada, H., Katano, T., Kanai, K., Ishida, A. and Steinhauer, L. C. 2002 Equilibrim analysis of a flowing two-fluid plasma. *Phys. Plasmas* **9**, 4605–4614.

[6] Kanki, T., Nagata, M., Uyama, T., Ikuno, S. and Kamitani, A. 1998 Partially relaxed magnetohydrodynamic equilibria with bias-flux leakage obtained in a helicity-driven spheromak. *J. Phys. Soc. Japan* **67**, 140–146.

[7] Kanki, T., Nagata, M. and Uyama, T. 2004 In: *Proceedings of the 12th International Congress on Plasma Physics*, Nice, France, 25–29 October 2004, P1-087 and http://hal.ccsd.cnrs.fr/ccsd-00001748.

https://doi.org/10.1017/S0022377806005927 Published online by Cambridge University Press