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Egyptian Mathematical Texts and Their Contexts

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Argument

The extant sources for ancient Egyptian mathematics are extremely limited. It is therefore necessary to read the few sources carefully and use additional information from further Egyptian sources in order to achieve the most detailed picture possible. Traditional approaches to Egyptian mathematics have provided only a superficial account of mathematical practices and almost no information about the role of mathematics within Egyptian culture. To enlarge our knowledge it is crucial to use a different methodological approach in the analysis of ancient mathematical techniques. In addition, it is indispensable to contextualize the mathematical problems with sources that are not specifically mathematical *per se*. In this article I discuss several possibilities for these additional sources, such as administrative texts, reliefs found in tombs, and other archaeological evidence. I exemplify the use of these sources with two problems from the Moscow mathematical papyrus.

I. Introduction

Twenty-five years ago, an article by Sabetai Unguru, "On the Need to Rewrite the History of Greek mathematics," appeared in the *Archive for History of Exact Sciences* (Unguru 1975). In this article, now regarded as a classic in the field, the author pointed out the dangers and pitfalls of applying modern mathematical symbolisms and terminology in the historiography of ancient mathematics.[1] Although it was meant to characterize the situation for Greek mathematics, his criticism holds as well for all but the most recent historiography of Egyptian and Mesopotamian mathematics.

In this article I will briefly describe traditional approaches to Egyptian mathematics and indicate their shortcomings. Then I will outline a new methodology for analyzing Egyptian mathematical problem texts. However, merely using a new formal interpretation of the texts is not enough. In addition, it is essential to consider the context of the mathematical problems. In contrast to the famous Greek mathematical texts that represent a deductive system of abstract mathematical theorems, Egyptian

[1] Unguru expanded his argument further in Unguru 1979; Unguru and Rowe 1981 and 1982; and most recently in Unguru and Fried 2001.

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mathematical problem texts (our major source for Egyptian mathematics) are embedded in daily-life practices. In the main part of this article I argue that the use of our knowledge of this context is indispensable for the analysis of Egyptian mathematical texts, and I provide two concrete examples from the Moscow mathematical papyrus. The conclusions combine results obtained through these new approaches and summarize briefly the role of mathematics within the Egyptian culture as represented by the mathematical texts and their contexts.

1. Traditional Historiography of Egyptian Mathematics

The study of ancient Egyptian mathematics started with the discovery of the Rhind mathematical papyrus at the end of the nineteenth century (Eisenlohr [1877] 1972). While literature on Egyptian mathematics has never ceased to be produced since then, for some time most scholars only repeated and discussed what had been written before. The reason for this stagnation is twofold: first, the available primary sources[2] have not increased significantly, and second, approaches to the various problems have not changed. The literature is written by scholars belonging to two communities, namely history of mathematics and Egyptology, with surprisingly little exchange between the two. Moreover, the upshot of the traditional approach is often similar to that ironically depicted in Unguru's article on the historiography of Greek mathematics: an anachronistic presentation of the supposed algebraic content hidden in the ancient texts.

The lack of new texts has led scholars to include administrative texts involving calculations in their presentations of Egyptian "mathematics texts" (e.g. Gillings 1972, 218–231, or Claggett 1999, 261–279). While it is certainly true that those sources can be useful for enlarging our knowledge on Egyptian mathematics, it must be kept in mind that they are not mathematical texts. As has been pointed out recently by Eleanor Robson in her work on Babylonian mathematics, there is a distinction between texts that reflect mathematical knowledge and texts that have been written about mathematics (Robson 1999, 7). *Mathematical texts* are simply texts that have been written to teach or learn mathematics.

In the extant Egyptian texts, two kinds of mathematical texts can be differentiated: problem texts and table texts. Table texts are collections of mathematical data structured in the form of tables that are used in solving mathematical problems. The extant Egyptian mathematical texts include tables used in fraction reckoning and tables for conversions of measures. Problem texts state a mathematical problem and then indicate the solution of that problem by step-by-step instructions, each step

[2] In order of their publication (if edited several times, all editions are listed): pRhind (BM 10057 and 58): Eisenlohr, 1877; Peet, 1923a; Chace/Bull/Manning, 1927/29; Robins/Shute, 1987. Lahun mathematical fragments: Griffith, 1898. pBerlin 6619: Schack-Schackenburg, 1900 and 1902. Cairo wooden boards (CG 25367 and 25368): Daressy, 1901 and Peet, 1923b. pMoscow (E4676): Struve, 1930. Mathematical leatherroll (BM 10250): Glanville, 1927. Ostracon Senmut 153: Hayes, 1942. Ostracon Turin 57170: López, 1980.

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usually consisting of one arithmetic operation. They can be characterized generally as suggested by Jim Ritter, as *numeric*, *rhetorical*, and *algorithmic* (Ritter 1989, 43–49). Numeric denotes the quality of always using concrete numbers. The texts are rhetorical in the sense that all operations are formulated without the use of symbolism, and they can be named algorithmic, since the solution is given as a sequence of instructions. The algorithmic trait of Egyptian (and also Babylonian)[3] mathematics had been recognized and stated from the beginning of the historiography of Egyptian mathematics (e.g. Neugebauer 1929, 4 and Neugebauer 1934, 113). But, until recently it has never been taken seriously in the understanding and interpretation of Egyptian problem texts. As in the historiography of Greek mathematics, Egyptian procedures have been taken merely as a clumsy way of stating mathematical facts. By "translating" Egyptian problems as algebraic equations all of the three characteristic features mentioned above are immediately lost. It is therefore not surprising that our knowledge about Egyptian mathematical practices remained rather superficial.

2. *New Approaches*

A close examination of the individual problem texts reveals a common structure. First, an introduction announces the title of the problem, names the data, and indicates the entity that is sought. This is followed by a series of instructions. These propose a sequence of arithmetical operations which, if carried out, will lead to the solution of the given problem. This part of the problem is grammatically marked by the use of imperatives (in the instructions) and the use of the verb form *sḏm.ḥr=f* (in the instructions and intermediate results). The function of this verb form can best be described as expressing a necessary sequence which results from the previously stated conditions (cf. Malaise and Winand 1999, 388–389 and Allen 2000, 304–306). As for announcing the result of a previously mentioned mathematical operation, the *sḏm.ḥr=f* expresses "mathematical facts" – if you add 2 and 2, the result will necessarily be 4. The use of the *sḏm.ḥr=f* in the instructions themselves can be interpreted as underlining their algorithmic aspect – given the stated mathematical problem, if a solution is to be found, it is necessarily those mathematical operations that have to be executed.

Some ten years ago, Jim Ritter proposed the utility of taking the algorithmic structure seriously (cf. Ritter 1989 [English translation Ritter 1995] and Ritter 1998). This approach enables a modern reader, at least to a certain degree, to walk in the shoes of those who created and worked with the sources. By applying this approach

[3] For Mesopotamian mathematics most explicitly Knuth 1972 (cf. also Knuth 1976). This has been largely ignored, presumably because Donald Knuth was a computing expert and had neither a background in history of mathematics nor in Assyriology. I would like to thank Eleanor Robson for this information.

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to the complete corpus of Egyptian problem texts, I have disclosed links between problems and have discerned substructures and refinements in the texts.[4]

Having acknowledged this important formal feature, the actual content of the problem texts must not be neglected. Today we know of around one hundred problems from the extant problem texts, most of which are found in the Rhind and Moscow papyri. These problems cover a variety of topics that are often situated in daily life, as for example problems 39 and 40 from the Rhind papyrus in which rations are to be calculated, and problems 41 to 46 from the same papyrus which deal with the calculation of granary volumes. A large group of problems, found in both the Rhind and the Moscow papyri, deal with calculations related to baking and brewing. This situation renders the "context" extremely significant to Egyptian problem texts.

This significance is evident on different levels. The translation of an Egyptian text, vital for a correct interpretation, often requires more than the knowledge of Egyptian grammar. The technical vocabulary is different from our own, therefore the knowledge of practical procedures may provide clues to concepts expressed in source texts. Moreover, due to the fragility of papyrus, relatively few papyri from pharaonic Egypt are still extant today (Parkinson and Quirke 1995, 51). All those papyri that ended up in the inhabited area neighboring the Nile are lost. The only survivors are some scattered examples that – by some chance – happened to be in the desert. In many cases this condition has been fulfilled in the circumstances of a burial which gave the advantage to texts that were supposedly useful for the afterlife of the deceased. The mathematical texts were obviously not favored by this selection (Ritter 2000, 115). It is therefore necessary to read other Egyptian texts, in order to fill lacunae in one type of document by information gained from another genre.

Furthermore, the context of the mathematical texts is indispensable for their deeper understanding and necessary to appreciate the role of mathematics within the Egyptian culture. The collections of mathematical problems known as mathematical texts most probably served as school textbooks in the education of scribes. They inform us therefore, not only about mathematical techniques but also about part of the professional life of an Egyptian employed in administration.

II. Egyptian Mathematical Texts and Their Contexts

The term "contextualization" has become very well appreciated in the history of mathematics. The range of contextual studies is large; for this reason, it may be reasonable to say which kind of "context" I refer to, and why. The following two sections will present an outline of the different contexts available for the study of Egyptian mathematics and the ways these contexts can be used to understand

[4] A detailed account of my algorithmic analysis can be found in Imhausen 2003. A shorter description of the method is given in Imhausen 2002.

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Egyptian mathematical texts. The third section will illustrate this with two examples of Egyptian problems.

1. *Various possibilities of "context"*

One of the characteristic features of Egyptian mathematical texts is their close relation to daily life: of the hundred or so problems, over half of them have a background in everyday life.[5] Within the Rhind mathematical papyrus (the only source that documents an ancient order of the problems), most of the "pure" mathematical problems are located at the beginning of the text. The "applied" problems (problems that contain an additional layer of knowledge from their practical background) are therefore thought to represent mathematics at a more advanced level. They contain problems referring to granaries, rations, baking and brewing, and herding. The mathematical exercises reflect primarily the theoretical side of these issues.

Information about these practical procedures can be gained from several sources. The most important among them are the so-called administrative or economic documents. They result from the actual work of scribes – work that scribes had been trained for with mathematical texts, among others. Moreover, they show the actual use of techniques that are taught in the mathematical texts. With this material we can greatly increase our understanding of the role of mathematics in the education of a scribe. Examples of these sources are the so-called ration texts. The Egyptian organization of work relied on a ration system. Quantities of bread and beer, the basic food items, were given as wages to the workers. The ration texts – in which at least sometimes mathematical techniques can be traced (cf. Guillemot 1992) – prove the importance of this kind of problem in the education of a scribe.

In addition to these very concrete examples, another type of background information is given by literary compositions referring to the profession of a scribe. For example, the *Satire of the Trades* compares the profession of a scribe to a range of other professions.[6] Against these professions, the scribe stands out as having a much more pleasant work environment as well as a higher rank in society. Another literary text referring to scribes is preserved as pAnastasi I, a literary letter of one scribe to his colleague, in which the sender accuses the addressee of not fulfilling the standards of the profession.[7] After describing his mistakes and failures, the author enumerates a number of situations in which the addressee will fail to be a good scribe.

Both of these literary compositions refer briefly to mathematics as part of the scribal profession. The *Satire of the Trades* includes sections on taxes that have to be paid (usually with great difficulty) by the different workers; the scribe in contrast, is

[5] An overview of the individual problems and their classification can be found in the introduction of Imhausen 2003. Out of 101 problems, 40 teach mathematical techniques; 58 have a practical background and 3 are too fragmentary to be assigned to either group.

[6] An English translation can be found in Lichtheim 1976, 168–175.

[7] The most recent translation (German) and commentary can be found in Fischer-Elfert 1986, 118–157.

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referred to as the one who records the taxes of the others.[8] pAnastasi I includes four situations that require calculations:

(1) calculating the number of bricks needed to build a ramp,
(2) calculating the number of persons needed to move an obelisk,
(3) calculating the number of persons to erect a colossal statue,
(4) calculating the rations of a military group of given size.

The mathematical problems that are stated in pAnastasi I have been analyzed by several scholars (see Fischer-Elfert 1986, 118–157 with further bibliographical references). Due to the fact that they don't contain all the necessary data to solve the sketched mathematical exercises, a variety of interpretations is possible. Their function in this text is obvious though: they come from a then-well-known body of scribal mathematical tasks so that the mention of the problem type alone was sufficient to recall in every Egyptian reader the group of problems referred to.

When using literary compositions to establish a "social setting" in which the scribes calculated and employed their mathematical training, it must not be forgotten that they are literary texts, written to entertain their readers and therefore not necessarily to be taken at face value.

The textual information is supplemented by evidence from paintings and reliefs in tombs as well as by archaeological finds. From the Old Kingdom on, scribes are depicted in tomb decorations. They are shown in scenes of accounting and measuring of all kinds.

In those cases where the mathematical problems are situated in the background of handicraft activities, these scenes of daily life provide an illustration of the underlying processes. Again, however, these illustrations cannot always be taken at face value, but have to be evaluated against evidence provided by archaeological findings (cf. Samuel 2000, 538).

Taking into account the various possibilities of sources available from the above listed contexts, a modern reader can fill in some of the blanks caused by the fragmentary textual evidence of the mathematical texts.

2. *Using the context to understand a mathematical problem*

The consideration of the context can contribute to the understanding of Egyptian mathematical problem texts in several ways. The first stage in dealing with ancient sources is their translation, which is often far from trivial. Apart from the fact that the manuscripts sometimes are in a deplorable state, some reduced to tiny fragments, the concepts expressed in them may be very different from those we use today. A

[8] "Only the scribe, it is he who calculates the taxes of all of them. You shall remember it" (pLansing 5,7; my translation). The hieratic text can be found in British Museum 1923, pl. 15–30.

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responsible translation has not only to ensure that the ancient concept is not lost, but has to observe at the same time that the translation itself is understandable to a modern reader. That this task is not easily achieved can be seen by the (often huge) philological commentaries added to translations.

The translation of ancient mathematical texts is no exception. Most Egyptian mathematical terminology was taken from "everyday" Egyptian. In addition, many of the problems have a practical setting. In these examples, a number of expressions and formulas have been directly adopted from the corresponding background. Knowing the "context," namely the administrational, economic, and handicraft practices of pharaonic Egypt is therefore a basic necessity for gaining access to the sources themselves and for enabling a translation of mathematical texts. As will be shown in one of the examples below, the mathematical interpretation of a problem is dependant on the understanding of the source text and its correct translation.

Besides this bare necessity, however, there are further reasons to consider the cultural background of the mathematical texts: A deeper understanding of a text can only be gained if it is taken into consideration that the creation of texts happens within specific conditions to fulfill a distinct intention. In the case of the Egyptian problem texts, this intention is rather obvious: teaching mathematics. As indicated previously, the preserved mathematical texts tell us even more. They show a large number of problems referring to a practical background. This reflects not only the group of persons the texts were intended to teach, the so-called scribes, but also the kind of mathematical techniques these scribes needed to learn for their profession.

Furthermore, the "practical character" of the Egyptian mathematical texts becomes understandable. The "applied" problems are not mathematical problems with an accidental practical setting. They are the actual training for handling the mathematics that an Egyptian scribe was confronted with in his daily work. Pivotal for our understanding of the significance of a mathematical problem in Egyptian mathematics – as it is displayed in the mathematical texts – is therefore its importance with regard to the actual work of the scribes.

3. *Individual Examples*

The following two examples illustrate the importance of considering the context. In the first case administrative documents and tomb representations provide the means to achieve an accurate translation of the problem text. The second case shows the understanding of the significance of a group of mathematical problems through the use of information from tomb representations.

The texts are given here in my hieroglyphic transcription. In addition, I have indicated references to the original hieratic texts. In several instances the translation makes use of Egyptian words. This indicates that the Egyptian term either does not

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have an equivalent English translation or that the specific meaning of the term is still open to discussion.

The Egyptian fraction notation used unit fractions only, with the sole exception of the fraction two-thirds (noted as $\overline{\overline{3}}$ in translations). They are written in the hieratic texts by the number that represents the denominator with a dot (to indicate the fraction) above it. This dot is conventionally rendered in translations as a horizontal line over the number, e.g. $\frac{1}{2}$ is written as $\overline{2}$.

3.1. The problem of a cobbler (pMoscow, no. 23)

Figure 1 shows the hieroglyphic transcription of problem 23 from the Moscow mathematical papyrus[9] with the translation by its editor, Wasili Struve, below.

Struve understood the text as a problem dealing with the wage of a cobbler who is normally paid twice within ten days and who does, in this problem, overtime-work for which the pay is to be calculated.

It turns out however, that this interpretation is incorrect in several aspects. The first objections against this interpretation were introduced in 1931 by the Egyptologist Thomas Eric Peet:

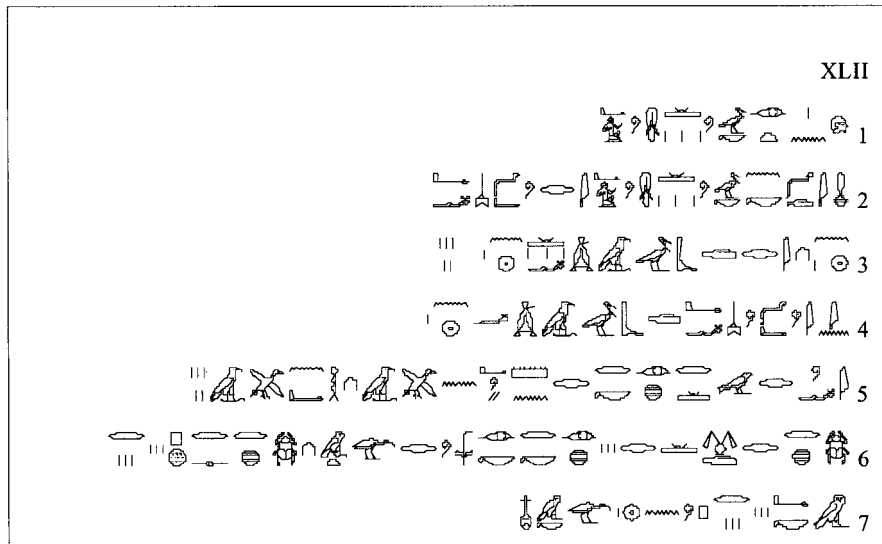
> S.'s interpretation of this exceedingly difficult problem is unconvincing for several reasons. In the first place, the words *mk* $3\frac{1}{3}$ *pw n hrw* in the last line cannot mean "It is $3\frac{1}{3}$ days" but only "It is $3\frac{1}{3}$ in a day." Secondly he introduces into the calculation a figure $\frac{2}{3}$ (relation of the ordinary working-day to the extraordinary) which is not in the data. Thirdly one of the data, 10, is, on his theory, entirely unnecessary to the problem. Fourthly, the Egyptian for "a day which is long," i.e. "a long day" is not *hrw jw.f r wrr*, which if it meant anything at all, would be definitely future.[10] Fifthly, *n hrw* 10 cannot express duration of time, "for 10 days," but only time within which, "in 10 days"; and lastly, S.'s translation completely fails to convey the sense of the problem such as he conceives it to be. (Peet 1931, 158)

In order to meet these objections, it is necessary to consider the background of the problem. The title of the problem indicates an administrative context. The Egyptian term used to denote the quantity that is to be calculated is "*bꜣkw*." This was understood by Struve as "Arbeiten" (produce). However, the term *bꜣkw*, which is derived from the verb *bꜣk* "to work," is also used as a technical term in a number of administrative documents. From the references to *bꜣkw* in these texts it has been

[9] The hieratic text can be found in Struve 1930, col. XLII.

[10] Here Peet has not read the German text correctly which is indeed written in future tense ("der groß sein wird").

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Translation by Wasili Struve (Struve 1930, 106–107):[11]

1. Form der Berechnung der Arbeiten eines Schusters.
2. Wenn man dir nennt die Arbeiten eines Schusters. Wenn er schneidet
3. für 10 Tage und wenn er belohnt wird für 5 Tage,
4. wird er nun schneiden und wird er belohnt werden für einen Tag,
5. der groß sein wird? Rechne du (zusammen) die beiden Hälften (wörtl. Seiten) dieser 10 mit dieser 5.
6. Es entstehen zusammen: 3 (Hälften). Rechne mit ihnen (wörtl. ihm), um zu finden 10. Es entsteht 3 1/3 mal.
7. Siehe: es sind 3 1/3 Tage. Du hast richtig gefunden.

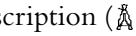
Fig. 1. pMoscow, problem 23, hieroglyphic transcription and Struve's translation.

[11] 1. Method of calculating the produce of a cobbler.
2. If you are told the produce of a cobbler. If he cuts
3. for 10 days, and if he is rewarded for 5 days,
4. will he cut and be rewarded for a day
5. which will be long? You calculate the two halves of this 10 with this 5.
6. There result together 3. Calculate with them to find 10. It results 3 1/3 times.
7. Look, it is 3 1/3 days. You have found correctly.

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possible to grasp its meaning: *b3kw* indicates the product of a working process, but includes also the obligation to deliver a fixed quantity of this product within a certain period of time.[12] Accordingly it may be translated as "work-rate."[13] The mention of *b3kw* suggests that the calculation refers to the amount of the cobbler's product which must be submitted in a fixed period of time.

The handicraft context of this problem is named in its setting: it is the work of a cobbler. We have access to this context through artifacts from settlement sites as well as depictions of handicraft processes in tombs.[14] The basic produce of an Egyptian cobbler, as can be seen from many depictions found in tombs, is sandals (e.g.  *tb.t*).[15] The production of sandals consists of the preparation of the leather, its cutting, and finally the finishing process of the shoe, which incorporates putting together the leather pieces and, eventually, applying some kind of decoration to the leather. This is the setting of problem 23 of the Mocow Papyrus.

The term *ḏb3* () which Struve took as a term for a payment made to the cobbler is significant in regard to this context. This term can be found in at least one inscription ( *ḏb3 tb.t*) that accompanies the depiction of the manufacturing of sandals in an Old Kingdom tomb (fig. 2).

In this depiction one of three cobblers is working on something lying in front of him. This inscription indicates therefore that *ḏb3* is used to denote the final part of the process of manufacturing sandals, after the leather has been cut. Thus, in the text of problem 23, *ḏb3* probably refers to part of the manufacturing process, and not to a payment made for the work.[16]

These lexical considerations lead to another interpretation of this problem that fits in well with the calculations found in the text:

1. Method of calculating the work rate (*b3kw*) of a cobbler:
2. If it is said to you "the work rate (*b3kw*) of a cobbler": if he cuts,
3. it is 10 per day; if he finishes, it is 5 per day.
4/5. If he cuts and finishes, how many will it be per day?
Then you calculate the parts of this 10 together with this 5.

[12] It must be stressed that the term and the underlying concept of *b3kw* is not restricted to craftsmen, there are also references to the *b3kw* of a scribe and in one inscription gold, silver, and copper – which are generally used to denote the exchange value – are indicated as *b3kw* of traders.

[13] I would like to thank Eleanor Robson for suggesting this translation to me, and also for referring me to the respective Mesopotamian (Old Babylonian) problems, e.g. IM 53961 (cf. Robson 1999, 94–96).

[14] The tomb depictions of craftsmen have been analyzed in Drenkhahn 1976. For leather manufacture, cf. also Hasanien 1997. These interpretations have to be read carefully, cf. now van Driel-Murray 2000, which contradicts several of the statements made in Hasanien 1997.

[15] Leather manufacture scenes from the Middle Kingdom can be found in Newberry 1894, pl. IV (tomb 15) and Newberry 1893, pl. XI (tomb 2). On production of sandals, see van Driel-Murray 2000, 312–316.

[16] This was already suggested (without further explanation) by Peet 1931, 158. Peet translates *ḏb3* tentatively as "to decorate."

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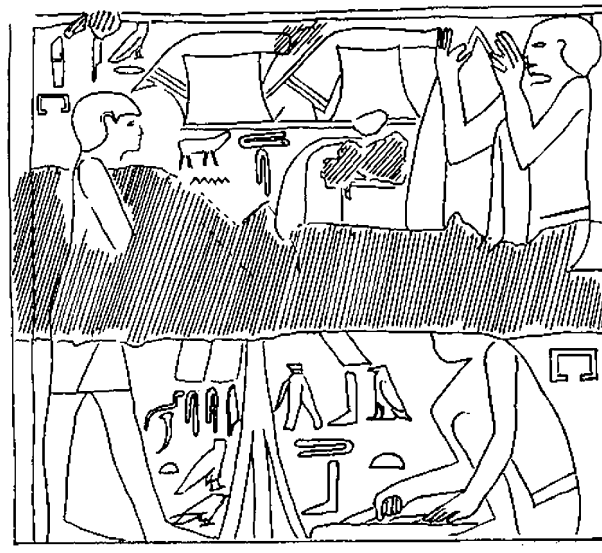


Fig. 2. Leather working in the tomb of *ʿnḫ-m-ʿ-Ḥr*.[17]

6. Then the sum results as 3. Then you divide 10 by this.
   Then 3 $\overline{3}$ times result.
7. Behold, it is 3 $\overline{3}$ per day.
   What has been found by you is correct.

The title of the problem (ln. 1) provides the information that the work-rate (*bȝkw*) of a cobbler is calculated. The following lines (ln. 2–3) state that the cobbler has to cut leather for ten pairs of sandals as his work-rate per day – if he only cuts the leather; and that he has to finish five pairs per day if the leather has already been cut. The next two lines (ln. 4–5) tell us what is looked for, namely the amount of his contribution (work-rate), given the condition that the cobbler does both parts of the sandal production, cutting and finishing the shoe. The following instructions given to solve the problem (ln. 4–6) suit this interpretation. The solution basically uses the fact that the contribution of the cobbler, if he only cuts leather, is twice the contribution if he finishes the sandals. If he has to cut ten pairs as his daily work-rate and he has to finish five, it will take three days work to amount to ten pairs of sandals. What was wanted was the number of sandals per day if he does both. This is obtained by dividing 10 by 3, which results in $3\frac{1}{3}$ pairs of sandals. The problem concludes with the notice "*gmj = k nfr*" "What has been found by you is correct."

The previous criticism of Struve's interpretation of this problem should not be taken as an attempt to demonstrate his incompetence. It rather shows the importance of considering the underlying procedures – and at the time of Struve's edition much

[17] My line drawing. A photo of the scene can be found in Capart 1907, pl. 33.

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less was known of these – and it demonstrates the problems every modern reader has to face in dealing with this kind of source.

3.2. The Bread and Beer Problems

Within the Egyptian mathematical problem texts, the so-called bread-and-beer problems hold a significant position due to their frequency.[18] The following presentation starts again from the text of one concrete mathematical problem (problem 16 of the Moscow papyrus). The first reading of the problem text (without being given the background of the problem) will lead to a number of questions that have to be answered in order to understand the problem and its solution. Using the context of this mathematical problem in form of tomb representations, archaeological evidence and other examples from the same group of mathematical problems is essential to our understanding. However, in order to comprehend all of the calculations executed, one needs to consider further contextual evidence in form of an administrative/economic text. Taking this document into account helps to comprehend factors and a relevant set phrase named in the problem.

Figure 3 shows the hieroglyphic text as well as my translation of pMoscow, no. 16.[19] This text starts, like the text of problem 23, with the title of the problem. This also renders one of the given data: "Method of calculating beer vessels of *psw* 2."[20] The given data are then introduced by the expression "if you are told . . .." They are:

- (beer vessels of) *psw* 2,
- "$\overline{2}\,\overline{4}$ of *bš3* for dates,"
- (emmer, measured as) $2\overline{\overline{3}}$

The entity that is sought in this problem is not explicitly stated, but the data mentioned in this introduction are already sufficient to indicate the problem. The introduction is followed by the second part of the problem, namely the procedure of its solution.

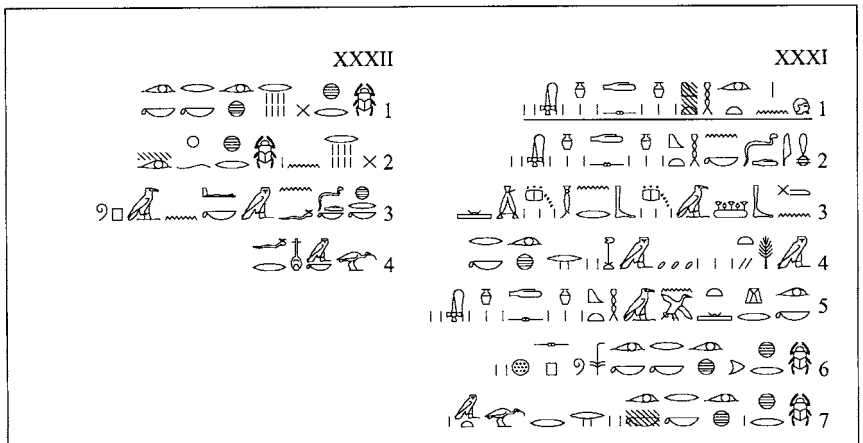
As usual, the text contains a series of instructions for arithmetical operations, followed by the individual results. These are sometimes supplemented by further explanations: "Then you calculate the need of these beer-vessels of *psw* 2. Then it results as $\overline{2}$ *ḥq3.t*. Then you calculate it times 2. Then 1 results. Then you divide 1 by $2\overline{\overline{3}}$. Then $\overline{4}\,\overline{8}$ results. Then you calculate $\overline{4}\,\overline{8}$ of 1. Then $\overline{4}\,\overline{8}$ *ḥq3.t* result." The end of

[18] Nine problems in the Rhind papyrus (problems 69–78) and eleven problems in the Moscow papyrus (problems 5, 8, 9, 12, 13, 15, 16, 20, 21, 22, and 24) belong to this group.

[19] The readings given here differ from the readings given by Struve in several places. An explanation for the readings given here can be found in Imhausen 1999.

[20] Unlike the previous problem, the heading of this problem uses red ink. Within the Moscow papyrus, only the first 16 problems each use red ink to indicate the heading of a new problem.

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XXXI

Method of calculating beer-vessels of *psw* 2

If you are told beer-vessels of *psw* 2, $\overline{2}\,\overline{4}$ of *bš3* for dates exchanged as emmer, measured as $2\overline{\overline{3}}$.

Then you calculate the need of these beer-vessels of *psw* 2.

Then it results as $\overline{2}$ *ḥq3.t*.

Then you calculate it times 2.

Then 1 results.

Then you divide 1 by $2\overline{\overline{3}}$.

XXXII

Then $\overline{4}\,\overline{8}$ results.

Then you calculate $\overline{4}\,\overline{8}$ of 1.

Then $\overline{4}\,\overline{8}$ *ḥq3.t* result.

Then you tell him:

Look: It is these.

What has been found by you is correct.

Fig. 3. pMoscow, problem 16 (Red ink in the source text is rendered as underlined in the hieroglyphic text and as **bold print** in the translation).

the text states that the result of the last step solved the problem, followed by a note that this solution is correct: "Then you tell him: Look: It is these. What has been found by you is correct."

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This example shows again very clearly the difficulties a modern reader encounters when reading an ancient text. Although all of the individual arithmetical steps can be followed easily, the sense of the problem remains obscure. The title of the problem states that beer shall be calculated, but the final result is given as $\overline{4}\,\overline{8}$ *ḥqꜣ.t*. This is a quantity; however, it is certainly not a quantity of beer: *ḥqꜣ.t* is the standard Egyptian measure for grain volumes, approximately 4.8 liters. Beer, by contrast, is usually counted by vessels (as is in fact indicated in the text of this problem). The translation includes, besides *ḥqꜣ.t*, the Egyptian words *psw* and *bšꜣ* which have to be explained. Finally, two factors are indicated, namely $\overline{2}\,\overline{4}$ and $2\overline{\overline{3}}$. Several calculations are executed with the final result of $\overline{4}\,\overline{8}$ *ḥqꜣ.t* which is identified as the correct solution.

Again, in order to understand what is going on, information about the problem's practical background is needed. "Understanding" not only refers to the individual calculations, but also to the problem's relevance within Egyptian mathematics. To understand the problem, the following questions have to be answered:

- What are the meanings of *psw* and *bšꜣ*?
- What does $\overline{2}\,\overline{4}$ of *bšꜣ* for dates imply?
- What is the relevance of the factor $2\overline{\overline{3}}$?
- What has been calculated in the end?

To answer these questions, a basic knowledge of baking and brewing in ancient Egypt is needed.

Egyptian grain and the Egyptian production of bread and beer

Two kinds of grain, barley and emmer (a variety of wheat), are proven to have been used in ancient Egypt. Both were used in the production of bread and beer. Besides these, another cereal called *bšꜣ* is mentioned in the Egyptian texts. Its exact meaning has not yet been established (cf. Faltings 1995 and Posener-Krièger 1977), but it is most probably a grain product used for beer brewing, presumably some sort of malt.

The production of bread and beer in Egypt can be traced by archaeological artifacts as well as by representations on tomb walls decorated with scenes from daily life (cf. Faltings 1998 and Samuel 2000).[21] There is a certain variety in details depicted in these scenes. Not every depiction shows the same sequence of steps. Some of them are represented only in a few instances. The following description is based on the work of Dina Faltings, who examined the available material from the Old Kingdom (Faltings 1998). Figure 4 shows the scenes of baking and brewing from the tomb of Nianchchnum and Chnumhotep.

[21] The most detailed tomb scenes found are during the Old Kingdom. This is a period prior to the mathematical texts that are dated to the Middle Kingdom. During the Middle Kingdom models representing the scenes from daily life become more prominent (see for example the models of bakers and brewers from the tomb of Meketre in Roehrig 2002, 11–12). These are mostly less detailed than the Old Kingdom tomb scenes. However, they show no significant change in the representation of baking and brewing, therefore the Old Kingdom scenes are used for further explications. A Middle Kingdom scene of baking and brewing can be found in the tomb of Intefiqer (cf. Davies and Gardiner 1920, pl. 8 and 11).

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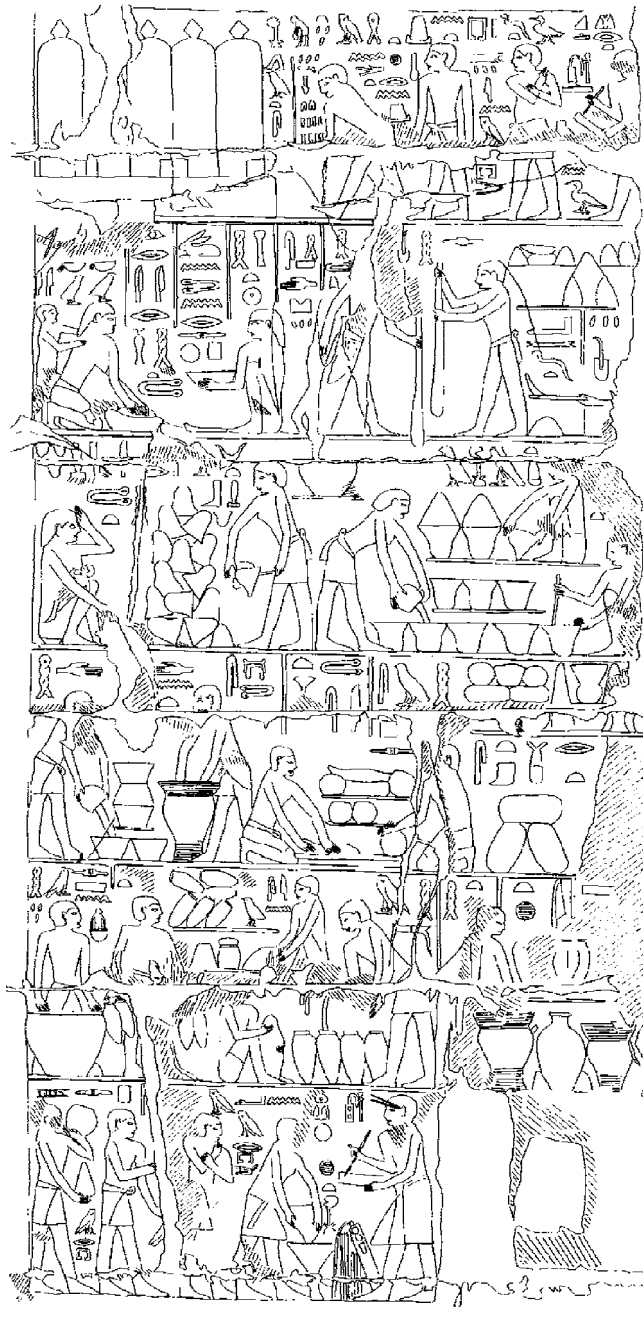


Fig. 4. Scenes of baking and brewing in the tomb of Nianchchnum and Chnumhotep.[22]

[22] My line drawing. A photo of the scene can be found in Moussa and Altenmüller 1977, Tafel 23.

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The scenes start with the depiction of workers taking grain out of the granary and handing it to the bakers. An official notes the amounts that are given to them (1st register). The production of bread starts with purifying and grinding the grain (2nd register). The flour is then made into dough (2nd register) which is distributed into pre-heated bread moulds and is then baked (3rd register). The 4th register shows the preparations of another type of bread. The preparation of the beer starts from bread that is mixed with dates and water in a large bin (5th register). The dates serve to fortify the taste as well as to bring yeast to the mixture for fermentation. The yeast organisms are found on the skins of the dates. Before further processing, the mixture is probably left alone for fermentation. This is followed by the procedure of straining the mixture. The beer is finally poured into vessels (5th register). The last scenes of the representations show the delivery of produced goods. A scribe is shown measuring the delivered quantities (6th register).

A connection between the production of bread and beer and the mathematical bread and beer problems can be traced by means of the terminology that is used. The representations of baking and brewing show the distribution of grain and the delivery of bread and beer. To be able to control the production, the distributed grain must have a specific equivalent in bread and beer, that is, a certain quantity of bread and beer of a known quality. It must be possible therefore to calculate how many loaves of bread can be baked with a given amount of grain. The entities and formulations used to describe this are found in the bread and beer problems of the mathematical texts.

The most important *terminus technicus* herein is the Egyptian word *psw*. The *psw* denotes how many bread loaves or vessels of beer are prepared from one *ḥqȝ.t* of grain.[23] Therefore it can be calculated as the quotient of the number of bread loaves and the amount of grain that have been used to produce them.

$$psw_{bread} \equiv \frac{\text{number of bread loaves (made from 1 } ḥqȝ.t \text{ of grain)}}{1\ ḥqȝ.t \text{ of grain}}$$

$$= \frac{\text{number of bread loaves}}{\text{amount of grain used for their production}}$$

$$psw_{beer} \equiv \frac{\text{number of vessels beer (made from 1 } ḥqȝ.t \text{ of grain)}}{1\ ḥqȝ.t \text{ of grain}}$$

$$= \frac{\text{number of vessels of beer}}{\text{amount of grain used for their production}}$$

[23] This relies on constant sizes of bread or vessels for beer. It should not be assumed that constant vessel or loaf sizes were in use all over Egypt, although regional standards are perfectly possible. Proof of this has been established in at least one case (cf. Bárta 1996). Average sizes of bread moulds can be found in Faltings 1998.

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The higher the *psw* of bread or beer, the lower is its actual content of grain. In the problems of the Moscow papyrus the *psw* of bread is constantly 20 (20 bread loaves were made from 1 *ḥqꜣ.t* of grain). The *psw* of beer varies between 2 and 6.

With this additional information, the mention of *psw* in problem 16 of the Moscow papyrus becomes understandable, as well as the first calculation (XXXI.5), in which the amount of grain needed per vessel is determined. If the *psw* is said to be 2, the amount of grain needed for one vessel of beer is $\frac{1}{2}$ (its inverse).

The role of *bšꜣ* and dates in the problem, as well as the introduction of emmer and the following calculations, are still unclear. At this point, it is helpful to consider another document from the context of the bread and beer problems. pBerlin 10.078 was written in the same period as the Moscow papyrus. Anthony Spalinger has established that this document contains the information that different types of grain are valued in standard proportions (Spalinger 1988). According to Spalinger, this can be rendered as follows:

	Barley	Emmer	Dates	*bšꜣ*
Barley	1	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{3}{2}$
Emmer	$\frac{8}{3}$	1	$\frac{16}{9}$	4
Dates	$\frac{3}{2}$	$\frac{9}{16}$	1	$\frac{9}{4}$
bšꜣ	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{4}{9}$	1

Each line of the table notes how many *ḥqꜣ.t* of the grain noted on top of each respective column equals one *ḥqꜣ.t* of the grain noted at the beginning of the line. For example, line two gives the information that 1 *ḥqꜣ.t* of emmer equals respectively $\frac{8}{3}$ *ḥqꜣ.t* of barley, 1 *ḥqꜣ.t* of emmer, $\frac{16}{9}$ *ḥqꜣ.t* of dates or 4 *ḥqꜣ.t* of *bšꜣ*. These value proportions are found in the Moscow papyrus as well, where barley is used as a kind of reference grain, as can be seen from problem 20. In this problem, a number of bread loaves made of emmer of a known *psw* are given, and the value of these bread loaves as an amount of barley is calculated.

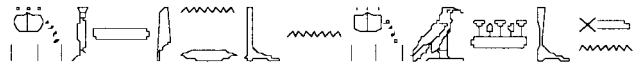
As a consequence, the quantities of barley mentioned in the problems of the Moscow papyrus have two possible meanings. Either they denote a quantity used for the actual baking/brewing, or they represent the value of (another) quantity of a different type of grain that is used for baking or brewing.

Taking this further information into account, the factor $2\overline{\overline{3}}$ mentioned in the text of our problem is understandable. It denotes the proportion of value between emmer and barley. This also indicates that the amount of grain (emmer) calculated is the value of the product. The respective calculations are found in our problem in column XXXI, line 7: the inverse of the given factor $2\overline{\overline{3}}$ is determined, since – as we will see

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later – an amount of barley must be transferred into the equivalent amount of emmer.

The last missing piece in our understanding is the meaning of the formula $\overline{2}\,\overline{4}$ of *bš3* for dates. As indicated above, *bš3* is a cereal, probably malt. *bš3* and dates are used in the production of beer. Within the problems relating to the production of bread and beer of the Moscow papyrus there are two possible formulas that mention *bš3* and dates:



(1)



(2)

The first formula, which has been translated above as "$\overline{2}\,\overline{4}$ of *bš3* for dates," is used in problems, no. 5, 8, 9, 13, 16, 22, and 24; the second one which can be translated as "*bš3* like dates" only in problem 12.

The occurrence of one of these formulas in the mathematical problems usually implies that amounts of grain are multiplied by a certain factor. The exact meaning of these formulas has been discussed since 1930, a discussion that I will not repeat here.[24] The first formula, "$\overline{2}\,\overline{4}$ of *bš3* for dates" could not yet be interpreted in a way that explains the calculations that usually follow it in the problem texts.

A close examination of the problems in which either of the two phrases are employed makes it evident that they mention the ingredients. The mention of *bš3* and dates therefore indicates that they are used for brewing. Furthermore, they include relative quantitative statements of the amount of *bš3* and dates. The explanation of those formulas must not only explain the factors that appear within them (namely 3:4 and 1:1) but also the respective factors that appear after their appearance in the mathematical problems.

The second formula, "*bš3* like dates" turns out to be easier to explain (cf. Spalinger 1988).[25] The formula indicates that the materials used for brewing are equal amounts of *bš3* and dates. While both ingredients determine the value of the resulting product, it is only the amount of *bš3* that is considered for the respective *psw*-value.

From this explanation follow a number of premises that can be used to explain the formula in our problem:

- the calculated entity designates the value of the named products,

[24] The relevant literature in order of its appearance is: Struve 1930, 59–79 and 90–92; Gardiner 1947, 226*; Nims 1958; Reineke 1964, 189–190; Spalinger 1988.

[25] Spalinger's interpretation of the other formula, however, uses only some of the available problem texts and needs serious emendations of the source texts. The problems not taken into account in his interpretation show that the explanation given by him cannot be correct.

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- the *psw*-value mentioned in the problem does not change; the amount of grain used for brewing is determined by the *psw*-value indicated at the beginning,[26]
- the *psw* refers only to the grain (*bš3*), dates are not concerned.
- the calculation of the value includes all the ingredients, grain and dates; the value is given (if not indicated otherwise) as an amount of barley.

After these preliminary observations, problem 9 from the Moscow papyrus can be used to determine the meaning of the "$\overline{2}\,\overline{4}$ of *bš3* for dates" – formula. This text includes the formula and its text is very detailed. It indicates that beer that is said to be "$\overline{2}\,\overline{4}$ of *bš3* for dates" has twice the value of beer made of barley alone. The value is calculated from the *psw* (indication of grain quantity used for brewing), the kind of grain (*bš3*), and the amount of dates. The result, given as an amount of grain (barley), denotes only the value. Since the amount and kind of grain are known, the factor 2, which is named in the problem text, can be used to determine the amount of dates. The total value of the mixture used for brewing consists of the value of the *bš3*, and the value of the dates. The *psw* indicates the amount of grain used for brewing. The text of some of the problems indicated that the value of the beer named "$\overline{2}\,\overline{4}$ of *bš3* for dates" is twice of that made of barley alone. Since the *psw* is given, the value can be calculated as well.

The value of *bš3*, as an amount of barley, is deducible from the amount of *bš3* and the factor to calculate an amount of *bš3* as an amount of barley given in the table of grain values. Thereby the value of *bš3* can be calculated. The only thing remaining to be determined is the part of dates.

With the entities known, the value of the dates can be expressed in relation to the amount of grain (*bš3*) used for brewing. This results in the relation 3:4. One possible explanation for the factor $\overline{2}\,\overline{4}$ (= 3:4) in the formula is therefore the statement of the relation of the amount of grain to the value of the dates.

With these explications, the text of problem 16 can be followed: The title "Method of calculating beer-vessels of *psw* 2" indicates the type of problem and already names one of the data (*psw* = 2). Then the data are introduced: "If you are told beer-vessels of *psw* 2, $\overline{2}\,\overline{4}$ of *bš3* for dates exchanged as emmer, measured as $2\overline{\overline{3}}$." Beer of *psw* 2 is named as the starting point. Then it is indicated that it should be the more expensive beer, characterized as "$\overline{2}\,\overline{4}$ of *bš3* for dates." The value of this beer has to be expressed as a quantity of emmer. Therefore the factor to convert grain from emmer to barley is given.

This introduction is followed by the algorithm to solve the problem: "Then you calculate the need of these beer-vessels of *psw* 2. Then it results as $\overline{2}$ *ḥq3.t*." The definition of the *psw* is used to calculate the amount of grain needed to brew each vessel. Then the superior quality is brought into play: "Then you calculate it times 2. Then 1 results." For the reasons stated above, the amount of grain is doubled. This is

[26] Spalinger assumes a change of the *psw*. From problem 12 it follows, however, that the *psw* is independent of further quality announcements.

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then converted into the equivalent amount of emmer: "Then you divide 1 by $2\overline{\overline{3}}$. Then $\overline{4}\,\overline{8}$ results. Then you calculate $\overline{4}\,\overline{8}$ of 1. Then $\overline{4}\,\overline{8}$ *ḥqȝ.t* result." Since the conversion is barley into emmer, the inverse of the given factor has to be calculated first. It results as $\overline{4}\,\overline{8}$. This is used to convert the calculated amount of barley into emmer. This is the solution of the problem: "Then you tell him: Behold: It is these. What has been found by you is correct."

III. Conclusions

It is now 25 years since Unguru published his article challenging historians of mathematics to rewrite the history of Greek mathematics. His article demonstrating fundamental pitfalls in the former traditional historiography of Greek mathematics is just as meaningful for historians of Egyptian mathematics. Those pitfalls are the major reason for the recent stagnation of the field. To get out of this dead-end, I have proposed a new approach to analyzing the sources (Imhausen 2002 and Imhausen 2003). Taking the algorithmic feature of the texts into account, this approach is appropriate to the internal structure of the extant mathematical texts. The inclusion of the context that Egyptian mathematical texts have been recognized to be deeply linked with, ensures first of all the correct interpretation of the actual source. Furthermore, if more than just a superficial account of "mathematical content" is to be achieved, the embedding of the texts into their background enables us to gain a deeper understanding of the use of mathematics in ancient Egypt and its role in society. Mathematical texts, as well as the reference to mathematics in Egyptian literature prove that mathematical training was an important element of the education of a scribe. The group of "scribes" undoubtedly consisted of persons from a range of social positions. The largest part of these (which is the group referred to in literary texts as "the scribes") were probably the ones doing basic daily accounting. It is from this group of persons that we find documents displaying mathematical techniques, namely economic and administrative documents, which allow us to follow mathematical practices in ancient Egypt.

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Sabetai Unguru's "On the Need to Rewrite the History of Greek Mathematics" was one of the first articles I ever read in history of mathematics, succeeded at a later time by the follow-up articles by Sabetai Unguru and David Rowe. These have deeply influenced my studies as well as my research, and I would like to express my gratitude to both authors. I would also like to thank Eleanor Robson and Jens Høyrup for their helpful suggestions as referees of this article.

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