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Complete dynamic modelling of a moving base 6-dof parallel manipulator

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SUMMARY

In this paper a new approach based on the generalized momentum is used to obtain the dynamic model of a six degrees-of-freedom (dof) parallel manipulator. First, the system dynamic equations are obtained supposing the manipulator base platform is fixed. Afterwards, the dynamic model is extended to the case of a moving base platform. This could be important in a macro/micro robotic application, where a small manipulator is attached in series to a big manipulator. Simulation results of a macro/micro robotic system are presented and the contribution of the base platform motion to the total actuating forces is shown.

KEYWORDS: Robotics; Parallel manipulators; Robot dynamics; Generalized momentum; Moving base.

1. Introduction

Dynamic models play an important role in parallel manipulators simulation and control. In the first case, the manipulator trajectory (position, velocity and acceleration) is to be computed for the given actuating forces or torques (direct dynamics). In the second case, the actuators forces or torques required to generate a given manipulator trajectory should be calculated (inverse dynamics). Mainly in this case, the efficiency of the computation for the manipulator dynamics is of paramount importance, as manipulator real-time control is usually necessary.[1]

The dynamic model of a parallel manipulator operating in free space can be mathematically represented, in Cartesian coordinates, by a system of nonlinear differential equations that may be written in matrix form as

$$\mathbf{I}(\mathbf{x}) \cdot \ddot{\mathbf{x}} + \mathbf{V}(\mathbf{x}, \dot{\mathbf{x}}) \cdot \dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}) = \mathbf{f}, \tag{1}$$

where $\mathbf{I}(\mathbf{x})$ is the inertia matrix, $\mathbf{V}(\mathbf{x}, \dot{\mathbf{x}})$ the Coriolis and centripetal terms matrix, $\mathbf{G}(\mathbf{x})$ a vector of gravitational generalized forces, $\mathbf{x}$ the generalized position of the moving platform (or end-effector) and $\mathbf{f}$ the controlled generalized force applied on the end-effector. Thus,

$$\mathbf{f} = \mathbf{J}^T(\mathbf{x}) \cdot \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{\tau}$ is the generalized force developed by the actuators and $\mathbf{J}(\mathbf{x})$ is a Jacobian matrix.

Dynamic modelling of parallel manipulators presents an inherent complexity, mainly due to system closed-loop structure and kinematic constraints. Several approaches have been applied to the dynamic analysis of parallel manipulators, the Newton–Euler and the Lagrange methods being the most popular ones. The Newton–Euler approach uses the free body diagrams of the rigid bodies. Do and Yang[2] and Reboulet and Berthomieu[3] used this method on the dynamic modelling of a Stewart platform. Ji[4] presented a study of the influence of leg inertia on the dynamic model of a Stewart platform. Dasgupta and Mruthyunjaya[5] used the Newton–Euler approach to develop a closed-form dynamic model of the Stewart platform. This method was also used by Khalil and Ibrahim,[6] Riebe and Ulbrich[7] and Guo and Li,[8] among others.

The Lagrange method describes the dynamics of a mechanical system from the concepts of work and energy. Nguyen and Pooran[9] used this method to model a Stewart platform, modelling the legs as point masses. Lebret *et al.*[10] followed an approach similar to the one used by Nguyen and Pooran.[9] Lagrange's method was also used by Gregório and Parenti-Castelli[11] and Caccavale *et al.*,[12] for example.

These methods use classical mechanics principles, as is the case for all the approaches found in the literature, namely the ones based on the principle of virtual work,[13,14] screw theory,[15] recursive matrix method[16] and Hamilton's principle.[17] Thus, all approaches are equivalent as they are describing the same physical system.[18] All methods lead to equivalent dynamic equations, although these equations present different levels of complexity and associated computational loads.[1] Minimizing the number of operations involved in the computation of the manipulator dynamic model has been the main goal of recent proposed techniques.[1,16,19,20,21]

In a previous work[22] it was presented a new approach to the dynamic modelling of a six degrees-of-freedom (dof) parallel manipulator: the use of the generalized momentum concept. The method was used to compute the analytic expressions of the inertia and Coriolis and centripetal terms matrices of the manipulator rigid bodies. These matrices can be added, as they are expressed in the same frame. The computational load of the dynamic model was evaluated. It was shown the model obtained using the proposed approach presents a very small computational load.

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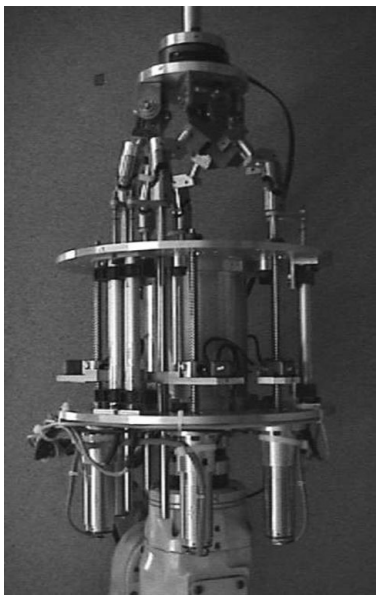


Fig. 1. Photography of the RCID parallel manipulator.

This paper extends the manipulator dynamic model to the case of a non-stationary base platform. First, the dynamic equations are obtained supposing the manipulator base platform is fixed. Afterwards, the dynamic model is extended to the case of a moving base platform. This is important, for example, in a macro/micro robotic application, where a small manipulator is attached in series to a big manipulator. The later performing large movements, while the former being responsible for the small motions.

The manipulator under study may be seen as a variation of the well known Stewart platform, with the uniqueness of having all the actuators fixed to the base platform and only capable to move in a direction perpendicular to that base. A prototype of this manipulator, the robotic controlled impedance device (RCID), was developed aiming a broad set of force-impedance control tasks.[23]

This paper is organized as follows. Section 2 presents a brief description of the parallel manipulator, the RCID. Section 3 presents the manipulator dynamic model, developed using the generalized momentum approach, considering the base platform is fixed. In Section 4 the dynamic model is extended, considering the base platform motion. Section 5 presents some simulation results for the inverse dynamic model. Conclusions are drawn in Section 6.

2. Parallel Manipulator Mechanical Structure

The RCID is a 6-dof parallel manipulator (Fig. 1). Parallel manipulators are well known for their high dynamic performances and low positioning errors.[24,25] In the last few years parallel manipulators have attracted great attention from researchers involved with robot manipulators, robotic end effectors, robotic devices for high-precision robotic tasks, machine-tools, simulators and haptic devices.[26–30]

The mechanical structure of the RCID comprises a (fixed) base platform and a moving (payload) platform, linked together by six independent, identical, open kinematic chains (Fig. 2). Each chain comprises two links: the first link (linear actuator) is always normal to the base and has a variable

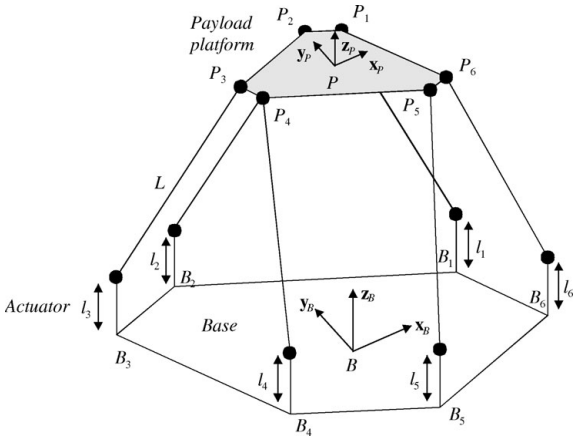


Fig. 2. A schematic view of the RCID mechanical structure.

length l_i with one of its ends fixed to the base and the other one attached, by a universal joint, to the second link; the second link (fixed-length link) has a fixed length L and is attached to the payload platform by a spherical joint. Points B_i and P_i are the connecting points to the base and payload platforms. They are located at the vertices of two semi-regular hexagons, inscribed in circumferences of radius r_B and r_P, that are coplanar with the base and payload platforms (Fig. 3).

Regarding kinematic modelling, a right-handed reference frame $\{\mathrm{B}\}$ is attached to the base. Its origin is located at point B, the centroid of the base. Axis $\mathbf{x}_B$ is normal to the line connecting points B_1 and B_6 and axis $\mathbf{z}_B$ is normal to the base, pointing towards the payload platform. The angles between points B_1 and B_3 and points B_3 and B_5 are set to 120°. The separation angles between points B_1 and B_6, B_2 and B_3, and B_4 and B_5 are denoted by $2\phi_B$ (Fig. 3). In a similar way, a right-handed frame $\{\mathrm{P}\}$ is assigned to the payload platform. Its origin is located at point P, the centroid of the payload platform. Axis $\mathbf{x}_P$ is normal to the line connecting points P_1 and P_6 and axis $\mathbf{z}_P$ is normal to the payload platform, pointing in a direction opposite to the base. The angles between points P_1 and P_3 and points P_3 and P_5 are set to 120°. The separation angles between points P_1 and P_2, P_3 and P_4, and P_5 and P_6 are denoted by $2\phi_P$ (Fig. 3). The main kinematic RCID parameters were adjusted in order to maximize the manipulator dexterity[31] inside a prescribed workspace: the payload platform may be positioned anywhere inside a sphere of radius 10 mm (centred at a point of the line containing axis $\mathbf{z}_B$) and rotate ±15° around any axis containing the payload platform centre. This requires the actuators displacement of $\Delta l_i = 70$ mm, approximately. The main kinematic parameters values are shown in Table I.

Table I. RCID main kinematic parameters.

Parameter	Value
r_B	80 mm
r_P	40 mm
ϕ_B	15°
ϕ_P	0°
L	97.98 mm
Δl_i	70 mm

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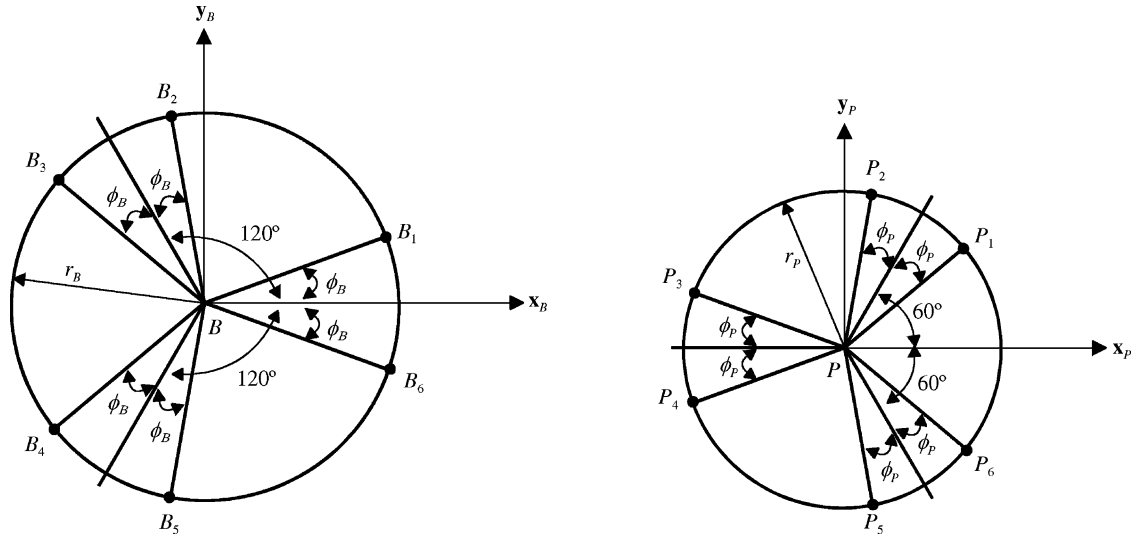


Fig. 3. Position of the connecting points to the base and payload platforms.

The RCID maximal height is approximately 530 mm (when the payload platform is at its farthest position). Maximum diameter is approximately 265 mm (corresponding to the fixed platform diameter), and total mass is about 9.7 kg. Maximum payload platform velocity along vertical direction is 220 mm/s and maximum payload capability is 5 kg, approximately.

For kinematic modelling purposes, attaching frames {P} and {B} to the payload and base platforms, respectively, the generalized position of frame {P} relative to frame {B} may be represented by the vector

$$
\begin{aligned}
{}^{B}\mathbf{x}_{P|_{B|E}} &= [x_P \quad y_P \quad z_P \quad \psi_P \quad \theta_P \quad \varphi_P]^T \\
&= \left[{}^{B}\mathbf{x}^T_{P\,(\mathrm{pos})|_B} \quad {}^{B}\mathbf{x}^T_{P\,(o)|_E}\right]^T,
\end{aligned} \tag{3}
$$

where ${}^{B}\mathbf{x}_{P(pos)|_B} \equiv {}^{B}\mathbf{p}_{P|_B} = [x_P \quad y_P \quad z_P]^T$ is the position of the origin of frame {P} relative to frame {B}, and ${}^{B}\mathbf{x}_{P(o)|E} = [\psi_P \quad \theta_P \quad \varphi_P]^T$ defines an Euler angles system representing orientation of frame {P} relative to {B}. The rotation matrix is given by

$$
{}^{B}\mathbf{R}_P = \begin{bmatrix}
C(\psi_P)C(\theta_P) & C(\psi_P)S(\theta_P)S(\varphi_P) - S(\psi_P)C(\varphi_P) & C(\psi_P)S(\theta_P)C(\varphi_P) + S(\psi_P)S(\varphi_P) \\
S(\psi_P)C(\theta_P) & S(\psi_P)S(\theta_P)S(\varphi_P) + C(\psi_P)C(\varphi_P) & S(\psi_P)S(\theta_P)C(\varphi_P) - C(\psi_P)S(\varphi_P) \\
-S(\theta_P) & C(\theta_P)S(\varphi_P) & C(\theta_P)C(\varphi_P)
\end{bmatrix}, \tag{4}
$$

$S(\cdot)$ and $C(\cdot)$ correspond to the sine and cosine functions, respectively.

The velocity kinematics is represented by the Euler angles Jacobian matrix $\mathbf{J}_E$ or the kinematic Jacobian $\mathbf{J}_C$. These Jacobians relate the velocities of the active joints, the actuators, to the generalized velocity of the moving platform:

$$
\dot{\mathbf{l}} = \mathbf{J}_E \cdot {}^{B}\dot{\mathbf{x}}_{P|_{B|E}} = \mathbf{J}_E \cdot \begin{bmatrix} {}^{B}\dot{\mathbf{x}}_{P(pos)|_B} \\ {}^{B}\dot{\mathbf{x}}_{P(o)|_E} \end{bmatrix}, \tag{5}
$$

$$
\dot{\mathbf{l}} = \mathbf{J}_C \cdot {}^{B}\dot{\mathbf{x}}_{P|_B} = \mathbf{J}_C \cdot \begin{bmatrix} {}^{B}\dot{\mathbf{x}}_{P(\mathrm{pos})|_B} \\ {}^{B}\boldsymbol{\omega}_{p|_B} \end{bmatrix} \tag{6}
$$

with

$$
\dot{\mathbf{l}} = [\dot{l}_1 \quad \dot{l}_2 \quad \cdots \quad \dot{l}_6]^T, \tag{7}
$$

$$
{}^{B}\boldsymbol{\omega}_{P|_B} = \mathbf{J}_A \cdot {}^{B}\dot{\mathbf{x}}_{P(o)|_E} \tag{8}
$$

and[32]

$$
\mathbf{J}_A = \begin{bmatrix}
0 & -S(\psi_P) & C(\theta_P)C(\psi_P) \\
0 & C(\psi_P) & C(\theta_P)S(\psi_P) \\
1 & 0 & -S(\theta_P)
\end{bmatrix}. \tag{9}
$$

Vectors ${}^{B}\dot{\mathbf{x}}_{P(\mathrm{pos})|_B} \equiv {}^{B}\mathbf{v}_{P|_B}$ and ${}^{B}\boldsymbol{\omega}_{P|_B}$ represent the linear and angular velocities of the moving platform relative to {B}, respectively, and ${}^{B}\dot{\mathbf{x}}_{P(o)|_E}$ represents the Euler angles time derivative.

3. Manipulator Dynamic Modelling Using the Generalized Momentum Approach

In this section the system dynamic equations are obtained supposing the manipulator base platform is fixed. First, the generalized momentum of each rigid body is determined and the inertial components of the generalized forces are computed. Then, the gravitational contribution of each rigid body is calculated.

The generalized momentum of a rigid body $\mathbf{q}_c$ may be computed from the following general expression:

$$
\mathbf{q}_c = \mathbf{I}_c \cdot \mathbf{u}_c. \tag{10}
$$

Vector $\mathbf{u}_c$ represents the generalized velocity (linear and angular) of the body and $\mathbf{I}_c$ is its inertia matrix. Vectors $\mathbf{q}_c$ and $\mathbf{u}_c$ and inertia matrix $\mathbf{I}_c$ must be expressed in the same frame of reference.

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Equation (10) may also be written as

$$\mathbf{q}_c = \begin{bmatrix} \mathbf{Q}_c \\ \mathbf{H}_c \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{c(tra)} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{c(rot)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_c \\ \boldsymbol{\omega}_c \end{bmatrix}, \tag{11}$$

where $\mathbf{Q}_c$ is the linear momentum vector due to rigid body translation and $\mathbf{H}_c$ is the angular momentum vector due to body rotation. $\mathbf{I}_{c(tra)}$ is the translational inertia matrix and $\mathbf{I}_{c(rot)}$ the rotational inertia matrix. $\mathbf{v}_c$ and $\boldsymbol{\omega}_c$ are the body linear and angular velocities.

The inertial component of the generalized force (force and moment) acting on the body can be obtained from the time derivative of Eq. (10):

$$\mathbf{f}_{c(ine)} = \dot{\mathbf{q}}_c = \dot{\mathbf{I}}_c \cdot \mathbf{u}_c + \mathbf{I}_c \cdot \dot{\mathbf{u}}_c \tag{12}$$

with force and moment expressed in the same frame and $\mathbf{f}_{c(ine)} = [\,\mathbf{F}^T_{c(ine)} \quad \mathbf{M}^T_{c(ine)}\,]^T$. Equivalently,

$$\mathbf{F}_{c(ine)} = \dot{\mathbf{Q}}_c = \mathbf{I}_{c(tra)} \cdot \dot{\mathbf{v}}_c + \dot{\mathbf{I}}_{c(tra)} \cdot \mathbf{v}_c, \tag{13}$$

$$\mathbf{M}_{c(ine)} = \dot{\mathbf{H}}_c = \mathbf{I}_{c(rot)} \cdot \dot{\boldsymbol{\omega}}_c + \dot{\mathbf{I}}_{c(rot)} \cdot \boldsymbol{\omega}_c. \tag{14}$$

3.1. Moving platform modelling

The linear momentum of the moving platform, written in frame $\{\mathrm{B}\}$, $\mathbf{Q}_{P|_B}$, may be obtained from the following expression:

$$\mathbf{Q}_{P|_B} = m_P \cdot {}^B\mathbf{v}_{P|_B} = \mathbf{I}_{P(tra)} \cdot {}^B\mathbf{v}_{P|_B}, \tag{15}$$

where $\mathbf{I}_{P(tra)}$ is the translational inertia matrix of the moving platform,

$$\mathbf{I}_{P(tra)} = \begin{bmatrix} m_P & 0 & 0 \\ 0 & m_P & 0 \\ 0 & 0 & m_P \end{bmatrix}, \tag{16}$$

m_p being its mass.

The time derivative of Eq. (15) results in

$${}^P\mathbf{F}_{P(ine)|_B} = \dot{\mathbf{Q}}_{P|_B} = m_P \cdot {}^B\dot{\mathbf{v}}_{P|_B}. \tag{17}$$

Here ${}^P\mathbf{F}_{P(ine)|_B}$ is the inertial component of the force acting on $\{\mathrm{P}\}$ due to the moving platform translation, expressed in frame $\{\mathrm{B}\}$.

The angular momentum about the mobile platform centre of mass $\mathbf{H}_{P|_B}$, also written in frame $\{\mathrm{B}\}$, is

$$\mathbf{H}_{P|_B} = \mathbf{I}_{P(rot)|_B} \cdot {}^B\boldsymbol{\omega}_{P|_B}, \tag{18}$$

where $\mathbf{I}_{P(rot)|_B}$ represents the rotational inertia matrix of the moving platform, expressed in the base frame $\{\mathrm{B}\}$.

The inertia matrix of a rigid body is constant when expressed in a frame that is fixed relative to that body. Furthermore, if the frame axes coincide with the body principal directions of inertia, then all inertia products are zero and the inertia matrix is diagonal. Therefore, the rotational inertia matrix of the moving platform, when expressed in frame $\{\mathrm{P}\}$ may be written as

$$\mathbf{I}_{P(rot)|_P} = \begin{bmatrix} I_{P_{xx}} & 0 & 0 \\ 0 & I_{P_{yy}} & 0 \\ 0 & 0 & I_{P_{zz}} \end{bmatrix}. \tag{19}$$

This inertia matrix can be written in frame $\{\mathrm{B}\}$ using the following transformation:[33]

$$\mathbf{I}_{P(rot)|_B} = {}^B\mathbf{R}_P \cdot \mathbf{I}_{P(rot)|_P} \cdot {}^B\mathbf{R}^T_P. \tag{20}$$

The time derivative of Eq. (18) will result in

$${}^P\mathbf{M}_{P(ine)|_B} = \dot{\mathbf{H}}_{P|_B} = \dot{\mathbf{I}}_{P(rot)|_B} \cdot {}^B\boldsymbol{\omega}_{P|_B} + \mathbf{I}_{P(rot)|_B} \cdot {}^B\dot{\boldsymbol{\omega}}_{P|_B}. \tag{21}$$

Here ${}^P\mathbf{M}_{P(ine)|_B}$ is the inertial component of the moment acting on $\{\mathrm{P}\}$ due to the moving platform rotation, expressed in frame $\{\mathrm{B}\}$.

The corresponding actuating forces $\boldsymbol{\tau}_{P(ine)}$ may be computed from the following relation:

$$\boldsymbol{\tau}_{P(ine)} = \mathbf{J}_C^{-T} \cdot {}^P\mathbf{f}_{P(ine)|_B}, \tag{22}$$

where

$${}^P\mathbf{f}_{P(ine)|_B} = \left[\, {}^P\mathbf{F}^T_{P(ine)|_B} \; {}^P\mathbf{M}^T_{P(ine)|_B} \,\right]^T. \tag{23}$$

3.2. Actuators modelling

As the RCID actuators can only move perpendicularly to the base plane, their angular velocity relative to frame $\{\mathrm{B}\}$ is always zero. So, each actuator can be modelled as a point mass located at its centre of mass.

Considering the vector $\boldsymbol{\tau}_{A_i}$ represents the total actuating force due to each actuator motion, it can be represented by the sum of an inertial and a gravitational component:

$$\boldsymbol{\tau}_{A_i} = \boldsymbol{\tau}_{A_i(ine)} + \boldsymbol{\tau}_{A_i(gra)}, \tag{24}$$

where

$$\boldsymbol{\tau}_{A_i(ine)} = [\cdots \quad 0 \quad \cdots \quad m_A \cdot \ddot{l}_i \quad \cdots \quad 0 \quad \cdots]^T, \tag{25}$$

where m_A is the mass and $\ddot{l}_i$ the acceleration of actuator i.

As the actuators acceleration is given by

$$\ddot{\mathbf{l}} = \dot{\mathbf{J}}_C \cdot {}^B\dot{\mathbf{x}}_{P|_B} + \mathbf{J}_C \cdot {}^B\ddot{\mathbf{x}}_{P|_B}, \tag{26}$$

the inertial component of the actuating forces $\boldsymbol{\tau}_{A_i(ine)}$ due to actuators translation will be

$$\boldsymbol{\tau}_{A_i(ine)} = [\cdots \; 0 \; \cdots \; m_A\left(\dot{\mathbf{J}}_{F_i} \cdot {}^B\dot{\mathbf{x}}_{P|_B} + \mathbf{J}_{F_i} \cdot {}^B\ddot{\mathbf{x}}_{P|_B}\right) \; \cdots \; 0 \; \cdots]^T, \tag{27}$$

where $\mathbf{J}_{F_i}$ is line i of Jacobian matrix, $\mathbf{J}_C$.

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It should be noted that this vector may be added to Eq. (22), because both represent actuators forces.

3.3. Fixed-length links modelling

Considering the centre of mass of each fixed-length link, cm_L, is located at a constant distance, b_{cm}, from the fixed-length link to moving platform connecting point (Fig. 4) then its position relative to frame {B} is

$$ {}^{B}\mathbf{p}_{L_i|_B} = {}^{B}\mathbf{x}_{P(pos)|_B} + {}^{P}\mathbf{p}_{i|_B} - \frac{b_{cm}}{L}\cdot\mathbf{a}_i. \tag{28} $$

Equation (28) may be successively rewritten as

$$ {}^{B}\mathbf{p}_{L_i|_B} = {}^{B}\mathbf{x}_{P(pos)|_B} + {}^{P}\mathbf{p}_{i|_B} - \frac{b_{cm}}{L} \cdot \left({}^{B}\mathbf{x}_{P(pos)|_B} - \mathbf{b}_i + {}^{P}\mathbf{p}_{i|_B} - \mathbf{d}_i\right), \tag{29} $$

$$ {}^{B}\mathbf{p}_{L_i|_B} = \left(1 - \frac{b_{cm}}{L}\right)\cdot {}^{B}\mathbf{x}_{P(pos)|_B} + \left(1 - \frac{b_{cm}}{L}\right) \cdot {}^{P}\mathbf{p}_{i|_B} + \frac{b_{cm}}{L}\cdot\mathbf{b}_i + \frac{b_{cm}}{L}\cdot\mathbf{d}_i, \tag{30} $$

where ${}^{B}\mathbf{p}_{L_i|_B}$ being a vector expressed in frame {B}.

The linear velocity of the fixed-length link centre of mass ${}^{B}\dot{\mathbf{p}}_{L_i|_B}$ relative to {B} and expressed in the same frame may be computed from the time derivative of Eq. (30):

$$ {}^{B}\dot{\mathbf{p}}_{L_i|_B} = \left(1 - \frac{b_{cm}}{L}\right)\cdot\left({}^{B}\dot{\mathbf{x}}_{P(pos)|_B} + {}^{B}\boldsymbol{\omega}_{P|_B}\times {}^{P}\mathbf{p}_{i|_B}\right) + \frac{b_{cm}}{L}\cdot\dot{\mathbf{d}}_i, $$

$$ = \left(1 - \frac{b_{cm}}{L}\right)\cdot\left({}^{B}\dot{\mathbf{x}}_{P(pos)|_B} + {}^{B}\boldsymbol{\omega}_{P|_B}\times {}^{P}\mathbf{p}_{i|_B}\right) + \frac{b_{cm}}{L}\cdot\dot{l}_i\cdot\mathbf{z}_B. \tag{31} $$

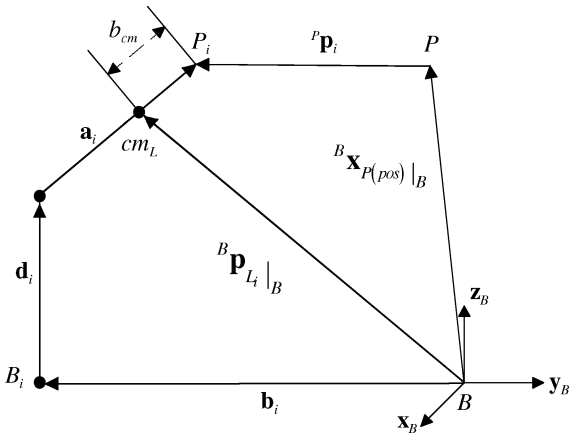


Fig. 4. Position of the centre of mass of a fixed-length link *i*.

Equation (31) can be rewritten as

$$ {}^{B}\dot{\mathbf{p}}_{L_i|_B} = \mathbf{J}_{B_i}\cdot\begin{bmatrix} {}^{B}\mathbf{v}_{P|_B} \\ {}^{B}\boldsymbol{\omega}_{P|_B} \end{bmatrix}, \tag{32} $$

where the Jacobian $\mathbf{J}_{B_i}$ is given by

$$ \mathbf{J}_{B_i} = \left(1 - \frac{b_{cm}}{L}\right)\cdot \begin{bmatrix} 1 & 0 & 0 & 0 & {}^{P}p_{i|_{Bz}} & -{}^{P}p_{i|_{By}} \\ 0 & 1 & 0 & -{}^{P}p_{i|_{Bz}} & 0 & {}^{P}p_{i|_{Bx}} \\ \dfrac{b_{cm}}{L-b_{cm}}J_{Ci1} & \dfrac{b_{cm}}{L-b_{cm}}J_{Ci2} & \dfrac{b_{cm}}{L-b_{cm}}J_{Ci3}+1 & {}^{P}p_{i|_{By}}+\dfrac{b_{cm}}{L-b_{cm}}J_{Ci4} & -{}^{P}p_{i|_{Bx}}+\dfrac{b_{cm}}{L-b_{cm}}J_{Ci5} & \dfrac{b_{cm}}{L-b_{cm}}J_{Ci6} \end{bmatrix}, \tag{33} $$

where J_{Cij} are the elements of line *i* column *j* of matrix $\mathbf{J}_C$.

The linear momentum of each fixed-length link $\mathbf{Q}_{L_i|_B}$ can be represented in frame {B} as

$$ \mathbf{Q}_{L_i|_B} = m_L\cdot {}^{B}\dot{\mathbf{p}}_{L_i|_B}, \tag{34} $$

where m_L is the fixed-length link mass.

Introducing Jacobian $\mathbf{J}_{B_i}$ in the previous equation, results in

$$ \mathbf{Q}_{L_i|_B} = m_L\cdot\mathbf{J}_{B_i}\cdot {}^{B}\dot{\mathbf{x}}_{P|_B}. \tag{35} $$

The inertial component of the force applied to the fixed-length link due to its translation and expressed in {B} can be obtained from the time derivative of Eq. (35):

$$ {}^{L_i}\mathbf{F}_{L_i(ine)(tra)|_B} = \dot{\mathbf{Q}}_{L_i|_B} = m_L\cdot\dot{\mathbf{J}}_{B_i}\cdot {}^{B}\dot{\mathbf{x}}_{P|_B} + m_L\cdot\mathbf{J}_{B_i}\cdot {}^{B}\ddot{\mathbf{x}}_{P|_B}. \tag{36} $$

When Eq. (36) is multiplied by $\mathbf{J}_{B_i}^T$, the inertial component of the generalized force (force and moment) applied to {P}, due to each fixed-length link translation, is obtained in frame {B}:

$$ {}^{P}\mathbf{f}_{L_i(ine)(tra)|_B} = m_L\cdot\mathbf{J}_{B_i}^T\cdot\dot{\mathbf{J}}_{B_i}\cdot {}^{B}\dot{\mathbf{x}}_{P|_B} + m_L\cdot\mathbf{J}_{B_i}^T\cdot\mathbf{J}_{B_i}\cdot {}^{B}\ddot{\mathbf{x}}_{P|_B}. \tag{37} $$

The corresponding actuating forces, $\boldsymbol{\tau}_{L_i(ine)(tra)}$, may be computed from the following relation:

$$ \boldsymbol{\tau}_{L_i(ine)(tra)} = \mathbf{J}_C^{-T}\cdot {}^{P}\mathbf{f}_{L_i(ine)(tra)|_B}. \tag{38} $$

Now, regarding rotation, the angular momentum of each fixed-length link can be represented in frame {B} as

$$ \mathbf{H}_{L_i|_B} = \mathbf{I}_{L_i(rot)|_B}\cdot {}^{B}\boldsymbol{\omega}_{L_i|_B}. \tag{39} $$

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It is convenient to express the inertia matrix of the rotating fixed-length link in a frame fixed to the fixed-length link itself, $\{L_i\} \equiv \{\mathbf{x}_{L_i},\ \mathbf{y}_{L_i},\ \mathbf{z}_{L_i}\}$. So,

$$\mathbf{I}_{L_i(rot)|_B} = {}^B\mathbf{R}_{L_i} \cdot \mathbf{I}_{L_i(rot)|_{L_i}} \cdot {}^B\mathbf{R}_{L_i}^T, \tag{40}$$

where ${}^B\mathbf{R}_{L_i}$ is the orientation matrix of each fixed-length link frame $\{\mathrm{L}_i\}$, relative to the base frame, $\{\mathrm{B}\}$.

Fixed-length links frames were chosen in the following way: axis $\mathbf{x}_{L_i}$ coincides with the fixed-length link axis and points from the fixed-length link to moving platform, meaning that it is coincident with vector $\mathbf{a}_i$; axis $\mathbf{y}_{L_i}$ is perpendicular to $\mathbf{x}_{L_i}$ and always parallel to the base plane, this condition being possible given the existence of a universal joint in the fixed-length link to actuator connecting point that negates any rotation along its own axis; axis $\mathbf{z}_{L_i}$ completes the frame according to the right-hand rule and its projection along axis $\mathbf{z}_B$ is always positive. With this choice matrix ${}^B\mathbf{R}_{L_i}$ becomes

$${}^B\mathbf{R}_{L_i} = [\mathbf{x}_{L_i} \quad \mathbf{y}_{L_i} \quad \mathbf{z}_{L_i}], \tag{41}$$

where

$$\mathbf{x}_{L_i} = \left[\frac{a_{ix}}{L} \quad \frac{a_{iy}}{L} \quad \frac{a_{iz}}{L}\right]^T, \tag{42}$$

$$\mathbf{y}_{L_i} = \left[-\frac{a_{iy}}{\sqrt{a_{ix}^2 + a_{iy}^2}} \quad \frac{a_{ix}}{\sqrt{a_{ix}^2 + a_{iy}^2}} \quad 0\right]^T, \tag{43}$$

$$\mathbf{z}_{L_i} = \mathbf{x}_{L_i} \times \mathbf{y}_{L_i}. \tag{44}$$

So, the inertia matrices of the fixed-length links can be written as

$$\mathbf{I}_{L_i(rot)|L_i} = \begin{bmatrix} I_{L_{xx}} & 0 & 0 \\ 0 & I_{L_{yy}} & 0 \\ 0 & 0 & I_{L_{zz}} \end{bmatrix}, \tag{45}$$

where $I_{L_{xx}}$, $I_{L_{yy}}$ and $I_{L_{zz}}$ are the fixed-length link moments of inertia expressed in its own frame.

The angular velocity of each fixed-length link can be obtained from the linear velocities of two points belonging to it. If these two points are taken as the fixed-length link to actuator and the fixed-length link to moving platform connecting points, the following expression results:

$${}^B\boldsymbol{\omega}_{L_i|_B} \times \mathbf{a}_i = {}^B\mathbf{v}_{P|_B} + {}^B\boldsymbol{\omega}_{P|_B} \times {}^P\mathbf{p}_{i|_B} - \dot{l}_i \cdot \mathbf{z}_B. \tag{46}$$

As the fixed-length link cannot rotate along its own axis the angular velocity along $\mathbf{x}_{L_i} = \hat{\mathbf{a}}_i$ is always zero and so vectors $\mathbf{a}_i$ and ${}^B\boldsymbol{\omega}_{L_i|_B}$ are always perpendicular. This property enables Eq. (46) to be rewritten as

$${}^B\boldsymbol{\omega}_{L_i|_B} = \frac{1}{L^2} \cdot \left[\mathbf{a}_i \times \left({}^B\mathbf{v}_{P|_B} + {}^B\boldsymbol{\omega}_{P|_B} \times {}^P\mathbf{p}_{i|_B} - \dot{l}_i \cdot \mathbf{z}_B\right)\right] \tag{47}$$

or

$${}^B\boldsymbol{\omega}_{L_i|_B} = \mathbf{J}_{D_i} \cdot \begin{bmatrix} {}^B\mathbf{v}_{P|_B} \\ {}^B\boldsymbol{\omega}_{P|_B} \end{bmatrix}, \tag{48}$$

where Jacobian $\mathbf{J}_{D_i}$ is given by

$$\mathbf{J}_{D_i} = \frac{1}{L^2} \cdot \begin{bmatrix} -a_{iy}J_{Ci1} & -a_{iy}J_{Ci2} - a_{iz} & a_{iy}(1 - J_{Ci3}) & a_{iy}\left({}^P p_{i\,|_{By}} - J_{Ci4}\right) + a_{iz}\,{}^P p_{i\,|_{Bz}} & -a_{iy}\left({}^P p_{i\,|_{Bx}} + J_{Ci5}\right) & -a_{iy}J_{Ci6} - a_{iz}\,{}^P p_{i\,|_{Bx}} \\ a_{iz} + a_{ix}J_{Ci1} & a_{ix}J_{Ci2} & -a_{iz}(1 - J_{Ci3}) & -a_{ix}\left({}^P p_{i\,|_{By}} - J_{Ci4}\right) & a_{iz}\,{}^P p_{i\,|_{Bz}} + a_{ix}\left({}^P p_{i\,|_{Bx}} + J_{Ci5}\right) & -a_{iz}\,{}^P p_{i\,|_{By}} + a_{ix}J_{Ci6} \\ -a_{iy} & a_{ix} & 0 & -a_{ix}\,{}^P p_{i\,|_{Bz}} & -a_{iy}\,{}^P p_{i\,|_{Bz}} & a_{ix}\,{}^P p_{i\,|_{Bx}} + a_{iy}\,{}^P p_{i\,|_{By}} \end{bmatrix}. \tag{49}$$

Introducing Jacobian $\mathbf{J}_{D_i}$ in Eq. (39) results in

$$\mathbf{H}_{L_i\,|_B} = \mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i} \cdot {}^B\dot{\mathbf{x}}_{P\,|_B}. \tag{50}$$

The inertial component of the moment applied to the fixed-length link due to its rotation and expressed in $\{\mathrm{B}\}$ can be obtained from the time derivative of Eq. (50):

$$\begin{aligned} {}^{L_i}\mathbf{M}_{L_i(ine)(rot)\,|_B} &= \dot{\mathbf{H}}_{L_i\,|_B} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i}\right) \cdot {}^B\dot{\mathbf{x}}_{P\,|_B} \\ &\quad + \mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i} \cdot {}^B\ddot{\mathbf{x}}_{P\,|_B}. \end{aligned} \tag{51}$$

When Eq. (51) is pre-multiplied by $\mathbf{J}_{D_i}^T$, the inertial component of the generalized force applied to $\{\mathrm{P}\}$, due to each fixed-length link rotation is obtained in frame $\{\mathrm{B}\}$:

$$\begin{aligned} {}^P\mathbf{f}_{L_i(ine)(rot)\,|_B} &= \mathbf{J}_{D_i}^T \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i}\right) \cdot {}^B\dot{\mathbf{x}}_{P\,|_B} \\ &\quad + \mathbf{J}_{D_i}^T \cdot \mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i} \cdot {}^B\ddot{\mathbf{x}}_{P\,|_B}. \end{aligned} \tag{52}$$

The corresponding actuating forces $\boldsymbol{\tau}_{L_i(ine)(rot)}$ may be computed from the following equation:

$$\boldsymbol{\tau}_{L_i(ine)(rot)} = \mathbf{J}_C^{-T} \cdot {}^P\mathbf{f}_{L_i(ine)(rot)|_B}, \tag{53}$$

and can be added to the actuating forces represented by Eqs. (22), (27) and (38).

3.4. Gravitational component of the manipulator dynamic model

Given a general frame $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$, with $\mathbf{z} \equiv -\hat{\mathbf{g}}$, the potential energy of a rigid body is given by

$$P_c = m_c \cdot g \cdot z_c, \tag{54}$$

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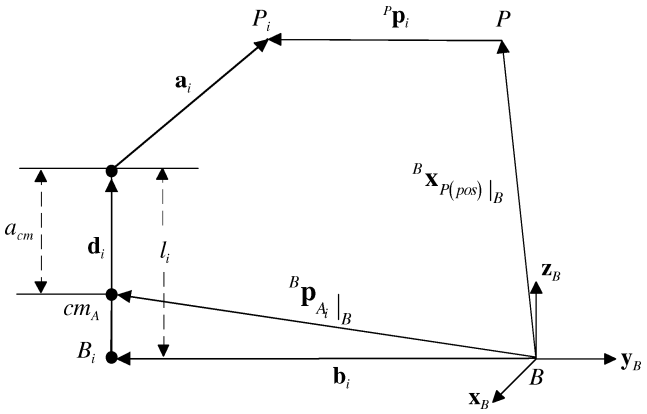


Fig. 5. Position of the centre of mass of an actuator.

where m_c is the body mass, g is the modulus of the gravitational acceleration and z_c the distance, along $\mathbf{z}$, from the frame origin to the body centre of mass.

The gravitational components of the actuating forces can be easily obtained from the potential energy of the different rigid bodies that compose the system:

$$\boldsymbol{\tau}_{P(gra)} = \mathbf{J}_E^{-T} \cdot \frac{\partial P_P\left({}^B\mathbf{x}_{P\,|_{B|E}}\right)}{\partial\, {}^B\mathbf{x}_{P\,|_{B|E}}}, \tag{55}$$

$$\boldsymbol{\tau}_{A_i(gra)} = \mathbf{J}_E^{-T} \cdot \frac{\partial P_{A_i}\left({}^B\mathbf{x}_{P\,|_{B|E}}\right)}{\partial\, {}^B\mathbf{x}_{P\,|_{B|E}}}, \tag{56}$$

$$\boldsymbol{\tau}_{L_i(gra)} = \mathbf{J}_E^{-T} \cdot \frac{\partial P_{L_i}\left({}^B\mathbf{x}_{P\,|_{B|E}}\right)}{\partial\, {}^B\mathbf{x}_{P\,|_{B|E}}}. \tag{57}$$

3.4.1. Moving platform potential energy. Using the base frame, {B}, as a datum and assuming $\mathbf{z}_B \equiv -\hat{\mathbf{g}}$, the moving platform potential energy can be written as

$$P_P = m_P \cdot g \cdot z_P. \tag{58}$$

3.4.2. Actuators potential energy. If all actuators are assumed to be equal and the centre of mass of each actuator cm_A is placed at a fixed distance a_{cm} from its actuator to fixed-length link connecting point (Fig. 5), the position of the centre of mass relative to frame {B} is

$${}^B\mathbf{p}_{A_i\,|_B} = \mathbf{b}_i + (l_i - a_{cm}) \cdot \mathbf{z}_B, \tag{59}$$

where ${}^B\mathbf{p}_{A_i\,|_B}$ is a vector expressed in {B}.

Knowing the position of the centre of mass of each actuator relative to frame {B} (Fig. 5), its potential energy, P_{A_i}, can be written as

$$P_{A_i} = m_A \cdot g \cdot (l_i - a_{cm}). \tag{60}$$

3.4.3. Fixed-length links potential energy. In order to find the potential energy of each fixed-length link the position of its centre of mass relative to frame {B} (Fig. 4) must be known. Therefore, using Eq. (30) for each fixed-length link

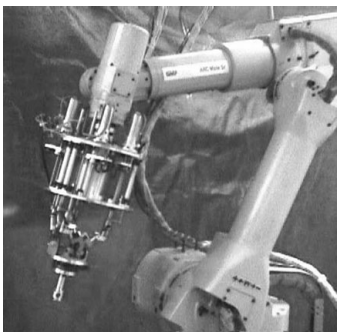


Fig. 6. Serial combination of the RCID and an industrial manipulator.

leads to

$$P_{L_i} = m_L \cdot g \cdot \left[\left(1 - \frac{b_{cm}}{L}\right) \cdot z_P + \left(1 - \frac{b_{cm}}{L}\right) \cdot {}^P p_{i\,|_{Bz}} + \frac{b_{cm}}{L} \cdot l_i\right]. \tag{61}$$

4. Dynamic Model of the Moving Base Manipulator

The RCID can be used working attached in series to a position controlled industrial robot: the latter one performs the large amplitude movements while the RCID is only used for the small and high bandwidth movements needed for force/impedance control (Fig. 6). A control strategy that improves the performance of current commercial industrial robots, increasing its flexibility when executing tasks that require some kind of force control was presented in Lopes and Almeida.[23]

The industrial manipulator movements impose a motion on the RCID base platform, which originates not only new inertial force components, but also the modification of the gravitational forces, due to changes in the base platform orientation.

In the next subsections the system dynamic equations are obtained supposing the manipulator base platform is moving. For each rigid body, the generalized momentum is determined, the inertial components of the generalized forces are computed, and the gravitational contributions are recalculated taking into account the base platform orientation.

4.1. Payload platform modelling

Considering the RCID base motion, that is, the motion of frame {B} relative to a fixed world frame, {W} $\equiv$ {$\mathbf{x}_W$, $\mathbf{y}_W$, $\mathbf{z}_W$}, the position of the payload platform {P}, relative to {W} and expressed in {W}, may be given by

$${}^W\mathbf{p}_{P\,|_W} = {}^W\mathbf{p}_{B\,|_W} + {}^B\mathbf{p}_{P\,|_W}, \tag{62}$$

where ${}^W\mathbf{p}_{B\,|_W}$ is the position of frame {B}, and ${}^B\mathbf{p}_{P\,|_W}$ represents the position of frame {P} relative to {B} and expressed in {W}.

The linear velocity of the payload platform {P}, relative to {W} and expressed in {W}, may be obtained taking the

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time derivative of the previous equation, that is,

$$^{W}\mathbf{v}_{P\,|_W} = {}^{W}\mathbf{v}_{B\,|_W} + {}^{B}\mathbf{v}_{P\,|_W} + {}^{W}\boldsymbol{\omega}_{B\,|_W} \times {}^{B}\mathbf{p}_{P\,|_W}, \quad (63)$$

where $^{W}\mathbf{v}_{B\,|_W}$ is the linear velocity of frame {B}, $^{B}\mathbf{v}_{P\,|_W}$ is the linear velocity of frame {P} as seen by an observer fixed in {B}, $^{W}\boldsymbol{\omega}_{B\,|_W}$ represents the angular velocity of frame {B} relative to {W} and $^{B}\mathbf{p}_{P\,|_W} \equiv {}^{B}\mathbf{x}_{P(pos)\,|_W}$ represents the position of {P} relative to {B} and expressed in {W}.

In the following analysis, knowledge of the generalized position of frame {B} relative to {W}, $^{W}\mathbf{x}_{B\,|_{W|E}} = [x_B\ y_B\ z_B\ \psi_B\ \theta_B\ \varphi_B]^T = [{}^{W}\mathbf{x}^T_{B(pos)\,|_W}\ \ {}^{W}\mathbf{x}^T_{B(o)\,|_E}]^T$, shall be assumed: $^{W}\mathbf{x}_{B(pos)|_W}$ represents the position vector expressed in frame {W}, and $^{W}\mathbf{x}_{B(o)|_E}$ represents orientation expressed in an Euler angles system. Knowledge of its first and second time derivatives shall also be supposed, that is, $^{W}\dot{\mathbf{x}}_{B|_{W|E}}$ and $^{W}\ddot{\mathbf{x}}_{B|_{W|E}}$, respectively. Therefore, the orientation matrix $^{W}\mathbf{R}_B$ of frame {B} relative to {W} can easily be computed, and the Jacobian $\mathbf{J}_G$ relating the angular velocity of the base frame relative to {W}, $^{W}\boldsymbol{\omega}_{B\,|_W}$, to the first time derivative of the Euler angles $^{W}\dot{\mathbf{x}}_{B(o)\,|_E}$ is given by

$$^{W}\boldsymbol{\omega}_{B\,|_W} = \mathbf{J}_G \cdot {}^{W}\dot{\mathbf{x}}_{B(o)\,|_E}. \quad (64)$$

In frame {B} the following equation can be written as

$$^{W}\mathbf{v}_{P\,|_B} = {}^{W}\mathbf{v}_{B\,|_B} + {}^{B}\mathbf{v}_{P\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{P\,|_B}. \quad (65)$$

The total linear momentum of {P} expressed in frame {B} will be

$$\mathbf{Q}_{P(tot)\,|_B} = m_P \cdot {}^{W}\mathbf{v}_{P\,|_B}. \quad (66)$$

Taking the time derivative of the previous equation results in

$$\dot{\mathbf{Q}}_{P(tot)\,|_B} = m_P \cdot {}^{W}\dot{\mathbf{v}}_{P\,|_B}. \quad (67)$$

Knowing that,

$$^{W}\boldsymbol{\omega}_{P\,|_B} = {}^{B}\boldsymbol{\omega}_{P\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B}, \quad (68)$$

$$^{W}\dot{\boldsymbol{\omega}}_{P\,|_B} = {}^{B}\dot{\boldsymbol{\omega}}_{P\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\boldsymbol{\omega}_{P\,|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B}, \quad (69)$$

result in

$$^{W}\dot{\mathbf{v}}_{P\,|_B} = {}^{W}\dot{\mathbf{v}}_{B\,|_B} + {}^{B}\dot{\mathbf{v}}_{P\,|_B} + 2{}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{v}_{P\,|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B} \times {}^{B}\mathbf{p}_{P\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times \left({}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{P\,|_B}\right) \quad (70)$$

The inertial part of the total force applied in {P}, due to the payload platform translation, expressed in frame {B} will be

$$^{P}\mathbf{F}_{P(ine)(tot)\,|_B} = \dot{\mathbf{Q}}_{P(tot)\,|_B}. \quad (71)$$

That is,

$$^{P}\mathbf{F}_{P(ine)(tot)\,|_B} = {}^{P}\mathbf{F}_{P(ine)(fix)\,|_B} + {}^{P}\mathbf{F}_{P(ine)(man)\,|_B}, \quad (72)$$

$$^{P}\mathbf{F}_{P(ine)(fix)\,|_B} = m_P \cdot {}^{B}\dot{\mathbf{v}}_{P\,|_B}, \quad (73)$$

$$^{P}\mathbf{F}_{P(ine)(man)|_B} = m_P \cdot \left({}^{W}\dot{\mathbf{v}}_{B\,|_B} + 2{}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{v}_{P\,|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B} \times {}^{B}\mathbf{p}_{P\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times \left({}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{P\,|_B}\right)\right), \quad (74)$$

where $^{P}\mathbf{F}_{P(ine)(fix)\,|_B}$ represents the inertial part of the force, considering the base platform is not moving, and $^{P}\mathbf{F}_{P(ine)(man)\,|_B}$ represents the inertial part of the force which results from the base motion.

On the other hand, the angular momentum of {P} expressed in frame {B} will be

$$\mathbf{H}_{P(tot)\,|_B} = \mathbf{I}_{P(rot)\,|_B} \cdot {}^{W}\boldsymbol{\omega}_{P\,|_B}. \quad (75)$$

Taking the time derivative of this equation results in

$$\dot{\mathbf{H}}_{P(tot)\,|_B} = \dot{\mathbf{I}}_{P(rot)\,|_B} \cdot {}^{W}\boldsymbol{\omega}_{P\,|_B} + \mathbf{I}_{P(rot)\,|_B} \cdot {}^{W}\dot{\boldsymbol{\omega}}_{P\,|_B}. \quad (76)$$

The inertial part of the total moment applied in {P}, due to the payload platform rotation, expressed in frame {B} will be

$$^{P}\mathbf{M}_{P(ine)(tot)\,|_B} = \dot{\mathbf{H}}_{P(tot)\,|_B}, \quad (77)$$

that is,

$$^{P}\mathbf{M}_{P(ine)(tot)\,|_B} = {}^{P}\mathbf{M}_{P(ine)(fix)\,|_B} + {}^{P}\mathbf{M}_{P(ine)(man)\,|_B}, \quad (78)$$

where

$$^{P}\mathbf{M}_{P(ine)(fix)\,|_B} = \mathbf{I}_{P(rot)\,|_B} \cdot {}^{B}\dot{\boldsymbol{\omega}}_{P\,|_B} + \dot{\mathbf{I}}_{P(rot)\,|_B} \cdot {}^{B}\boldsymbol{\omega}_{P\,|_B}, \quad (79)$$

$$^{P}\mathbf{M}_{P(ine)(man)\,|_B} = \dot{\mathbf{I}}_{P(rot)\,|_B} \cdot {}^{W}\boldsymbol{\omega}_{B\,|_B} + \mathbf{I}_{P(rot)\,|_B} \cdot \left({}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\boldsymbol{\omega}_{P\,|_B}\right). \quad (80)$$

$^{P}\mathbf{M}_{P(ine)(fix)\,|_B}$ represents the inertial part of the moment, considering the base platform is not moving, and $^{P}\mathbf{M}_{P(ine)(man)\,|_B}$ represents the inertial part of the moment which results from the base motion.

The total inertial component of the generalized force applied to {P} and expressed in {B} will be

$$^{P}\mathbf{f}_{P(ine)(tot)\,|_B} = \left[{}^{P}\mathbf{F}^T_{P(ine)(tot)\,|_B} \quad {}^{P}\mathbf{M}^T_{P(ine)(tot)\,|_B}\right]^T. \quad (81)$$

The inertial component of the actuating forces due to the base motion $\boldsymbol{\tau}_{P(ine)(man)}$ will be

$$\boldsymbol{\tau}_{P(ine)(man)} = \mathbf{J}_C^{-T} \cdot {}^{P}\mathbf{f}_{P(ine)(man)\,|_B}. \quad (82)$$

It should be noted that using this formulation, the forces $\boldsymbol{\tau}_{P(ine)(man)}$ can be added to the corresponding forces resulting from the model that considers a fixed base (Eq. (22)).

On the other hand, regarding the gravitational part of the generalized force, if the base platform orientation is changing, then the force applied to {P} and expressed in {B} results in

$$^{P}\mathbf{f}_{P(gra)\,|_B} = {}^{W}\Re^T_B \cdot {}^{P}\mathbf{f}_{P(gra)\,|_W}, \quad (83)$$

where

$$^{W}\Re_B = \begin{bmatrix} {}^{W}\mathbf{R}_B & \mathbf{0} \\ \mathbf{0} & {}^{W}\mathbf{R}_B \end{bmatrix} \quad (84)$$

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and ${}^{P}\mathbf{f}_{P(gra)\,|_W}$ is the gravitational force applied to {P} and expressed in {W}. This force can be computed using the model that considers both a non-moving base platform and frame {B} parallel to {W}.

The gravitational component of the actuating forces due to the moving platform $\boldsymbol{\tau}_{P(gra)}$ is given by

$$\boldsymbol{\tau}_{P(gra)} = \mathbf{J}_C^{-T} \cdot {}^{P}\mathbf{f}_{P(gra)\,|_B}. \tag{85}$$

4.2. Actuators modelling

The linear velocity of each actuator relative to {W} and expressed in {W} may be expressed by

$${}^{W}\mathbf{v}_{A_i\,|_W} = {}^{W}\mathbf{v}_{B\,|_W} + {}^{B}\mathbf{v}_{A_i\,|_W} + {}^{W}\boldsymbol{\omega}_{B\,|_W} \times {}^{B}\mathbf{p}_{A_i\,|_W}, \tag{86}$$

where ${}^{B}\mathbf{v}_{A_i\,|_W}$ is the actuator linear velocity as seen by an observer fixed in {B} and ${}^{B}\mathbf{p}_{A_i\,|_W}$ represents the position of the centre of mass of each actuator relative to {B} and expressed in {W}.

Expressing the previous equation in the base frame gives

$${}^{W}\mathbf{v}_{A_i\,|_B} = {}^{W}\mathbf{v}_{B\,|_B} + {}^{B}\mathbf{v}_{A_i\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{A_i\,|_B}, \tag{87}$$

and, taking the time derivative of this equation, results in

$$\begin{aligned} {}^{W}\dot{\mathbf{v}}_{A_i|_B} = {}^{W}\dot{\mathbf{v}}_{B|_B} + {}^{B}\dot{\mathbf{v}}_{A_i|_B} + 2{}^{W}\boldsymbol{\omega}_{B|_B} \times {}^{B}\mathbf{v}_{A_i|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B|_B} \\ \times {}^{B}\mathbf{p}_{A_i|_B} + {}^{W}\boldsymbol{\omega}_{B|_B} \times \left({}^{W}\boldsymbol{\omega}_{B|_B} \times {}^{B}\mathbf{p}_{A_i|_B}\right), \end{aligned} \tag{88}$$

$$\begin{aligned} {}^{W}\dot{\mathbf{v}}_{A_i|_B} = {}^{W}\dot{\mathbf{v}}_{B|_B} + \ddot{l}_i\,\mathbf{z}_B + 2{}^{W}\boldsymbol{\omega}_{B|_B} \times \dot{l}_i\,\mathbf{z}_B + {}^{W}\dot{\boldsymbol{\omega}}_{B|_B} \\ \times {}^{B}\mathbf{p}_{A_i|_B} + {}^{W}\boldsymbol{\omega}_{B|_B} \times \left({}^{W}\boldsymbol{\omega}_{B|_B} \times {}^{B}\mathbf{p}_{A_i|_B}\right), \end{aligned} \tag{89}$$

The inertial part of the total force applied in each actuator, due to the actuator motion, expressed in frame {B} will be

$${}^{A_i}\mathbf{F}_{A_i(ine)(tot)|_B} = m_A \cdot {}^{W}\dot{\mathbf{v}}_{A_i|_B}, \tag{90}$$

that is,

$${}^{A_i}\mathbf{F}_{A_i(ine)(tot)|_B} = {}^{A_i}\mathbf{F}_{A_i(ine)(fix)|_B} + {}^{A_i}\mathbf{F}_{A_i(ine)(man)|_B}, \tag{91}$$

$${}^{A_i}\mathbf{F}_{A_i(ine)(fix)|_B} = m_A \cdot \ddot{l}_i \cdot \mathbf{z}_B, \tag{92}$$

$$\begin{aligned} {}^{A_i}\mathbf{F}_{A_i(ine)(man)|_B} = m_A \cdot \big({}^{W}\dot{\mathbf{v}}_{B|_B} + 2{}^{W}\boldsymbol{\omega}_{B|_B} \times \dot{l}_i\,\mathbf{z}_B + {}^{W}\dot{\boldsymbol{\omega}}_{B|_B} \\ \times {}^{B}\mathbf{p}_{A_i|_B} + {}^{W}\boldsymbol{\omega}_{B|_B} \times \left({}^{W}\boldsymbol{\omega}_{B|_B} \times {}^{B}\mathbf{p}_{A_i|_B}\right)\big). \end{aligned} \tag{93}$$

${}^{A_i}\mathbf{F}_{A_i(ine)(fix)|_B}$ represents the inertial part of the force, considering the base platform is not moving, and ${}^{A_i}\mathbf{F}_{A_i(ine)(man)|_B}$ represents the inertial part of the force which results from the base motion.

As each actuator only moves in the $\mathbf{z}_B$ direction, all the force components in the $\mathbf{x}_B\mathbf{y}_B$ plane are absorbed by the mechanical structure. Thus, it will be sufficient to consider the component along the $\mathbf{z}_B$ direction of vector ${}^{A_i}\mathbf{F}_{A_i(ine)(man)|_B}$.

The inertial part of the actuating forces given the base motion will be

$$\boldsymbol{\tau}_{A(ine)(man)} = \left[{}^{A_1}F_{A_1(ine)(man)\,|_{Bz}}\ {}^{A_2}F_{A_2(ine)(man)\,|_{Bz}}\ \cdots\ {}^{A_6}\ F_{A_6(ine)(man)\,|_{Bz}}\right]^T. \tag{94}$$

These forces can be added to the corresponding forces resulting from the model, which considers a fixed base.

Regarding now the gravitational part of the force applied to each actuator, attaching to each actuator a frame with $\mathbf{z}_{A_i}$ axis parallel to $\mathbf{z}_B$, the orientation matrix of the actuator frame relative to {W} might be represented by

$${}^{W}\mathbf{R}_{A_i} = \left[\vdots\ \ \vdots\ \ {}^{W}\mathbf{z}_B\right], \tag{95}$$

where ${}^{W}\mathbf{z}_B$ being the third column of matrix ${}^{W}\mathbf{R}_B$.

Considering that each actuator weight, expressed in frame {W} is given by ${}^{A_i}\mathbf{F}_{A_i(gra)\,|_W} = [0\ 0\ m_A \cdot g]^T$, then the force applied to each actuator expressed in {B} is

$${}^{A_i}\mathbf{F}_{A_i(gra)\,|_B} = {}^{W}\mathbf{z}_B^T \cdot [0 \quad 0 \quad m_A \cdot g]^T \cdot \mathbf{z}_B. \tag{96}$$

The actuating forces due to the actuators weight will be

$$\boldsymbol{\tau}_{A(gra)} = \left[{}^{A_1}F_{A_1(gra)\,|_{Bz}} \quad {}^{A_2}F_{A_2(gra)\,|_{Bz}} \quad \cdots \quad {}^{A_6}F_{A_6(gra)\,|_{Bz}}\right]^T. \tag{97}$$

4.3. Fixed-length links modelling

The position of the fixed-length link i, relative to {W} and expressed in {W}, may be expressed by

$${}^{W}\mathbf{p}_{L_i\,|_W} = {}^{W}\mathbf{p}_{B\,|_W} + {}^{B}\mathbf{p}_{L_i\,|_W}. \tag{98}$$

The linear velocity of the fixed-length link, relative to {W} and expressed in {W}, may be obtained taking the time derivative of the previous equation, that is,

$${}^{W}\mathbf{v}_{L_i\,|_W} = {}^{W}\mathbf{v}_{B\,|_W} + {}^{B}\mathbf{v}_{L_i\,|_W} + {}^{W}\boldsymbol{\omega}_{B\,|_W} \times {}^{B}\mathbf{p}_{L_i\,|_W}, \tag{99}$$

where, ${}^{B}\mathbf{v}_{L_i\,|_W}$ is the linear velocity of frame $\{L_i\}$ as seen by an observer fixed in {B}, and ${}^{B}\mathbf{p}_{L_i\,|_W}$ represents the position of $\{L_i\}$ relative to {B} and expressed in {W}.

In frame {B} the following equation can be written as

$${}^{W}\mathbf{v}_{L_i\,|_B} = {}^{W}\mathbf{v}_{B\,|_B} + {}^{B}\mathbf{v}_{L_i\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{L_i\,|_B}. \tag{100}$$

The total linear momentum of the fixed-length link, expressed in frame {B} will be

$$\mathbf{Q}_{L_i(tot)\,|_B} = m_L \cdot {}^{W}\mathbf{v}_{L_i\,|_B}. \tag{101}$$

Taking the time derivative of the previous equation results in

$$\dot{\mathbf{Q}}_{L_i(tot)\,|_B} = m_L \cdot {}^{W}\dot{\mathbf{v}}_{L_i\,|_B}, \tag{102}$$

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where

$$ {}^{W}\dot{\mathbf{v}}_{L_i\,|_B} = {}^{W}\dot{\mathbf{v}}_{B\,|_B} + {}^{B}\dot{\mathbf{v}}_{L_i\,|_B} + 2\,{}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{v}_{L_i\,|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B} \times {}^{B}\mathbf{p}_{L_i\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times \left({}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{L_i\,|_B}\right). \quad (103) $$

The inertial part of the total force applied in $\{L_i\}$, due to the fixed-length link translation, expressed in frame $\{B\}$ will be

$$ {}^{L_i}\mathbf{F}_{L_i(ine)(tot)\,|_B} = \dot{\mathbf{Q}}_{L_i(tot)\,|_B}. \quad (104) $$

That is,

$$ {}^{L_i}\mathbf{F}_{L_i(ine)(tot)\,|_B} = {}^{L_i}\mathbf{F}_{L_i(ine)(fix)\,|_B} + {}^{L_i}\mathbf{F}_{L_i(ine)(man)\,|_B}, \quad (105) $$

$$ {}^{L_i}\mathbf{F}_{L_i(ine)(fix)\,|_B} = m_L \cdot {}^{B}\dot{\mathbf{v}}_{L_i\,|_B} = m_L \cdot \dot{\mathbf{J}}_{B_i} \cdot {}^{B}\dot{\mathbf{x}}_{P\,|_B} + m_L \cdot \mathbf{J}_{B_i} \cdot {}^{B}\ddot{\mathbf{x}}_{P\,|_B}, \quad (106) $$

$$ {}^{L_i}\mathbf{F}_{L_i(ine)(man)\,|_B} = m_L \cdot \left({}^{W}\dot{\mathbf{v}}_{B\,|_B} + 2{}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{v}_{L_i\,|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B} \times {}^{B}\mathbf{p}_{L_i\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times \left({}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\mathbf{p}_{L_i\,|_B}\right)\right), \quad (107) $$

where ${}^{L_i}\mathbf{F}_{L_i(ine)(fix)\,|_B}$ represents the inertial part of the force, considering the base platform is not moving, and ${}^{L_i}\mathbf{F}_{L_i(ine)(man)\,|_B}$ represents the inertial part of the force which results from the base motion.

When Eq. (104) is pre-multiplied by $\mathbf{J}_{B_i}^T$, the inertial component of the generalized force applied to $\{P\}$, due to each fixed-length link translation is obtained in frame $\{B\}$:

$$ {}^{P}\mathbf{f}_{L_i(ine)(tot)(tra)\,|_B} = \mathbf{J}_{B_i}^T \cdot {}^{L_i}\mathbf{F}_{L_i(ine)(tot)\,|_B}. \quad (108) $$

The inertial component of the actuating forces due to the base translation $\boldsymbol{\tau}_{L_i(ine)(man)(tra)}$ will be

$$ \boldsymbol{\tau}_{L_i(ine)(man)(tra)} = \mathbf{J}_C^{-T} \cdot {}^{P}\mathbf{f}_{L_i(ine)(man)(tra)\,|_B}. \quad (109) $$

On the other hand, the angular momentum of $\{L_i\}$ expressed in frame $\{B\}$ will be

$$ \mathbf{H}_{L_i(tot)\,|_B} = \mathbf{I}_{L_i(rot)\,|_B} \cdot {}^{W}\boldsymbol{\omega}_{L_i\,|_B}. \quad (110) $$

Taking the time derivative of this equation results in

$$ \dot{\mathbf{H}}_{L_i(tot)\,|_B} = \dot{\mathbf{I}}_{L_i(rot)\,|_B} \cdot {}^{W}\boldsymbol{\omega}_{L_i\,|_B} + \mathbf{I}_{L_i(rot)\,|_B} \cdot {}^{W}\dot{\boldsymbol{\omega}}_{L_i\,|_B}, \quad (111) $$

where

$$ {}^{W}\boldsymbol{\omega}_{L_i\,|_B} = {}^{B}\boldsymbol{\omega}_{L_i\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B}, \quad (112) $$

$$ {}^{W}\dot{\boldsymbol{\omega}}_{L_i\,|_B} = {}^{B}\dot{\boldsymbol{\omega}}_{L_i\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\boldsymbol{\omega}_{L_i\,|_B} + {}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B}. \quad (113) $$

The inertial part of the total moment applied in $\{L_i\}$, due to the fixed-length link rotation, expressed in frame $\{B\}$ will be

$$ {}^{L_i}\mathbf{M}_{L_i(ine)(tot)\,|_B} = \dot{\mathbf{H}}_{L_i(tot)\,|_B}, \quad (114) $$

that is,

$$ {}^{L_i}\mathbf{M}_{L_i(ine)(tot)\,|_B} = {}^{L_i}\mathbf{M}_{L_i(ine)(fix)\,|_B} + {}^{L_i}\mathbf{M}_{L_i(ine)(man)\,|_B}, \quad (115) $$

where

$$ {}^{L_i}\mathbf{M}_{L_i(ine)(fix)\,|_B} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i}\right) \cdot {}^{B}\dot{\mathbf{x}}_{P\,|_B} + \mathbf{I}_{L_i(rot)\,|_B} \cdot \mathbf{J}_{D_i} \cdot {}^{B}\ddot{\mathbf{x}}_{P\,|_B}, \quad (116) $$

$$ {}^{L_i}\mathbf{M}_{L_i(ine)(man)\,|_B} = \dot{\mathbf{I}}_{L_i(rot)\,|_B} \cdot {}^{W}\boldsymbol{\omega}_{B\,|_B} + \mathbf{I}_{L_i(rot)\,|_B} \cdot \left({}^{W}\dot{\boldsymbol{\omega}}_{B\,|_B} + {}^{W}\boldsymbol{\omega}_{B\,|_B} \times {}^{B}\boldsymbol{\omega}_{L_i\,|_B}\right). \quad (117) $$

${}^{L_i}\mathbf{M}_{L_i(ine)(fix)\,|_B}$ represents the inertial part of the moment, considering the base platform is not moving, and ${}^{L_i}\mathbf{M}_{L_i(ine)(man)\,|_B}$ represents the inertial part of the moment which results from the base motion.

When Eq. (114) is pre-multiplied by $\mathbf{J}_{D_i}^T$, the inertial component of the generalized force applied to $\{P\}$, due to each fixed-length link rotation is obtained in frame $\{B\}$:

$$ {}^{P}\mathbf{f}_{L_i(ine)(tot)(rot)\,|_B} = \mathbf{J}_{D_i}^T \cdot {}^{L_i}\mathbf{M}_{L_i(ine)(tot)\,|_B}. \quad (118) $$

The inertial component of the actuating forces due to the base rotation $\boldsymbol{\tau}_{L_i(ine)(man)(rot)}$ will be

$$ \boldsymbol{\tau}_{L_i(ine)(man)(rot)} = \mathbf{J}_C^{-T} \cdot {}^{P}\mathbf{f}_{L_i(ine)(man)(rot)\,|_B}. \quad (119) $$

Now, with reference to the gravitational part of the generalized force, if the base platform orientation changes, then the force applied in $\{P\}$ and expressed in $\{B\}$ results in

$$ {}^{P}\mathbf{f}_{L_i(gra)\,|_B} = {}^{W}\Re_B^T \cdot {}^{P}\mathbf{f}_{L_i(gra)\,|_W}, \quad (120) $$

where ${}^{P}\mathbf{f}_{L_i(gra)\,|_W}$ is the gravitational force applied in $\{P\}$ and expressed in $\{W\}$. This force can be computed using the model that considers both a non-moving base platform and frame $\{B\}$ parallel to $\{W\}$.

The gravitational component of the actuating forces due to each fixed-length link, $\boldsymbol{\tau}_{L_i(gra)}$, is given by

$$ \boldsymbol{\tau}_{L_i(gra)} = \mathbf{J}_C^{-T} \cdot {}^{P}\mathbf{f}_{L_i(gra)\,|_B}. \quad (121) $$

It should be noted that the base platform motion originates new inertial contributions to the parallel manipulator dynamic model, expressed by Eqs. (82), (94), (109) and (119). These contributions should the added to the corresponding ones resulting from the model that considers a fixed base: Eqs. (22), (27), (38) and (53). Regarding the gravitational part of the dynamic model, the base platform motion modifies the gravitational force components given by Eqs. (55), (56) and (57), resulting in the Eqs. (85), (97) and (121).

Computational efficiency of the proposed model has been evaluated by counting the number of scalar operations (additions, multiplications and divisions) needed for the calculations. The results were then compared with the ones obtained by using the Lagrange's formulation. As

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Table II. Number of arithmetic operations involved in the computation of the inertial part of the generalized forces.

	Lagrange			Generalized momentum		
	Addition	Multiplication	Division	Addition	Multiplication	Division
Contribution of the mobile platform	342	637	0	240	370	0
Contribution of each translating fixed-length link	730	1508	8	189	315	5
Contribution of each rotating fixed-length link	2205	3577	11	448	780	13
Total operations	17952	31147	114	4062	6940	108

Table III. Parallel manipulator parameters.

Parameter	Value	Parameter	Value	Parameter	Value
r_B	80 mm	m_P	1.356 kg	I_{Pzz}	10.8×10^{-4} kg·m^2
r_P	40 mm	m_A	0.316 kg	I_{Lxx}	4.2×10^{-6} kg·m^2
L	97.98 mm	m_L	0.083 kg	I_{Lyy}	6.8×10^{-5} kg·m^2
ϕ_B	15°	I_{Pxx}	8.2×10^{-4} kg·m^2	I_{Lzz}	6.8×10^{-5} kg·m^2
ϕ_P	0°	I_{Pyy}	8.2×10^{-4} kg·m^2	b_{cm}	48.99 mm

the largest difference between the two methods rests on the computation of the inertial part of the generalized forces produced by the moving platform and the fixed-length links, the number of arithmetic operations involved in the computation of these components was tabulated and compared. The results, presented in Table II, were obtained using the Maple® software package. As can be seen from the total number of operations (which include the mobile platform and the six fixed-length links), the generalized momentum approach is much more efficient. This can be a great advantage if real-time simulation and control is needed.

5. Numerical Simulation

A 6-dof parallel manipulator, presenting the kinematic and dynamic parameters shown in Table III, was considered for simulation purposes.

A desired trajectory was specified, in Cartesian space, for the moving platform relative to the base, {B}. The moving platform initial position is $P_1 = [0, 0, 500, 0, 0, 0]$ (mm; deg), it is then displaced to the next point $P_2 = [-10, -10, 510, 15, -15, 15]$ (mm; deg), and finally returns to point P_1.

Third-order trigonometric splines were interpolated between the three specified points, in order to obtain continuous and smooth trajectories. The trajectory was chosen in order to displace the moving platform to a point in the border of the parallel manipulator workspace. The trajectory time length requires actuators velocities near their maximum values (Fig. 7).

Similarly, a linear trajectory in Cartesian space was specified for the base platform relative to the world frame {W}. This is equivalent to specify an industrial robot end-effector Cartesian trajectory. The base platform initial position is $B_1 = [500, 0, 1000, 0, 0, 0]$ (mm; deg), it is then displaced to the next point $B_2 = [200, 200, 700, 45, -45, -20]$ (mm; deg), and finally it stops in point $B_3 = [0, 200, 500, 0, 0, 0]$ (mm; deg).

Linear interpolation between the three specified points was used in order to obtain continuous and smooth trajectories. The trajectory was chosen in order to displace the base platform (the RCID) in a wide region of the industrial robot workspace. The trajectory time length requires high velocities and accelerations (Fig. 8). Figure 9 shows the three-dimensional spatial trajectory of the base platform.

Figures 10–12 show the developed actuators forces necessary to follow the specified payload platform Cartesian trajectory. Figure 10 represents the contribution to the total actuators forces considering the base platform is fixed. Figure 11 shows the corresponding contribution considering the base platform is moving. Figure 12 presents the total force developed by the actuators.

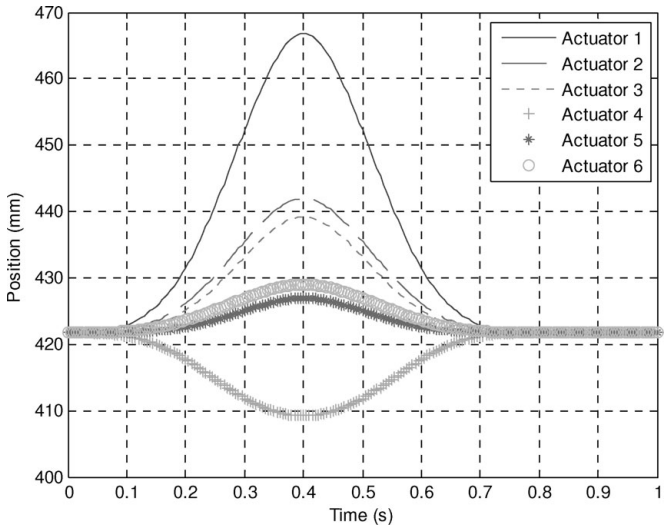


Fig. 7. Actuators trajectories: position.

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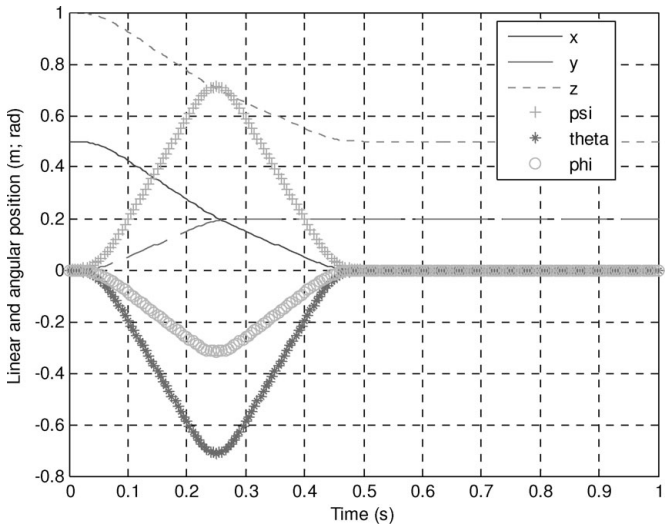


Fig. 8. Base platform trajectory: position.

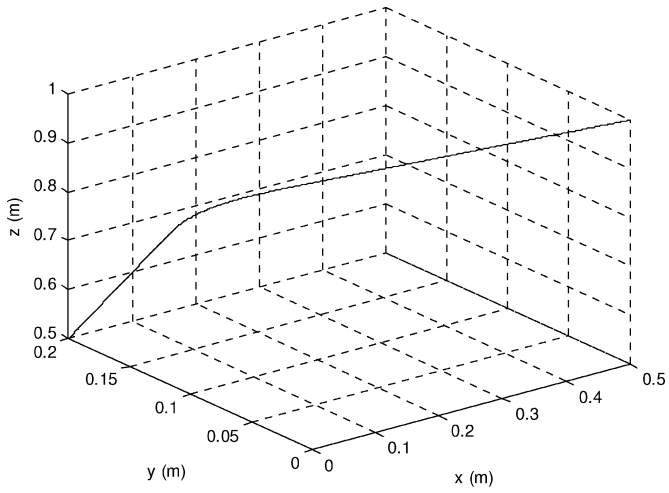


Fig. 9. Base platform trajectory: position.

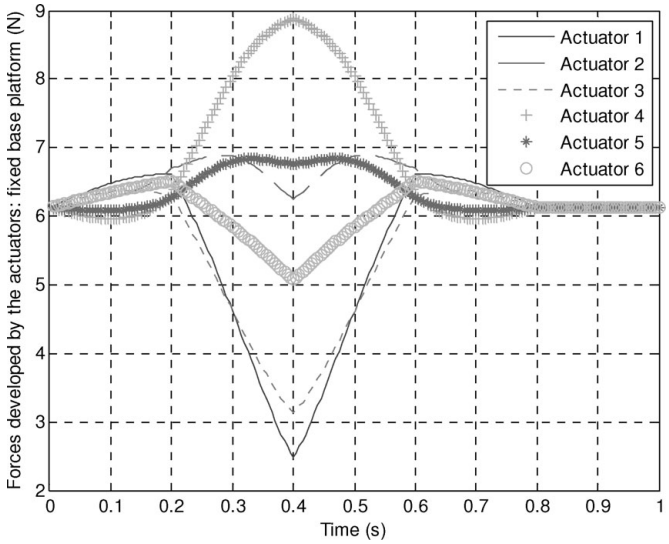


Fig. 10. Forces developed by the actuators: fixed base platform.

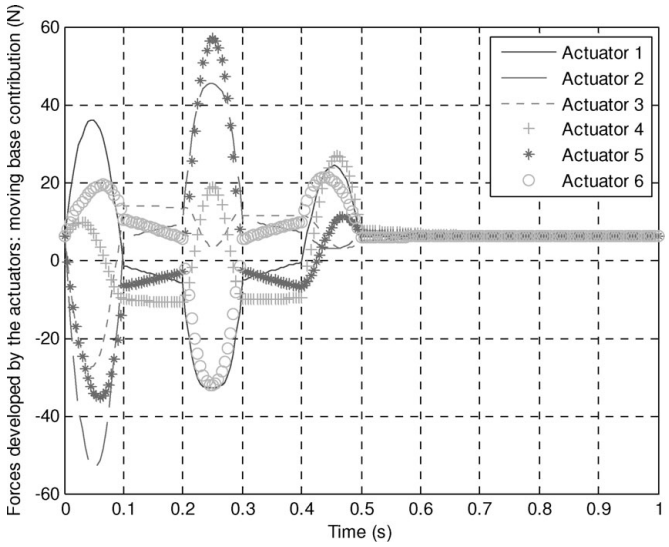


Fig. 11. Forces developed by the actuators: moving base contribution.

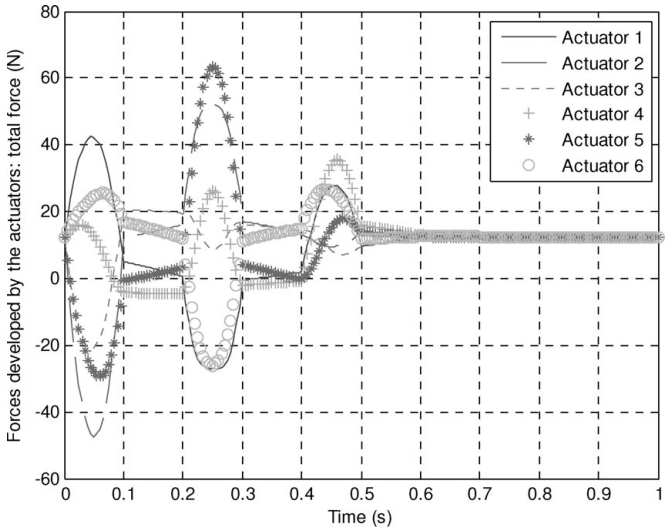


Fig. 12. Forces developed by the actuators: total force.

It is important to note that base platform motion gives important contributions to the parallel manipulator dynamic model. These contributions are highly dependent on the velocities and accelerations of the base platform and, for the presented simulation, they are about one order of magnitude higher than the contributions obtained when the base platform is fixed. Therefore, these components should not be neglected as they are in several related works presented in the literature.

6. Conclusion

A parallel manipulator is a complex multi-body dynamic system having several closed loops. Typically, it is composed of a fixed base platform and a moving payload platform, connected by at least two independent open kinematic chains. When compared to their serial counterparts, parallel manipulators usually present high dynamic performances, high stiffness, high load capacity, low positioning errors and small workspace.

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Dynamic modelling of parallel manipulators presents an inherent difficulty. Despite the intensive study in this topic of robotics, mostly conducted in the last two decades, additional research still has to be done in this area.

In this paper an approach based on the manipulator generalized momentum was explored and applied to the dynamic modelling of a variant of the well-known Stewart platform. First, the system dynamic equations are obtained supposing the manipulator base platform is fixed. Afterwards, the dynamic model is extended to the case of a moving base platform.

It is shown the base platform motion originates new inertial forces contributions to the parallel manipulator dynamic model. Compact analytic expressions for these contributions are presented and it is shown they can be added to the corresponding ones resulting from the model that considers a fixed base. On the other hand, regarding the gravitational part of the dynamic model, the base platform motion modifies the gravitational force components derived when considering a fixed base platform. Analytic expressions for these components are also presented. A numerical simulation illustrates that the base platform motion can give important contributions to the parallel manipulator dynamic model and, therefore, these components should not be neglected.

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