1695	Richard Morden	A1	1.19
1745	John Elphinstone	A0	1.22
1769	Emanuel Bowen	A0	1.22
1803	Charles F Delamarche	A0	1.22
1821	James Playfair	A0	1.20
1840	Henry Teesdale	A1	1.20
1852	Henry D Betts	A0	1.19
1893	J A Harvie-Brown & J G Bartholomew	A1	1.18
1936	C S Hammond	A0	1.21

TABLE 1: Details of the maps used and their resulting divider dimension.

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10.1017/mag.2019.120

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103.39 A lemma to solve Langley's problem

Introduction

Langley's problem, first proposed in 1922 [1], is also known as the adventitious angles problem. Many math lovers also call it the ‘world's hardest easy geometry problem’ or the ‘most annoying elementary geometry problem’. Professor Ross Honsberger of University of Waterloo, Canada, and Professor Zhengang Xiao of Hunan Institute of Technology called it a minor gem [2, 3].

There are more than 40 known solutions to this special geometry problem. [4] (2 solutions), [5] (7 solutions), [6] (12 solutions), [7] (7 solutions), [8] (9 solutions), [9] (8 solutions), [10] (3 solutions), [11] (2 solutions), [12] (3 solutions), [13] (6 solutions), [14] (10 solutions) and [15] (13 solutions).

This Note provides one lemma that results in two new solutions.

https://doi.org/10.1017/mag.2019.121 Published online by Cambridge University Press

Langley's problem

Langley's problem states: ABC is an isosceles triangle with $CA = CB$, and $\angle C = 20°$. The point D is on AC and E is on BC such that $\angle ABD = 60°$, and $\angle BAE = 50°$. Prove $\angle EDB = 30°$ (Figure 1).

Lemma:

Given that ABC is an isosceles triangle in which $CA = CB$, $\angle C = 20°$, $\angle CAB = \angle CBA = 80°$ and D is on AC and E is on BC such that $\angle ABD = 60°$ and $\angle BAE = 50°$. If $DF \parallel AE$ with F on BC, show that $CF = AB = BE$ (Figure 2).

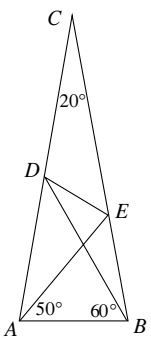


FIGURE 1

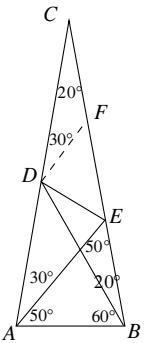


FIGURE 2

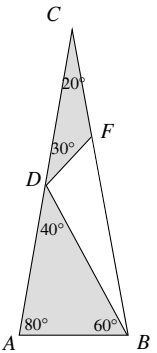


FIGURE 3

Proof of Lemma: We simplify Figure 2 into Figure 3. We know that $\angle CFD = 180° - 30° - 20° = 130°$. We also know that $CD = DB$.

Applying the Sine Rule to ΔCFD, we have:

$$\frac{CD}{CF} = \frac{\sin \angle CFD}{\sin \angle CDF} = \frac{\sin 130°}{\sin 30°} = \frac{\sin 50°}{\frac{1}{2}} = 2 \cos 40°. \qquad (1)$$

Applying the Sine Rule to ΔABD, we have:

$$\frac{DB}{AB} = \frac{\sin \angle DAB}{\sin \angle ADB} = \frac{\sin 80°}{\sin 40°} = \frac{2 \sin 40° \cos 40°}{\sin 40°} = 2 \cos 40°. \qquad (2)$$

From (1) and (2), we see that

$$\frac{CD}{CF} = \frac{DB}{AB}.$$

Since $CD = BD$, therefore $CF = AB$.

Solving Langley's problem using the lemma

Method 1:

Draw $DF \parallel AE$ from D to meet CB at F. $\angle CDF = \angle CAE = 30°$. From the lemma, $CF = AB = EB$. Therefore $\Delta CFD \equiv \Delta BED$ (S.A.S). So, $\angle EDC = \angle FDC = 30°$.

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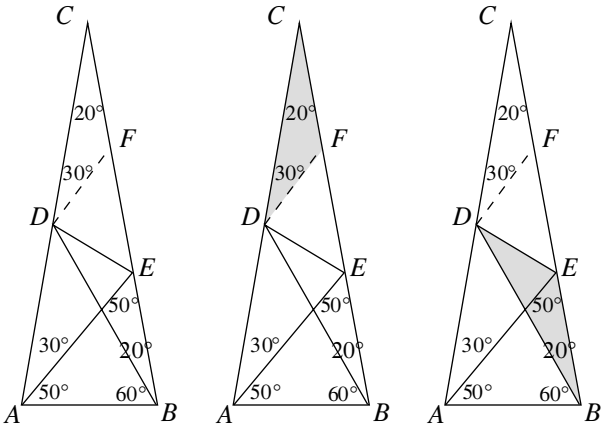


FIGURE 4

Method 2:

Draw $DF \parallel AE$ from D to meet CB at F. $\angle CDF = \angle CAE = 30°$. From the lemma, $CF = AB = EB$. Draw DG, the angle bisector of $\angle ADB$, to meet AE at G. Since $\angle BDG = \angle DBE = 20°$, therefore $DG \parallel BE \parallel FE$. Thus, $DFEG$ is a parallelogram with $DG = EF$.

We then see that $\Delta CAE \sim \Delta DAG$. So, $\frac{CA}{CE} = \frac{DA}{DG}$. By the proportional property, we have

$$\frac{CA}{CE} = \frac{DA}{DG} = \frac{CA - DA}{CE - DG} = \frac{CD}{CE - FE} = \frac{CD}{CF} = \frac{BD}{BE}.$$

We also see $\angle ACE = \angle DBE = 20°$, thus $\Delta CAE \sim \Delta BDE$, and $\angle EDB = \angle EAC = 30°$.

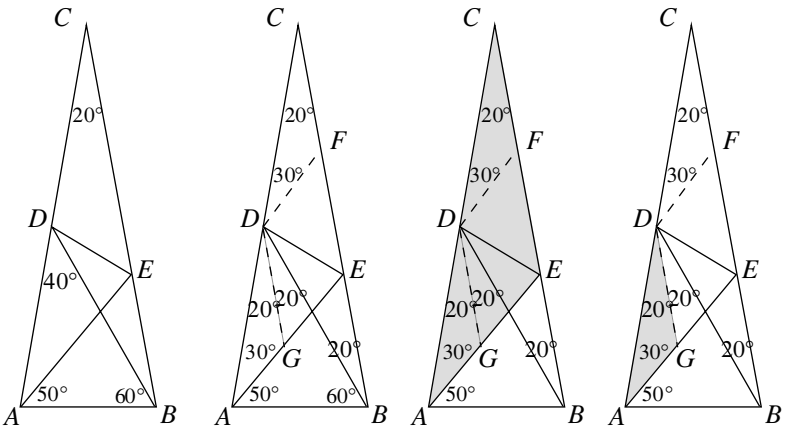


FIGURE 5

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Conclusion

The author proposes a lemma that can be used to solve Langley's problem. The lemma was proved using a trigonometric method without drawing auxiliary lines, although the author believes there exist purely geometric proofs.

Based on the lemma, two new solutions are added to the garden of existing solutions for Langley's problem.

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10.1017/mag.2019.121

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https://doi.org/10.1017/mag.2019.121 Published online by Cambridge University Press